AMRITA VISHWA VIDYAPEETHAM
(University established u/s 3 of UGC Act 1956)
Amrita Entrance Examination – Engineering 2009

PHYSICS, CHEMISTRY & MATHEMATICS

Question booklet version code	**B**	Question booklet no:	**208209**	Time	: 3 hrs
Number of pages	**20**	Number of questions	**120**	Max. Marks	: 360

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INSTRUCTIONS TO THE CANDIDATES

GENERAL

1. Any malpractice or attempt to commit malpractice in the examination hall will lead to disqualification of the candidate.
2. Candidates are not allowed to carry any textual material, printed or written bits of paper, Mathematical and Physical Tables, electronic gadgets like calculator, cell phone etc. into the examination hall.
3. Candidates shall possess the University Hall Ticket which should be produced on demand.
4. Candidates shall occupy the respective seats bearing their registration numbers on time.
5. **Candidates are not permitted to leave the hall before the end of the examination.**
6. **Candidates are required to handover the ANSWER SHEET and the QUESTION BOOKLET to the invigilator before leaving the hall.**

QUESTION BOOKLET

7. **DO NOT OPEN THE SEALED QUESTION BOOKLET UNTIL THE INVIGILATOR ANNOUNCES TO DO SO.**
8. Before opening the Question Booklet, write the Registration Number, Name and Signature using ball pen in the space provided at the top of this page.
9. Immediately after opening the booklet, the candidate should examine whether it contains all the 120 questions in serial order and 20 pages as mentioned at the top of this page. In case of unprinted, torn or missing pages in the booklet, the matter should be reported to the invigilator immediately.
10. Rough work may be done on the space provided in this booklet.

(Continued on the last page of this question booklet)

PHYSICS

Data:
Acceleration due to gravity = 10m/s^2, Mass of electron = 0.511MeV/c^2
Charge of electron= 1.6 x 10^{-19} C, Velocity of light in vacuum, c= 3 x 10^8 m/s
Boltzman constant, k =1.38x10^{-23} J K^{-1}=8.67x10^{-5} eV K^{-1}

1. An electric dipole is placed in a non-uniform electric field. It experiences
   a) a force but no torque   b) no force, no torque
   c) a force and a torque   d) no force but a torque

2. A battery of emf 10 V is connected across a 1 Ω resistor. The voltage across the 1Ω resistor is 5V. The internal resistance of the battery is
   a) 2 Ω   b) 0.5 Ω   c) 1 Ω   d) 4 Ω

3. Which unit is appropriate for specifying magnetic induction?
   a) N C^{-1} b) N C^{-1} m^{-1} s   c) J C^{-1} m^{-1} s   d) A m^2

4. Four masses of 1 kg each are placed at four corners of a square of side 2m placed symmetrically in xy plane. The square is set in rotation around z-axis with angular velocity ω = 2 rad s^{-1}. The angular momentum of this system in motion in SI units is
   a) 32   b) 16   c) 8   d) 64

5. At what distance from the point of equilibrium, the kinetic energy equals the potential energy for a simple harmonic oscillator of amplitude A?
   a) A/2   b) A/√2   c) A/4   d)A/(2√2)

6. If the tension along a stretched string is doubled, the speed of sound along it will
   a) increase by a factor 2   b) increase by a factor 4
   c) increase by a factor √2   d) remain unchanged

7. The kinetic energy of an atom in helium gas held at temperature -200° C is of the order of
   a) 10eV   b)10^{-3} eV   c)10^{-6} eV   d) 10^{-5} eV

8. The resistivity of copper is 0.01Ω m^{-1}. What is the resistance of a wire 50 cm long and having circular cross section area 0.1 mm^2
   a) 50 KΩ   b) 5 K Ω   c) 10 K Ω   d) 15 K Ω

9. An alpha particle is accelerated by a potential difference of 4 volt. The energy acquired is
   a) 6.4 eV   b) 4 eV   c) 8 eV   d) 3.2 eV

Rough work

10. A straight infinitely long thin wire along the z axis carries current 2 A. At a point (3,4,5) the magnitude of magnetic field intensity($\mathbf{H}$) in SI units is

a) $\dfrac{1}{5\pi}$ b) $\dfrac{2}{50\pi}$ c) $\dfrac{1}{25\pi}$ d) $\dfrac{1}{\sqrt{50\pi}}$

11. Twenty seven mercury drops of equal radii and having equal charges are combined to form a big drop. The ratio of the capacitance of the bigger drop to each individual drop is

a) 9:1   b)1:9   c) 3:1   d) 1:3

12. In its ground state, the Ar^+ ion has

   a) 6 electrons in 1p state   b) 5 electrons in 3p state
   c) 4 electrons in 3d state   d) one electron in 3s state

