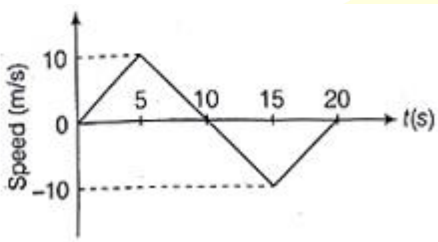
Physics

Single correct answer type:

1. The one-dimensional motion of a point particle is shown in the figure. Select the correct statement.



(A) The total distance travelled by the particle is zero
(B) The total displacement of the particle is zero
(C) The maximum acceleration of the particle is $\frac{1}{2}ms^{-2}$
(D) The total distance travelled by the particle at the end of 10s is $100m$
(E) At the fifth second, the acceleration of the particle is $2ms^{-2}$
Solution: (B)
Given, figure shows the relation between speed and time. So, area of the figure will be displacement.
Therefore,
area of first triangular $(d_1) = \frac{1}{2} \times 10 \times 10 = 50m$, area of second triangular

$$(d_2) = \frac{1}{2} \times (-10) \times 10 = -50m$$

Therefore, total area $(d) =$ total displacement of particle
$\Rightarrow d_1 + d_2 = 50 - 50 = 0$

2. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum and g is the acceleration due to gravity. The length is measured using a meter scale which has 2000 divisions. If the measured value L is $50\ cm$, the accuracy in the determination of g is 1.1% and the time taken for 100 oscillations is 100 seconds, what should be the resolution of the clock (in milliseconds)?
(A) 1 (B) 2 (C) 5 (D) 0.25 (E) 0.1
Solution: (C)
Given,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or $T^2 = 4\pi^2\frac{l}{g}$

$\therefore \quad 2\frac{\Delta T}{T} = \frac{\Delta l}{l} + \frac{\Delta g}{g} \qquad \ldots(i)$

Now, $l = 50cm, \Delta l = 2mm = 0.2\ cm$

$\frac{\Delta g}{g} = 1.1\% = \frac{1.1}{100}$

Put these values in Equation. (i), then we get

$\frac{\Delta T}{T} = \frac{1}{2}\left[\frac{0.2}{50} + \frac{1.1}{100}\right]$

$= 7.5 \times 10^{-3} s$ or $7.5\ ms$

$\because$ In 100s, resolution of clock is $7.5\ ms$.

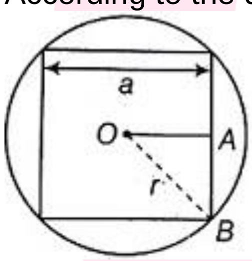
$\therefore$ In 60s resolution of clock is $\frac{7.5\times 60}{100} \approx 5ms$

3. From a circular card board of uniform thickness and mass M, a square disc of maximum possible area is cut. If the moment of inertia of the square with the axis of rotation at the centre and perpendicular to the plane of the disc is $\frac{Ma^2}{6}$, the radius of the circular card board is

(A) $\sqrt{2}a$ (B) $\frac{a}{\sqrt{2}}$ (C) $2a$ (D) $\frac{1}{2a}$ (E) $2\sqrt{2}a$

Solution: (B)

According to the question,



In the triangular OAB,

$(OB)^2 = (OA)^2 + (AB)^2$

Or $r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$

$r = \sqrt{\frac{2a^2}{4}} = \frac{a}{\sqrt{2}}$

4. The length is measured using a vernier system whose main scale is 30 cm long with 600 divisions. If 19 divisions of the main scale coincide with 20 divisions of the vernier scale, then its least count is

(A) $0.25\ cm$ (B) $0.025\ cm$
(C) $0.25\ mm$ (D) $0.025\ mm$
(E) $0.0025\ mm$

Solution: (D)

$\because$ Least count $= MSD - VSD$

$\because MSD = \frac{\left(\frac{30}{100}\right)}{600} = 5 \times 10^{-4} m$

and $VSD = \frac{19}{20} \times MSD$

$= \frac{19}{20} \times 5 \times 10^{-4} = 4.75 \times 10^{-4}$

$\therefore$ Least count $= (5 - 4.75) \times 10^{-4} = 0.025 mm$

5. A particle of mass m is moving along the $X-\text{axis}$ under the potential $V(x) = \frac{kx^2}{2} + \frac{\lambda}{x}$, where k and x are positive constants of appropriate dimensions. The particle is slightly displaced from its equilibrium position. The particle oscillates with the angular frequency (ω) given by

(A) $3\frac{k}{m}$ (B) $3\frac{m}{k}$ (C) $\sqrt{\frac{k}{m}}$ (D) $\sqrt{3\frac{m}{k}}$ (E) $\sqrt{3\frac{k}{m}}$

Solution: (C)

Given, $V(x) = \frac{kx^2}{2} + \frac{\lambda}{x}$

$\because$ In electrical analogy,

$\omega = \frac{1}{\sqrt{LC}}$

but in mechanical analogy L and C

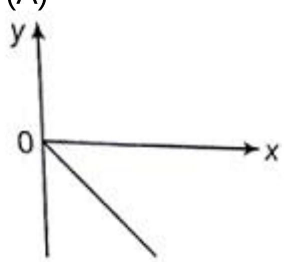
ω will be transformed into m and $\frac{1}{k}$.

Here, m is mass of particle.

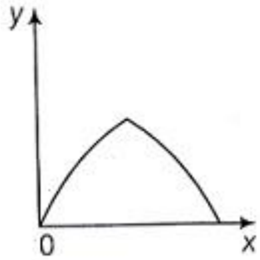
Hence, angular frequency, $\omega = \frac{1}{\sqrt{m\left(\frac{1}{k}\right)}}$ or $\omega = \sqrt{\frac{k}{m}}$

6. Two particles of mass m and $2m$ have their position vectors as a function of time as $r_1(t) = t\hat{i} + t^3\hat{j} + 2t^2\hat{k}$ and $r_1(t) = t\hat{i} + t^3\hat{j} + t^3\hat{k}$ respectively (where t is the time). Which one of the following graphs represents the path of the centre of mass?

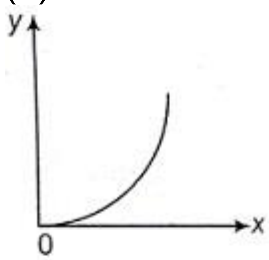
(A)



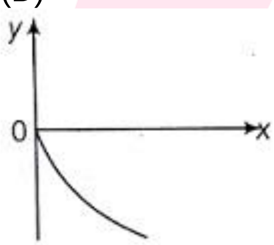
(B)



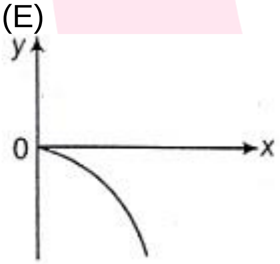
(C)



(D)



(E)



Solution: (E)

Path of the centre of mass in a two particle system,

$$r(t) = \left[\frac{m_1 r_1(t) + m_2 r_2(t)}{m_1 + m_2}\right]$$

or $r(t) = \left[\frac{m\left(t\hat{i} - t^3\hat{j} + 2t^2\hat{k} + 2m\left(t\hat{i} - t^3\hat{j} - t^2\hat{k}\right)\right)}{m + 2m}\right]$

$$\Rightarrow \; r(t) = \frac{3t\hat{i} - 3t^3\hat{j}}{3} = t\hat{i} - t^3\hat{j}$$

7. Two planets A and B have the same average density. Their radii R_A and R_B are such that $R_A : R_B = 3:1$. If g_A and g_B are the acceleration due to gravity at the surfaces of the planets, the $g_A : g_B$ equals

(A) $3:1$ (B) $1:3$ (C) $9:1$ (D) $1:9$ (E) $\sqrt{3}:1$

Solution: (A)

Given,
$\frac{R_A}{R_B} = \frac{3}{1}$ .....(i)
and $\rho_A = \rho_B$ ....(ii)
$\because$ Average density, $\rho = \frac{3g}{4\pi RG}$
$\therefore$ From Equation (ii),
$$\Rightarrow \quad \frac{3g_A}{4\pi R_A G} = \frac{3g_B}{4\pi R_B G} \quad \Rightarrow \quad \frac{g_A}{g_B} = \frac{R_A}{R_B}$$
From Equation (i),
$$\frac{g_A}{g_B} = \frac{3}{1}$$

8. The magnetic induction field has the dimensions of
(A) Force
(B) Force constant
(C) Surface tension
(D) $\frac{\text{Surface tension}}{\text{Current}}$
(E) Force constant λ current

Solution: (D)
$$\because \quad F = qvB$$
$$\therefore \quad B = \frac{F}{qv} = \frac{F}{q\left(\frac{d}{t}\right)} = \frac{F}{Id} = \frac{T}{I}$$
$$\therefore \quad B = \frac{\text{Surface tension}}{\text{Current}}$$

9. Einstein was awarded the Nobel Prize for his work on
(A) Photoelectric effect
(B) Special theory of relativity
(C) Brownian motion
(D) General theory of relativity
(E) Quantum theory

Solution: (A)
Einstein was awarded the Nobel Prize for his work in Photoelectric effect.

10. A thin circular ring of mass m and radius R is rotating about its axis perpendicular to the plane of the ring with a constant angular velocity ω. Two point particles each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates, with an angular velocity $\frac{\omega}{2}$. Then, the ratio $\frac{m}{M}$ is

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) $\sqrt{2}$

(E) $\frac{1}{\sqrt{2}}$

Solution: (B)

$\because$ Momentum before particle is attached to the ring, $= I\omega = mR^2\omega$

Momentum after two particle is attached to the ring, $= \frac{I\omega}{2} + (\mu x^2)\frac{\omega}{2}$

Here, $\mu = \frac{M \cdot M}{M+M} = \frac{M}{2}$ and $x = 2R$

So, $I\frac{\omega}{2} + (\mu x^2)\frac{\omega}{2} = \left[mR^2 + \frac{M}{2}(4R^2)\right]\frac{\omega}{2}$

$= [mR^2 + 2MR^2]\dfrac{\omega}{2}$

According law of conservation of momentum, (momentum before) = (momentum after)

$\Rightarrow \; mR^2\omega = (mR^2 + 2MR^2)\dfrac{\omega}{2}$

$\Rightarrow \; 2mR^2 = mR^2 + 2MR^2$

So $\frac{m}{M} = 2$

11. A body of mass $m = 1kg$ is moving in a medium and experiences a fractions force $F = -kv$, where v is the speed of the body. The initial speed is $v_0 = 10\, ms^{-1}$ and after 10s, its energy becomes half of initial energy. Then, the value of k is

(A) $10 \ln \sqrt{2}$

(B) $\ln \sqrt{2}$

(C) $\frac{\ln 2}{20}$

(D) $10 \ln 2$

(E) $\ln 2$

Solution: (C)

$\because$ According to the question,

$\dfrac{1}{2}mv_f^2 = \dfrac{1}{2} \times \dfrac{1}{2}mv_i^2$

$\left(v_i, v_f = \text{initial and final speeds of the body}\right)$

or $v_f^2 = \frac{v_i^2}{2}$ or $v_f = \frac{10}{\sqrt{2}}$

Given, $f = -kv$

or $ma = -kv \Rightarrow \frac{mdv}{dt} = -kv$

$$\Rightarrow \int_{10}^{\frac{10}{\sqrt{2}}} \frac{1}{v} dv = -\int_{0}^{10} kdt \quad (\because \quad m = 1kg)$$

$$\Rightarrow \quad (\ln v)_{10}^{\frac{10}{\sqrt{2}}} = -k(10)$$

$$\Rightarrow \quad \ln \frac{10}{\sqrt{2}} - \ln 10 = -k(10)$$

$$\Rightarrow \quad k = \frac{1}{10} \ln \left(\frac{10}{\frac{10}{\sqrt{2}}} \right) = \frac{1}{10} \ln \sqrt{2} = \frac{\ln 2}{20}$$

12. The position vector of the particle is $r(t) = a \cos \omega t \hat{\imath} + a \sin \omega t \hat{\jmath}$, where a and ω are real constants of suitable dimensions. The acceleration is
(A) Perpendicular to the velocity
(B) Parallel to the velocity
(C) Directed away from the origin
(D) Perpendicular to the position vector
(E) Always along with the direction of $\hat{\imath}$

Solution: (A)
Given that, $r(t) = a \cos \omega t \hat{\imath} + a \sin \omega t \hat{\jmath}$

$$\because v = \frac{dr(t)}{dt} = -a\omega \sin \omega t \ \hat{\imath} + a\omega \cos \omega t \ \hat{\imath} j$$

$$a = \frac{dv}{dt} = -a\omega^2 \cos \omega t \hat{\imath} - a\omega^2 \cos \omega t \hat{\jmath}$$

$\because a \cdot v = (-a\omega \sin \omega t \hat{\imath} + a\omega \cos \omega t \hat{\jmath})$
$(-a\omega^2 \cos \omega t \ \hat{\imath} - a\omega^2 \sin \omega t \ \hat{\jmath})$
$a \cdot v = a^2\omega^3 \sin \omega t \cos \omega t - a^2\omega^3 \sin \omega t \cos \omega t$
$\Rightarrow a \cdot v = 0 \ [\because a \cdot v = |a||v| \cos \theta]$
Above result implies that acceleration is perpendicular to velocity.

13. Some of the following equations are kinetic equations, where the symbols have their usual meaning. The $\mathrm{work} - \mathrm{energy}$ theorem is represented by
(A) $v = u + at$
(B) $s = ut$
(C) $s = ut + \frac{1}{2}at^2$
(D) $v^2 = \frac{u^2}{2} + as$
(E) $v^2 = u^2 + 2as$

Solution: (E)
According work-energy theorem, net work = change in kinetic energy = final KE-initial KE

So $F \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$\Rightarrow \quad ma \cdot s = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad v^2 - u^2 = 2as \ \Rightarrow \ v^2 = u^2 + 2as$$

14. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following do not change with time?

(A) $\frac{aT}{v}$

(B) $aT + 2\pi v$

(C) $a^2T^2 + 4\pi^2v^2$

(D) aT

(E) vT

Solution: (A)

$\because$ In option $(a), \frac{aT}{v} = \frac{\frac{m}{s^2} \times s}{\left(\frac{m}{s}\right)} = \frac{m \times s^2}{m \times s^2} = [M^0L^0T^0]$

So, according to the result above relation does not depend on time.

