Resonance
Educating for better tomorrow

SET C

ISAT 2012

Date : 22-04-2012 | **Duration : 3 Hours** | **Max. Marks : 225**

Please read the instructions carefully. You are allotted 5 minutes

INSTRUCTIONS

1. This question booklet contains 75 questions.

2. Each question is provided with multiple options : (A), (B), (C) and (D). Only ONE of them is correct.

3. All questions carry equal mark.
   * Three (3) marks for a correct answer.
   * Minus One (–1) mark for a wrong answer.

4. Multiple answers for a question will be regarded as a wrong answer.

5. A separate OMR answer sheet is provided.

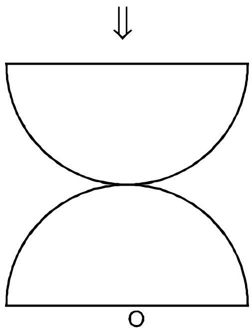
6. Read the instructions on the OMR Answer Sheet Carefully.
   * Write (with pen) your roll number and darken (with HB pencil) the appropriate ovals in Item No. 1 of the OMR answer Sheet.
   * Write your name (with pen) in Item No. 2 of the OMR Answer Sheet.
   * Put your signature (with pen) in Item No. 3 of the OMR Answer Sheet.
   * Question booklets are labelled as SET A, SET B, SET C, SET D or SET E on the right hand top corner. Write (with pen) this label and darken (with HB pencil) the appropriate ovals in Item No. 5 of the OMR Answer sheet)
   * Use HB pencil to darken the ovals corresponding to your answers in Item No. 6 of the OMR Answer Sheet.

7. Space available in the Question Booklet alone should be used for rough work.

8. At the end of the examination, the OMR Answer Sheet should be returned to the invigilator.

PHYSICS

1. Two hemispheres made of glass ($\mu = 1.5$) are kept as shown in the figure. The radius of each hemisphere is R. The image of point O when looked at from the top is at a distance y from O. The value of y is given by :



(A) 0 (B) R/3 (C) 2R/3 (D) 4R/3

Ans. (C)

Sol.

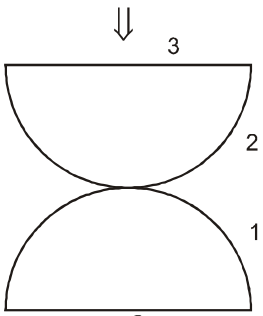


Image will be formed after 3 refractions

For surface 1

$u = -R$, $\mu_1 = 1.5$, $\mu_2 = 1$, radius = $-R$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{-R} = \frac{1-1.5}{-R}$$

$$\frac{1}{v} + \frac{1.5}{R} = \frac{0.5}{R}$$

$$\frac{1}{v} = \frac{0.5-1.5}{R}$$

$$v = \frac{R}{-1} = -R$$

For surface 2

$u = -R$, $\mu_1 = 1$, $\mu_2 = 1.5$, radius = $+R$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-R} = \frac{1.5-1}{R}$$

$$\frac{1.5}{v} + \frac{1}{R} = \frac{0.5}{R}$$

$$\frac{1.5}{v} = \frac{-0.5}{R}$$

$v = -3R$

for surface 3

$u = -4R$, $\mu_1 = 1.5$, $\mu_2 = 1$, $R = \infty$

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

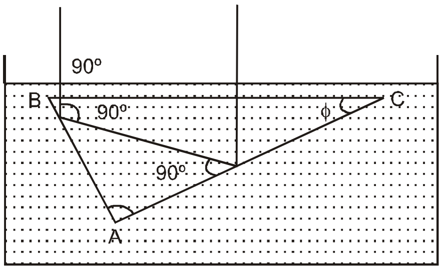
$$\frac{1}{v} = -\frac{1.5}{4R}$$

$$v = \frac{-4R}{1.5}$$

$$v = -\frac{8}{3}R$$

distance of image from point O is $\frac{8}{3}R - 2R = \frac{2}{3}R$

2. A right-angled prism ABC ($\angle C < \angle B$) made of a material of refractive index μ_0 is immersed in a medium of refractive index μ, as shown in the figure. The minimum value of ϕ for which a light ray incident normally on face BC emerges out parallel with the same intensity is :

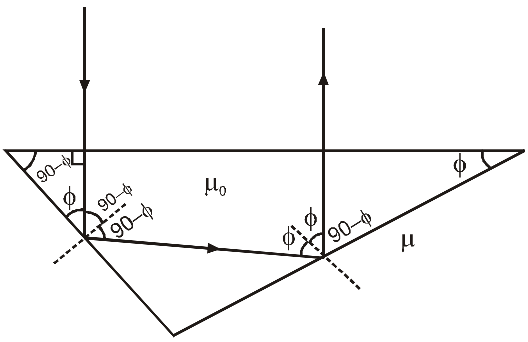


(A) $\frac{1}{2}\sin^{-1}(2\mu/\mu_0)$ (B) $\cos^{-1}(\mu/\mu_0)$

(C) $\tan^{-1}(\mu/\mu_0)$ (D) $\sin^{-1}(\mu/\mu_0)$

Ans. (D)

Sol.



