INDIAN INSTITUTE OF SCIENCE
BANGALORE - 560012

ENTRANCE TEST FOR ADMISSIONS - 2009

Program : Integrated Ph.D

Entrance Paper : Physical Sciences

Paper Code : PS

Day & Date

SUNDAY, 26TH APRIL 2009

Time

2.00 P.M. TO 5.00 P.M.

Integrated Ph.D. (Physical Sciences)

General Instructions

- This question paper consists only of multiple-choice questions.
- Answers are to be marked in the OMR sheet provided.
- For each question darken the appropriate bubble to indicate your answer. Use only HB pencils for bubbling answers.
- Mark only one bubble per question. If you mark more than one bubble, the question will be evaluated as incorrect.
- If you wish to change your answer, please erase the existing mark completely before marking the other bubble.

All 50 multiple-choice questions in this test should be answered. Each question carries **TWO MARKS**. For each of these questions, four answers are provided; of these only **ONE** is correct.

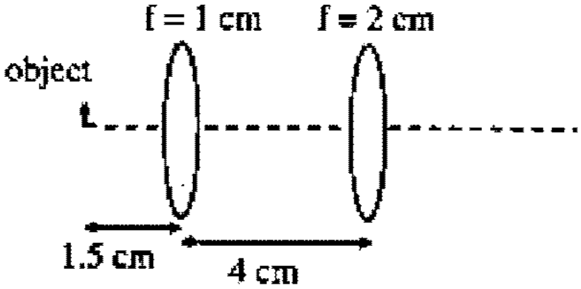
N.B. There is negative marking: 1/2 mark will be deducted for each incorrect answer.

Some information that may be useful in answering this test is given below:

Acceleration due to gravity	$g = 9.8$ m/s^2
Molar gas constant	$R = 8.314$ J/mol K
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J s
Speed of light in vacuum	$c = 3 \times 10^8$ m/s
Gravitational constant	$G = 6.672 \times 10^{-11}$ N m^2/kg^2
Mass of electron	$m_e = 9.1095 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.6725 \times 10^{-27}$ kg
Mass of neutron	$m_n = 1.6742 \times 10^{-27}$ kg
Coulomb constant	$k = 1/(4\pi\epsilon_0) = 8.988 \times 10^9$ N m^2/C^2
Charge of electron	$e = 1.602 \times 10^{-19}$ C
Permittivity of vacuum	$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
Permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}$ T m/A
Energy conversion (eV to J)	1 eV $= 1.602 \times 10^{-19}$ J

PHYSICAL SCIENCES

1. Two thin convex lenses L_1 and L_2 with focal lengths 1 cm and 2 cm respectively are separated by a distance of 4 cm along their axis as shown below:



An object is placed at a distance of 1.5 cm before the first lens. The ratio of the final image size to the object size is

(A) 1

(B) 2

(C) 3

(D) 4

2. An incoherent beam of light strikes a glass plate at normal incidence. It undergoes multiple reflections at the two edges before exiting the plate. The intensity reflection coefficient at each interface is R and the glass has negligible absorption. Neglecting interference effects, the ratio of the total transmitted intensity to the incident intensity is given by

(A) $2(1-R)$

(B) $\dfrac{1-R}{1+R}$

(C) $\dfrac{(1-R)^2}{(1+R)^2}$

(D) $(1-R)^2$

3. A Young's double slit apparatus has a slit separation of 0.5 mm and a screen placed at a distance of 100 cm from the slits. When illuminated by a 632 nm Helium-Neon laser, the position of the first intensity minimum with respect to the central maximum will be

(A) 31.6 μm

(B) 63.2 μm

(C) 316 μm

(D) 632 μm

4. A half-wave plate is an anisotropic optical element which introduces a retardation of $\lambda/2$ in the optical path length for the electric field component parallel to its optical axis, where λ is the wavelength of the incident radiation. If linearly polarized light is incident on a half-wave plate with its polarization at 45° to the optical axis, the transmitted light is (compared to the initial polarization)

(A) linearly polarized and rotated by 45°

(B) linearly polarized and rotated by 90°

(C) left circularly polarized

(D) right circularly polarized

5. The dimensionless fine structure constant α is the ratio of the electron speed in the Bohr model of the hydrogen atom to the speed of light. If e is the electron charge, $\hbar \equiv h/2\pi$, ϵ_0 the permittivity of vacuum, and c the speed of light, then α is given by

