2009

BOOKLET NO. TEST CODE : **SIA**

Forenoon

Questions : 30	**Time : 2 hours**

Write your Name, Registration Number, Test Centre, Test Code and the Number of this booklet in the appropriate places on the answersheet.

This test contains 30 questions in all – 20 in Group A and 10 in Group B. For each of the 30 questions in both groups, there are four suggested answers. In Group A, *only one* of the suggested answers is correct, while in Group B, *either one or two* are correct. In either case, you will need to identify *all* the correct answers and *only* the correct answers in order to get full credit for that question. Indicate your choice of the correct answer(s) by putting cross mark(s) ($\times$) in the appropriate box(es) $\Box$ on the answersheet.

You will get
4 marks for each correctly answered question,
0 marks for each incorrectly answered question and
1 mark for each unattempted question.

ALL ROUGH WORK MUST BE DONE ON THIS BOOKLET ONLY.
YOU ARE NOT ALLOWED TO USE CALCULATORS IN ANY FORM.

WAIT FOR THE SIGNAL TO START.

Group A

Each of the following questions have exactly one correct option and you have to identify it.

1. If k times the sum of the first n natural numbers is equal to the sum of the squares of the first n natural numbers, then $\cos^{-1}\left(\frac{2n-3k}{2}\right)$ is

   (A) $\frac{5\pi}{6}$. (B) $\frac{2\pi}{3}$. (C) $\frac{\pi}{3}$. (D) $\frac{\pi}{6}$.

2. Two circles touch each other at P. The two common tangents to the circles, none of which pass through P, meet at E. They touch the larger circle at C and D. The larger circle has radius 3 units and CE has length 4 units. Then the radius of the smaller circle is

   (A) 1. (B) $\frac{5}{7}$. (C) $\frac{3}{4}$. (D) $\frac{1}{2}$.

3. Suppose $ABCDEFGHIJ$ is a ten-digit number, where the digits are all distinct. Moreover, $A > B > C$ satisfy $A + B + C = 9$, $D > E > F$ are consecutive even digits and $G > H > I > J$ are consecutive odd digits. Then A is

   (A) 8. (B) 7. (C) 6. (D) 5.

4. Let ABC be a right angled triangle with $AB > BC > CA$. Construct three equilateral triangles BCP, CQA and ARB, so that A and P are on opposite sides of BC; B and Q are on opposite sides of CA; C and R are on opposite sides of AB. Then

   (A) $CR > AP > BQ$. (B) $CR < AP < BQ$.

   (C) $CR = AP = BQ$. (D) $CR^2 = AP^2 + BQ^2$.

5. The value of $(1 + \tan 1°)(1 + \tan 2°) \cdots (1 + \tan 44°)$ is

   (A) 2. (B) a multiple of 22.

   (C) not an integer. (D) a multiple of 4.

6. Let $y = x/(1+x)$, where

$$x = \omega^{2009^{2009^{\cdots \text{upto 2009 times}}}}$$

and ω is a complex cube root of 1. Then y is

(A) ω. (B) $-\omega$. (C) ω^2. (D) $-\omega^2$.

7. The number of solutions of θ in the interval $[0, 2\pi]$ satisfying

$$\left(\log_{\sqrt{3}} \tan\theta\right) \sqrt{\log_{\tan\theta} 3 + \log_{\sqrt{3}} 3\sqrt{3}} = -1$$

is

(A) 0. (B) 2. (C) 4. (D) 6.

8. A building with ten storeys, each storey of height 3 metres, stands on one side of a wide street. From a point on the other side of the street directly opposite to the building, it is observed that the three uppermost storeys together subtend an angle equal to that subtended by the two lowest storeys. The width of the street is

(A) $6\sqrt{35}$ metres. (B) $6\sqrt{70}$ metres.

(C) 6 metres. (D) $6\sqrt{3}$ metres.

9. A collection of black and white balls are to be arranged on a straight line, such that each ball has at least one neighbour of different colour. If there are 100 black balls, then the maximum number of white balls that allows such an arrangement is

(A) 100. (B) 101. (C) 202. (D) 200.