13. Positions of two masses m=1 and M=4 are given by the vectors $\mathbf{r_1}$= 3 $\mathbf{i}$ -4 $\mathbf{j}$ and $\mathbf{r_2}$= -7$\mathbf{i}$- 4$\mathbf{j}$ .All numbers are in SI units. Distance of their center of mass from M is

   a) 1   b) 2   c) 4   d) 5

14. A mass m is undergoing uniform circular motion in xy plane with constant speed 40 ms^{-1} around the origin. At t=0 its position coordinate is (6,8). What is the time period?

   a) π s   b)2π s   c) π/2 s   d) 20π/7 s

15. At t=0, a projectile of mass 1 kg is projected with speed 10 ms^{-1} and making an angle 30° with the horizontal. The x-component of its velocity at t=1 s is

   a) 8.66 ms^{-1} b) 5 ms^{-1} c) 10 ms^{-1} d) 7.07 ms^{-1}

16. The sides and mass of a solid cube of uniform density are measured each with accuracy $\pm$ 1%. The accuracy of its density D calculated using this data and using the formula D=M/L^3 is approximately

   a) $\pm$ 1%   b) $\pm$ 2%   c) $\pm$ 3%   d) $\pm$ 4%

17. Assume the earth to be a sphere of constant density and radius R. What is the acceleration due to gravity at a distance R/4 from the center of the earth?

   a) 10 ms^{-2} b) 4 ms^{-2} c) 2.5 ms^{-2} d) 5 ms^{-2}

18. A unit charge is placed at point (0,3) in xy plane. The direction of electric field at the point (4,0) is along

   a) unit vector $\mathbf{i}$ b) unit vector($\mathbf{i}$ - $\mathbf{j}$)/$\sqrt{2}$
   c) unit vector (4$\mathbf{i}$-3$\mathbf{j}$)/5   d) unit vector (-4$\mathbf{i}$ +3$\mathbf{j}$)/5

19. A charge Q is set in a field of constant magnetic induction $\mathbf{B}$= B $\mathbf{k}$ with a velocity $\mathbf{v}$ = 4 $\mathbf{i}$ + $\mathbf{k}$. Its trajectory will trace a

   a) straight line   b) circle   c) ellipse   d) helix

Rough work

20. A proton and an alpha particle are accelerated by a constant electric field. Their accelerations will be in the ratio
    a) 1:2   b) 1:1   c) 4:1   d) 2:1

21. A particle of rest mass m and momentum p is moving with a velocity very close to the velocity of light, c. Its total energy is approximately
    a) mc^2 b) $p^2/2m$ c) p^2/m d) pc

22. Neutrino is a
    a) chargeless, fermion of negligible mass
    b) chargeless, massless, spinless boson
    c) massless, chargeless fermion of spin 3/2
    d) massless fermion of charge (1/3)e

23. In a circuit containing a capacitor and a resistance, as the frequency of the applied alternating current increases, the impedance
    a) decreases   b) increases
    c) remains unchanged   d) first increases and then decreases

24. Two electric bulbs having resistances in the ratio 2:1 are connected in parallel to a constant voltage source. The power dissipated in them has the ratio
    a) 1:2   b) 1:4   c) 2:1   d) 4:1

25. Which of the following exhibits perfect diamagnetism?
    a) insulator   b) conductor
    c) semiconductor   d) superconductor

26. The critical angle for total internal reflection from a medium to air is 30°. What is the velocity of light in the medium?
    a) 3×10^8 ms^{-1} b) 1.5×10^8 ms^{-1}
    c) 2×10^8 ms^{-1} d) 1.732×10^8 ms^{-1}

27. Two interfering waves have amplitudes in the ratio 5:1. The ratio of the maximum to the minimum intensity is
    a) 25:1   b) 4:9   c) 6:4   d) 9:4

28. The optical path length across the thickness of a transparent slab is 10 cm. Its refractive index is 1.4. The thickness of the slab is
    a) 14 cm   b) 10 cm   c) 7.14 cm   d) 19.6 cm

29. A telescope has an objective lens of diameter 10cm. What is its angular resolution (in radians) for a wavelength of 600nm?
    a) 3.66×10^{-6} b) 7.32×10^{-6} c) 7.32×10^{-5} d) 7.32×10^{-7}

30. The ground state energy of hydrogen atom is -13.6 eV. What is the first excited state energy of He$^+$ ion?
    a) -13.6 eV   b) -6.8 eV   c) -27.2 eV   d) -19.2 eV

