2016

Booklet No.

TEST CODE: PEA
Forenoon

Questions: 30

Time: 2 hours

- On the answer booklet write your Name, Registration number, Test Centre, Test Code and the Number of this Booklet in the appropriate places on the Answer-sheet.
- This test has 30 questions. **ANSWER ALL QUESTIONS.** All questions carry equal (4) marks.
- For each of the 30 questions, there are four suggested answers. Only one of the suggested answers is correct. You will have to identify the correct answer to get full credit for that question. Indicate your choice of the correct answer by darkening the appropriate oval completely on the answer-sheet.
- You will get:

  4 marks for each **correctly** answered question,

  0 marks for each **incorrectly** answered question, and

  1 mark for each **unanswered** question.

1. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are two functions such that $f(x) = ax+b$ and $g(x) = cx+d$, then $f(g(x)) = g(f(x))$ holds if and only if

   A. $f(a) = g(c)$ B. $f(b) = g(b)$ C. $f(d) = g(b)$ D. $f(c) = g(a)$

2. A box contains 90 good and 10 defective screws. If 10 screws are drawn without replacement, the probability that none of them is defective is

   A. $\frac{^{90}C_{10}}{^{100}C_{10}}$ B. $\frac{90^{10}}{100^{10}}$ C. $\frac{^{90}P_{10}}{^{100}P_{10}}$ D. None of the above

3. The number of ordered pairs of integers (x, y) satisfying the equation

   $$x^2 + 6x + y^2 = 4$$

   is

   A. 2 B. 4 C. 6 D. 8

4. The value of $\lim_{x\to-\infty} \frac{3x^2-Sin(5x)}{x^2+2}$ is

   A. 0 B. 1 C. 2 D. 3

5. The smallest integer that produces remainders of $2, 4, 6$ and 1 when divided by $3, 5, 7$ and 11 respectively is

   A. 104 B. 1154 C. 419 D. None of the above

6. Given thirty people, the probability that among the twelve months there are six containing two birthdays each and six containing three each is

   A. $\frac{30!}{2^6 6^6} \times^{12} C_6 \times 12^{-30}$ B. $^{12}C_6 \times^{30} C_{12}$ C. $\frac{30!}{2^6 6^6} \times^{12} C_6 \times \frac{1}{^{30}C_{12}}$ D. None of the above

7. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. Then the number of solutions of $|x^2 - [x]| = 1$ is

   A. 0 B. 1 C. 2 D. 3

8. Let $a, b, c \in \mathbb{R}$, $a^2 + b^2 + c^2 = 1$, and $A = ab + bc + ca$. Then

   A. $-\frac{1}{2} < A < 1$ B. $-1 < A < 1$ C. $-\frac{1}{2} < A \leq 1$ D. $-\frac{1}{2} \leq A \leq 1$

9. Two dice are rolled. If the two faces are different, the probability that one is a six is

   A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{1}{3}$

10. The minimum value of $\frac{x^2+2}{\sqrt{x^2+1}}$ (where x is a real number) is

    A. 1 B. 2 C. $\sqrt{2}$ D. None of the above

11. If $f(x) = (\frac{a+x}{b+x})^{a+b+2x}$, then $f'(0)$ equals

    A. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{a+b})^{a+b}$ B. $(2\log(\frac{a}{b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ C. $(2\log(\frac{a}{a+b}) + \frac{b^2-a^2}{ab}) \times (\frac{a}{b})^{a+b}$ D. None of the above

12. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random without replacement. Then the probability that none of the balls drawn is blue is

    A. 10/21 B. 11/21 C. 2/7 D. 5/7

13. The letters of the word COCHIN are permuted and all the permutations are arranged lexicographically (i.e., in alphabetical order as in an English dictionary). The number of words that appear before the word COCHIN is

    A. 96 B. 360 C. 192 D. 48

14. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable, and $f(0) = 1$. Assume that $f'(x) \geq 1$ for all $x \in \mathbb{R}$. The smallest possible value of $f(5)$ is

    A. 0 B. 1 C. 5 D. 6

15. A math contest is made up of 64 multiple choice questions each worth either 0 (if wrong or no answer) or 1 (if right). Then the number of students who must write the test to be sure that at least 32 have the same final score is