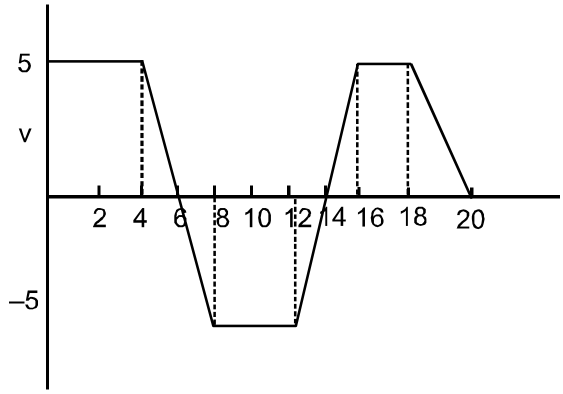
For given condition to be possible both ϕ and 90° – ϕ should be greater than critical angle

$\Rightarrow$ $\mu_0 \sin\phi \geq \mu \sin 90°$

$$\sin\phi \geq \frac{\mu}{\mu_0}$$

$$\phi \geq \sin^{-1}\frac{\mu}{\mu_0}$$

3. Velocity $\vec{v}$ (m/s) versus time graph of a cyclist moving along the x-axis is shown below from time t = 0 to t = 20 seconds.

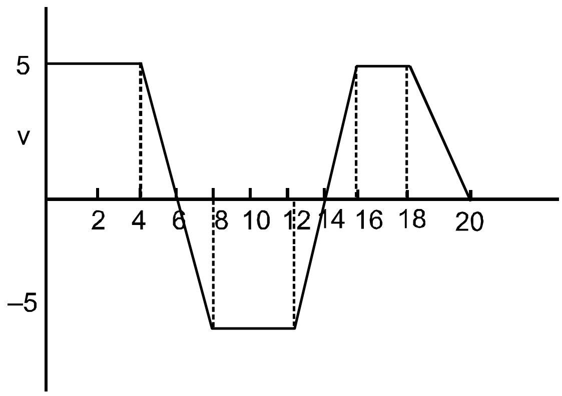


The average velocity and the average speed on the cyclist during this time interval are (in m/s)

(A) $\frac{3}{4}\hat{i}, \frac{15}{4}$ (B) $\frac{3}{4}\hat{i}, \frac{3}{4}$ (C) $0, \frac{15}{4}$ (D) $\frac{15}{4}\hat{i}, \frac{15}{4}$

Ans. **(A)**

Sol.



$$<\vec{v}> = \frac{\vec{s}}{\Delta t} = \frac{\text{Area of vt graph}}{\Delta t}$$

$$= \frac{5\times4+\frac{1}{2}\times5\times2-\frac{1}{2}\times5\times2-5\times4-\frac{1}{2}\times5\times2+\frac{1}{2}\times5\times2+5\times2+\frac{1}{2}\times5\times2}{20}$$

$$= \frac{15}{20} = \frac{3}{4} \text{ m/s } \hat{i}$$

$$<v> = \frac{\text{distance travelled}}{\Delta t}$$

$$= \frac{\text{Area of speed time graph}}{\Delta t}$$

$$= \frac{5\times4+\frac{1}{2}\times5\times2+\frac{1}{2}\times5\times2+5\times4+\frac{1}{2}\times5\times2+\frac{1}{2}\times5\times2+5\times2+\frac{1}{2}\times5\times2}{20}$$

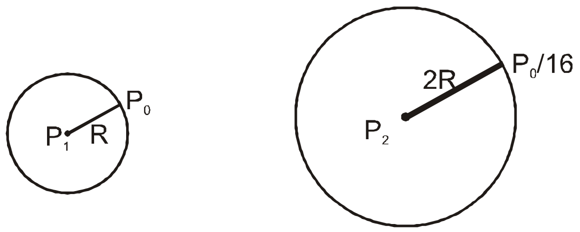
$$= \frac{75}{20} = \frac{15}{4} \text{ m/s}$$

4. In a closed container filled with air at a pressure P_0 there is an air bubble of radius R. As the pressure in the container is reduced isothermally to $P_0/16$; the bubble expands and its radius becomes 2R. The surface tension of the bubble is :

(A) $\frac{1}{24}P_0R$ (B) $\frac{3}{8}P_0R$ (C) $\frac{5}{8}P_0R$ (D) $\frac{7}{8}P_0R$

Ans. (A)

Sol.



$P_1 - P_0 = \frac{4T}{R}$ ...(i)

$P_2 - \frac{P_0}{16} = \frac{4T}{2R}$ ...(ii)

$P_1\frac{4}{3}\pi R^3 = P_2\frac{4}{3}\pi(2R)^3$

$P_1 = 8P_2$ ...(iii)

Put P_1 and P_2 from I and II into III

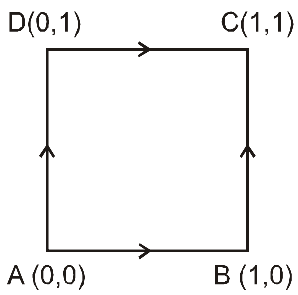
$$\left(\frac{4T}{R}+P_0\right) = 8\left(\frac{4T}{2R}+\frac{P_0}{16}\right)$$

$$\frac{4T}{R}+P_0 = \frac{16T}{R}+\frac{P_0}{2}$$

$$\frac{P_0}{2} = \frac{12T}{R}$$

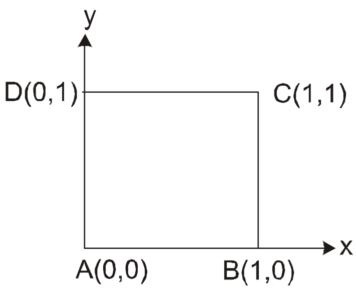
$$T = \frac{P_0R}{24}$$

5. A particle is moving in a force field given by $\vec{F} = y^2\hat{i} - x^2\hat{j}$. Starting from A the particle has to reach C either along ABC or ADC. Let the work done along the two paths be W_1 and W_2 respectively. Then (W_1, W_2) are.



(A) (–1, 1) (B) (–1, 0) (C) (1, 1) (D) (–1, –1)

Ans. (A)

Sol. 