(A) $\frac{e^2}{4\pi\epsilon_0\hbar c}$

(B) $\frac{e^2\hbar}{4\pi\epsilon_0 c}$

(C) $\frac{4\pi\epsilon_0 e^2}{\hbar c}$

(D) $\frac{e^2\hbar c}{4\pi\epsilon_0}$

6. A box of resistors is labelled $R \pm x\%$ where R is the mean value and x is the tolerance i.e. the rms deviation given as a percentage of the mean value. If two resistors from a box labelled $10\,\mathrm{K}\Omega \pm 5\%$ are connected in series, the equivalent resistance can be labelled $20\,\mathrm{K}\Omega \pm x\%$ where x is given by

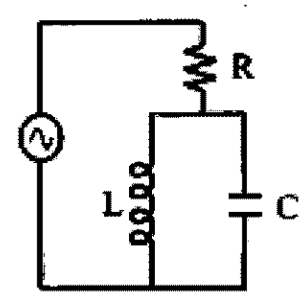
(A) $5/\sqrt{2}$

(B) 5

(C) $5\sqrt{2}$

(D) 10

7. The LCR circuit shown below is driven by an ideal AC voltage source.



At the resonant frequency $f = 1/2\pi\sqrt{LC}$,

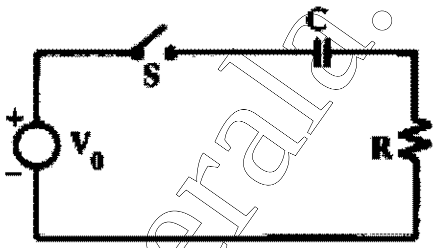
(A) the current through R is infinite

(B) the current through R is zero

(C) the current through L is zero

(D) the current through C is zero

8. An RC circuit is connected to a DC voltage source through a switch as shown below.



The switch is initially open and the capacitor uncharged. At time $t = 0$, the switch is closed. The current through the circuit for $t > 0$ is given by

(A) $\frac{V_0}{R}\exp(-tR/C)$

(B) $\frac{V_0}{R}[1 - \exp(-tR/C)]$

(C) $\frac{V_0}{R}\exp(-t/RC)$

(D) $\frac{V_0}{R}[1 - \exp(-t/RC)]$

9. An optical pyrometer gives an output voltage proportional to the incident radiation intensity. For a black body at 600 K, it reads 1.5 V. If the temperature changes by 2 K, the change in the output signal is

(A) 5 mV

(B) 10 mV

(C) 20 mV

(D) 33 mV

10. A particle of mass M which is at rest in the laboratory, decays into two particles, one of which has a mass $M/2$ while the other is massless. What is the magnitude of the momentum of each of the particles in the laboratory frame?

(A) $3Mc/8$

(B) $Mc/8$

(C) $3Mc/4$

(D) $Mc/4$

11. Given that j is the total angular momentum, which set of quantum numbers is allowed for an electron state in the hydrogen atom?

(A) $n = 3, l = 2, j = 5/2$

(B) $n = 3, l = 2, j = 3$

(C) $n = 3, l = 3, j = 5/2$

(D) $n = 3, l = 3, j = 3$

12. A radioactive sample of 1 gm at 9:00 hrs decays to 0.5 g by 10:00 hrs. Another 0.5 g is now added to it. At what time will 0.1 g of the radioactive material be left?

(A) 12 hrs 19 min

(B) 12 hrs 32 min

(C) 13 hrs 19 min

(D) 13 hrs 32 min

13. The ratio of energies of the Balmer-γ ($n = 5 \to n = 2$) to the Lyman-γ ($n = 4 \to n = 1$) is

(A) 28 : 125

(B) 3 : 5

(C) 3 : 10

(D) 21 : 10

14. What is the ground state energy of six non-interacting electrons in a 2-dimensional, isotropic, simple harmonic oscillator characterized by ω?

(A) $3\hbar\omega$

(B) $5\hbar\omega$

(C) $10\hbar\omega$

(D) $12\hbar\omega$

15. Given that the binding energy per nucleon is 1.1 MeV for deuteron and 7.0 MeV for an α-particle, how much energy is released in deuteron fusion?

(A) 5.9 MeV

(B) 11.8 MeV

(C) 25.8 MeV

(D) 23.6 MeV

16. What is the maximum electron energy in neutron β-decay?

(A) 783 eV

(B) 783 keV

(C) 783 GeV

(D) 783 TeV

17. Which of these is a doubly magic nucleus?

(A) ^{14}C

(B) ^{12}C

(C) ^{14}O

(D) ^{16}O

18. Given that when light is shone on an aluminium target (work function for Aluminium is 4.08 eV) leads to the emission of electrons with maximum kinetic energy of 5.0 eV, what was the frequency of the light?