10. Let $f(x)$ be a real-valued function satisfying $af(x) + bf(-x) = px^2 + qx + r$, where a and b are distinct real numbers and p, q and r are non-zero real numbers. Then $f(x) = 0$ will have real solution when

(A) $(\frac{a+b}{a-b})^2 \leq \frac{q^2}{4pr}$. (B) $(\frac{a+b}{a-b})^2 \leq \frac{4pr}{q^2}$.

(C) $(\frac{a+b}{a-b})^2 \geq \frac{q^2}{4pr}$. (D) $(\frac{a+b}{a-b})^2 \geq \frac{4pr}{q^2}$.

11. A circle is inscribed in a square of side x, then a square is inscribed in that circle, a circle is inscribed in the latter square, and so on. If S_n is the sum of the areas of the first n circles so inscribed, then, $\lim_{n\to\infty} S_n$ is

(A) $\frac{\pi x^2}{4}$. (B) $\frac{\pi x^2}{3}$. (C) $\frac{\pi x^2}{2}$. (D) πx^2.

12. Let $1, 4, \ldots$ and $9, 14, \ldots$ be two arithmetic progressions. Then the number of distinct integers in the collection of first 500 terms of each of the progressions is

(A) 833. (B) 835. (C) 837. (D) 901.

13. Consider all the 8-letter words that can be formed by arranging the letters in BACHELOR in all possible ways. Any two such words are called *equivalent* if those two words maintain the same relative order of the letters A, E and O. For example, BACOHELR and CABLROEH are equivalent. How many words are there which are equivalent to BACHELOR?

(A) $\binom{8}{3} \times 3!$. (B) $\binom{8}{3} \times 5!$. (C) $2 \times \binom{8}{3}^2$. (D) $5! \times 3! \times 2!$.

14. The limit

$$\lim_{n\to\infty} \left(\frac{1}{6} + \frac{1}{24} + \frac{1}{60} + \frac{1}{120} + \cdots + \frac{1}{n^3 - n} \right)$$

equals

(A) 1. (B) $\frac{1}{2}$. (C) $\frac{1}{4}$. (D) $\frac{1}{8}$.

15. Let a and b be real numbers satisfying $a^2 + b^2 \neq 0$. Then the set of real numbers c, such that the equations $al + bm = c$ and $l^2 + m^2 = 1$ have real solutions for l and m is

(A) $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$. (B) $[-|a+b|, |a+b|]$.

(C) $[0, a^2 + b^2]$. (D) $(-\infty, \infty)$.

16. Let f be an onto and differentiable function defined on $[0,1]$ to $[0,T]$, such that $f(0)=0$. Which of the following statements is necessarily true?

(A) $f'(x)$ is greater than or equal to T *for all* x.

(B) $f'(x)$ is smaller than T *for all* x.

(C) $f'(x)$ is greater than or equal to T *for some* x.

(D) $f'(x)$ is smaller than T *for some* x.

17. The area of the region bounded by $|x|+|y|+|x+y| \leq 2$ is

(A) 2. (B) 3. (C) 4. (D) 6.

18. Let f and g be two positive valued functions defined on $[-1,1]$, such that $f(-x)=1/f(x)$ and g is an even function with $\int_{-1}^{1} g(x)dx = 1$. Then $I = \int_{-1}^{1} f(x)g(x)dx$ satisfies

(A) $I \geq 1$. (B) $I \leq 1$. (C) $\frac{1}{3} < I < 3$. (D) $I = 1$.

19. How many possible values of (a,b,c,d), with a,b,c,d real, are there such that $abc=d$, $bcd=a$, $cda=b$ and $dab=c$?

(A) 1. (B) 6. (C) 9. (D) 17.

20. What is the maximum possible value of a positive integer n, such that for any choice of seven distinct elements from $\{1,2,\ldots,n\}$, there will exist two numbers x and y satisfying $1 < x/y \leq 2$?

(A) 2×7. (B) $2^7 - 2$. (C) $7^2 - 2$. (D) $7^7 - 2$.

Group B

Each of the following questions has either one or two correct options and you have to identify all the correct options.