    A. 2016 B. ${}^{52}C_{39}$ C. 39^{52} D. None of the above

16. Player 1 and Player 2 both start with 100 rupees. Each round of a game consists of the following:
    Both players choose a number randomly and independently from 1 to 5. If both players choose the same number, then Player 1 gives rupees 10 to Player 2. Otherwise, Player 2 gives rupees 10 to Player 1. Then the expected amount of money Player 1 will be left with after playing 10 rounds of this game is

    A. 120 B. 100 C. 50 D. 160

17. Let $f : (0, 1) \to \mathbb{R}$ be a function defined by

$$f(x) = \left\{ \begin{array}{ll} x^2, & \text{if } x \text{ is rational;} \\ 2 - x^2, & \text{otherwise.} \end{array} \right\}$$

    Then f is continuous at

    A. no point in (0, 1) B. exactly one point in (0, 1) C. exactly two points in (0, 1) D. more than two points in (0, 1)

18. The last digit of 432^{43567} is

A. 2 B. 4 C. 6 D. 8

19. Two squares are chosen at random on a chessboard (with 64 squares). Then the probability that they have a side in common is

A. 1/9 B. 1/27 C. 1/18 D. None of the above

20. Let $f : [0, 1] \to \mathbb{R}$ be a function such that $f(x) = \frac{x}{x-2}$. Then f is

A. one-one B. onto C. one-one and onto D. None of the above

21. Let

$$A = \begin{bmatrix} 1+a_1 & a_2 & \dots & a_n \\ a_1 & 1+a_2 & \dots & a_n \\ \dots & \dots & \dots & \dots \\ a_1 & a_2 & \dots & 1+a_n \end{bmatrix}.$$

The determinant of A is

A. $a_1+a_2+\ldots+a_n$ B. $1+a_1a_2\ldots a_n$ C. $n+a_1a_2\ldots a_n$ D. $1+a_1+a_2+\ldots+a_n$

22. The ratio of boys to girls at birth in Singapore is 1.09:1 Then the proportion of Singaporean families with exactly 6 children who will have at least 3 boys is

A. 0.696 B. 0.315 C. 0.521 D. 0.455

23. Let

$$X = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$

Then

A.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 0 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

B.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & 0 & 0 \\ -2 & 4 & 0 \\ 3 & -6 & 4 \end{pmatrix}$$

C.
$$X^{-1} = 1/8 \begin{pmatrix} 4 & -2 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 4 \end{pmatrix}$$

D. X^{-1} does not exist

24. Suppose the probability of having a girl is 1/2 and so is the probability of having a boy. Now consider a family with two children. Then the probability that both the children are girls given that at least one of them is a girl is

A. 1/4 B. 2/3 C. 1/3 D. 1/2

25. Let $U : \mathbb{R}_+ \to \mathbb{R}$ be a strictly increasing function such that $U(x) \neq -1$ for all $x \in \mathbb{R}_+$, where $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. Then the function $V : \mathbb{R}_+ \to \mathbb{R}$, defined by $V(x) = \frac{U(x)}{1+U(x)}$, is

A. necessarily strictly increasing B. necessarily strictly decreasing C. necessarily constant D. None of the above

26. A box contains three coins: two regular coins and one fake two-headed coin (i.e., $P(H) = 1$). Bagha picks a coin at random and tosses it, and gets head. Then the probability that it is the two-headed coin is

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. None of the above

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(x)f'(x) < 0$ for all $x \in \mathbb{R}$. Then A. $f(x)$ is an increasing function B. $|f(x)|$ is an increasing function C. $f(x)$ is a decreasing function D. $|f(x)|$ is a decreasing function

28. Let X and Y be two independent discrete random variables with the CDFs F_X and F_Y. Then the CDF of $W = \min\{X, Y\}$ is

A. $F_W(w) = \frac{F_X(w)+F_Y(w)}{2}$ B. $F_W(w) = \min\{F_X(w), F_Y(w)\}$ C. $F_W(w) = F_X(w)F_Y(w)$ D. None of the above

29. Two dice are thrown simultaneously. Then the probability of getting two numbers whose product is even is

A. 1/8 B. 1/4 C. 3/4 D. None of the above

30. Let $[x]$ denote the greatest integer less than or equal to x for any real number x. The range of the function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \frac{\sin(\pi[x])}{x^2+5}$, is

A. $(-1, 1)$ B. $[-1, 1]$ C. $\{-1, 1\}$ D. $\{0\}$