(A) 1.2×10^{15} Hz

(B) 9.9×10^{14} Hz

(C) 2.2×10^{15} Hz

(D) 2.2×10^{14} Hz

19. Replace in a hydrogen atom, the electron with a muon, which has the same charge, but is 210 times as heavy. The ionization potential of this atom is approximately

(A) 13.6 eV

(B) 2.9 keV

(C) 65.7 meV

(D) 27.2 eV

20. Consider a collection of molecules of an ideal gas at temperature T. The ratio of the rms speed to the most probable speed of the molecule is

(A) $\sqrt{3} : \sqrt{2}$

(B) $\sqrt{3} : \sqrt{8}$

(C) $3 : 8$

(D) $3 : 2$

21. A monoatomic ideal gas of N atoms undergoes isothermal reversible expansion from volume V_1 to V_2. The change in entropy of the gas is

(A) 0

(B) $Nk_B \ln \frac{V_1}{V_2}$

(C) $2Nk_B \ln \frac{V_2}{V_1}$

(D) $Nk_B \ln \frac{V_2}{V_1}$

22. Two vessels separately contain two ideal gases A and B at the same temperature, the pressure of A being twice that of B. Under these conditions, the density of A is found to be one and half times the density of B. The ratio of molecular weights of A and B is

(A) 1/2

(B) 2/3

(C) 3/4

(D) 2

23. A system has two energy levels 0 and $\epsilon > 0$. The energy levels are populated according to the Boltzmann's distribution law. In the limit $T \to \infty$ the entropy per particle is

(A) $+\infty$

(B) $k_B \ln 2$

(C) $2k_B$

(D) $k_B \ln \epsilon^2$

24. Keeping density and sound speed constant, doubling the pressure implies that the ratio of specific heats C_p/C_v

(A) doubles

(B) halves

(C) remains constant

(D) quadruples

25. Considering fermionic particles A and B with chemical potentials μ and $-\mu$ respectively moving in a box at temperature T. The number density of particle A is

(A) greater than that of B

(B) less than that of B

(C) equal to that of B

(D) zero

26. Let z_1 and z_2 be two non-zero complex numbers. If

$$|z_1 + z_2| = |z_1| + |z_2|,$$

which of the following is true

(A) $\mathrm{Re}(z_1\bar{z}_2) < 0$, and $\mathrm{Im}(z_1\bar{z}_2) = 0$

(B) $\mathrm{Re}(z_1\bar{z}_2) > 0$, and $\mathrm{Im}(z_1\bar{z}_2) = 0$

(C) $\mathrm{Re}(z_1\bar{z}_2) > 0$, and $\mathrm{Im}(z_1\bar{z}_2) > 0$

(D) $\mathrm{Re}(z_1\bar{z}_2) < 0$, and $\mathrm{Im}(z_1\bar{z}_2) < 0$

27. Let M be a 3×3 Hermitian matrix which satisfies the matrix equation

$$M^2 - 5M + 6I = 0$$

where I refers to the identity matrix. Which of the following are possible eigenvalues of M

(A) $\{1, 2, 3\}$

(B) $\{2, 2, 3\}$

(C) $\{2, 3, 5\}$

(D) $\{5, 5, 6\}$

28. Given two dice of which one is unbiased while the other is such that the probability of obtaining an even number is 1/4 and the probabilities of obtaining odd numbers are the same. In a simultaneous throw of both the dice what is the probability of obtaining an outcome of 4

(A) $\frac{1}{12}$

(B) $\frac{1}{8}$

(C) $\frac{7}{24}$

(D) $\frac{5}{72}$

29. The number of times that a circle of radius π, centered at the origin intersects the curve $y = \tan x$ is

(A) 2

(B) 4

(C) 6

(D) 8

30. The solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{3}\frac{y}{x} = 0$$

with the condition $y = 2$ when $x = 3$ is given by

(A) $x^2y^3 = 72$

(B) $x^2y^3 = 108$

(C) $x^3y^2 = 108$

(D) $x^3y^2 = 72$

31. A point particle is moving in the (x, y) plane on a trajectory given in polar coordinates by the equation

$$r^2 - 2r\sin\left(\theta + \frac{\pi}{4}\right) - 3 = 0$$

The trajectory of the particle is

(A) a parabola

(B) a straight line

(C) a circle

(D) a hyperbola

32. The n-th term of a sequence is given by

$$t_n = \left(x + \frac{y}{n}\right)^n.$$

where x, y are real numbers. Which of the following is true

(A) If $x = -1$ then $\lim_{n\to\infty} t_n = e^{-x}$

(B) If $x > 1$ then $\lim_{n\to\infty} t_n = e^{y/x}$

(C) If $x = 1$ then $\lim_{n\to\infty} t_n = e^{y}$

(D) If $x < 1$ then $\lim_{n\to\infty} t_n = e^{y/x}$

33. For the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \cdots$$

which of the following is true?