15. A rubber cord of density d, Young's modulus Y and length L is suspended vertically. If the cord extends by a length $0.5L$ under its own weight, then L is

(A) $\frac{Y}{2dg}$

(B) $\frac{Y}{dg}$

(C) $\frac{2Y}{dg}$

(D) $\frac{dg}{2Y}$

(E) $\frac{dg}{Y}$

Solution: (B)

$\because$ Young's modulus $Y = \frac{\text{Stress}}{\text{Strain}}$

$Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$ ……(i)

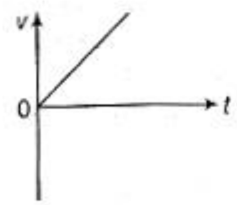
$\because$ Force exerted due to its own weight $= \frac{mg}{2}$

$$\therefore \quad Y = \frac{\left(\frac{mg}{2}\right)}{A\left(\frac{0.5}{L}\right)} = \left(\frac{mg}{AL}\right)$$

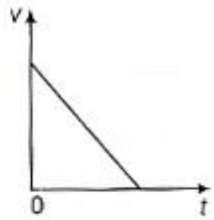
So $L = \frac{Y}{\left(\frac{mg}{AL}\right)} = \frac{Y}{dg} \qquad \left(\because \ \frac{m}{AL} = d\right)$

16. Which of the following graphs represents the speed v of a projectile as a function of time t?

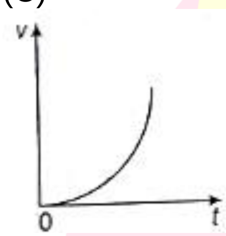
(A)



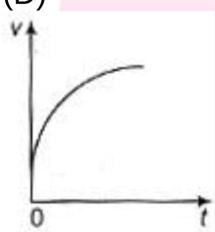
(B)



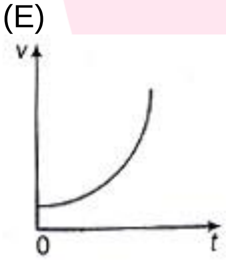
(C)



(D)



(E)



Solution: (B)
In a projectile motion, speed (v) of a projectile decreases with time (t).
Hence, graph in option (B) is correct.

17. A body P floats in water with half its volume immersed. Another body Q floats in a liquid of density $\frac{3}{4}th$ of the density of water with two-third of the volume immersed. The ratio of density of P to that of Q is
(A) $1 : 2$
(B) $1 : 1$
(C) $2 : 1$

(D) $2:3$
(E) $3:4$

Solution: (B)
Let, for body P, volume $= V_P$
Given, Immersed volume $= \frac{V_P}{2}$
For body Q volume $= V_Q$
Immersed volume $= \frac{2}{3}V_Q$
According to Archimedes principle,
$\because$ Weight of body = weight of fluid displaced
For body P,

$$V_P\rho_Q g = \left(\frac{1}{2}V_P\right)\rho_w g$$

So, $\frac{\rho_P}{\rho_w} = \frac{1}{2}$ .....(i)
For body Q,

$$V_Q\rho_Q g = \left(\frac{2}{3}V_Q\right)\left(\frac{3}{4}\right)\rho_w g$$

$$\left(\because \text{ density of liquid} = \frac{3}{4}\rho_w\right)$$

So, $\frac{\rho_Q}{\rho_w} = \frac{1}{2}$ .....(ii)
From Equations (i) and (ii),
$\frac{\rho_P}{\rho_Q} = 1$ or $\rho_P : \rho_Q = 1:1$

18. A pipe of $1m$ length is closed at one end. Taking the speed of sound in air as $320ms^{-1}$, the air column in the pipe cannot resonate for the frequency (in Hz)
(A) 80
(B) 160
(C) 240
(D) 560
(E) 720

Solution: (B)
Given, $v = 320ms^{-1}, l = 1m$
For an organ pipe whose one end is closed, only odd harmonics containing odd multiples of fundamental frequency are present.
Resonate frequency of first mode, $n_1 = \frac{v}{4l'}$
Second mode, $n_2 = 3n_1$,
Third mode, $n_3 = 5n_1$,
Fourth mode, $n_4 = 7n_1$,
Fifth mode, $n_5 = 9n_1$
So, $n_1 = \frac{320}{4} = 80Hz$
$n_2 = 3 \times 80 = 240\ Hz$
$n_3 = 5 \times 80 = 400\ Hz$

$n_4 = 7 \times 80 = 560\ Hz$
$n_5 = 9 \times 80 = 720\ Hz$

So according result, option (B) as multiple of cannot a resonating frequency of the pipe.

19. A wave pulse in a string is described by the equation $y_1 = \frac{5}{(3x-4t)^2+2}$ and another wave pulse in the same string is described by $y_2 = \frac{-5}{(3x+4t-6)^2+2}$. The values of y_1, y_2 and x are in metres and t in seconds.
Which of the following statement is correct?

(A) y_1 travels along$-x$-direction and y_2 along $+x$-direction
(B) Both y_1 and y_2 travel along $-x$-direction
(C) Both y_1 and y_2 travel along $+x$-direction
(D) At $x = 1m, y_1$ and y_2 always cancel
(E) At time $t = 1s, y_1$ and y_2 exactly cancel everywhere

Solution: (D)
$\because$ Given,

$$y_1 = \frac{5}{(3x-4t)^2+2}, y_2 = \frac{-5}{(3x+4t-6)^2+2}$$

According option (d), at $x = 1$

$$y_1 = \frac{5}{(3-4t)^2+2}$$

$$y_2 = \frac{-5}{(3+4t-6)^2+2}$$

$$= \frac{-5}{(3-4t)^2+2}$$

Both wave pulse equation are existing in same string therefore resultant equation of wave pulse.
$y = y_1 + y_2 = 0$
Hence, option At $x = 1m, y_1$ and y_2 always cancel is correct.

20. The maximum transverse velocity and maximum transverse acceleration of a harmonic wave in a $\text{one} - \text{dimensional}$ string are $1ms^{-1}$ and $1\ ms^{-2}$ respectively. The phase velocity if the wave is $1ms^{-1}$. The waveform is
(A) $\sin(x-t)$
(B) $\sin(2x-t)$
(C) $\sin(x-2t)$
(D) $\sin\left(\frac{x}{2}-t\right)$
(E) $\sin\left(x-\frac{t}{2}\right)$

Solution: (A)
$\because$ Wave equation, $y = A_0 \sin(kx - \omega t)$ ....(i)

where, k = angular wave number $= \frac{2\pi}{\lambda}$
A_0 = amplitude
$\because \;\; v_{max} = a\omega$
$\therefore \;\; \omega = 1$
$\left(\because \text{given that } v_{max} = 1\frac{m}{s}, a = 1\frac{m}{s^2}\right)$
$\because \;\; k = 1$ (given)
$\therefore$ From Equation (i),
$y = \sin(x - t)$

21. Two particles A and B of same mass have their de-Broglie wavelengths in the ratio $X_A : X_B = K : 1$. Their potential energies $U_A : U_B = 1 : K^2$. The ratio of their total energies $E_A : E_B$ is
(A) $K^2 : 1$
(B) $1 : K^2$
(C) $K : 1$
(D) $1 : K$
(E) $1 : 1$

Solution: (B)
According to question, $\frac{X_A}{X_B} = \frac{K}{1}$ …(i)
$\frac{U_A}{U_B} = \frac{1}{K^2}$
So, $U_A = y$ and $U_B = K^2 y$
According to de-Broglie wavelength,
$\lambda = \frac{h}{\sqrt{2mK_1}}$ …..(ii)
Here, K_1 is kinetic energy.
So, from Equations (i) and (ii),
$$\Rightarrow \frac{\left(\frac{h}{\sqrt{2mK_{1A}}}\right)}{\left(\frac{h}{\sqrt{2mK_{1B}}}\right)} = \frac{K}{1} \Rightarrow \sqrt{\frac{K_{1B}}{K_{1A}}} = \frac{K}{1}$$
$$\Rightarrow \frac{K_{1B}}{K_{1A}} = K^2$$
So, $K_{1B} = K^2 x$ and $K_{1A} = x$
$\because$ Total energy, $E = K_1 + U$
So, $E_A = K_{1A} + U_A$
and $E_B = K_{1B} + U_B$
or $\frac{E_A}{E_B} = \frac{K_{1A}+U_A}{K_{1B}+U_B} = \frac{x+y}{K^2x+K^2y}$
$\Rightarrow$ So, $\frac{E_A}{E_B} = \frac{1}{K^2}$ (or) $E_A : E_B = 1 : K^2$

22. A particle is moving along with X-axis such that its acceleration is proportional to the displacement from the equilibrium position and they are in the same direction. The displacement $x(t)$ is given by

(A) $\sin \omega t, \omega > 0$

(B) $\sin \omega t + \cos \omega t, \omega > 0$

(C) $e^{\omega t}, \omega > 0$

(D) $e^{\omega t} + \sin \omega t, \omega > 0$

(E) $e^{\omega t} + e^{\omega 2t}, \omega_1$ and $\omega_2 > 0$

Solution: (C)

Given, $a \propto x(t)$ .....(i)

According option (a),

$x(t) = \sin \omega t, \omega > 0$

$\therefore \quad u(t) = \frac{dx(t)}{dt} = \frac{d}{dt} \sin \omega t = \omega \cos \omega t$ and a

$$a = \frac{du(t)}{dt} = \frac{d}{dt} (\omega \cos \omega t) = -\omega^2 \sin \omega t$$

or $a = -\omega^2 x(t)$ ....(ii)

According option (b),

$x(t) = \sin \omega t + \cos \omega t, \omega > 0$

$$\therefore \ v(t) = \frac{dx(t)}{dt} = \omega \cos \omega t - \omega \sin \omega t$$

and $a = \frac{dv(t)}{dt} = -\omega^2 \sin \omega t - \omega^2 \cos \omega t$

or $a = -(\omega^2 \sin \omega t + \omega^2 \cos \omega t)$ or $a = -\omega^2 x(t)$ .....(iii)

According option (c),

$x(t) = e^{\omega t}, \omega > 0$

$$\therefore \ v(t) = \frac{dx(t)}{dt} = \omega e^{\omega t}$$

And $a = \frac{dv(t)}{dt} = \omega^2 e^{\omega t}$

or $a = \omega^2 x(t)$ .....(iv)

According option (d),

$x(t) = e^{\omega t} + \sin \omega t \ \omega > 0$

$$\therefore \ v(t) = \frac{dx(t)}{dt} = \omega e^{\omega t} + \omega \cos \omega t$$

and $a = \frac{dv(t)}{dt} = \omega^2 e^{\omega t} - \omega^2 \sin \omega t$

or $a = \omega^2 (e^{\omega t} - \sin \omega t)$ .....(v)

According option (e),

$x(t) = e^{\omega_1 t} + e^{-\omega_2 t}, \omega_1$ and $\omega_2 > 0$

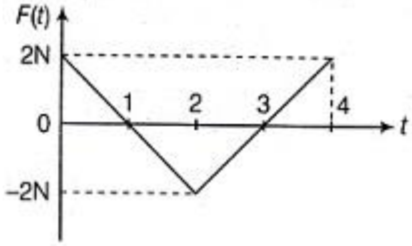
$$\therefore \ v(t) = \frac{dx(t)}{dt} = \omega_1 e^{\omega_1 t} - \omega_2 e^{-\omega_2 t}$$

And $a = \frac{dv(t)}{dt} = \omega_1^2 e^{\omega_1 t} + \omega_2^2 e^{-\omega_2 t}$

Or $a = \omega_1^2 \, e^{\omega_1 t} + \omega_2^2 e^{-\omega_2 t}$ ...(vi)

Hence by comparing Equations. (ii), (iii), (iv), (v) and (vi) with Equation (i), option (c) is correct.

23. A block of mass $1kg$ is free to move along the X-axis. It is at rest and from time $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in figure. The kinetic energy of the block at $t = 4$s is



(A) $1J$
(B) $2J$
(C) $3J$
(D) $0J$
(E) $4J$

Solution: (D)
$\because$ Total momentum of block = Area of given graph
$\therefore P = A_1 + A_2 + A_3$
Or $P = \frac{1}{2} \times 1 \times 2 - \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 2$
Or $P = 0$ or $mv = 0$ or $v = 0$,
Hence, kinetic energy at $(t = 45) = \frac{1}{2}mv^2 = 0$

24. Consider a wire with density (d) and stress (σ). For the same density, if the stress increases 2 times, the speed of the transverse waves along the wire changes by
(A) $\sqrt{2}$
(B) $\frac{1}{\sqrt{2}}$
(C) 2
(D) $\frac{1}{2}$
(E) 4

Solution: (A)
$\because$ Speed of transverse wave
$v = \sqrt{\frac{\text{Stress}}{\text{density of wire}}}$ or $v = \sqrt{\frac{\sigma}{d}}$
According to question, if σ increases 2 times
then $v_1 = \sqrt{\frac{2\sigma}{d}} = \sqrt{2}\sqrt{\frac{\sigma}{d}} = \sqrt{2}v$
Hence, speed of transverse waves along with wire changes by $\sqrt{2}$ times.

25. Two soap bubbles of radii $3mm$ and $4\,mm$ confined in vacuum coalesce isothermally to form a new bubble. The radius of the bubble formed $(\text{in } mm)$ is
(A) 3

(B) 3.5
(C) 4
(D) 5
(E) 7

Solution: (D)
Surface area new bubble = surface area of first bubble + surface area of second bubble
$\Rightarrow A = A_1 + A_1$
$\because$ Surface area of a bubble, $A = 8\pi R^2$
$\therefore$ $8\pi R^2 = 8\pi R_1^2 + 8\pi R_2^2$
$\Rightarrow$ $R^2 = R_1^2 + R_2^2$
(give that $R_1 = 3mm, R_2 = 4mm$)
$\Rightarrow$ $R^2 = (3)^2 + (4)^2$
$\Rightarrow$ $R = 5mm$

26. An oscillator circuit contains an inductor $0.05H$ and a capacitor of capacity $80\mu F$. When the maximum voltage across the capacitor is $200V$, the maximum current (in amperes) in the circuit is
(A) 2
(B) 4
(C) 8
(D) 10
(E) 16

Solution: (C)
Given $L = 0.05H, C = 80\mu F$
$V_{\max} = 200V$
$\because$ Voltage equation $V(t) = V_m \sin \omega t$
$\therefore$ Current (i) $= \frac{cdv}{dt} = c\frac{d}{dt}V_m \sin \omega t = CV_m \omega \cos \omega t$
$\because \omega = \dfrac{1}{LC}$
$\therefore$ $i = V_m\sqrt{\dfrac{C}{L}}\cos \omega t$
$\therefore$ Maximum current $(i_m), V_m\sqrt{\frac{C}{L}} = 200 \times \sqrt{\frac{80\mu}{0.05}}$
$i_m = 8A$

27. The displacement y of a particle, if given by $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\,t)$. This expression may be considered to be a result of the superposition of how many simple harmonic motions?
(A) 4
(B) 3
(C) 2

(D) 5
(E) 6

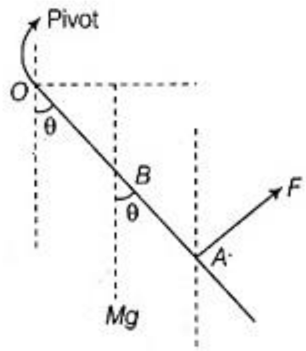
Solution: (B)
∵ Equation of displacement of particle is
$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000t)$ .....(i)
Or $y = 4\left[\left(\frac{1+\cos t}{2}\right)\sin 1000t\right]$
$\left(\because \cos^2 x = \dfrac{1+\cos 2x}{2}\right)$
Or $y = 3\sin 1000t + 2\cos t \sin 1000t$
Or $y = 2\sin 1000t + \sin 1001t + \sin t\ 999t$
$[2\sin A\cos B = \sin(A+B) + \sin(A-B)]$
Hence, equation (i) is the result of the superposition of three simple harmonic motion.