Rough work

CHEMISTRY

31. The electronic configuration of the element X is [Ar]$4s^2 3d^{10}$. Which one of the following is the most suitable formula for its oxide?

a) X_2O b) X_2O_3 c) XO d) X_2O_5

32. In which of the following pairs does nitrogen exhibit a valency of +1 and −1 respectively?

a) Nitrous oxide and nitric oxide   b) Nitrous oxide and hydroxylamine

c) Hydroxylamine and hydrazine   d) Nitric oxide and hydroxylamine

33. Acetylene is dissolved in acetone at increased pressure and is transported. This is based on

a) Boyle's law   b) Charles' law

c) Henry's law   d) Dalton's law.

34. 200 mL of 1.0 N, 400 mL of 0.5 N and 400 mL of 0.25 N of a solution are mixed together. The normality of the resultant solution will be

a) 0.5   b) 1.0   c) 0.1   d) 0.25

35. The volume of one molal solution of potassium chloride increases by 1.5% when its temperature is raised from 25 to $30^{\circ}C$. The molality of the solution will

a) increase by 1.5%   b) remain the same

c) increase by 3.0%   d) decrease by 1.5%

36. The increase in bond strength when fluorine is bonded to arsenic is due to

a) $p\pi - p\pi$ bonding   b) $d\pi - d\pi$ bonding

c) $p\pi \rightarrow d\pi$ donation   d) $d\pi \rightarrow p\pi$ donation

37. For a zero order reaction, the unit for the equilibrium constant is

a) s^{-1} b) $(mol/L)^{-1} s^{-1}$

c) no unit   d) $mol \, L^{-1} s^{-1}$

38. Lanthanides and Actinides exhibit a common oxidation state of

a) + 2   b) + 5   c) − 4   d) + 3

39. The catalyst used in an automobile car's exhaust system to oxidize carbon monoxide to carbon dioxide is

a) homogeneous type   b) mixed type

c) heterogeneous type   d) enzyme type

40. The percentage of empty space in a face centered cubic (FCC) unit cell is

a) 2.6   b) 26   c) 74   d) 7.4

Rough work

41. By passing certain quantity of electricity through a solution of copper sulphate, 5g of copper is deposited on the cathode. The same quantity of electricity is passed through brine solution in a divided cell. What is the amount of caustic soda in g formed in the cathode compartment? Atomic weights of copper and sodium are 63.5 and 23 respectively.
   a) 63.0   b) 6.3   c) 0.63   d) inadequate data

42. A hydrocarbon of molecular formula C_6H_{12} (A) was subjected to ozonolysis which gave compounds B and C. B on reduction with lithium aluminium hydride gave a primary alcohol C_3H_8O. C on reduction with zinc amalgam and hydrochloric acid gave a hydrocarbon C_3H_8. C does not respond to Tollen's reagent. What can be A?
   a) 2-methyl pent-2-ene   b) 3-methyl pent-2-ene
   c) hexene-2   d) hexene-3

43. 22.4 mL of hydrogen gas combines with 11.2 mL of oxygen at NTP. What is the number of molecules of water vapour formed?
   a) 22.4   b) 6.023×10^{23} c) 6.023×10^{20} d) 6.023×10^{17}

44. Chlorine dioxide is formed when
   a) Chlorate ion reacts with a reducing agent
   b) Chlorate ion reacts with an oxidizing agent
   c) Chlorate ion reacts with hypochlorite ion
   d) Chlorite ion reacts with chlorine

45. The correct order of crystal field splitting energy of the following ligands is
   a) $H_2O < C_2O_4^{2-} < NH_3 < CN^-$
   b) $NH_3 < C_2O_4^{2-} < H_2O < CN^-$
   c) $C_2O_4^{2-} < H_2O < NH_3 < CN^-$
   d) $N^- < NH_3 < C_2O_4^{2-} < H_2O$

46. m- dinitrobenzene on treatment with ammonium sulphide gives
   a) m-diaminobenzene   b) m-nitroaniline
   c) m-amino nitrosobenzene   d) benzene

47. Reaction of ethyl benzene with N-bromosuccinimide at room temperature produces
   a) 1-bromo-1-phenyl ethane   b) 1-bromo-2-phenyl ethane
   c) p-bromo ethyl benzene   d) o-bromo ethyl benzene

48. What is the emf of the following cell at 25°C?
   Ni (s) | Ni^{++} ‖ Ni^{++} | Ni (s)
              0.05M   1.6M
   a) 44.4 mV   b) 444 mV   c) 4.44 V   d) 0.0 V