21. Which of the following are roots of the equation $x^7 + 27x = 0$?

(A) $-\sqrt{3}i$.
(B) $\frac{\sqrt{3}}{2}(-1 + \sqrt{3}i)$.
(C) $-\frac{\sqrt{3}}{2}(1 + i)$.
(D) $\frac{\sqrt{3}}{2}(\sqrt{3} - i)$.

22. The equation $|x^2 - x - 6| = x + 2$ has

(A) two positive roots.
(B) two real roots.
(C) three real roots.
(D) none of the above.

23. If $0 < x < \pi/2$, then

(A) $\cos(\cos x) > \sin x$.
(B) $\sin(\sin x) > \sin x$.
(C) $\sin(\cos x) > \cos x$.
(D) $\cos(\sin x) > \sin x$.

24. Suppose $ABCD$ is a quadrilateral such that the coordinates of A, B and C are $(1, 3)$, $(-2, 6)$ and $(5, -8)$ respectively. For which choices of coordinates of D will $ABCD$ be a trapezium?

(A) $(3, -6)$.
(B) $(6, -9)$.
(C) $(0, 5)$.
(D) $(3, -1)$.

25. Let x and y be two real numbers such that $2\log(x - 2y) = \log x + \log y$ holds. Which of the following are possible values of x/y?

(A) 4.
(B) 3.
(C) 2.
(D) 1.

26. Let f be a differentiable function satisfying $f'(x) = f'(-x)$ for all x. Then

(A) f is an odd function.
(B) $f(x) + f(-x) = 2f(0)$ for all x.
(C) $\frac{1}{2}f(x) + \frac{1}{2}f(y) = f(\frac{1}{2}(x + y))$ for all x, y.
(D) If $f(1) = f(2)$, then $f(-1) = f(-2)$.

27. Consider the function

$$f(x) = \begin{cases} \dfrac{\max\{x, \frac{1}{x}\}}{\min\{x, \frac{1}{x}\}}, & \text{when } x \neq 0, \\ 1, & \text{when } x = 0. \end{cases}$$

Then

(A) $\lim\limits_{x \to 0+} f(x) = 0$.

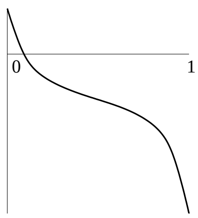
(B) $\lim\limits_{x \to 0-} f(x) = 0$.

(C) $f(x)$ is continuous for all $x \neq 0$.

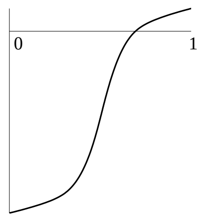
(D) $f(x)$ is differentiable for all $x \neq 0$.

28. Which of the following graphs represent functions whose derivatives have a maximum in the interval $(0, 1)$?

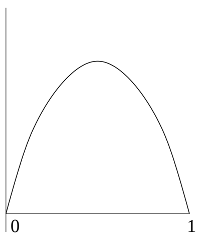
(A)



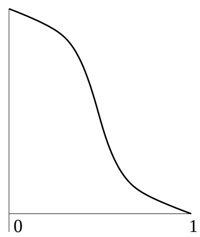
(B)



(C)



(D)



29. A collection of geometric figures is said to satisfy *Helly property* if the following condition holds:

> for any choice of three figures A, B, C from the collection satisfying $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$ and $C \cap A \neq \emptyset$, one must have $A \cap B \cap C \neq \emptyset$.

Which of the following collections satisfy Helly property?

(A) A set of circles.

(B) A set of hexagons.

(C) A set of squares with sides parallel to the axes.

(D) A set of horizontal line segments.

30. Consider an array of m rows and n columns obtained by arranging the first mn integers in some order. Let b_i be the maximum of the numbers in the i-th row and c_j be the minimum of the numbers in the j-th column. If

$$b = \min_{1 \leq i \leq m} b_i \qquad \text{and} \qquad c = \max_{1 \leq j \leq n} c_j,$$

then which of the following statements are necessarily true?

(A) $m \leq c$. (B) $n \geq b$. (C) $c \geq b$. (D) $c \leq b$.