$W_{ABC} = W_{AB} + W_{BC}$

$= \int_{x=0}^{x=1} (y^2\hat{i} - x^2\hat{j}).dx\hat{i} + \int_{y=0}^{y=1} (y^2\hat{i} - x^2\hat{j}).dy\hat{j}$

for AB y = 0 and for BC x = 1

$= \int_{x=0}^{x=1} (0\hat{i} - x^2\hat{j}).dx\hat{i} + \int_{y=0}^{y=1} (y^2\hat{i} - \hat{j}).dy\hat{j}$

$= 0 + -\int_0^1 dy$

$= -1$

$W_{ADC} = W_{AD} + W_{DC}$

$= \int_{y=0}^{y=1} (y^2\hat{i} - x^2\hat{j}) \,.\, dy\hat{j} + \int_{x=0}^{x=1} (y^2\hat{i} - x^2\hat{j}) \,.\, dx\,\hat{i}$

for AD x = 0 and for DC y = 1

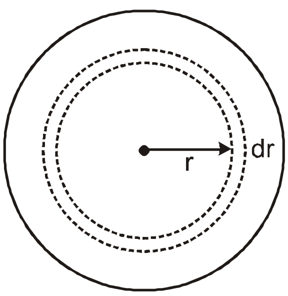
$= \int_0^1 (y^2\hat{i} - 0)dy\hat{j} + \int_0^1 (1\hat{i} - x^2\hat{j})dx\hat{i}$

$= 0 + \int_0^1 dx = 1$

6. The mass density of a dusty planet of radius R is seen to vary from its center as p(r) = $P_0(1 - r/R)$. The ratio of the forces experienced by a test particle of mass m at distances r = R/2 and r = 3R/2 is given by :

(A) 45/16 (B) 45/64 (C) 16/45 (D) 64/45

Ans. (A)

Sol. 

$dV = 4\pi r^2\, dr$

$dM = \rho dV$

$dM = \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr$

$$M = \int_0^x dm$$

$$= \int_0^x \rho_0\left(1-\frac{r}{R}\right)4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^x \left(r^2 dr - \frac{r^3}{R}dr\right)$$

$$= 4\pi\rho_0\left[\frac{x^3}{3}-\frac{x^4}{4R}\right]$$

Case-1 $x = \frac{R}{2}$

$$M = 4\pi\rho_0\left[\frac{R^3}{24}-\frac{R^4}{64R}\right]$$

$$= 4\pi\rho_0 \frac{8R^3-3R^3}{64\times 3} = \frac{5\pi\rho_0 R^3}{48}$$

$$F_1 = \frac{GMm}{(R/2)^2} = \frac{\frac{G5\pi\rho_0 R^3 m}{48}}{\frac{R^2}{4}} = \frac{5G\pi m R\rho_0}{12}$$

Case-2 $x = R$

$$M = 4\pi\rho_0\left[\frac{R^3}{3}-\frac{R^3}{4R}\right]$$

$$= 4\pi\rho_0 \frac{R^3}{12}$$

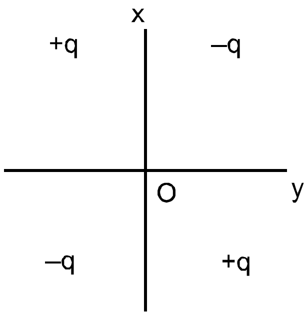
$$= \frac{\pi\rho_0 R^3}{3}$$

$$F_2 = \frac{GMm}{(3R/2)^2}$$

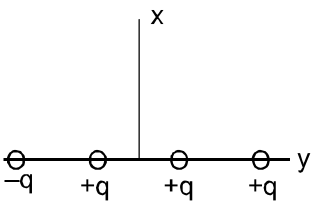
$$= \frac{\frac{G\pi\rho_0 R^3 m}{3}}{\frac{9R^2}{4}} = \frac{4Gm\pi\rho_0 R}{27}$$

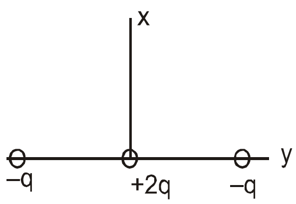
$$\frac{F_1}{F_2} = \frac{5/12}{4/27} = \frac{5\times 27}{4\times 12} = \frac{45}{16}$$

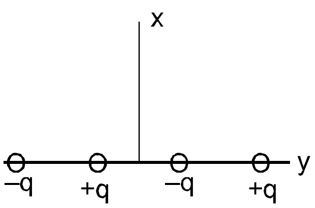
7. Two positive and two negative charges of magnitude q are kept on the x – y plane as shown in the figure. Consider the electric field at some point P on the z-axis very far from the origin O.

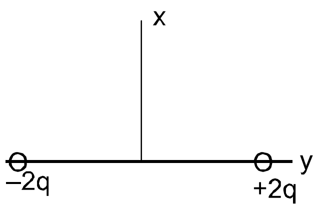


Which of the following configurations will have the electric field closest to the above configuration at P ?

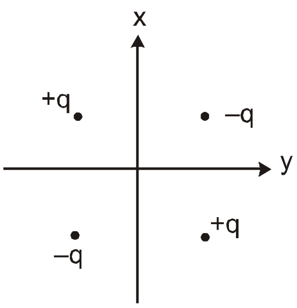
(A)


(B)


(C)


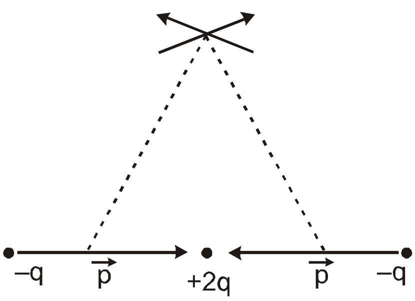
(D)


Ans. **(B)**

Sol.