28. A cylindrical tube, open at both the ends has fundamental frequency n. If one of the ends is closed, the fundamental frequency will become
(A) $\frac{n}{2}$
(B) $2n$
(C) n
(D) $4n$
(E) $3n$

Solution: (A)
For a cylindrical tube
If both the ends is closed then
Fundamental frequency, $n = \frac{v}{2l}$ .....(i)
If one of the ends is closed then fundamental frequency, $n_1 = \frac{v}{4l}$ ....(ii)
from equation (i) and (ii)
$$n_1 = \frac{n}{2}$$

29. A uniform bar of mass m is supported by a pivot at its top about which the bar can swing like a pendulum. If a force F is applied perpendicular to the lower end of the bar as shown in figure, what is the value of F in order to hold the bar in equilibrium at an angle (θ) from the vertical
(A) $2mg\sin\theta$
(B) $mg\sin\theta$
(C) $mg\cos\theta$
(D) $\frac{mg}{2}\sin\theta$
(E) $\frac{mg}{2}\cos\theta$

Solution: (D)
Let the length of bar is l, then according to the problem.



For equilibrium of the bar,
$F_{\text{ext}}^{\text{net}} = 0$ and $\tau_{\text{ext}}^{\text{net}} = 0$
taking torque about pivot O,
$OB \times Mg + OA \times F = 0$
$\Rightarrow -OB\ Mg\sin\theta + OA.F\sin 90^o = 0$
(clockwise) (anti-clockwise)
$\therefore\ \ IF = \frac{1}{2}Mg\sin\theta$
$\Rightarrow\ \ F = \frac{Mg\sin\theta}{2}$

30. A particle of rest mass m_0 is travelling, so that its total energy is twice its rest mass energy. It collides with another stationary particle of rest mass m_0 to form a new particle. What is the rest mass of the new particle?
(A) $\sqrt{6}\,m_0$
(B) $2m_0$
(C) $2\sqrt{3}m_0$
(D) $\sqrt{3}m_0$
(E) $3m_0$

Solution: (E)
$\because$ Total energy of particle $= mc^2 - m_0c^2$ rest mass energy of particle $= m_0c^2$ according question,
$\Rightarrow\ \ mc^2 - m_0c^2 = 2m_0c^2$
Or $m = 3m_0$
When a particle collides with another particle then net mass of new particle will be $m = 3m_0$

31. The dimensions of ε_0 (permittivity in free space) is
(A) $ML^2T^4A^2$
(B) $ML^{-3}T^2A^2$
(C) $M^{-1}L^3T^4A^2$
(D) $ML^3T^2A^2$
(E) $M^{-1}L^{-3}T^4A^2$

Solution: (E)

Electrical force between two charge

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

dimension of $\varepsilon_0 = \frac{[A^2T^2]}{[MLT^{-2}][L^2]}$

$= [M^{-1}L^{-3}T^4A^2]$

32. The displacement of a wave is represented by $y = 0.6 \times 10^{-3}\sin(500t - 0.05x)$, where all the quantities are in their proper units. The maximum particle velocity $(\text{in } ms^{-1})$ of the medium is

(A) 0.5

(B) 0.03

(C) 0.150

(D) 0.75

(E) 0.3

Solution: (E)

∵ The displacement of wave

$y = 0.6 \times 10^{-3}\sin(500t - 0.05x)$

$\therefore \quad v = \frac{dy}{dt} = 0.6 \times 10^{-3} \times 500\sin(500t - 0.05x)$

$v = 0.3\sin(500t - 0.05x)$

For maximum particle velocity,

$\sin(500t - 0.05x) = 1$

Hence, $V_{\max} = 0.3ms^{-1}$

33. The electric field of certain radiation is given by the equation $E = 200\,\{\sin(4\pi \times 10^{10})t + \sin(4\pi \times 10^{15})t\}$ falls in a metal surface having work function $2.0\,eV$. The maximum kinetic energy $(\text{in } eV)$ of the photoelectrons is [use Planck's constant $(h) = 6.63 \times 10^{-34}J - s$ and electron charge $(E) = 1.6 \times 10^{-19}C$]

(A) 3.3

(B) 4.3

(C) 5.3

(D) 6.3

(E) 7.3

Solution: (D)

∵ According Einstein's photoelectric equation,

$\frac{1}{2}mv_{\max}^2 = hv - w_0$ .....(i)

Here, w_0 is work function and v is frequency of photon.

According question equation, electric field representing given as, is

$E = 200\,\{\sin(4\pi \times 10^{10})t + \sin(4\pi \times 10^{15})t\}$

Here, fundamental frequency of above equation will be LCM of both component frequency.

Hence, the fundamental frequencies

$\omega = 4\pi \times 10^{15}$ rad

$\because \omega = 2\pi v$ or $h, v = \frac{4\pi \times 10^{15}}{2\pi} = 2\pi \times 10^{15} Hz$

Put the value of h, v and W_0 in Equation (i)

$KE_{\max} = (6.63 \times 10^{-34} \times 2\pi \times 10^{15} - 2 \times 1.6 \times 10^{19}) + \hat{j}$

Hence $KE_{\max} = 6.3\ e.V$

34. The de-Broglie wavelength λ_n of the electron in the n^{th} orbit of hydrogen atom is

(A) Inversely proportional to n

(B) Proportional to n^2

(C) Proportional to n

(D) Inversely proportional to n^2

(E) Inversely proportional to radius of the orbit in the n^{th} state

Solution: (A)

$\because$ de-Broglie wavelength λ_n of the electron in the n^{th}orbit of hydrogen atom

$$\lambda = \frac{2\pi}{nh} \times hr$$

$$\Rightarrow \quad \lambda_n = \frac{2\pi r}{n}$$

$$\left[\begin{array}{c} \text{As}, \lambda = \frac{h}{p} = \frac{h}{mv} \\ \text{or } mvr = \frac{hr}{\lambda} = \frac{nh}{2\pi}, \lambda = \frac{2\pi}{nh} \times hr \end{array}\right]$$

$$\lambda_n \times \frac{1}{n}$$

Hence, λ_n is inversely proportional to n.

35. In a thermodynamic system, Q represents the energy transferred to or from a system by heat and W represents the energy transferred to or from a system by work

(I) $Q > 0$ and $W = 0$

(II) $Q < 0$ and $W = 0$

(III) $W > 0$ and $Q = 0$

(IV) $W < 0$ and $Q = 0$

Which of the above will lead to an increase in the internal energy of the system?

(A) I only

(B) II only

(C) I and IV only

(D) II and III only

(E) II and IV only

Solution: (C)

According first law of thermodynamics,

$Q = \Delta V + W$

Or $\Delta V = Q - W$

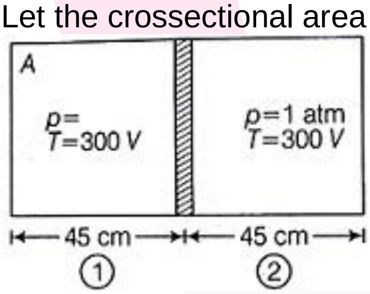
According given conditions

I. $Q > 0$ and $W = 0$

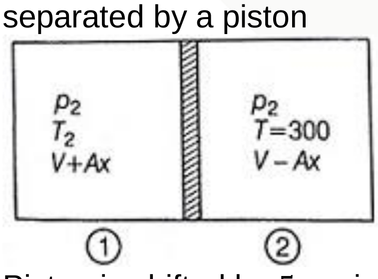
or $\Delta V = Q$
Here $Q > 0$, therefore ΔV will be increase.
II. $Q < 0$ and $W = 0$
or $\Delta V = Q$
Here $Q > 0$, therefore ΔV will be decrease.
III. $W > 0$ and $Q = 0$
or $\Delta V = -W$
Here $W > 0$, therefore ΔV will be decrease.
IV. $W < 0$ and $Q = 0$
or $\Delta V = -W$
Here $W < 0$, therefore ΔV will be increase.
Hence, condition I and IV will lead to an increase in the internal energy of the system.
Therefore, option I and IV only is correct.

36. A cylinder closed at both ends is separated into two equal parts (45 cm each) by a piston impermeable to heat. Both the parts contain the same masses of gas at a temperature of $300K$ and a pressure of $1\ atm$. How much the gas should be heated in one part of the cylinder to shift the piston by $5cm$ and the pressure of the gas after shifting the piston?
(A) $T = 365K$and $P = 1.125\ atm$
(B) $T = 350K$and $P = 1.125\ atm$
(C) $T = 375K$and $P = 2.125\ atm$
(D) $T = 350K$and $P = 2.125\ atm$
(E) $T = 375K$and $P = 1.125\ atm$

Solution: (E)
Let the crossectional area of the cylinder is A. Initially



Representing a closed cylinder
separated by a piston



Piston is shifted by $5\ cm$ in the cylinder
For 1, $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \Rightarrow \frac{pV}{300} = \frac{p_2(V+Ax)}{T_2}$ .....(i)
For 2, $p_1V_1 = p_2V_2 \Rightarrow pV = p_2(V - Ax)$

$p_2 = \frac{pV}{(V-Ax)}$ ....(ii)

From Equations (i) and (ii),

$$\frac{pV}{300} = \frac{pV}{(V-Ax)} \cdot \frac{(V+Ax)}{T_2}$$

$\Rightarrow \; T_2 = \frac{V+Ax}{V-Ax} 300 = \frac{A45+A_S}{A45-AS} \times 300 = \frac{50}{40} 300$

$T_2 = 375K$

Putting T_2 in Equation (i),

$$P_2 = \frac{PV}{(V+Ax)} \times \frac{375}{300} = \frac{1 \times A \times 45}{A(450+5)} \times \frac{375}{300}$$

$= 1.125 \; atm$

37. Five moles of an ideal monoatomic gas with an initial temperature of $150^o C$ expand and in the process absorb $1500J$ of heat and does $2500J$ of work. The final temperature of the gas in $^o C$ is (ideal gas constant, $R = 8.314 \; JK^{-1} mol^{-1}$).

(A) 134

(B) 126

(C) 144

(D) 166

(E) 174

Solution: (A)

According to thermodynamics first law, $\Delta Q = \Delta U + \Delta W$

Given, that

$\Delta Q = +1500J$

$\Delta W = +2500J$

$\therefore \; 1500 = \Delta V + 2500$

$\Rightarrow \; \Delta V = 1000J$

$\because \; \Delta V = mC_V \Delta T$ ....(i)

$\because$ For monoatomic gas,

$$C_V = \frac{3}{2} R = 1.5 \times 8.314$$

$C_V = 12.471$

$\therefore$ From Equation (i),

$\therefore -1000 = 5 \times 12.471 \times (T_2 - T_1)$

Or $T_2 = 150 - \frac{1000}{5 \times 12.471} = 134^o C$

38. The temperature of an ideal gas is increased from $100K$ to $400K$. If the rms speed of the gas molecule is v at $100K$, then at $400K$ it becomes

(A) $2v$

(B) $4v$

(C) $0.5v$

(D) $0.25v$

(E) v

Solution: (A)

$\because$ rms speed of molecule $V_{\text{rms}} \propto \sqrt{T}$

for $T = 100K$

$(V_{\text{rms}})_1 \propto \sqrt{100}$

For $T = 400K$

$(V_{\text{rms}})_2 = \sqrt{400}$

therefore, $\frac{(V_{\text{rms}})_1}{(V_{\text{rms}})_2} = \frac{\sqrt{100}}{\sqrt{400}}$

or $\frac{V}{(V_{\text{rms}})_2} = \frac{1}{2}$

or $(V_{\text{rms}})_2 = 2v$

39. A uniform copper rod of $50\ cm$ length is insulated on the sides and has its ends exposed to ice and steam respectively. If there is a layer of water $1mm$ thick at each end, the temperature gradient $(\text{in }^{\text{o}}Cm^{-1})$ in the bar is (assume that the thermal conductivity of copper is $400\ \text{Wm}^{-1}K^{-1}$ and water is $0.4\ \text{Wm}^{-1}K^{-1}$)

(A) 60

(B) 40

(C) 50

(D) 55

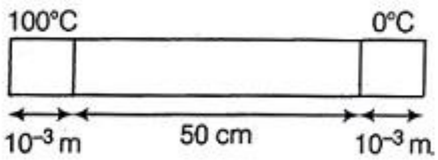
(E) 65

Solution: (B)

Given, $K_c = 400WM^{-1}K^{-1}$,

$K_w = 0.4\ WM^{-1}K^{-1}$

$\theta_1 = 100^oC, \theta_2 = 0^oC$



In steady state, flow of heat will be same throughout the whole system

$$\frac{K_wA(100-\theta)}{10^{-3}} = \frac{K_cA(\theta-\theta')}{\left(\frac{50}{100}\right)}$$

Or $(100-\theta) = \frac{K_c}{K_w} \times 10^{-3} \times \frac{(\theta-\theta')}{0.5}$

$$= \frac{400}{0.4} \times 10^{-3} \times \frac{(\theta-\theta')}{0.5}$$

$(100-\theta) = 2(\theta-\theta')$

$\Rightarrow\ (100-\theta) = 2(\theta-100+\theta) \qquad (\because 100-\theta = \theta'\text{given})$