Rough work

49. 4.0 liters of 0.8M sulphuric acid is prepared from 98% sulphuric acid of specific gravity 1.84 by dilution with water. What is the specific gravity of the diluted solution? Equivalent weight of sulphuric acid is 49.
   a) 10.37   b) 1.037   c) 0.1037   d) 0.01037

50. 70g of ammonium chloride is mixed with 560 mL of ammonia (NH_3) and the mixture is made up to one liter with water. Ionization constant of ammonium hydroxide is 1.8×10^{-5} at 25°C. Atomic weights of nitrogen and chlorine are 14 and 35.5 respectively. Density of liquid ammonia is $0.8 gcm^{-3}$. Ionic product of water is 1×10^{-14}. What is the pH of this solution?
   a) 1.056   b) 3.44   c) 10.56   d) 7.12

51. Polymer Dispersity Index of a polymer refers to
   a) ratio between number average and viscosity average molecular weights
   b) ratio between weight average and number average molecular weights
   c) ratio between number average molecular weight and chain length
   d) ratio between viscosity average molecular weight and density of polymer solution.

52. Antacid (gelusil) contains
   a) sodium hydroxide and aluminium hydroxide
   b) calcium hydroxide and magnesium hydroxide
   c) aluminium hydroxide and magnesium hydroxide
   d) aluminium hydroxide and calcium hydroxide

53. Which one of the following is the correct statement?
   a) Nucleophilic aromatic substitution occurs selectively at para position to nitro group.
   b) Nucleophilic aromatic substitution occurs selectively at ortho position to nitro group.
   c) Nucleophilic aromatic substitution occurs at ortho and para positions to nitro group.
   d) Nitro group is substituted by the incoming group.

54. Ethyl fluoride is formed by heating ethyl chloride with mercurous fluoride. This type of reaction is called
   a) Finkelstein reaction   b) Friedel-Crafts fluorination
   c) Swarts reaction   d) Sandmeyer reaction

55. In methyl cyanide C-H bond is longer and weaker than C-N bond because
   a) sp hybridization leads to the formation of shorter and stronger bond
   b) sp^3 hybridization leads to the formation of shorter and stronger bond
   c) sp^2 hybridization leads to the formation of longer and stronger bond
   d) dsp^2 hybridization leads to the formation of shorter and weaker bond

Rough work

56. The order of reactivity of hydrogen halides for cleavage of ethers at high temperature is
   a) HBr > HI > HCl
   b) HI > HCl > HBr
   c) HI > HBr > HCl
   d) HCl > HBr > HI

57. The coordinate compound iron carbonyl has the following structure.
   a) tetrahedral
   b) square planar
   c) octahedral
   d) trigonal bipyramidal

58. What will be the energy in MeV released in the fusion of deuterium to form one mol of helium gas? Masses of deuterium and helium are 2.014 m_u and 4.003 m_u respectively. One m_u is 1.66057×10^{-27} kg and velocity of light is 3.0×10^8 ms^{-1}
   a) 23.44
   b) 234.4
   c) 2344
   d) 0.2344

59. The activation energy of the forward reaction of the following is 250 kJ.
$$A + B \leftrightarrow C + D + 400 \text{ kJ}$$
   The activation energy for the backward reaction for the same will be
   a) 250 kJ
   b) 400 kJ
   c) 150 kJ
   d) 650 kJ

60. Concentrated sulphuric acid is stored and transported in
   a) lead containers
   b) PVC tanks
   c) polypropylene containers
   d) mild steel tanks

Rough work

MATHEMATICS

61. If $z = (\lambda + 3) + i\sqrt{5 - \lambda^2}$, then the locus of z is
    (a) $x^2 + y^2 = 25$
    (b) $x^2 + y^2 = 9$
    (c) $x^2 + y^2 - 6x + 4 = 0$
    (d) $x^2 + y^2 - 6x + 25 = 0$

62. If $\cos\theta + i\sin\theta$ is a root of the equation $a_0 x^n + a_1 x^{n-1} + \ldots\ldots\ldots + a_n = 0$, then the value of $a_0 + a_1 \cos\theta + a_2 \cos 2\theta + \ldots\ldots + a_n \cos n\theta$ is
    (a) 0
    (b) n
    (c) $\cos(n+1)\theta$
    (d) $\sin(n+1)\theta$

63. If $\cos A + \cos B + \cos C = 0 = \sin A + \sin B + \sin C$, then the value of $\cos(A - B) + \cos(B - C) + \cos(C - A)$ is
    (a) $1/2$
    (b) -3
    (c) $3/2$
    (d) $-3/2$