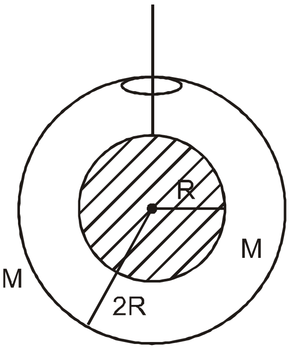
net $\vec{E}$ due to all charges at a point on z- axis is zero.

system B will produce zero $\vec{E}$ at a far point on z axis



This system is behaving like two equal and opposste dipoles. So $\vec{E}$ of both the dipoles will cancle each other.

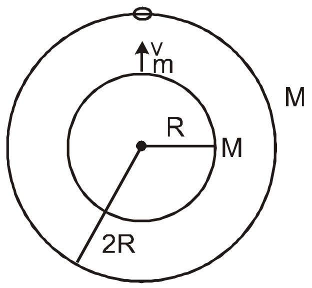
8. A sphere of mass M and radius R is surrounded by a shell of the same mass and radius 2R. A small hole is made in the shell to allow a test mass to pass through when ejected out from the surface of the inner sphere. The escape velocity for the test mass is.



(A) $\sqrt{\frac{3GM}{2R}}$ (B) $\sqrt{\frac{GM}{R}}$ (C) $\sqrt{\frac{2GM}{R}}$ (D) $\sqrt{\frac{3GM}{R}}$

Ans. **(D)**

Sol.



$U_i + KE_i = U_f + KE_f$

$$\frac{-GM}{R}m - \frac{-GM}{2R}m + \frac{1}{2}mv^2 \qquad = 0 + 0$$

$$V^2 = \frac{2GM}{R} + \frac{GM}{R}$$

$$V = \sqrt{\frac{3GM}{R}}$$

9. Two physicists 'A' and 'B' calculate the efficiency of a Carnot engine running between two heat reservoir by measuring their temperatures. 'A' measures the temperature of higher temperature reservoir to (900 ± 10) K and that of the lower reservoir very accurately (almost no error) to be 300 K. 'B' measure the temperature of the high temperature reservoir accurately to be 900 K but that of the cooler one be (300 ± 10) K. Percentage error in the efficiency η_A and η_B b 'A' and 'B' respectively are

(A) $\frac{10}{9}$ and $\frac{10}{3}$ (B) $\frac{10}{9}$ and $\frac{5}{3}$ (C) $\frac{5}{9}$ and $\frac{10}{3}$ (D) $\frac{5}{9}$ and $\frac{5}{3}$

Ans. (D)

Sol. $\eta = \frac{T_2 - T_1}{T_2}$

$\ln \eta = \ln\ (T_2 - T_1) - \ln T_2$

$$\frac{d\eta}{\eta} = \frac{d(T_2 - T_1)}{T_2 - T_1} - \frac{dT_2}{T_2}$$

$= \frac{10}{600} - \frac{10}{900}$

$= \frac{1}{60} - \frac{1}{90}$

$= \frac{3-2}{180}$

$= \frac{1}{180}$

$\frac{d\eta}{\eta} \times 100 = \frac{1}{180} \times 100 = \frac{5}{9}$ for A

$\frac{d\eta}{\eta} = \frac{10}{600} + 0$

$= \frac{1}{60}$

$\frac{d\eta}{\eta} \times 100 = \frac{1}{60} \times 100$

$= \frac{5}{3}$ for B

10. An electron traveling with velocity $\vec{v} = 3\hat{i} + 5\hat{j}$ in an electric field $\vec{E}$ and magnetic field $\vec{B} = 6\hat{i} + 4\hat{j} - \hat{k}$ goes undeflected. Then $\vec{E}$ is

(A) $-5\hat{i} + 3\hat{j} - 18\hat{k}$ (B) $3\hat{i} - 5\hat{j} - 18\hat{k}$ (C) $-3\hat{i} + 5\hat{j} - 18\hat{k}$ (D) $5\hat{i} - 3\hat{j} + 18\hat{k}$

Ans. **(D)**

Sol. $\vec{F}_{net} = 0$

$\vec{F}_E + \vec{F}_M = 0$

$\Rightarrow \vec{F}_E = -\vec{F}_M$

$\Rightarrow q\vec{E} = -q(\vec{V} \times \vec{B})$

$\Rightarrow \vec{E} = -(\vec{V} \times \vec{B})$

$\vec{E} = -[(3\hat{i} + 5\hat{j}) \times (6\hat{i} \times 4\hat{j} - \hat{k})]$

$\vec{E} = 5\hat{i} - 3\hat{j} + 18\hat{k}$

Comprehensive (11 –12)

According to the Bohr model, the energy levels of a Hydrogen atom can be found by making two assumptions: (i) the electrons move in a circular orbit and (ii) the angular momentum of the electron in the n^{th} energy level is quantized to have a value $n\frac{h}{2\pi}$. The levels calculated with a nuclear charge Ze, with a single electron in the orbit are called Hydrogenic levels, Assume that the two electrons in the ground state of a Helium atom occupy the corresponding lowest Hydrogenic level.