Or $(100-\theta) = 4\theta - 200$

$\Rightarrow\ 5\theta = 300 \Rightarrow \theta = 60^oC$

$\therefore\ \theta' = 100-\theta = 100-60 = 40^oC$

Hence, temperature gradient along the bar

$$= \frac{\theta-\theta'}{0.5} = \frac{60-40}{0.5} = 40^o cm$$

40. A Carnot engine whose low temperature reservoir is at $350K$ has an efficiency of 50%. It is desired to increase this to 60%. If the temperature of the low temperature reservoir remains constant, then the temperature of high temperature reservoir must be increased by how many degrees?
(A) 15
(B) 175
(C) 100
(D) 50
(E) 120

Solution: (B)
Efficiency of Carnot engine,
$$\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \times 100$$
When $\eta = 50\%$
then $\frac{50}{100} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \Rightarrow 0.5 = 1 - \frac{350}{T_{\text{hot}}}$
or $T_{\text{hot}} = 700K$
When $\eta = 60\%$
then $\frac{60}{100} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \Rightarrow 0.6 = 1 - \frac{350}{T_{\text{hot}}}$
or $T_{\text{hot}} = 875K$
Hence, the temperature of high reservoir is increased by
$T'_{\text{hot}} - T_{\text{hot}} = 875K - 700K = 175K$

41. Two identical systems, with heat capacity at constant volume that varies as $C_v = bT^3$ (where b is a constant) are thermally isolated. Initially, one system is at a temperature $100\,K$ and the other is at $200K$. The systems are then brought to thermal contact and the combined system is allowed to reach thermal equilibrium. The final temperature (in K) of the combined system will be
(A) 171
(B) 141
(C) 150
(D) 180
(E) 125

Solution: (A)
Energy given by a system = Energy taken by another system
$dQ_1 = dQ_2$
$$\Rightarrow \int_{100}^{T} mc_v dT = -\int_{200}^{T} mc_v dT$$
$$\Rightarrow \int_{100}^{T} bT^3 dT = -\int_{200}^{T} bT^3 dT \Rightarrow (T^4)_{10}^{T} = -(T^4)_{200}^{T}$$

$\Rightarrow \quad T^4 - 100^4 = -(T^4 - 200^4)$
$\Rightarrow \quad 2T^4 = 17 \times 10^8 \Rightarrow T = 171K$

42. Water flows steadily through a horizontal pipe of a variable cross-section. If the pressure of the water is p at a point, where the speed of the flow is v, what is the pressure at another point, where the speed of the flow is $2v$? Let the density of water be
(A) $\rho + \left(\frac{3}{2}\right)\rho\, v^2$
(B) $\rho - 2\rho v^2$
(C) $\rho + 2\rho v^2$
(D) $\rho - 3\rho v^2$
(E) $\rho - \left(\frac{3}{2}\right)\rho v^2$

Solution: (E)
Two point are situated in a pipe and their height from ground is zero $(h = 0)$.
$\because$ According Bernoulli's theorem,
$p + \rho g h + \frac{1}{2}\rho v^2 =$ constant
At a point pressure of water is p and speed of flow is v
therefore, $p + \frac{1}{2}\rho v^2 = c$ .....(i)
At another point speed of flow is $2v$ therefore,
$p_1 + \frac{1}{2}\rho(2v)^2 = c$ ......(ii)
To find pressure p_1 at another point, equating the equation (i) and (ii)
$$\Rightarrow \quad p + \frac{1}{2}\rho v^2 = p_1 + \frac{1}{2}\rho(4v^2)$$
$$\Rightarrow \quad p_1 = p - \frac{3}{2}\rho v^2$$

43. A soap bubble of radius r is blown upto form a bubble of radius $2r$ under isothermal conditions. If $\pi\sigma$ is the surface tension of soap solution, the energy spent in doing so is
(A) $6\pi\sigma r^2$
(B) $3\pi\sigma r^2$
(C) $24\pi\sigma r^2$
(D) $12\pi\sigma r^2$
(E) $9\pi\sigma r^2$

Solution: (C)
Energy spent = Work done $= T\Delta A$
Here, A is total surface area
$\therefore A = 8\pi r^2$ for soap bubble
Hence, energy spent $= \sigma[8\pi(2r)^2 - 8\pi r^2]$
$= \sigma(32\pi r^2 - 8\pi r^2)$
$= 24\pi\sigma r^2$

44. The mean momentum of a nucleon in a nucleus with mass number A varies as

(A) A^3
(B) A^2
(C) $A^{-\frac{2}{3}}$
(D) $A^{-\frac{1}{3}}$
(E) $A^{\frac{1}{3}}$

Solution: (E)

$\because$ Mean momentum of a nucleon in a nucleus is proportional to $A^{\frac{1}{3}}$.

45. A decay chain of the nucleus ${}^{238}_{92}U$ involves eight α-decays and six β-decays. The final nucleus at the end of the process will be
(A) $Z = 76; A = 200$
(B) $Z = 84; A = 206$
(C) $Z = 84; A = 224$
(D) $Z = 82; A = 206$
(E) $Z = 82; A = 200$

Solution: (D)
(i) When α-particle is emitted from a radioactive nucleus then atomic number decreases by 2 and atomic mass decreases by 4.
(ii) When β-particle emitted from a radioactive nucleus then atomic number is increased by 1 and atomic mass unaffected.
Here, atomic number $(Z) = 92$
After emitting 8 α-particles and 6β-particles,
$Z = 92 - 2 \times 8 + 6 = 82$
and atomic mass, $A = 238$
$\therefore A = 238 - 4 \times 8 = 206$

46. A flat mirror revolves at a constant angular velocity making $n = 0.4$ revolutions per second. With what velocity (in ms^{-1}) will a light spot move along a spherical screen with a radius of 15 metres, if the mirror is at the centre of curvature of the screen?
(A) 37.7
(B) 60.3
(C) 68.7
(D) 75.4
(E) 90.4

Solution: (D)
$\because$ Angular velocity of mirror $= 0.4\ \text{rev/s}$
$= 0.4 \times 2\pi = 0.8\pi\ \text{rad/s}$
$\therefore$ Angular velocity of reflected ray
$= 2 \times 0.8\pi = 1.6\pi\ \text{rad/s}$
Hence, velocity of light spot over the screen
$v = rw = 15 \times 1.6\pi = 75.4\ \text{m/s}$

47. A parallel beam of light of wavelength $4000\,\text{Å}$ passes through a slit of width $5 \times 10^{-3} m$. The angular spread of the central maxima in the diffraction pattern is
(A) 1.6×10^{-3} rad
(B) 1.6×10^{-4} rad
(C) 1.2×10^{-3} rad
(D) 3.2×10^{-3} rad
(E) 3.2×10^{-4} rad

Solution: (B)
Angular spread of the central maxima in the diffraction pattern $= \frac{2\lambda}{e}$ (e = width of the slit)
Here, $e = 5 \times 10^{-3} m, \lambda = 4000\text{Å}$
Hence, $2\,\theta = 2\left(\frac{4000\times10^{-10}}{5\times10^{-3}}\right)$
$= 1600 \times 10^{-7}$ rad
or $2\theta = 1.6 \times 10^{-4}$ rad

48. A wire made of aluminium having resistivity $\rho = 2.8 \times 10^{-8} m$ with a circular cross-section and has a radius of $2 \times 10^{-3} m$. A current of $5A$ flows through the wire. If the voltage difference between the ends is $1V$, what is the length of the wire in meters?
(A) 50
(B) 60
(C) 90
(D) 120
(E) 110

Solution: (C)
Given, resistivity $(\rho) = 2.8 \times 10^{-8} \Omega - m,$
Radius $(r) = 2 \times 10^{-3} m,$
Current $I = 5A$
Voltage difference $(V) = 1v$
$\because$ Resistivity $(\rho) = \frac{RA}{l}$

$$\therefore\ 2.8 \times 10^{-8} = \frac{R\pi(2 \times 10^{-3})^2}{l}$$

$$\Rightarrow\ l = \frac{R(\pi \times 4 \times 10^{-6})}{2.8 \times 10^{-8}}$$

$$\Rightarrow\ l = \frac{\left(\frac{V}{I}\right)4\pi\times10^{-6}}{2.8\times10^{-8}} \quad (\text{from Ohm's law}, V = IR)$$

$$l = \frac{1}{5} \times \frac{4\pi \times 10^{-6}}{2.8 \times 10^{-8}} = 0.897 \times 10^2 = 89.7m$$

$\approx 90m$

49. When two capacities are connected in parallel the resulting combination has capacitance $10\mu F$. The same capacitors when connected in series results in a capacitance $0.5\mu F$. The respective values of individual capacitors are

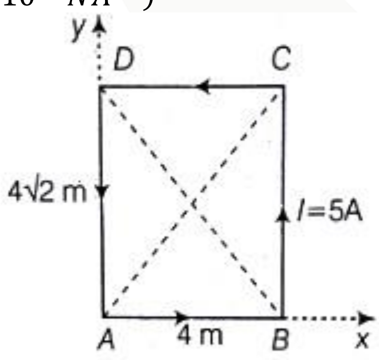
(A) $1.9\ \mu F$ and $0.2 \mu F$
(B) $(8+2\sqrt{5}) \mu F$ and $(2-2\sqrt{5}) \mu F$
(C) $(5+2\sqrt{5}) \mu F$ and $(5-2\sqrt{5}) \mu F$
(D) $12 \mu F$ and $17 \mu F$
(E) $5 \mu F$ and $2\ \mu F$

Solution: (C)
When capacitors are connected in parallel then $C_{eq} = C_1 + C_2 = 10 \mu F$ .....(i)
When capacitors are connected in series
then $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 0.5 \mu F$
or $C_1 C_2 = 0.5\ (C_1 + C_2)$
or $C_1 C_2 = 5 (\mu F)^2$ .....(ii)
$(\because\ C_1 + C_2 = 10 \mu F)$
$\because\ (C_1 + C_2)^2 = C_1^2 + C_2^2 - 2C_1C_2$ ......(iii)
$(C_1 - C_2)^2 = C_1^2 + C_2^2 - 2C_1C_2$ ...(iv)
Put the value from Equations (i) and (ii) in the equation (iii)
Hence, $C_1^2 + C_2^2 = (10\mu)^2 - 2 \times 5\mu^2 = 90\mu^2$
Put the above value in Equation (iv)
$(C_1 - C_2)^2 = 90\mu^2 - 2 \times 5\mu^2 = 80\mu^2$
$\therefore\ C_1 - C_2 = \sqrt{5}\mu$ .....(v)
Now from Equations (i) and (v),

$$C_1 = \frac{10 + 4\sqrt{5}}{2} = (5 + 2\sqrt{5})\ \mu F$$

$$C_2 = \frac{10 - 4\sqrt{5}}{2} = (5 - 2\sqrt{5})\ \mu F$$

50. A rectangular conducting loop of length $4\sqrt{2}m$ and breadth $4m$ carrying a current of $5A$ in the anti-clockwise direction is placed in the xy-plane. The magnitude of the magnetic induction field vector B at the intersection of the diagonal is $(\text{use}\ \mu_0 = 4 \times 10^{-7} NA^{-2})$



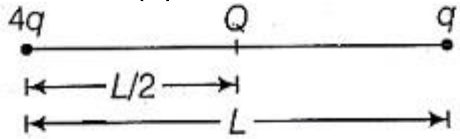
(A) $1.2 \times 10^{-6} T$
(B) $1.2 \times 10^{-5} T$
(C) $2.4 \times 10^{-6} T$
(D) $2.4 \times 10^{-5} T$

(E) $1.2 \times 10^{-7} T$

Solution: (A)
Magnetic field due to a straight current carrying conductor of finite length
$B = \frac{\mu_0}{4\pi d} \frac{I}{d} (\sin\theta_1 + \sin\theta_2)$
(i) Magnetic field due to conductor DA.
Here, $d = \frac{4}{2} = 2m$
$\theta_1 = \theta_2 = \tan^{-1}\left(\frac{2\sqrt{2}}{2}\right) = 54.73^o$
$\therefore\ \sin\theta_1 = \sin\theta_2 = \sin 54.73^o = 0.816$
and $I = 5A$ (given)
From Equation (i),
$B_1 = \frac{\mu_0}{4\pi} \times \frac{5}{2}(0.816 + 0.816)$
$B_1 = 4.08 \times 10^{-7} T$
Similarly, magnetic field due to conductor BC
$B_2 = 4.08 \times 10^{-7} T.$
(ii) Magnetic field due to conductor AB.
Here, $d = \frac{4\sqrt{2}}{2} = 2\sqrt{2}m$
$\theta_1 = \theta_2 = \tan^{-1}\left(\frac{2}{2\sqrt{2}}\right) = 35.26^o$
$\therefore\ \sin\theta_1 = \sin\theta_2 = \sin 35.26^o = 0.577$
and $I = 5A$ (given)
From Equation (i),
$B_3 = \frac{\mu_0}{4\pi} \times \frac{5}{2\sqrt{2}}(0.57 + 0.57)$
$B_3 = 2.04 \times 10^{-7} T$
Similarly magnetic field due to conductor CD
$B_4 = 2.04 \times 10^{-7} T$
Hence, magnitude of induction field vector B at the intersection of the diagonals
$B = B_1 + B_2 + B_3 + B_4$
$B = (4.08 + 4.08 + 2.04 + 2.04) \times 10^{-7}$
$B = 12.24 \times 10^{-7} T$
Or $B = 1.2 \times 10^{-6} T$

51. Three point charges $4q, Q$ and q are placed in a straight line of length L at points $0, \frac{L}{2}$ and L respectively. The net force on charge q is zero. The value of Q is
(A) $4q$
(B) $-q$
(C) $-0.5q$
(D) $-2q$
(E) $3q$

Solution: (B)



$\because$ Net force on charge, $q = 0$

$\Rightarrow \; f_1 + f_2 = 0$ .....(i)

Here, f_1 is the force on charge q due to charge $4q$ f_2 is the force on charge q due to charge Q

$\therefore$ From equation (i),

$\Rightarrow \; \frac{K(4q)q}{L^2} + \frac{KQq}{\left(\frac{L}{2}\right)^2} = 0$

$\Rightarrow \; 4q^2 + 4Qq = 0$

$\Rightarrow \; Q = -q$

52. A particle of charge Q moves with a velocity $v = a\hat{\imath}$ in a magnetic field $B = b\hat{\jmath} + c\hat{k}$ where a, b and c are constants. The magnitude of the force experienced by the particle is