64. Given that $\sin A$, $\cos A$ and $\tan A$ are in G.P., the value of $\cot^6 A - \cot^2 A$ is
    (a) -1
    (b) 0
    (c) 1
    (d) 2

65. If $\sin^{-1}\left(\dfrac{2a}{1+a^2}\right) - \cos^{-1}\left(\dfrac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$, then 'x' is
    (a) $\dfrac{a+b}{1+ab}$
    (b) $\dfrac{a-b}{1+ab}$
    (c) $\dfrac{a-b}{1-ab}$
    (d) $\dfrac{a+b}{1-ab}$

66. The value of $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$ is
    (a) $xyz(x+y+z)$
    (b) xyz
    (c) $(x+y+z)^3$
    (d) $x^2 y^2 z^2 (x+y+z)$

67. The positive solution of the equation $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$ is
    (a) 3
    (b) 9
    (c) 12
    (d) 6

Rough work

68. If $A^{-1} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$, then Adj(A) is

(a) $\begin{pmatrix} -2 & 0 & -1 \\ 9 & -2 & 3 \\ 6 & -1 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 0 & 1 \\ -9 & -2 & -3 \\ -6 & -1 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 0 & -1 \\ -9 & 2 & 3 \\ -6 & 1 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{pmatrix}$

69. The system of equations $x + 2y - z = 2$; $5y - 5z = 3$; $2x - y + \lambda z = \mu$ has infinitely many solutions if the pair $\{\lambda, \mu\}$ is

(a) $\{3, 1\}$ (b) $\{1, 3\}$ (c) $\{-3, 1\}$ (d) $\{-1, 3\}$

70. It is given that x, y, z not all zero satisfy the equations $x = cy + bz$, $y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2$ is

(a) abc (b) $abc - 1$ (c) $1 - 2abc$ (d) $1 + 2abc$

71. In a plane, a set of 15 parallel lines intersect another set of 20 parallel lines to form parallelograms. The number of such parallelograms formed is

(a) 19850   (b) 19750   (c) 19000   (d) 19950

72. If $|\bar{a}| = 5$, $|\bar{b}| = 7$ and $|\bar{a} - \bar{b}| = 12$, then $|\bar{a} + \bar{b}|$ is equal to

(a) 2   (b) 4   (c) 12   (d) $\sqrt{74}$

73. If $\bar{a}$ and $\bar{b}$ are non collinear vectors, then $\dfrac{\bar{a}}{|\bar{a}|} + \dfrac{\bar{b}}{|\bar{b}|}$ is

(a) a unit vector

(b) in the plane of $\bar{a}$ and $\bar{b}$

(c) perpendicular to $\bar{a}$ and $\bar{b}$

(d) parallel to $\bar{a}$ and $\bar{b}$

Rough work

74. Forces acting on a particle having magnitudes 3, 2, 1 units act in the directions of the vectors $2\hat{i} + 4\hat{j} + 4\hat{k}$, $4\hat{i} - 4\hat{j} + 2\hat{k}$ and $4\hat{i} - 4\hat{j} - 2\hat{k}$ respectively. The work done by the forces in displacing the particle from the point $A(2, -1, 6)$ to the point $B(5, -1, 3)$ is

    (a) 2 units   (b) 4 units   (c) 6 units   (d) 3 units

75. $\underset{n \to \infty}{Lt}\ 4^{n-1} \sin\left(\dfrac{a}{4^n}\right)$ is equal to

    (a) $-a$ (b) $a/2$ (c) $a/4$ (d) $-a/4$

76. If $f(x)$ is a continuous function satisfying $f(x) f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$ and $f(1) > 0$, then $\underset{x \to 1}{Lt}\ f(x)$ is equal to

    (a) 2   (b) 1   (c) 3   (d) $3/2$

77. If $y = \log \sqrt{e \log x}$, $\dfrac{dy}{dx}$ at $x = e$ is

    (a) $\dfrac{1}{\sqrt{e \log e}}$ (b) $\dfrac{1}{2e \log e}$ (c) $\dfrac{e \log e}{2}$ (d) $\sqrt{e \log e}$

78. The derivative of $\cos^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$ with respect to $\tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$ is

    (a) $1/3$ (b) $-1/3$ (c) $-2/3$ (d) $2/3$

79. If α, β are the roots of the equation $x^2 - 3x + 7 = 0$, then the equation whose roots are $\alpha^2 + 2, \beta^2 + 2$ is

    (a) $y^2 + y + 43 = 0$ (b) $y^2 - y + 43 = 0$ (c) $y^2 + y - 43 = 0$ (d) $y^2 - y - 43 = 0$