11. The minimum repulsive energy between the two electrons would then be

(A) 3.4 eV (B) 6.8 eV (C) 13.6 eV (D) 27.2 eV

Ans. **(D)**

Sol. U = 2E

$$\frac{-K(Ze^2)}{r} = 2\,(-54.4) \qquad \Rightarrow \qquad \frac{Ke^2}{r} = 54.4 \text{ eV}$$

Minimum repulsive energy between the two electrons

$$= \frac{Ke^2}{2r} = \frac{54.4}{2} = 27.2\text{eV}$$

12. If the Hydrogen atom ionization temperature is T, the temperature at which He atoms ionize completely (both electrons having left the atom) would be

(A) 8T (B) 4T (C) 6T (D) 2T

Ans. **(A)**

Sol. $\frac{5}{2}KT = 13.6$ ...(i)

energy required to remove two e^- from He must be greater than 54.4 eV.

$$\frac{3}{2}KT' > 54.4 \qquad \text{...(ii)}$$

From (i) and (ii)

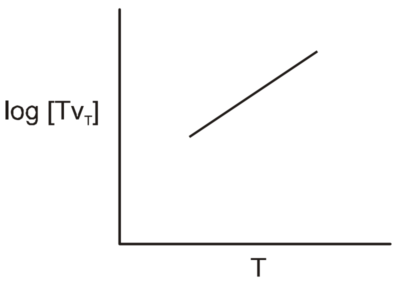
$$\frac{3T'}{5T} > \frac{54.4}{13.6}$$

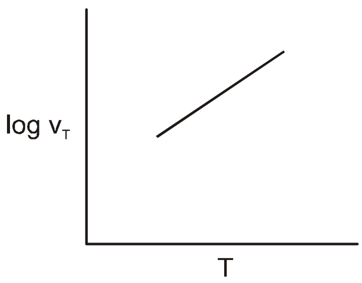
$$T' > \frac{20}{3}T$$

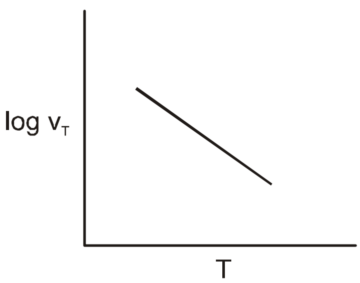
T' > 6.66 T

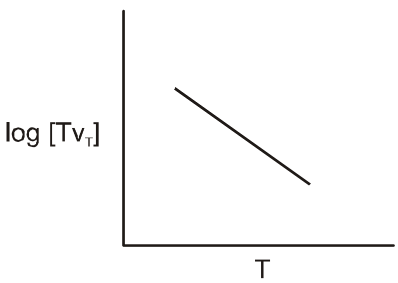
T' can be 8T **Ans. (A)**

13. The coefficient of viscosity of fluid is known to vary with temperature (in a certain range) as $\eta(T) = CT \exp(-T/T_0)$. In an experiment to verify this, the terminal velocity v_t (T) of a spherical pebble dropped in the fluid is measured as a function of the fluid temperature. The correct graph that verifies the behavior is

(A)


(B)


(C)


(D)


Ans. **(A)**

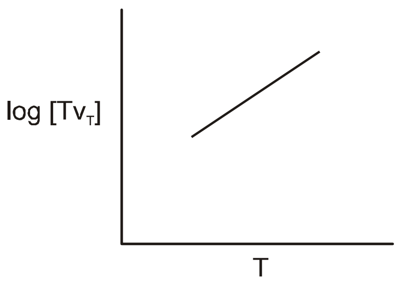
Sol. $V_T = \frac{2}{9}\frac{(\rho_1 - \rho_2)r^2 g}{\eta}$

$V_T = \frac{2}{9}\frac{(\rho_1 - \rho_2)r^2 g}{CT\exp(-T/T_0)}$

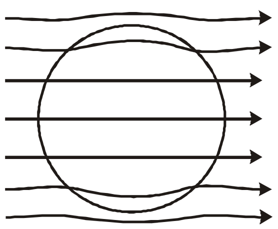
$TV_T = Ke^{(T/T_0)}$

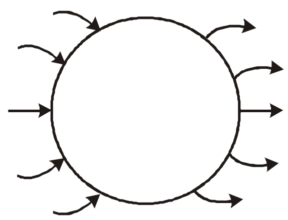
$\log(TV_T) = \frac{T}{T_0} + \log K$

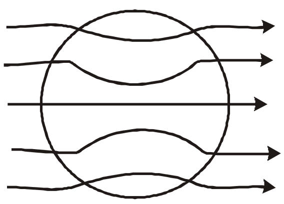
Hence graph should be like this

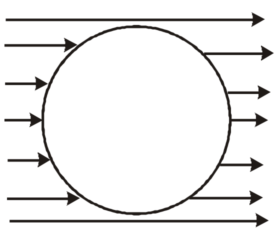


14. A metal sphere is kept in a uniform electric field as shown. What is the correct representation of the field lines?

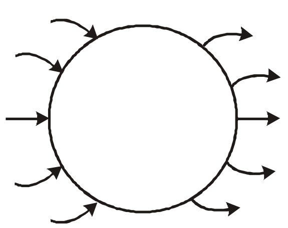
(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. 

Electric field lines are always perpendicular to surface of conductor and absent inside the conductor.

15. A particle moves in a force field of the form $\vec{F} = k\frac{\vec{r}\times\vec{L}}{r^2}$ where $\vec{r}$ is the positive vector from the origin $\vec{L}$ the angular momentum about origin and k is a positive constant. Then,
(A) angular momentum of the particle is conserved
(B) magnitude of angular momentum decreases exponentially, but its direction remains unchanged
(C) magnitude of angular momentum increases exponentially, but its direction remains unchanged
(D) magnitude of angular momentum remains constant, but its direction changes.