(A) $Q(b + c)$

(B) Zero

(C) $Q\sqrt{(ab)^2 + (ac)^2}$

(D) $Q\sqrt{(b^2 + c^2)}$

(E) $Qa(b - c)$

Solution: (C)

$\because$ Lorentz force, $F = q(v \times B)$ ....(i)

Here $v = a\hat{\imath}, B = b\hat{\jmath} + c\hat{k}, q = Q$

From Equation (i),

$F = Q\left(a\hat{\imath} \times b\hat{\jmath} + c\hat{k}\right)$

$F = Q\left(ab\hat{k} - ac\hat{\jmath}\right)$

(According cross production),

$\left(\because \hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\imath} \times \hat{k} = -\hat{\jmath}\right)$

$\therefore$ Magnitude of the force,

$|F| = Q\sqrt{(ab)^2 + (ac)^2}$

or $F = Qa\sqrt{(b^2 + c^2)}$

53. A point charge $+Q$ is held at rest at a point P. Another point charge $-q$, whose mass is m, moves at a constant velocity v in a circular orbit of radius R_1, around P. The work required to increase the radius of revolution of $-q$ from R_1, to another orbit R_2 is $(R_2 > R_1)$

(A) $\frac{Qq}{2}\left[\frac{1}{R_2} + \frac{1}{R_1}\right]$

(B) $\frac{Qq}{3}\left[\frac{1}{R_2}\right]$

(C) $KQq\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$
(D) $-KQq\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$
(E) $2KQq\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$

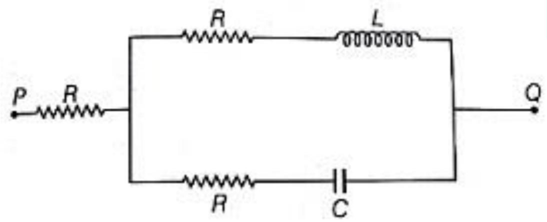
Solution: (D)
The force due to point charge $+Q$ on charge $-q$ is $F=\frac{kQ(-q)}{R^2}$
Therefore work required to increase the radius of revolution of $-q$ from R_1 to R_2 is

$$W=-\int_{R_1}^{R_2} F.dR$$

$$=-\int_{R_1}^{R_2} -k\frac{Qq}{R^2}dR = kQq\left[\frac{1}{R}\right]_{R_1}^{R_2}$$

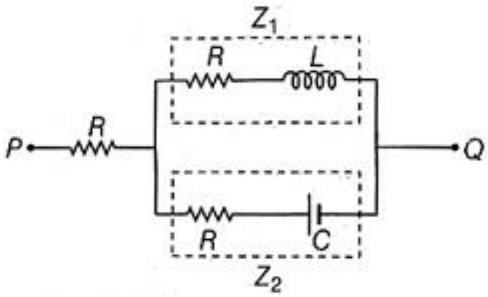
$$=-kQq\left[\frac{1}{R_2}-\frac{1}{R_1}\right]$$

54. A voltage $V_{PQ}=V_0\cos\omega t$ (where, V_0 is a real amplitude) is applied between the points P and Q in the network shown in the figure. The values of capacitance and inductance are
$C=\frac{1}{\omega R\sqrt{3}}$ and $L=\frac{R\sqrt{3}}{\omega}$
Then, the total impedance between P and Q is



(A) $1.5\ R$
(B) $2R$
(C) $3R$
(D) $4R$
(E) $2.5R$

Solution: (C)
∵ Given that, $C=\frac{1}{\omega R\sqrt{3}}, L=\frac{R\sqrt{3}}{\omega}$



In the above figure,

$$Z_1 = R + j\omega L = R + j\omega\left(\frac{R\sqrt{3}}{\omega}\right) = R + jR\sqrt{3}$$

$$Z_2 = R - j\frac{1}{\omega c} = R - j\frac{1}{\omega\left(\frac{1}{\omega R\sqrt{3}}\right)} = R - jR\sqrt{3}$$

$\because$ Impedance Z_1 and Z_2 are in parallel,

So, $Z_{\mathrm{eq}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

$$= \frac{\left(R + jR\sqrt{3}\right)\left(R - jR\sqrt{3}\right)}{R + jR\sqrt{3} + R - jR\sqrt{3}}$$

$$= \frac{R^2 + R^2\left(\sqrt{3}\right)^2}{2R}$$

$(\because (a-b)(a+b) = a^2 - b^2)$

$$Z_{\mathrm{eq}} = \frac{4R^2}{2R} = 2R$$

So, total impedance between P and Q is

$Z_{PQ} = R + Z_{\mathrm{eq}}$

$Z_{PQ} = R + 2R = 3R$

55. Two particles A and B of same mass have their total energies E_A and E_B in the ratio $E_A : E_B = 1 : 2$. Their potential energies U_A and U_B are in the ratio $U_A : U_B = 1 : 2$. If λ_A and λ_B are their de-Broglie wavelengths, then $\lambda_A : \lambda_B$ is

(A) $1 : 2$

(B) $2 : 1$

(C) $1 : \sqrt{2}$

(D) $\sqrt{2} : 1$

(E) $1 : 1$

Solution: (D)

Given,

$$\frac{E_A}{E_B} = \frac{1}{2}, \frac{U_A}{U_B} = \frac{1}{2}$$

So $E_A = x, E_B = 2x$

And $U_A = y, U_B = 2y$

$\because$ $E_A = U_A + K_A$

And $E_B = U_B + K_B$
here K_A and K_B are kinetic energy of particles A and B
So $K_A = E_A - U_A = (x - y)$ .....(i)
$K_B = E_B - U_B = 2(x - y)$ ....(ii)
∵ de-Broglie wavelength,
$$\lambda = \frac{h}{\sqrt{2mk}}$$
So $\lambda_A = \frac{h}{\sqrt{2mK_A}}, \lambda_B = \frac{h}{\sqrt{2mK_B}}$
∴ $\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{K_B}{K_A}}$ .....(iii)
From Equation (i), (ii) and (iii),
$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{2(x-y)}{(x-y)}} = \frac{\sqrt{2}}{1}$

56. The electrical conductivity of a metal is
(A) Directly proportional to the mean free path
(B) Directly proportional to the mass of electron
(C) Inversely proportional to the relaxation time
(D) Inversely proportional to the mean free path
(E) Directly proportional to the average speed of free electrons

Solution: (A)
∵ Electrical conductivity, $G \propto \lambda$
Here, λ is mean free path.

57. A $2\ MeV$ neutron is emitted in a fission reactor. If it looses half of its kinetic energy in each collision with a moderator atom, how many collisions must it undergo to achieve thermal energy of $0.039\ eV$?
(A) 20
(B) 26
(C) 30
(D) 42
(E) 48

Solution: (B)
$2\ MeV$ neutron looses half of its kinetic energy in each collision. So, making geometric progression of each collision $3\ MeV, 1MeV, 5MeV$ ......$0.039\ eV$.
In this geometric progression,
$a = 2MeV$
$$\gamma = \frac{1}{2}$$
nth term,
$a_n = ar^{n-1}$
$$\therefore\ 0.039 = 2 \times 10^6 \left(\frac{1}{2}\right)^{n-1}$$

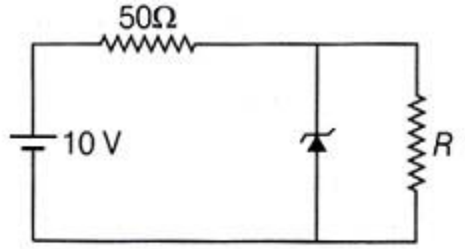
$\Rightarrow \quad 1.95 \times 10^{-8} = \left(\frac{1}{2}\right)^{n-1}$

Taking log both sides,

$\Rightarrow \quad (n-1)\ln\left(\frac{1}{2}\right) = \ln(1.95 \times 10^{-8})$

Number of collision $(n) = 26.6 \approx 26$ collision

58. The $6V$ Zener diode is shown in figure has negligible resistance and a knee current of $5mA$. The minimum value of R (in Ω) so that the voltage across it does not fall below $6V$ is



(A) 40
(B) 60
(C) 72
(D) 80
(E) 120

Solution: (D)
In Zener diode,

$\Rightarrow \quad \frac{V_{\text{max}} - V_Z}{R} \geq I_{\text{knee}} + I_2$

$\Rightarrow \quad \frac{10-6}{50} \geq 5m + I_R$

$\Rightarrow \; I_R \leq 80m - 5m$

$\because I_{R_{\text{max}}} = 75mA$ and $I_{R_{\text{min}}} = 80mA$

59. An electron is moving with a velocity $2 \times 10^6\, m/s$ along positive x-direction in the uniform electric field of $8 \times 10^7\, V/m$ applied along positive y-direction. The magnitude and direction of a uniform magnetic field (in tesla) that will cause the electrons to move undeviated along its original path is
(A) 40 in $-ve\ z$-direction
(B) 40 in $+ve\ z$-direction
(C) 4 in $+ve\ z$-direction
(D) 4 in $-ve\ z$-direction
(E) 8 in $+ve\ z$-direction

Solution: (A)
For electrons to move undeviated along its original path,
⇒ Electric force = Magnetic force
Or $qE = qvB$

Or $B = \frac{E}{v} = \frac{8\times10^7}{2\times10^6}$
$B = 40T$
Direction of magnetic field will be $-ve\ z$-direction because of negative charge.

60. What is minimum thickness (in nm) of a soap film $(n = 1.3)$ that results in constructive interference in reflected light if the film is illuminated with light whose wavelength in free space is $620\ nm$
(A) 100
(B) 120
(C) 160
(D) 240
(E) 180

Solution: (B)
For constructive interference,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

$$\Rightarrow \quad t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n} = \frac{\left(0 + \frac{1}{2}\right)\times 620 \times 10^{-9}}{2 \times 1.3} = 120\ nm$$

61. Three variable Boolean expression
$PQ + PQR + \bar{P}Q + P\bar{Q}R$ can be written as
(A) $\bar{Q} + \bar{P}R$
(B) $\bar{P} + \bar{Q}R$
(C) $Q + PR$
(D) $Q + \bar{P}R$
(E) $P + QR$

Solution: (C)
$\because$ Given, boolean expression is
$= PQ + PQR + \bar{P}Q + P\bar{Q}R$
$= PQ(1 + R) + \bar{P}Q + P\bar{Q}R$
$= PQ + \bar{P}Q + P\bar{Q}R \quad (\because 1 + R = 1)$
$= Q(P + \bar{P}) + P\bar{Q}R$
$= Q + PR\bar{Q} \qquad (\because P + \bar{P} = 1)$
$= (Q + PR)\,(Q + \bar{Q})$
$= Q + PR \quad (\because Q + \bar{Q} = 1)$

62. A prism is made up of material of refractive index $\sqrt{2}$. The angle of the prism is A. If the angle of minimum deviation is equal to the angle of the prism, the value of A is
(A) 30^o
(B) 45^o
(C) 60^o
(D) 75^o

(E) 90^o

Solution: (E)

$$\because\ \mu = \frac{\sin\left[\frac{(A+\delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

Here, μ is refractive index δ_m is minimum deviation angle, A is angle of prism.

$$\therefore\quad \sqrt{2} = \frac{\sin\left[\frac{A+A}{2}\right]}{\sin\frac{A}{2}}$$

$$\Rightarrow\quad \sqrt{2}\sin\frac{A}{2} = \sin A$$

Hence, $A = 90^o$

63. Consider a cylindrical conductor of length L and area of cross-section A. The specific conductivity varies as $\sigma(x) = \sigma_0 \frac{L}{\sqrt{x}}$ where x is the distance along the axis of the cylinder from one of its ends. The resistance of the system along the cylindrical axis is

(A) $\frac{2\sqrt{L}}{3A\sigma_0}$

(B) $\frac{3\sqrt{L}}{2A\sigma_0}$

(C) $\frac{\sqrt{L}}{3A\sigma_0}$

(D) $\frac{2\sqrt{L}}{A\sigma_0}$

(E) $\frac{4\sqrt{L}}{3A\sigma_0}$

Solution: (A)

Given, $\sigma(x) = \sigma_0 \frac{1}{\sqrt{x}}$

$\because$ Resistance of the system along the cylindrical axis,

$$R = \int_0^L \frac{\rho(x)}{A} dx$$

$$= \int_0^L \frac{\left(\frac{1}{\sigma_0 \frac{L}{\sqrt{x}}}\right)}{A} dx \qquad \left[\because \rho(x) = \frac{1}{\sigma(x)}\right]$$

$$= \int_0^L \frac{\sqrt{x}}{\sigma_0 AL} dx = \frac{1}{\sigma_0 AL}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^L = \frac{2}{3}\frac{1}{\sigma_0 AL}\left(L^{\frac{3}{2}} - O\right)$$

$$= \frac{2}{3}\cdot\frac{1}{\sigma_0 AL} \times L^{\frac{3}{2}}, R = \frac{2}{3}\cdot\frac{\sqrt{L}}{A\sigma_0}$$

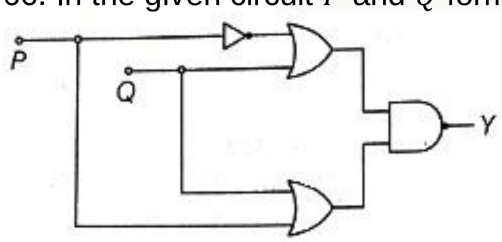
64. If the emission rate of blackbody at $0^o C$ is R, then the rate of emission at $273^o C$ is

(A) $2R$
(B) $4R$
(C) $8R$
(D) $16R$
(E) $32R$

Solution: (D)
According Stefan's law, emission rate of a ideal blackbody
$E \propto T^4$
For $T = 0^o C = 273K$
$E_1 \propto (273)^4$ ...(i)
For $T = 273^o C = 273 + 273 = 546K$
$E_2 \propto (543)^4$ .....(ii)
From Equations (i) and (ii),
$\Rightarrow \frac{E_1}{E_2} = \left(\frac{273}{546}\right)^4 \Rightarrow \frac{R}{E_2} = \left(\frac{1}{2}\right)^4 \quad (\because \; E_1 = R)$
Hence, $E_2 = 16R$

65. For any material, if R, T and A represent the reflection coefficient, transparent coefficient and absorption coefficient respectively, then for a blackbody which one of the following is true
(A) $R = 1, T = 0, A = 0$
(B) $R = 1, T = 1, A = 0$
(C) $R = 0, T = 1, A = 1$
(D) $R = 0, T = 0, A = 1$
(E) $R = 0, T = 1, A = 0$