80. If one of the roots of $ax^2 + bx + c = 0$ is the 4th power of the other, then the value of $\left(ac^4\right)^{1/5} + \left(a^4 c\right)^{1/5}$ is

    (a) 0   (b) 1   (c) $-b$ (d) b

Rough work

81. If $\log 2,\ \log(2^x - 1),\ \log(2^x + 3)$ are in arithmetic progression, then the value of 'x' is

   (a) $\log_5 2$ (b) 2 (c) $\log_e 2$ (d) $\log_2 5$

82. If a, b, c are in harmonic progression and if, $x = \dfrac{c}{a+b},\ y = \dfrac{b}{a+c},\ z = \dfrac{a}{c+b}$,

   then $\dfrac{1}{x} + \dfrac{1}{z}$ is

   (a) $\dfrac{2}{y}$ (b) $\dfrac{1}{y}$ (c) $\dfrac{3}{y}$ (d) $-\dfrac{1}{y}$

83. If $\sum\limits_{r=1}^{n}(3r + 2)(r - 5) = an^3 + bn^2 + cn$, then the values of a, b, c are

   (a) $1,\ -5,\ -16$ (b) $-5,\ 1,\ -16$ (c) $-16,\ 1,\ 5$ (d) $-5,\ -16,\ 1$

84. If $z = (\cos 2 + i\sin 2 + 1)^n$, then $|z|$ is

   (a) $2^n \cos 1$ (b) $2^n \cos n$ (c) $2^n \sin n$ (d) $2^n \cos^n 1$

85. The number of times the digit '5' will be written when listing the numbers from 1 to 1000 (assuming that a single digit number is written as 00x and a double digit number as 0xy) is

   (a) 109 (b) 300 (c) 271 (d) 250

86. If $\int \dfrac{dx}{\sqrt{x(1 - 4x)}} = K\sin^{-1}(8x - 1) + C$, then K is equal to

   (a) $1/\sqrt{2}$ (b) $-1/\sqrt{2}$ (c) $-1/2$ (d) $1/2$

87. Let $\dfrac{d}{dx}F(x) = \dfrac{e^{\sin x}}{x},\ x > 0$. If $\int\limits_a^b \dfrac{2e^{\sin x}}{x}dx = F(k) - F(l)$, then one of the possible set of values of 'k' and 'l' is respectively

   (a) $\dfrac{b-a}{2},\ \dfrac{b+a}{2}$ (b) $\dfrac{b+a}{2},\ \dfrac{b-a}{2}$ (c) b^2, a^2 (d) a^2, b^2

Rough work

88. The solution of the differential equation $e^{\log\frac{dy}{dx}} = e^{2x} + y - 1$, $y(0) = 1$ is
   (a) $y = e^{2x} + e^x + 1$
   (b) $y = e^{2x} - e^x$
   (c) $y = e^{2x} - e^x + 1$
   (d) $y = e^{2x} + e^{-2x} + 1$

89. The arithmetic mean of n observations is 'm'. If two observations 0 and m are added, then the new mean is
   (a) m
   (b) $\dfrac{n}{m+1}$
   (c) $\dfrac{mn}{n+2}$
   (d) $\dfrac{m(n+1)}{n+2}$

90. Two events A and B have probabilities 0.20 and 0.40 respectively. The probability that both A and B occur simultaneously is 0.15. Then the probability that neither A nor B occurs is
   (a) 0.60
   (b) 0.40
   (c) 0.45
   (d) 0.55

91. The equations $ax + by + c = 0$ and $dx + ey - f = 0$ represent the same straight line if and only if
   (a) $a = d$, $b = e$
   (b) $\dfrac{a}{d} = -\dfrac{c}{f}$
   (c) $\dfrac{a}{d} = \dfrac{b}{e}$
   (d) $\dfrac{a}{d} = -\dfrac{c}{f} = \dfrac{b}{e}$

92. The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if
   (a) $\sqrt{a^2(1+m^2)} < c$
   (b) $\sqrt{a^2(1-m^2)} < c$
   (c) $\sqrt{a^2(1+m^2)} \geq c$
   (d) $\sqrt{a^2(1-m^2)} \geq c$

93. The foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide. Then the value of b^2 is
   (a) 1
   (b) 5
   (c) 9
   (d) 7

94. If ω is the cube root of unity, then the value of $(1-\omega)(1-\omega^2) + (2-\omega)(2-\omega^2) + \ldots\ldots\ldots + (n-\omega)(n-\omega^2)$ is
   (a) $\dfrac{n}{3}(n^2 + 3n - 5)$
   (b) $\dfrac{n}{3}(n^2 - 3n + 5)$
   (c) $\dfrac{n}{3}(n^2 + 3n + 5)$
   (d) $\dfrac{n}{3}(n^2 - 3n - 5)$