(A) The series is divergent

(B) The series is absolutely convergent

(C) The series converges to $\ln 2$

(D) The series converges to $\pi/2$

34. The normal to the surface given by the equation

$$z = \cos x \cosh y$$

at the point $x = \frac{\pi}{2}$ and $y = 0$ lies in

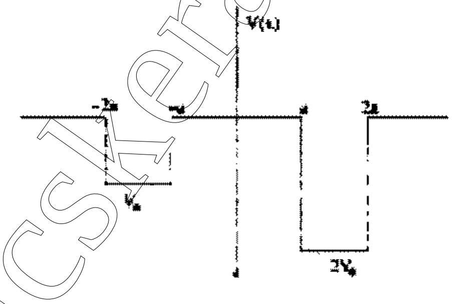
(A) (x, y) plane

(B) (x, z) plane

(C) (y, z) plane

(D) On the plane given by the equation $x + y + z = \frac{\pi}{2} + 1$

35. Consider the wave function of a bound state in the one dimensional potential shown below.



Which of the following is true

(A) $\int_{-a}^{a} |\Psi(x,t)|^2 dx < 1$

(B) $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx$ is time dependent

(C) $\int_{-2a}^{2a} |\Psi(x,t)|^2 dx = 1$

(D) $\int_{-2a}^{-a} |\Psi(x,t)|^2 dx + \int_{a}^{2a} |\Psi(x,t)|^2 dx = 1$

36. Let $\Psi_1(x,t)$ and $\Psi_2(x,t)$ be two distinct solutions of the Schrödinger equation

$$H\Psi(x,t) = -i\frac{\partial\Psi(x,t)}{\partial t}$$

for a given Hamiltonian H. Which of the following can never be a solution of the above equation?

(A) $\Psi_1(x,t) + e^{i\phi}\Psi_2(x,t)$ where ϕ is a constant

(B) $\frac{1}{\sqrt{2}}(\Psi_1(x,t) + \Psi_2(x,t))$

(C) $\frac{1}{\sqrt{2}}(\Psi_1(x,t) - \Psi_2(x,t))$

(D) $|\Psi_1(x,t)|^2 + |\Psi_2(x,t)|^2$

37. A hydrogen beam is prepared in the state

$$\Psi(\vec{r},t) = \frac{\sqrt{3}}{2}\exp\left(i\frac{E_1 t}{\hbar}\right)\Psi_1(\vec{r}) + \frac{1}{2}\exp\left(i\frac{E_2 t}{\hbar}\right)\Psi_2(\vec{r})$$

Where E_1 and E_2 are the energies of the ground state and the first excited state of the hyrogen atom and $\Psi_1(\vec{r})$ and $\Psi_2(\vec{r})$ are their wave functions respectively. The beam is incident on a detector which measures their energy. Let E_0 be the ionization energy of the hydrogen atom. The average energy measured by the detector is given by

(A) $-E_0$

(B) $-\frac{13}{16}E_0$

(C) $-\frac{3}{4}E_0$

(D) $-\frac{15}{16}E_0$

38. The uncertainity in the measurement of the momentum of a particle of mass m confined to move in one dimension at time t is given by

$$\Delta p = \sqrt{\frac{m\hbar}{2t}}.$$

where p is the momentum of the particle. Which of the following values can be a possible uncertainity in measurement of the position x at the same time t.

(A) $\Delta x = \sqrt{\frac{t\hbar}{m}}$

(B) $\Delta x = \frac{1}{2}\sqrt{\frac{t\hbar}{m}}$

(C) $\Delta x = \frac{1}{2}\sqrt{\frac{t\hbar}{2m}}$

(D) $\Delta x = \sqrt{\frac{t\hbar}{3m}}$

39. A plane parallel capacitor is charged up to a surface charge density σ. What is the attractive force per unit area between the two plates?

(A) $\frac{2\sigma^2}{\epsilon_0}$

(B) $\frac{\sigma}{\epsilon_0}$

(C) $\frac{\sigma}{2\epsilon_0}$

(D) $\frac{\sigma^2}{\epsilon_0}$

40. A spherically symmetric charge distribution $\rho(r)$ has zero net charge. For an arbitrarily chosen coordinate system, which of the following statements is true?