Solution: (D)
$\because$ Relation between R, T and A is
$A + T + R = 1$ ...(i)
$\because$ For ideal blackbody,
$A = 1$
$\therefore$ According Equation (i),
$R = 0, T = 0$ and $A = 1$

66. In the given circuit P and Q form the inputs. The output Y is


(A) $Y = \bar{P}$
(B) $Y = P\bar{Q}$
(C) $Y = P + Q$

(D) $Y = \bar{Q}$
(E) $Y = \bar{P} + Q$

Solution: (D)
From the given circuit, the output Y is
$Y = (\bar{P} + q)\,(P + q)$
$Y = \bar{P}.P + \bar{P}Q + Q.P + Q.Q$
$Y = Q + PQ + \bar{P}Q \quad (\because\ \bar{P}.P = 0)$
$Y = Q(1 + P + \bar{P}) \quad (\because 1 + A = 1)$
$Y = \bar{Q}$

67. A radio transmitter sends out $60W$ of radiation. Assuming that the radiation is uniform on a sphere with the transmitter at its centre, the intensity $(\text{in } W/m^2)$ of the wave at a distance $12\ km$ is
(A) 5.33×10^{-8}
(B) 3.33×10^{-6}
(C) 2.12×10^{-8}
(D) 6.66×10^{-8}
(E) 3.33×10^{-8}

Solution: (E)

Power of transmitted radiation, $P_T = 60W$
$\therefore$ Intensity of wave, $I = \frac{P_T}{A}$
$\because \quad A = 4\pi R^2$
$4\pi(12Km)^2 \quad (\because R = 12km \text{ given})$
$\Rightarrow \quad A = 4\pi \times 144 \times 10^6$
$\therefore\ I = \dfrac{60}{4\pi \times 144 \times 10^6}$
$=\ 3.31 \times 10^{-8}\,W/M^2$

68. Consider a system of gas of a diatomic molecule in which the speed of sound at 0^oC is $1260\,ms^{-1}$. Then, the molecular weight of the gas is (given, the gas constant R is $8.314\ J/mol\ K$)
(A) $2g$
(B) $2mg$
(C) $4g$
(D) $10g$
(E) $20g$

Solution: (A)

$\because$ Speed of sound, $V_{\text{sound}} = \sqrt{\frac{xRT}{m}}$

$\therefore\ = \dfrac{\gamma\,RT}{V_{\text{sound}}^2}$

For diatomic gas, $\gamma = 1.40$

$\therefore\ M = \dfrac{1.4 \times 8.314 \times 273}{(1260)^2} = 2 \times 10^{-3} kg = 2gm$

69. A satellite is orbiting the Earth in a circular orbit of radius R. Which one of the following statements is true?

(A) Angular momentum varies as $\frac{1}{\sqrt{R}}$

(B) Linear momentum varies as $\sqrt{R}$

(C) Frequency of revolution varies as $\frac{1}{R^2}$

(D) Kinetic energy varies as $\frac{1}{R}$

(E) Potential energy varies as R

Solution: (D)

$\because$ Kinetic energy of satellite,

$KE = \dfrac{GMm}{2R}$

$\therefore\ KE \propto \dfrac{1}{R}$

$\therefore$ Option Kinetic energy varies as $\frac{1}{R}$ is correct.

70. The magnitude of a magnetic field at the centre of a circular coil of radius R, having N turns and carrying a current I can be doubled by changing

(A) I to $2I$ and N to $2N$ keeping R unchanged

(B) N to $\frac{N}{2}$ and keeping I and R unchanged

(C) N to $2N$ and R to $2R$ keeping/unchanged

(D) R to $2R$ and I to $2l$ keeping N unchanged

(E) I to $2I$ and keeping N and R unchanged

Solution: (E)

$\because$ Magnetic field due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$

B can be doubled by changing I to $2I$ and Keeping N and R unchanged.

$\therefore$ Options I to $2I$ and keeping N and R unchanged is correct.

71. An alternating voltage $V = V_0 \sin \omega t$ is applied across a circuit and as a result, a current $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ flows in it. The power consumed per cycle is

(A) $I_0 V_0$

(B) $0.5 I_0 V_0$

(C) $0.7\, I_0 V_0$

(D) $1.41 I_0 V_0$

(E) 0

Solution: (E)

Given that, $V = V_0 \sin \omega t$

$I = I_0 \sin(\omega t + II_2)$
∵ Power consumed per cycle
$P_{av} = V_{rms} \cdot I_{rms} \cos$
Here $\phi = \frac{\pi}{2}$
$\therefore \; P_{av} = V_{rms} \cdot I_{rms} \cos \frac{\pi}{2} = 0\,W$

72. An electromagnetic wave of intensity I is incident on a non-reflecting surface. If C is the speed of light in free space, then the ratio I/C is same as
(A) Momentum
(B) Force
(C) Pressure
(D) Pressure per unit area
(E) Force x-area

Solution: (C)
∵ Intensity, $I = \frac{P}{A} = \frac{watt}{m^2}$
Speed of light, $c = \frac{m}{s}$

$$\therefore \quad \frac{I}{c} = \frac{\text{watt}}{m^2 \times \frac{m}{\text{sec}}}$$

$$= \frac{\left(\frac{\text{newton} \times m}{s}\right)}{\left(\frac{m^2 \times m}{s}\right)} = \frac{\text{newton}}{m^2}$$

Hence, $\frac{I}{C}$ =pressure.

Chemistry

Single correct answer type:

1. Which element has the highest first ionization potential?
(A) N
(B) Ne
(C) He
(D) H
(E) Li

Solution: (C)
Due to small size $(n = 1)$ and fully filled inert gas configuration. He show highest IE.

2. Which statement (s) is (are) false for the periodic classification of elements?
(A) The properties of the elements are the periodic functions of their atomic numbers
(B) Non-metallic elements are lesser in number than the metallic elements
(C) The first ionization energies of the elements along a period do not vary in a regular manner with increase in atomic number
(D) For transition elements, the d-electrons are filled monotonically with increase in atomic number
(E) Both The first ionization energies of the elements along a period do not vary in a regular manner with increase in atomic number and For transition elements, the d-electrons are filled monotonically with increase in atomic number

Solution: (D)
The option (For transition elements, the d-electrons are filled monotonically with increase in atomic number) is false as d-electrons do not filled monotonically with the increase in atomic number.

3. The electronegativities of N, C, Si and P are in the order
(A) $P < Si < C < N$
(B) $Si < P < N < C$
(C) $Si < P < C < N$
(D) $P < Si < N < C$
(E) Difficult to predict

Solution: (C)
Electronegativity increases on moving left to right in a period and decreases for the period below it $(\because n = \text{increases})$
$\because$ $(N \text{ and } C)$ and $(Si \text{ and } P)$ respectively belongs to $(n = 2)$ and $(n = 3)$
$\because$ Electronegativity of $N >$ electronegativity of C and electronegativity of $P >$ electronegativity of Si

Hence, correct order is:
$Si < P < C < N$

4. $Gd(64)$ has __ unpaired electrons with sum of spin __.
(A) $7, 3.5$
(B) $8, 3$
(C) $6, 3$
(D) $8, 4$
(E) $9, 3.5$

Solution: (D)
The element $Gd(Z = 64)$ show electronic configuration $= [Xe]_{54} \cdot 4f^7, 5d^1, 6s^2$ Thus, it has 8 –unpaired electrons ($4f^7$and $6d^1$) and its sum of spin is 4.

5. When SO_2 gas is passed into aqueous Na_2CO_3 the product (s) formed is (are)
(A) $NaHSO_4$
(B) Na_2SO_4
(C) $NaHSO_3$
(D) Na_2SO_3 and $NaHSO_3$
(E) $NaHSO_4$ and Na_2SO_4

Solution: (C)
On passing the $SO_2(g)$ in the aqueous solution of Na_2CO_3 we get $NaHSO_3$ as product.
$Na_2CO_3 + H_2O + 2SO_2(g) \rightarrow 2NaHSO_3 + CO_2$

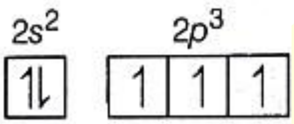
6. Portland cement does not contain
(A) $CaSiO_4$
(B) $CaSiO_3$
(C) $Ca_3Al_2O_6$
(D) $Ca_3(PO_4)_2$
(E) Both $Ca_3Al_2O_6$ and $Ca_3(PO_4)_2$

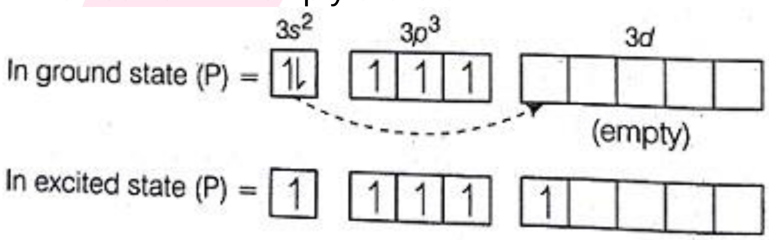
Solution: (D)
Main components of Portland–cement are
$(i)\ CaSiO_3 \quad (ii)\ CaSiO_4 \quad (iii)\ CaAl_2O_6$
and some other substance but it does not contain $Ca(PO_4)_2$.

7. $Al_2(SO_4)_3$ is used in the following but not
(A) As a coagulant in treating drinking water and sewage
(B) In plastics industry
(C) As a mordant in dyeing
(D) In paper industry
(E) Both As a mordant in dyeing and In paper industry

Solution: (B)
$Al_2(SO_4)_3$ is not used in plastic industry.

8. Maximum number of covalent bonds formed by N and P are
(A) $3, 5$
(B) $3, 6$
(C) $3, 4, 5$
(D) $3, 4, 6$
(E) None of these

Solution: (E)
The outermost orbital of N-atom contain 3 unpaired electrons and has $n = 2$


Thus, it is not able to expand its octet and can only form 3 covalent bonds.
It can also form one coordinate bond with the help of one lone−pair of electrons over N −atom ($\because$ total $= 4$)
On the other hand, the outer most orbital for P is $n = 3$
Which contain $3d$ empty orbitals.


(has 5−unpaired electrons)
Thus, none of the given are correct.
∴ Correct choice is None of these.

9. Consider the following statements concerning N_2H_4:
(i) It is an exothermic compound.
(ii) It burns in air with the evolution of heat.
(iii) It has kinetic stability.
(iv) In reduces Fe^{3+} to Fe^{2+} in acidic medium.
Which of the following combination is correct?
(A) (ii) and (iii) are correct
(B) (i) and (ii) are correct
(C) All are correct
(D) (iii) and (iv) are correct
(E) (ii), (iii) and (iv) are correct

Solution: (B)
Only the following statements are correct for N_2H_4.
(i) It is an exothermic compound.
(ii) It burns with evolution of heat.
∴ (i) and (ii) are correct is the correct choice.

10. Consider the following species:
(i) $[O_2]^{2-}$
(ii) $[CO]^+$
(iii) $[O_2]^+$
Among these sigma bond alone is present in
(A) (i) alone
(B) (ii) alone
(C) (iii) alone
(D) (i) and (ii)
(E) (i), (ii) and (iii)

Solution: (A)
$\because$ Among the given species, all are di$-$atomic species. Only $[O_2]^{2-}$ contain one σ bond between two bonded atoms.
$\therefore$ (i) alone is the correct option

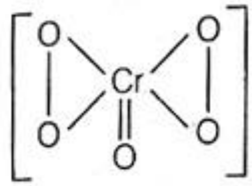
11. Select the correct option (s) for the following statements:
$1.\ Cl_2O$ and ClO_2 are used as bleaching agents.
$2.\ OCl^-$ salts are used as detergents.
$3.\ OCl^{-1}$ disproportionates in alkaline medium.
$4.\ BrO_3^-$ is oxidized in acidic medium.
(A) $1, 2, 3$ correct
(B) $2, 3, 4$ correct
(C) $1, 2, 4$ correct
(D) $1,3,4$ correct
(E) All are correct

Solution: (A)
Among the given statements:
$1.\ Cl_2O$ and ClO_2 give nascen oxygen, thus behave as bleaching agent.
2. Salts of OCl^- are used as detergents.
3. Oxidation state of Cl in OCl^- is $(+1)$, thus it shows disproportionation reaction in acidic medium:
$$5\underset{+1}{ClO^-}(5HClO) \rightarrow 2\underset{[0]}{Cl_2} + \underset{[+5]}{ClO_3^-}(HClO_3) + 2H_2O$$
$4.\ BrO_3^-$ does not oxidise in acidic medium.
Thus $1, 2, 3$ correct is the correct answer.

12. When H_2O_2 is added to an acidified $K_2Cr_2O_7$ solution
(A) A green colour solution is obtained
(B) A yellow solution is obtained
(C) A blue-violet solution is obtained
(D) A green precipitate is formed

(E) A yellow precipitate is formed

Solution: (C)
On adding H_2O_2 in acidic solution of $K_2Cr_2O_7, H_2O_2$ oxidise it to CrO_5 (chromic penta oxide/chromic per oxide) and gives the blue-violet solution.
$K_2Cr_2O_7 + H_2SO_4 + 4H_2O_2 \rightarrow K_2SO_4 + 2CrO_5 + 5H_2O$



(CrO_5)

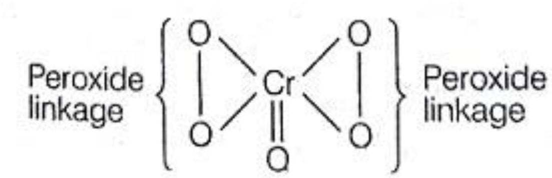
CrO_5 on decomposition gives oxygen
$4CrO_5 + 6H_2SO_4 \rightarrow Cr_2(SO_4)_3 + 6H_2O + 7O_2$

13. Consider the following compounds:
1. $(NH_4)_2Cr_2O_7$
2. NH_4NO_2
3. NH_4VO_3
4. NH_4NO_3
Which compound (s) yield nitrogen gas upon heating?
(A) 1 and 2
(B) 2 and 3
(C) 3 and 4
(D) 1 and 4
(E) All

Solution: (A)
Only $(NH_4)_2Cr_2O_7$ and NH_4NO_2 give N_2 on heating.
(i) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + 4H_2O + N_2$
(ii) $NH_4NO_2 \xrightarrow[\Delta]{\text{Thermal decoposition}} H_2O + N_2$
$\therefore$ 1 and 2 is the correct option.

14. How many peroxy linkages are present in CrO_5?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Solution: (B)
The structure of CrO_5 is as follows:



$\therefore$ It has two per-oxide linkage.