Rough work

95. In a triangle ABC, $\tan(A/2)$, $\tan(B/2)$, $\tan(C/2)$ are in H.P. and the sides 'a' and 'c' are given by 5 and 9 units, then the side 'b' is
(a) 6   (b) 8   (c) 7   (d) 11

96. The bases of two towers subtend an angle of $120°$ at a point on the ground which is at 10m distance from each of the bases. A bird sitting at the top of the higher tower starts flying at a constant speed along a straight path inclined at an angle of $45°$ to the tower and reaches the other top in 5 sec. The speed of the flight (in m/s) is
(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

97. The area bounded by the curve $y^2 = 4ax$ and the line $y = 2a$ and the y-axis is (in square units)
(a) $\frac{1}{3}a^2$ (b) $\frac{2}{3}a^2$ (c) $\frac{4}{3}a^2$ (d) $\frac{3}{4}a^2$

98. A solution of the equation $y\dfrac{dx}{dy} = x(\log x - \log y + 1)$ is
(a) $y = xe^{cx}$ (b) $x^2 = cy\log y$ (c) $x = ye^{cy}$ (d) $\log x = cy$

99. The solution of $\dfrac{dy}{dx}\tan y = \sin(x + y) + \sin(x - y)$ is
(a) $\sec y = C - 2\cos x$ (b) $y = C - 2\cos x$
(c) $\tan y = C - \sin x$ (d) $\cos y = C + 2\cos x$

100. The integrating factor of the linear differential equation $\left(\sin^2 y + x\cot y\right)\dfrac{dy}{dx} = 1$ is
(a) $\cosec y$ (b) $\sin y$ (c) $\tan y$ (d) $\cos y$

101. The variance of the first n natural numbers is
(a) $\dfrac{n(n+1)(2n+1)}{12}$ (b) $\dfrac{n^2-1}{12}$ (c) $\sqrt{\dfrac{n^2-1}{12}}$ (d) $\sqrt{\dfrac{n^2+1}{12}}$

Rough work

102. For a distribution, the coefficient of variation is 22.5% and the value of the arithmetic average is 7.5. Then the value of the standard deviation is
(a) 2   (b) 1.68   (c) 2.5   (d) 1

103. The mean of 10 numbers is 6 and their standard deviation is 2. Then the sum of the squares of these numbers is
(a) 600   (b) 300   (c) 100   (d) 400

104. A class has 10 boys and 5 girls. Three students are selected at random one after the other. The probability that first two are boys and the third is a girl is
(a) $\dfrac{2}{45}$ (b) $\dfrac{5}{91}$ (c) $\dfrac{15}{91}$ (d) $\dfrac{21}{91}$

105. A fair coin is tossed repeatedly. If the tail appears on first three tosses, then the probability of the head appearing on the fourth toss is
(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{8}$ (c) $\dfrac{7}{8}$ (d) $\dfrac{1}{16}$

106. If $P(X \le 4) = 0.8$ and $P(X = 4) = 0.2$, then $P(X \ge 4)$ is
(a) 0.2   (b) 0.4   (c) 0.5   (d) 0.6

107. If in a Binomial distribution $n = 4$, $P(X = 0) = \dfrac{81}{625}$, then $P(X = 4)$ is
(a) $\dfrac{3}{5}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{32}{625}$ (d) $\dfrac{16}{625}$

108. The points $(3, 3)$, $(-h, 0)$, $(0, k)$ are collinear if
(a) $\dfrac{1}{h} + \dfrac{1}{k} = \dfrac{1}{2}$ (b) $\dfrac{1}{h} + \dfrac{1}{k} = \dfrac{1}{3}$ (c) $\dfrac{1}{h} - \dfrac{1}{k} = \dfrac{1}{3}$ (d) $\dfrac{1}{h} + \dfrac{1}{k} = \dfrac{-1}{3}$

109. The foot of the perpendicular from the point $(1, 2)$ upon $x + y = 1$ is
(a) $(0, 1/2)$ (b) $(0, 1)$ (c) $(1, 0)$ (d) $(1/2, 1/2)$

110. If a, b, c are in H.P., then the line $\dfrac{x}{a} + \dfrac{y}{b} - \dfrac{1}{c} = 0$ always passes through the point
(a) $(-1, -2)$ (b) $(-1, 2)$ (c) $(1, -2)$ (d) $(1, -1)$