Ans. **(B)**

Sol. $\vec{F} = k\frac{\vec{r}\times\vec{L}}{r^2}$

$\vec{r}\times\vec{F} = k\frac{\vec{r}\times(\vec{r}\times\vec{L})}{r^2}$

$\vec{\tau} = k\frac{\vec{r}\,.\,\vec{L} - r^2\vec{L}}{r^2}$

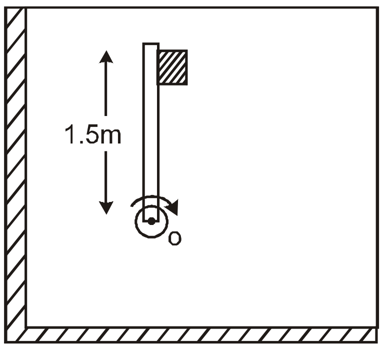
$\frac{d\vec{L}}{dt} = -k\vec{L}$ (as $\vec{r}$ perpendicular $\vec{L}$)

$\Rightarrow \int_{L_0}^{L}\frac{dL}{L} = -k\int_0^t dt$

$\Rightarrow \ell n\left(\frac{L}{L_0}\right) = -kt$

$\Rightarrow L = L_0e^{-kt}$, $k > 0$

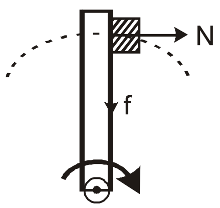
16. A small block is kept on frictionless horizontal table. A wooden plank pivoted at O but otherwise free to rotate pushes the block by applying a constant torque. Initially the angular speed $\omega = 0$. The coefficient of friction between the plank and the block is 0.2. If the block starts slipping 10s later, the angular speed of the block at that instant is



(A) 0.1 rad/s (B) 0.2 rad/s
(C) 0.02 rad/s (D) 0.01 rad/s

Ans. **(C)**

Sol.



$N = ma_t = mr\alpha$ $\omega = 0 + \alpha t$

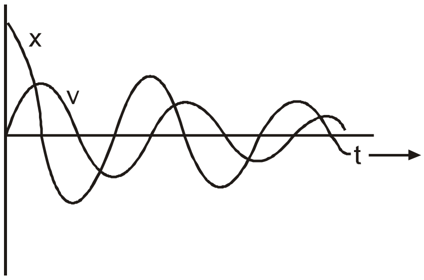
$f = mr\omega^2$ $\alpha = \frac{\omega}{t}$

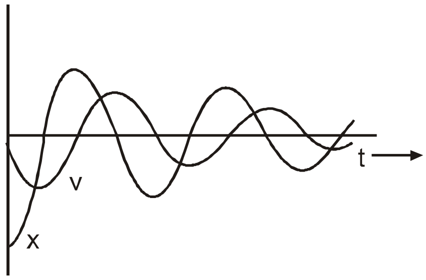
$\Rightarrow \mu\,(mr\alpha) = mr\omega^2$

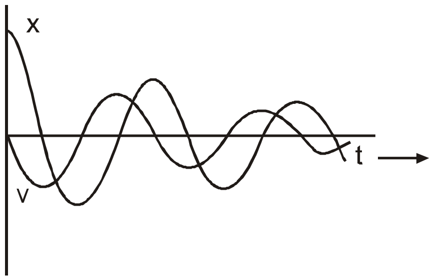
$\Rightarrow \omega = \frac{\mu}{t} = \frac{0.2}{10}$

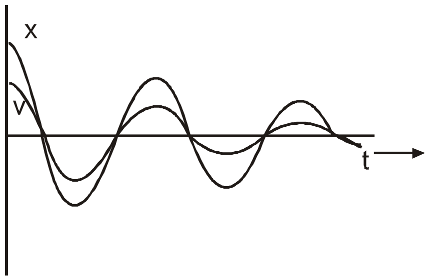
$\omega = 0.02$ rad/ses

17. Consider a damped simple harmonic oscillator given by the equation of motion $\frac{d^2x}{dt^2} = -\mu x - \nu\frac{dx}{dt}$, where ν and μ are both positive constants. The time evolution of its position and velocity are best described by

(A) 

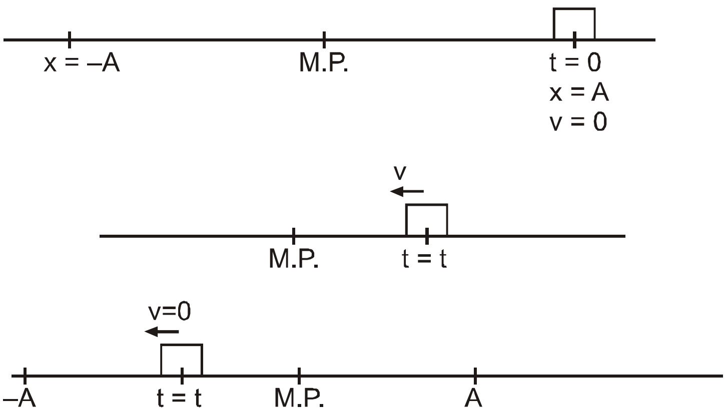
(B) 

(C) 

(D) 

Ans. **(C)**

Sol.



Alternate :

v is the slope of the x-t graph

18. A capacitor made of two parallel circular plates of area A holds a charge Q_0 initially. Suppose that it discharges as $Q(t) = Q_0e^{-\lambda t}$. During the discharge, the ratio $\in_B/\in_E$ of the magnetic field energy $\in_B$ to the electric field energy $\in_E$ between the two plates is given by

(A) $\frac{1}{16}\frac{\in_0\mu_0}{\pi}(A\lambda^2)$

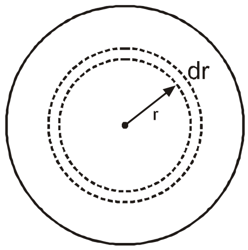
(B) $\frac{1}{8}\frac{\in_0\mu_0}{\pi}(A\lambda^2)$