15. More than four bonds are made by how many elements in carbon family?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Solution: (E)
The elements belong to carbon family are:
Carbon – (C)
Silicon – (Si)
Germanium –(Ge)
Tin – (Sn)
Lead – (Pb)
Flerovium – (Fl)
Except carbon all other elements contain empty d-orbitals in their outermost shell, thus can form more than four−bonds.
$\therefore$ 5 is the correct answer.

16. The effective nuclear charge of an element with three valence electrons is 2.60. What is the atomic number of the element?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Solution: (E)
$\because$ Effective nuclear charge $= 2.60$ and valency shell contain 3 electrons. Thus, minimum number of main shells for the given element are two i.e. $(n = 2)$ and its configuration will be $1s^2, 2s^2, 2p^1$.
Thus, the given element has 5 electrons in all.
Also, for a neutral atom.
$\because$ No. of electrons = Atomic number
Thus, atomic number of the element is 5.

17. The elution sequence of a mixture of compounds containing chlorobenzene, anthracene and p-cresol developed of an alumina column using a solvent system of progressively increasing polarity is

(A) Anthracene chlorobenzene p-cresol
(B) Anthracene p-cresol chlorobenzene
(C) Chlorobenzene p-cresol anthracene
(D) Chlorobenzene anthracene p-cresol
(E) p-cresol anthracene chlorobenzene

Solution: (A)

Alumina column having suitable solvent elutes the species based on their nature of polarity. Less polar species absorb first and more polar thereafter. Thus the correct order is:

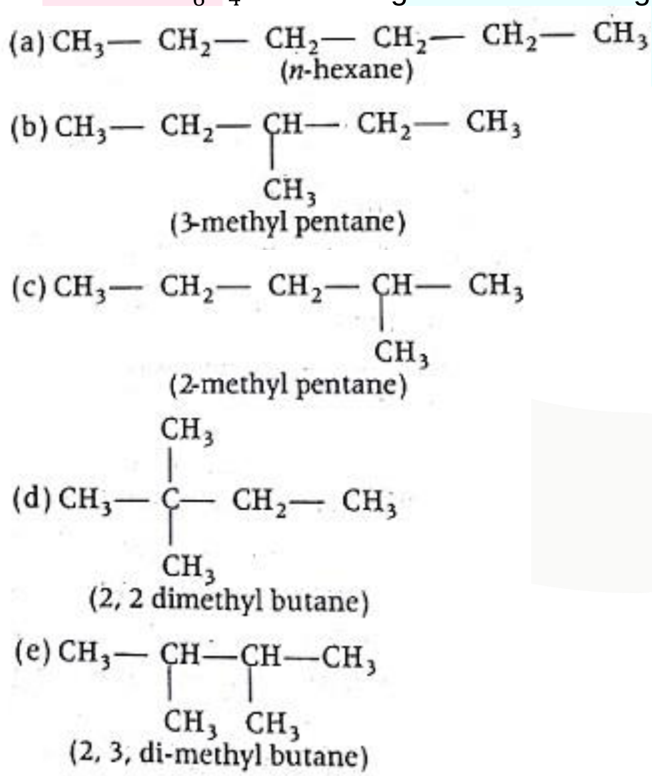
Anthracene $\rightarrow$ chlorobenzene $\rightarrow$ p-cresol

18. Number of constitutional isomers of alkane with formula C_6H_{14} is

(A) 3
(B) 2
(C) 5
(D) 10
(E) 8

Solution: (C)

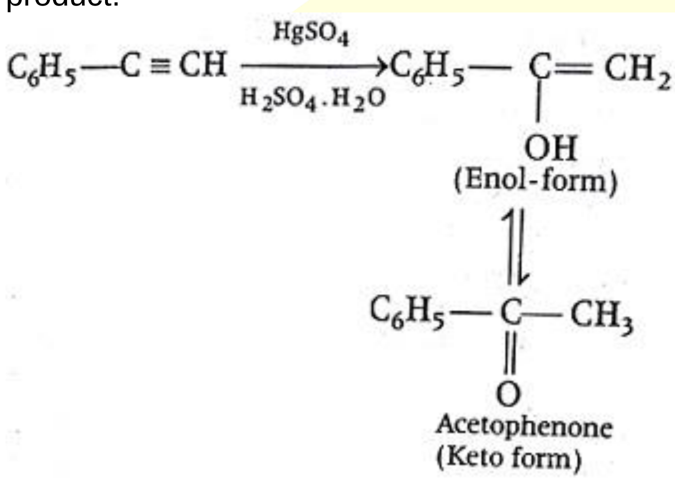
The formula C_6H_4 of alkane gives the following structural isomers-



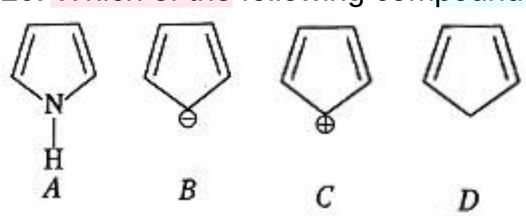
$\therefore$ Total $= 5$ –isomers

19. Phenylacetylene on treatment with $HgSO_4/H_2SO_4, H_2O$ produces
(A) Acetophenone
(B) Phenylacetaldehyde
(C) Phenylacetic acid
(D) 1-Phenylethanol
(E) 2-phenyletharol

Solution: (A)
On reaction with $HgSO_4/H_2SO_4 \cdot H_2O$, phenyl acetylene gives acetophenone as a product:

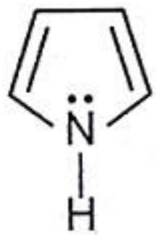


20. Which of the following compounds are aromatic?



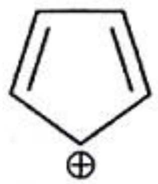
(A) A, B
(B) A, B, C
(C) B, C
(D) B, C, D
(E) A, B, D

Solution: (A)
(i) Structure (a) has two π-bonds and one lone pair of electrons over N-atom and has planer structure.
Thus, is an aromatic compound.



(ii) The structure (b) also contain two π-bonds and one lone pair of electrons. Thus, follow Huckel's rule. The structure is cyclic and planer.
$\therefore$ (b) is also an aromatic compound.

(iii) The structure (c) has only 2π-bonds and do not follow Huckel's rule. So, it not an aromatic compound.



(iv) Structure (d) also contain only two π $-$bonds, thus do not follow Huckel's rule and is not an aromatic compound.

21. Aromatic electrophilic substitution reaction that is reversible is
(A) Nitration
(B) Chlorination
(C) Sulphonation
(D) Alkylation
(E) Acylation

Solution: (C)
Among the aromatic electrophilic substitution reactions. Sulphonation is an example of reversible reaction.

22. Which one of the following statements is false?
(A) R and S configurations correspond to the enantiomers of an optically active compound
(B) The process of converting an optically active compound into a racemate is called racemization
(C) A molecule containing a plane of symmetry can be optically active
(D) Optical isomers that are not enantiomers are called diastereoisomers
(E) All chiral objects are asymmetric

Solution: (C)
$R-S-$ configuration is related to the enantiomers, which are optically active. $(\therefore \text{true})$

(ii) The mixing of two optically-active compounds $(d \text{ and } l - \text{type})$ in equimolar quantity is called racemization $(\therefore \text{true})$
(iii) A molecule containing plane of symmetry does not show optical$-$activity.
Hence, the given statement is false.
(iv) Optical isomers that are not enantiomers are called diastereoisomers (is true).
(v) All chiral objects are asymmetric (true)
$\therefore$ A molecule containing a plane of symmetry can be optically active is the correct answer.

23. Neopentyl bromide, undergoes dehydrohalogenation to give alkenes even though it has no hydrogen. This is due to
(A) E_2 mechanism
(B) E_1 mechanism
(C) Rearrangement of carbocations by E_1 mechanism
(D) E_1cB mechanism
(E) E_1 mechanism

Solution: (C)
Neopentyl bromide give a carbocation as intermediate which undergo for rearrangement and show E_1 mechanism (even has no $\beta - H$ atom).
Thus Rearrangement of carbocations by E_1 mechanism is the correct option.

24. The compound which does not lead to nitrile by substitution with $NaCN/DMSO$ is
(A) Benzyl chloride
(B) Ethyl chloride
(C) Isopropyl chloride
(D) Chlorobenzene
(E) Isobutyl chloride

Solution: (D)
Chlorobenzene does not lead to nitrile by substitution with $NaCN/DMSO$ due to resonance and double bond character between Cl and carbon $[C - \text{atom}]$ of the benzene ring.

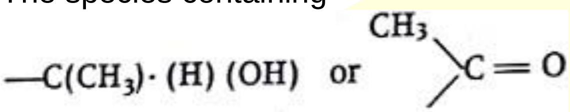
25. Oxidation of 1π alcohols to aldehydes is very successful for the alcohols like
(A) Pent-2-yn-1-ol
(B) 1-hexanol
(C) n-propyl alcohol
(D) 1-pentanol
(E) 1-octanol

Solution: (C)
The oxidation of n-propyl alcohol (among the 1^o alcohols) is very successful because of least steric hindrance in the given molecule.

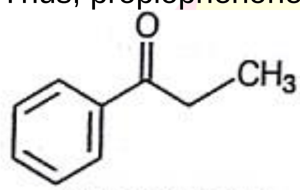
Bio-waste give the most efficient and clean fuel thus known as Green-fuel.

26. The compound that does not undergo halo form reaction is
(A) Acetaldehyde
(B) Ethanol
(C) Acetone
(D) Acetophenone
(E) Propiophenone

Solution: (E)
The species containing



will show haloform reaction.
Thus, propiophenone does not show haloform reaction.

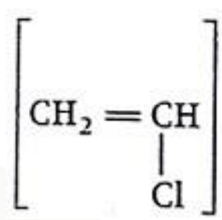


(Propiophenone)

27. The halogen compound which will not react with phenol to give ethers is
(A) Ethyl chloride
(B) Methyl chloride
(C) Benzyl chloride
(D) Vinyl chloride
(E) Allyl chloride

Solution: (D)

Due to resonance structure

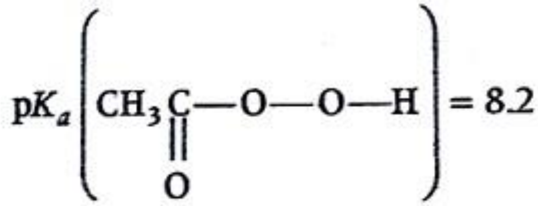


(vinyl chloride) will not react with phenol to give ethers.

28. The weakest among the following acids is
(A) Peroxyacetic acid
(B) Acetic acid
(C) Chloroacetic acid
(D) Trichloroacetic acid
(E) Propanoic acid

Solution: (A)
As pK_a value of
$pK_a(CH_3COOH) = 4.76$

$pK_a(CH_2Cl.COOH) = 2.75$
$pK_a(CCl_3COOH) = 0.65$



and more be the value of pK_a, weaker be the acid. Hence, peroxyacetic acid is the weakest acid.

29. The nitrosation of N, N-dimethylaniline takes place through the attack of electrophile
(A) Nitronium ion
(B) Protonated nitrous acid
(C) Nitrous acid
(D) Nitrite ion
(E) Nitrosonium ion

Solution: (E)
On attacking at
$N - N$ –dimethyl aniline by $\overset{+}{N}O_2$, electrophilic nitration takes place and the process is called Nitrosation. Here $\overset{+}{N}O_2$ act as an attacking electrophilic agent. It is produced as follows:
$HNO_3 + 2H_2SO_4 \rightarrow \overset{+}{N}O_2 + 2HSO_4^- + H_2O$

30. The nitrogeneous base present only in RNA is
(A) Guanine
(B) Adenine
(C) Cytosine
(D) Uracil
(E) Thymine

Solution: (D)
In *RNA*, nitrogen-base uracil is present in place of thymine (which is present in DNA).
∴ Uracil is the correct answer.

31. Green fuel is the fuel obtained from
(A) Bio-waste
(B) Metal waste
(C) Plastic waste
(D) Chemical waste
(E) Electronic waste

Solution: (A)

104. Barbiturates are potent
(A) Hypnotics
(B) Antimicrobials
(C) Antacids
(D) Antiseptics
(E) Antiallergics

Solution: (A)
Barbiturates are derivatives of barbituric acids. These are potent hypnotics.

105. $1\,mol$ of $FeSO_4$ (atomic weight of Fe is $55.84 g\ mol^{-1}$) is oxidized to $Fe_2(SO_4)_3$. Calculate the equivalent weight of ferrous ion
(A) 55.84
(B) 27.92
(C) 18.61
(D) 111.68
(E) 83.76

Solution: (A)
Atomic mass of $Fe = 55.84$
$\because$ Equivalent mass $= \frac{\text{Atomic mass}}{\text{Change in oxidation state}}$
For the charge, $Fe^{2+} \rightarrow Fe^{3+}$ i.e. $(3 - 2 = 1)$
the equivalent mass $= \frac{55.84}{1} = 55.84$

106. Mass % of carbon in ethanol is
(A) 52
(B) 13
(C) 34
(D) 90
(E) 80

Solution: (A)
Molecular mass of $C_2H_5OH = 46.00$
$$\because \begin{bmatrix} \text{Atomic mass of } C = 12.00 \\ \text{Atomic mass of } O = 16.00 \\ \text{Atomic mass of } H = 1.00 \end{bmatrix}$$
Also,
$\because 46.00$ of C_2H_5OH contain, $C = 24g$.
$\because 100g$ of C_2H_5OH contain, $C = \frac{24 \times 100}{46}$
$(52.17\% = 52\%)$

107. One mole of ethanol is produced reacting graphite, H_2 and O_2 together. The standard enthalpy of formation is $-277.7\ kJ\ mol^{-1}$. Calculate the standard enthalpy of the reaction when 4 moles of graphite is involved
(A) -277.7

(B) -555.4
(C) -138.85
(D) -69.42
(E) -1110.8

Solution: (B)
The related equation for the formation of ethanol is.
$$2C(\text{graphite}) + 3H_2(g) + \frac{1}{2}O_2(g) \rightarrow C_2H_5OH(l); \Delta H_f = -277.7\ kJ/mol$$
When 2 moles of graphite is involved the standard enthalpy of reaction is $-277.7\ kJ/mol$
∴ When 4 moles of graphite is involved, standard enthalpy of reaction is $2 \times (-277.7\ kJ/mol)$
$= (-)\ 555.4\ kJ/mol$

108. Which of the following process best describes atomization of $CH_4(g)$?
(A) Exothermic
(B) Endothermic
(C) Non-spontaneous
(D) Spontaneous
(E) Both Endothermic and Non-spontaneous

Solution: (E)
The energy required to separate one mole CH_4 molecule into its gaseous atoms is known as heat of atomization.
In this process, heat is absorbed i.e. (endothermic) and the said reaction is non-spontaneous.
∴ Both Endothermic and Non-spontaneous is the correct answer.