Rough work

111. If the sum of the slopes of the lines given by $x^2 - 2kxy + 9y^2 = 0$ is 6 times their product, then k has the value
(a) 2   (b) -2 (c) -3 (d) 3

112. The equation of the circle, if its centre is $(4, 5)$ and the circumference passes through the centre of the circle $x^2 + y^2 + 4x - 6y = 12$ is
(a) $x^2 + y^2 - 8x - 10y + 1 = 0$ (b) $x^2 + y^2 + 8x - 10y + 1 = 0$
(c) $x^2 + y^2 - 8x + 10y + 1 = 0$ (d) $x^2 + y^2 - 8x - 10y - 1 = 0$

113. The extremities of the diameter of a circle have coordinates $(-4, 3)$ and $(6, -2)$. Then the length of the intercept which the circle makes on the y-axis is
(a) $\sqrt{124}$ (b) 12   (c) 11   (d) $\sqrt{136}$

114. The eccentricity of the hyperbola $\dfrac{x^2}{5} - \dfrac{y^2}{5} = \dfrac{1}{\sqrt{1999}}$ is
(a) 2   (b) $\sqrt{2}$ (c) 4   (d) $2\sqrt{2}$

115. Let $\bar{a} = 2\hat{i} + \hat{k}$, $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{c} = 4\hat{i} - 2\hat{j} + 7\hat{k}$, then the vector $\bar{r}$ such that $\bar{r} \times \bar{b} = \bar{c} \times \bar{b}$ and $\bar{r} \cdot \bar{a} = 0$ is
(a) $\hat{i} + 7\hat{j} + 2\hat{k}$ (b) $-\hat{i} - 7\hat{j} + 2\hat{k}$
(c) $7\hat{i} + \hat{j} + 2\hat{k}$ (d) $\hat{i} + \hat{j} + 7\hat{k}$

116. If $f(x) = |\cos x|$, then $f'\left(\dfrac{3\pi}{4}\right)$ is equal to
(a) $-1/\sqrt{2}$ (b) $1/\sqrt{2}$ (c) 1   (d) -1

Rough work

117. Let $f(x) = \sin^4 x + \cos^4 x$, $0 < x < \dfrac{\pi}{2}$. Then the minimum value of $f(x)$ is

(a) $1/2$ (b) $-1/2$ (c) $1/4$ (d) *does not exist*

118. $\int \sqrt{x}\left(\sqrt[4]{1+x^{3/2}}\right)dx$ equals

(a) $\dfrac{4}{15}\left(1+x^{3/2}\right)^{5/4} + C$ (b) $\dfrac{8}{15}\left(1+x^{3/2}\right)^{5/4} + C$

(c) $\dfrac{8}{15}\left(1+x^{3/2}\right)^{5/2} + C$ (d) $\dfrac{15}{4}\left(1+x^{3/2}\right)^{5/4} + C$

119. $\int e^{\tan^{-1}x}\left(1+\dfrac{x}{1+x^2}\right)dx$ is equal to

(a) $x e^{\tan^{-1}x} + C$ (b) $\dfrac{x}{2}e^{\tan^{-1}x} + C$ (c) $\dfrac{1}{2}e^{\tan^{-1}x} + C$ (d) $e^{\tan^{-1}x} + C$

120. The value of $\displaystyle\int_0^{\pi/2} \dfrac{\sin x}{\sin x + \cos x}dx$ is

(a) π (b) $\pi/2$ (c) $\pi/4$ (d) 2π

Rough work

(Continued from the first page)

OMR ANSWER SHEET

11. Use the OMR answer sheet carefully; no spare sheet will be issued under any circumstance.

12. Do not fold or make any stray mark on the OMR sheet.

13. Use HB Pencil for shading the bubbles and black ball pen for writing.

14. In the OMR answer sheet, make the following entries

    a. Write the Registration number, Question Booklet Number and Question Booklet Version code.

    b. Fill the ovals corresponding to the Registration Number, Question Booklet Number and Question Booklet Version Code.

    c. Write your Name and Sign in the column provided.

15. Rough work should not be done on the answer sheet.

ANSWERING AND EVALUATION

16. For each question, four answers are suggested of which only one is correct / most appropriate. Mark the correct / most appropriate answer by darkening the corresponding bubble using HB pencil.

17. In case the candidate wishes to change the choice already shaded, he/she may erase the marking completely and thereafter shade the alternative bubble.

18. If more than one bubble is darkened against a question, it will be treated as an incorrect answer.

19. For each correct answer, three marks will be awarded.

20. **For each incorrect answer, one mark will be deducted from the total score.**

21. If any smudge is left on the OMR sheet, evaluation will become imperfect.

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