(C) $\frac{1}{16}\frac{1}{\in_0\mu_0}(A\lambda^2)$

(D) $\frac{1}{8}\frac{1}{\in_0\mu_0}(A\lambda^2)$

Ans. **(B)**

Sol. $\oint \vec{B} \cdot d\vec{s} = \mu_0 \in_0 \dfrac{d\phi}{dt}$



$$B2\pi r = \mu_0 \in_0 \frac{d}{dt}(E\pi r^2)$$

$$B2\pi r = \mu_0 \in_0 \pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \in_0 r}{2} \frac{d}{dt}\left[\frac{Q}{A \in_0}\right]$$

$$B = \frac{\mu_0 \in_0 r}{2} \frac{1}{A \in_0} \frac{dQ}{dt}$$

$$B = \frac{\mu_0 r}{2A} Q_0(-\lambda)e^{-\lambda t}$$

$$B = \frac{-\lambda \mu_0 Q_0}{2A} e^{-\lambda t} r$$

$$U_B = \int_0^R \frac{B^2}{2\mu_0} \, 2\pi r dr \, \ell$$

$$= \frac{\pi \ell}{\mu_0} \int_0^R \frac{\lambda^2 \mu_0^2 Q_0^2}{4A^2} \, e^{-2\lambda t} r^2 r dr$$

$$= \frac{\pi \ell}{\mu_0} \frac{\lambda^2 \mu_0^2 Q_0^2}{4A^2} e^{-2\lambda t} \frac{R^4}{4}$$

$$= \frac{\pi \ell \lambda^2 Q_0^2 \mu_0}{16A^2} R^4 e^{-2\lambda t} = \frac{\ell \lambda^2 Q_0^2 \mu_0}{16\pi} e^{-2\lambda t}$$

$$U_E = \frac{Q^2}{2C}$$

$$= \frac{Q_0^2 e^{-2\lambda t}}{2 \in_0 \pi R^2} \ell$$

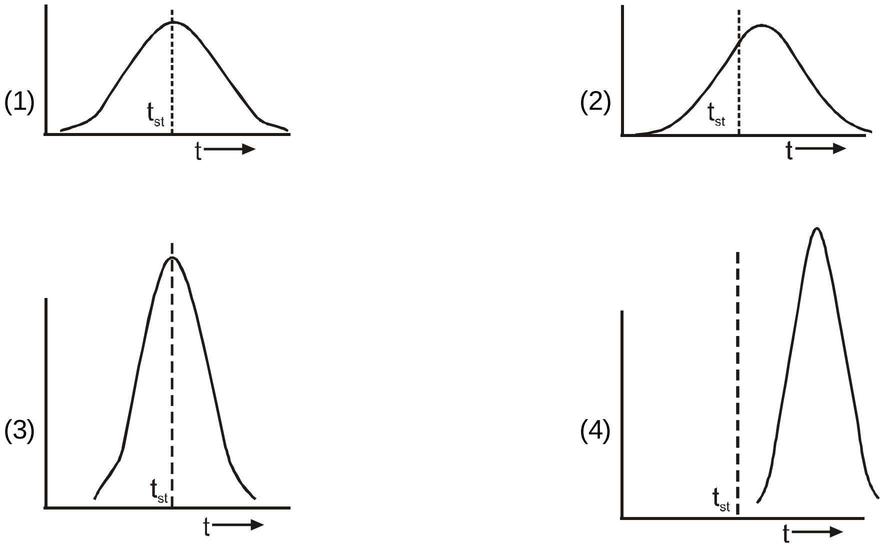
$$= \frac{Q_0^2 \ell e^{-2\lambda t}}{2 \in_0 \pi R^2} e^{-2\lambda t}$$

$$= \frac{\dfrac{\ell \lambda^2 Q_0^2 \mu_0}{16\pi} e^{-2\lambda t}}{\dfrac{Q_0^2 \ell}{2 \in_0 \pi R^2} e^{-2\lambda t}}$$

$$= \frac{\lambda^2 \mu_0 \in_0}{8} R^2$$

$$= \frac{\lambda^2 \mu_0 \in_0 A}{8\pi}$$

19. Four screw gauges are to be calibrated to the standard thickness t_{st} of a wire. Series of measurements were performed by each instrument to obtain a distribution of the measured values as shown in the figure.



Which of the following statements is correct ?

(A) Screw gauge 1 is less precise but more accurate than screw gauge 4

(B) Screw gauge 2 is more precise but less accurate than screw gauge 3

(C) Screw gauge 1 is more precise and more accurate than screw gauge 3

(D) Screw gauge 2 is less precise and less accurate than screw gauge 4

Ans. (A)

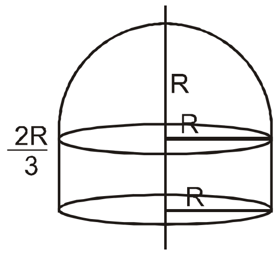
Sol. (1) is more accurate than (2) and (4)

(3) is more accurate than (2) and (4)

(3) is more precise than (1) and (2)

(4) is more precise than (1) and (2)

20. A solid hemisphere of radius R of some material is attached on top of a solid cylinder of radius R and height 2R/3 made of the same material, as shown in the figure. Find the moment of inertia of the solid body about the symmetry axis if its mass is M



(A) $\frac{11}{20}MR^2$ (B) $\frac{6}{5}MR^2$

(C) $\frac{9}{20}MR^2$ (D) $\frac{4}{5}MR^2$

Ans. (C)

Sol. $$\left[\frac{2}{3}\pi R^3 + \pi R^2\left(\frac{2R}{3}\right)\right]\rho = M \qquad \text{...(i)}$$

$$I = \left[\pi R^2\left(\frac{2}{3}R\right)\rho\right]\frac{R^2}{2} + \frac{2}{5}\left(\frac{2}{3}\pi R^3\rho\right)R^2$$