109. Consider the equilibrium $X_2 + Y_2 \rightleftharpoons P$. Find the stoichiometric coefficient of the P using the data given in the following table:

$X_2/mol\ L^{-1}$	$Y_2/mol\ L^{-1}$	$P/mol\ L^{-1}$
1.14×10^{-2}	0.12×10^{-2}	2.52×10^{-2}
0.92×10^{-2}	0.22×10^{-2}	3.08×10^{-2}

(A) 1
(B) 2
(C) 3
(D) 0.5
(E) 4

Solution: (B)
Given,
$X_2 + Y_2 \rightarrow ?\, P$
Let the coefficient of P be x.
Now, we know that

$K = \frac{[P]^x}{[X_2][Y_2]}$
According to the data given in question,
$K_1 = \frac{(2.52 \times 10^{-2})^x}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})}$
Also, $K_2 = \frac{(3.08 \times 10^{-2})^x}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})}$
Now, substitute the values of x given in option one by one.
$(a)\ K_1 = \frac{(2.52 \times 10^{-2})}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})} = 1842.10$
$K_2 = \frac{(3.08 \times 10^{-2})}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})} = 1521.73$
$\because K_1 \neq K_2$, thus option 1 is incorrect.
$(b)\ K_1 = \frac{(2.52 \times 10^{-2})^2}{(1.14 \times 10^{-2}) \times (0.12 \times 10^{-2})} = 46.42$

$K_2 = \frac{(3.08 \times 10^{-2})^2}{(0.92 \times 10^{-2}) \times (0.22 \times 10^{-2})} = 46.36$
$\because K_1 \neq K_2$
$\therefore$ Option 2 is correct

110. Which of the following can help predict the rate of a reaction if the standard Gibbs free energy of reaction ($\Leftarrow G\pi$) is known?
(A) Equilibrium constant
(B) $\Leftarrow H\pi$
(C) $\Leftarrow U\pi$
(D) Heat liberated during the course of reaction in calorimeter
(E) Both (Equilibrium constant) and ($\Leftarrow H\pi$)

Solution: (A)
$\because \Delta G^o$ (Gibbs free energy) is related to K (equilibrium constant) as follows:
$\Delta G^o = -2.303RT \log_{10} K$
where, $R =$ Gas constant
$T =$ Temperature in Kelvin
By knowing the value of K we can find out rate of a reaction.
Equilibrium constant is the correct answer.

111. Calculate the molarity of a solution containing $5g$ of $NaOH$ dissolved in the product of a $H_2 - O_2$ fuel cell operated at $1A$ current for 595.1 hours.
(Assume $1F = 96500\ C/mol$ of electrons and molecular weight of $NaOH$ as $40g\ mol^{-1}$)
(A) $0.05M$
(B) $0.025M$
(C) $0.1M$
(D) $0.075M$
(E) $1M$

Solution: (B)
Total charge produced by cell
$= 1A \times (5951 \times 3600)s$
$= 2142360C$
$\because \; 96500C = 1mol$
$\therefore \; 2142360C = \dfrac{2142360}{96500}$
$= 22.20mol$ of e^-
Now, in $H_2 - O_2$ fuel cell following reaction occurs,
At mode $2H_2(g) + 4O^-H(aq) \rightarrow 4H_2O(l) + 4e^-$
At cathode $O_2(g) + 2H_2O(l) + 4e^- \rightarrow 4OH^-(aq)$
Overall reaction $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$
Thus, from above reaction it is clear that
$2\,mol$ of $e^- \equiv 1\,mol$ of H_2O
$\because 22\,mol$ of $e^- \equiv 11\,mol$ of H_2O
$= (11 \times 18)g$ of H_2O
$\left[\because \text{No. of moles} = \dfrac{\text{Weight}}{\text{Molecular weight}}\right]$
$= 198g$ or mL of H_2O
Now, number of mol of $NaOH = \frac{5}{40} = 0.125\,mol$
We know that,
$\text{Molarity} = \dfrac{\text{No. of moles of solute}}{\text{Volume of solution (in L)}}$
$= \dfrac{0.125}{198} \times \; 1000 = 0.63M$

112. If $1mol$ of $NaCl$ solute is dissolved into the $1kg$ of water, at what temperature will water boil at 1.013 bar?
$(K_b \text{ of water is } 0.52K\;kg\;mot^{-1})$
(A) $373.15K$
(B) $373.67K$
(C) $374.19K$
(D) $373.19K$
(E) $375K$

Solution: (B)
$\because \Delta T = K_b \dfrac{n}{W_A}$
Given, $K_b = 0.52K\;kg\;mol^{-1}$
$n = 1, w_A = 1kg$
$\therefore \; \Delta T = \dfrac{0.52 \times 1}{1} = 0.52K$
$\because$ Boiling point of pure water at 1.013 bar (i.e. $1atm$) is $373.15\,K$
$\therefore$ Boiling point of the solution $= 373.15 + 0.52 = 373.67K$

113. Consider the electrochemical reaction between $Ag(s)$ and $Cl_2(g)$ electrodes in $1L$ of $0.1\,M\,KCl$ aqueous solution. Solubility product of $AgCl$ is $1.8\lambda 10^{-10}$ and $F = 96500\ C/mol$. At $1 \geq 10^{-6}A$ current, calculate the time required to start observing the $AgCl$ precipitation in the galvanic cell

(A) $173\ s$

(B) $346\ s$

(C) $125\ \lambda 10^{6}\ s$

(D) $1.25\ \lambda\ 10^{5} s$

(E) $101\ s$

Solution: (A)

The electrochemical reaction between $Ag(s)$ and $Cl_2(g)$ is as follows:

$AgCl \rightleftharpoons Ag^{+} + Cl^{-}$

Given, $K_{\text{sp}} = 1.8 \times 10^{-10}, [Cl^{-}] = 0.1\ M$

$\therefore\ K_{\text{sp}} = [Ag^{+}][Cl^{-}]$

$1.8 \times 10^{-10} = [Ag^{+}] \times 0.1$

$\therefore [Ag^{+}] = \dfrac{1.8 \times 10^{-10}}{0.1} = 1.8 \times 10^{-9} M$

$\therefore$ $1L$ of solution contains 1.8×10^{-9} moles of Ag^{+}.

Quantity of electricity required

$= 1.8 \times 10^{-9} \times 96500$

$= 1.73 \times 10^{-4} C$

$\therefore$ Time required $(t) = \frac{1.73 \times 10^{-4}}{1 \times 10^{-6}} = 173 s$

114. The voltage of the cell consisting of $Li\,(s)$ and $F_2(g)$ electrodes is $5.92V$ at standard condition at $298K$. What is the voltage if the electrolyte consists of $2M\ LiF$.

$(\ln 2 = 0.693, R = 8.314 JK^{-1}mol^{-1} \text{and } F = 96500C\ \text{mol}^{-1})$

(A) $5.90V$

(B) $5.937V$

(C) $5.88V$

(D) $4.9V$

(E) $4.8V$

Solution: (A)

Now, the cell reaction is

$$Li(s) + \frac{1}{2}F_2(g) \rightarrow Li^{+} + F^{-}$$

We know that,

$$E_{\text{cell}} = E^{o}_{\text{cell}} = \frac{RT}{nF}\ln\frac{\text{Product}}{\text{Reactant}}$$

$$= E^{o}_{\text{cell}} - \frac{2.303RT}{nF}\log[Li^{+}][F^{-}]$$

$$= 5.92 - \frac{2.303 \times 8.314 \times 298}{1 \times 96500}\log(2 \times 2)$$

$= 5.92 - \frac{0.059}{1} \times 2 \log 2$
$= 5.92 - 0.035 = 5.88V$

115. Consider the galvanic cell. $Pt(s)|H_2(1 \text{ bar})|HCl\ (aq)(1M)|Cl_2(1 \text{ bar})|Pt(s)$. After running the cell for sometime, the concentration of the electrolyte is automatically raised to $3M\ HCl$. Molar conductivity of the $3M\ HCl$ is about $240S\ cm^2 mol^{-1}$ and limiting molar conductivity of HCl is about $420S\ cm^2 mol^{-1}$. If $K_b$of water is $0.52K\ kg\ mol^{-1}$, calculate the boiling point of the electrolyte at the end of the experiment
(A) $375.6K$
(B) $376.3K$
(C) $378.1K$
(D) $380.3K$
(E) $381.6K$

Solution: (A)
Given, $\Lambda_m = 240S\ cm^2 mol^{-1}$,
$\Lambda_m^{\infty} = 240Scm^2 mol^{-1}$
$K_b = 0.52K\ kg\ mol^{-1}$
Degree of ionization, $\alpha = \frac{\Lambda_m}{\Lambda_m^{\infty}}$
$= \frac{240\ Scm^2 mol^{-1}}{420Scm^2 mol^{-1}} = 0.57$
Van't Hoff factor
$i = 1 + (n-1)\alpha$ [for ionization]
$= 1 + (2-1)0.57$ [for $HCl, n = 2$]
$= 1.57$
Elevation in boiling,
$\Delta T_b = iK_b \times \text{molality } (m)$
$= 1.57 \times 0.52K\ kg\ mol^{-1} \times 3\ mol\ kg^{-1}$
$= 2.45K$
Since, water boils at $373.15K$ at 1 bar pressure, therefore the boiling point of solution will be
$T_b = T_b^o + \Delta T_b = 373.15 + 2.45$
$= 375.6K$

116. The data given below are for the reaction of A and D_2 to form product at $295K$. Find the correct rate expression for this reaction

$\frac{D_2}{mol\ L^{-1}}$	$\frac{A}{mol\ L^{-1}}$	$\frac{\text{Initial rate}}{mol\ L^{-1} s^{-1}}$
0.05	0.05	$1\lambda 10^{-3}$
0.15	0.05	$3\lambda 10^{-3}$
0.05	0.15	$9\lambda 10^{-3}$

(A) $k[D_2]^1[A]^2$
(B) $k[D_2]^2[A]^1$

(C) $k[D_2]^1[A]^1$
(D) $k[D_2]^2[A]^2$
(E) $k[D_2]^1[A]^0$

Solution: (A)
From the data (1) and (2)

$$\frac{[r_{D_2}]_2}{[r_{D_2}]_1} = \frac{K[D_2]_2}{K[D_2]_1}$$

i.e $\frac{3\times10^{-3}}{1\times10^{-3}} = \frac{[0.15]}{[0.05]}$
$(3) = [3]^n$
$\because$ Rate $(r) \propto [\text{concentration}]^n$, where $n =$ order
$\therefore r(D_2) = [3]^1$
$\therefore$ Order of $[D_2] = 1$
Similarly, from data (1) and (3)

$$\frac{(r_A)_3}{(r_A)_1} = \frac{k[A]_3}{k[A]_1}$$

$$\Rightarrow \quad \frac{9\times10^{-3}}{1\times10^{-3}} = \frac{0.15}{0.05} = 3$$

$\because$ rate $(r) = [\text{conc.}]^n$
$\therefore \quad a = [3]^2$
or order for $[A] = 2$
Hence, rate expression $= k[D_2]^1 \cdot [A]^2$
$\therefore \; k[D_2]^1[A]^2$ is the correct answer.

117. Find the unit of the rate constant of a reaction represented with a rate equation, rate $= k[A]^{\frac{1}{2}}\,[B]^{\frac{3}{2}}$
(A) $mol^{-1}Ls^{-1}$
(B) s^{-1}
(C) $mol\;L^{-1}s^{-1}$
(D) $mol^{-2}L^2s^{-1}$
(E) $mol^{-3}L^3s^{-1}$

Solution: (A)
$\because$ Rate $(r) = k\,[\text{concentration}]^n$

$$\therefore \quad k = \frac{\text{rate}}{[\text{concentration}]^n}$$

or, $k = \frac{[mol\;L^{-1}]\cdot s^{-1}}{[mol\;L^{-1}]^2\cdot[mol\;L^{-1}]^{\frac{3}{2}}}$

$$k = \frac{[mol\;L^{-1}]\cdot s^{-1}}{[mol\;L^{-1}]^2}$$

$$k = mol^{-1}Ls^{-1}$$

$\therefore \; mol^{-1}Ls^{-1}$ is the correct answer.

118. Under what condition the order of the reaction,
$2HI \rightarrow H_2(g) + I_2(g)$, is zero
(A) At high temperature
(B) At high partial pressure of HI
(C) At low partial pressure of HI
(D) At high partial pressure of H_2
(E) At high partial pressure of I_2

Solution: (B)
The reaction,
$2HI \xrightarrow{\Delta,\ \text{catalyst}} H_2(g) + I_2(g)$
is of zero-order in which HI is present at high partial pressure.

119. Which of the following statement is true about the adsorption?
(A) $\Leftarrow H < 0$ and $\Leftarrow S < 0$
(B) $\Leftarrow H > 0$ and $\Leftarrow S < 0$
(C) $\Leftarrow H < 0$ and $\Leftarrow S > 0$
(D) $\Leftarrow H = 0$ and $\Leftarrow S < 0$
(E) $\Leftarrow H = 0$ and $\Leftarrow S > 0$

Solution: (A)
In adsorption the adsorbed particles show strong force of attraction with the surface on which they adsorb and therefore their randomness also decreases.
∴ $\Delta H < 0$ and $\Delta S < 0$
Hence, $\Leftarrow H < 0$ and $\Leftarrow S < 0$ is the correct answer.

120. In NH_3synthesis by Haber's process, what is the effect on the rate of the reaction with the addition of Mo and CO, respectively?
(A) Increases and decreases
(B) Decreases and decreases
(C) Decreases and increases
(D) Both Mo and CO increases the rate
(E) Both Mo and CO does not affect the rate

Solution: (A)
In formation of NH_3 by Haber's process.
$N_2(g) + 3H_2(g) \xrightarrow{\Delta,\ \text{catalyst}} 2NH_3(g)$
(i) When Mo is used as catalyst, it increase the rate of formation of NH_3 because it behaves as promoter.
(ii) But, when CO is used as catalyst it decreases the formation of NH_3 because it behaves as poisoning agent.
∴ Increases and decreases is the correct answer.