(A) Dipole moment is the first non-zero moment

(B) Quadrupole moment is always non-zero

(C) All moments are zero

(D) The monopole and the dipole moments are always zero

41. An ensemble of identical harmonic oscillators with a Hamiltonian

$$H = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

have energies between E and $E + \Delta E$. If the oscillators are initially, uniformly distributed in the phase space then which of the following is false?

(A) More particles are at the maximum amplitude than at the origin

(B) The mean displacement is zero

(C) Mean kinetic energy is equal to the mean potential energy

(D) More particles are at the origin than at maximum amplitude

42. A particle is moved quasi-statically from the point (-3.0) to $(3,0)$, along a path $y = x^2 - 9$, in an external force field given by $\boldsymbol{F} = y\boldsymbol{i} + 3y\boldsymbol{j}$. Given that all physical quantitites are in SI units, the magnitude of the work done on the particle is given by

(A) 36 J

(B) 18 J

(C) 9 J

(D) 0

43. Consider the solution to the one dimensional wave equation:

$$\Psi(x,t) = \Psi_0 \exp i(3x + 147t)$$

where x is in meters and t in seconds. The velocity of the wave is

(A) 7 m/s

(B) -1/7 m/s

(C) -7 m/s

(D) 1/7 m/s

44. A charge is placed at the vertex of a cube. The electric flux through a face that does not touch the charge is given by

(A) $\frac{q}{24\epsilon_0}$

(B) $\frac{q}{6\epsilon_0}$

(C) $\frac{q}{18\epsilon_0}$

(D) 0

45. The electric field due to an unknown charge distribution is given by

$$\boldsymbol{E} = \frac{q}{r^2}\exp(-4r)\,\hat{\boldsymbol{r}}$$

The total integrated charge over all space $\int \rho dV$ is given by

(A) 0

(B) q

(C) $q/4$

(D) q/π

46. A horizontally placed hollow tube has a crossectional area A at the beginning of the tube that gradually tapers off to $A/2$ at the end. An incompressible, ideal fluid of density ρ enters the tube with a velocity v at the beginning of the tube. What is the difference in pressure at the two ends of the tube?

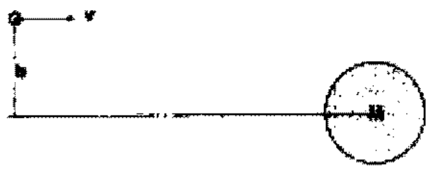
(A) $\rho v^2/2$

(B) $3\rho v^2/2$

(C) $2\rho v^2/2$

(D) $4\rho v^2/2$

47. A small asteroid is approaching a massive star with a speed v, from a very large distance, at an impact parameter b as shown below.



If the mass of the star is M and its radius is R, then what is the minimum value of b such that the asteroid will miss the star?

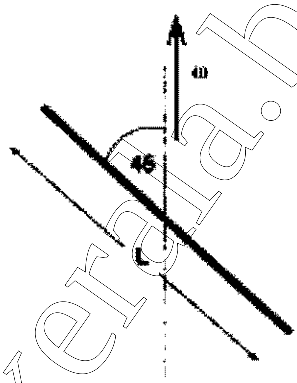
(A) $R\sqrt{1+2GM/v^2R}$

(B) $R\sqrt{1-2GM/v^2R}$

(C) R

(D) $R/2$

48. A thin rod is inclined to the vertical at an angle $\theta = 45$ degree as shown below.



The length of the rod is L and the angular velocity ω is in the vertical direction. What is the magnitude of the angular momentum?

(A) $ML^2\omega/12$

(B) $ML^2\omega/12\sqrt{2}$

(C) $ML^2\omega/6$

(D) $ML^2\omega/24$

49. A point particle at rest is released from the top of a sphere of radius R and slides down frictionlessly under gravity. At what angle from the vertical does it leave the sphere?

(A) $\cos^{-1}(1/3)$

(B) $\cos^{-1}(2/3)$

(C) $\sin^{-1}(1/\sqrt{2})$

(D) $\cos^{-1}(1/\sqrt{2})$

50. A ball of mass m is dropped from a height H on a table with a restitution coefficient $\eta \ll 1$. How much distance does the ball travel before it comes to a halt? (Hint: At each bounce the speed after the bounce is η times the initial speed)

(A) $\left(\frac{1+\eta^2}{1-\eta^2}\right) H$

(B) $\left(\frac{1-\eta^2}{1+\eta^2}\right) H$

(C) $\left(\frac{1+\eta}{1-\eta}\right) H$

(D) $\left(\frac{1-\eta}{1+\eta}\right) H$

*** End of the question paper ***