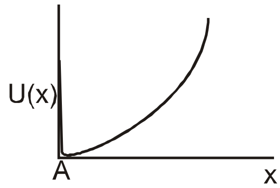
$$= \frac{1}{3}\pi R^5\rho + \frac{4}{15}\pi R^5\rho$$

$$I = \frac{9}{5}\pi R^5\rho \qquad \text{...(ii)}$$

(i) and (ii)

$$I = \frac{9}{20}MR^2$$

21. A model potential between two molecules A and B in a solid is shown in the figure,, where x give the distance of B with respect to A (fixed at the origin). The potential becomes very large ($\to \infty$) a x $\to$ 0 and varies as $\frac{1}{2}kx^2$ for x > 0. Taking the mean kinetic energy and the potential energy of molecule to be equal at all temperatures, the coefficient of linear thermal expansion by kinetic theory is proportional to



(A) $\frac{1}{\sqrt{T}}$ (B) $T^{-3/2}$ (C) $\frac{1}{T}$ (D) independent of T

Ans. **(C)**

Sol. $U = \frac{1}{2}kx^2 = \frac{3}{2}k'T$

$x^2 \propto T$

$x = c\sqrt{T}$

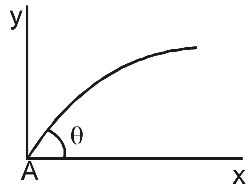
$\frac{dx}{dT} = c\frac{1}{2}\sqrt{T}$

$\frac{dx}{dT} = \frac{x}{\sqrt{T}}\frac{1}{2\sqrt{T}}$

$dx = \frac{xdT}{2T}$

$\alpha = \frac{1}{2T}$

22. A particle of mass m is projected in the vertical plane (taken to be the x-y plane) with speed v at an angle θ in the Earth's gravitational field (taken to be uniform). its angular momentum with respect to the point of projection of time is given by :

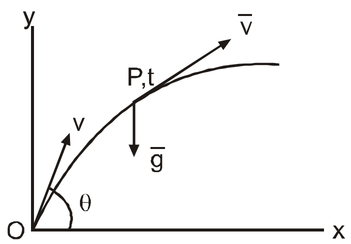


(A) -0.5 mv cosθ gt^2 $\hat{k}$ (B) $+0.5$ mv cosθ gt^2 $\hat{k}$

(C) $+ 0.5$ mv sinθ gt^2 $\hat{k}$ (D) -0.5 mv sinθ gt^2 $\hat{k}$

Ans. **(A)**

Sol.



$\vec{v} = \vec{u} + \vec{g}t$

$\overrightarrow{OP} = \vec{r} = \vec{u}t + \frac{1}{2}\vec{g}t^2$

$\vec{L} = \vec{r} \times m\vec{v} = (\vec{u}t + \frac{1}{2}\vec{g}t^2) \times m(\vec{u} + \vec{g}t)$

$= 0 + (\vec{u} \times m\vec{g})t^2 + \frac{1}{2}m(\vec{g} \times \vec{u})t^2$

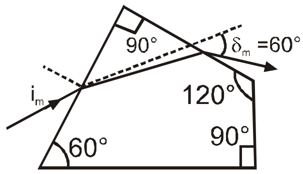
$= m(\vec{u} \times \vec{g})t^2 - \frac{1}{2}m(\vec{u} \times \vec{g})t^2$

$$= \frac{mt^2}{2}\left[v\cos\theta\hat{i} + v\sin\theta\hat{j}\right] \times (-g\hat{j})$$

$$= \frac{-mt^2}{2} vg\cos\theta\hat{k}$$

$$= -0.5\ mv\cos\theta\ gt^2\ \hat{k}$$

23. For the prism shown in the figure, the angle of incidence is adjusted such that the emergent ray has the angle of minimum deviation, $\delta_m = 60°$. The refractive index for the material is



(A) $2(\sqrt{3}-1)$ (B) $\frac{\sqrt{3}+1}{2}$

(C) $\sqrt{2}$ (D) $\frac{3}{2}$

Ans. **(B)**

Sol. $\delta = i + e - A$, $r + r' = A$

For minimum deviation $r = r' = 45°$

& $i = e$ $\therefore$ $60° = 2i - 90°$

$\therefore$ $i = 75°$

$\therefore$ $\mu = \frac{\sin 75°}{\sin 45°} = \frac{\sin(45° + 30°)}{\sin 45°}$

$\mu = \frac{\sqrt{3}+1}{2}$

24. A vertical resonance pipe is filled with water and resonates with a tuning fork at minimum air column length of 30 cm. When air in the pipe is replaced by a homogeneous mixutre of V_1 volume of helium and V_2 volume of Neon gas, the minimum resonance length changes to 42 cm. The ratio $V_1 : V_2$ is close to (molar weight of air $\approx$ 28 gm; $\gamma_{air} = 1.4$

(A) 4 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 2

Ans. No Answer is correct, Correct answer is 1 : 4

Sol. M_g = molecular mass of mixture of gases

$$= \frac{n_1M_1 + n_2M_2}{n_1 + n_2} = \frac{v_1M_1 + v_2M_2}{v_1 + v_2}$$

$M_g = \frac{x4 + 20}{x+1}$ where $x = \frac{v_1}{v_2}$

$$\frac{v_a}{4\ell_1} = \frac{v_g}{4\ell_2} \Rightarrow \frac{v_a}{v_g} = \frac{\ell_1}{\ell_2} = \frac{30}{42} = \frac{5}{7}$$

$$\frac{\frac{1.4RT}{28}}{\frac{5}{3}\frac{RT(x+1)}{4(x+5)}} = \frac{25}{49}$$

$$\frac{1}{20} \times \frac{3\times 4}{5} \times \left(\frac{x+5}{x+1}\right) = \frac{25}{49}$$

$$\frac{x+5}{x+1} = \frac{25\times 25}{3\times 49}$$

625 x + 625 = 147x + 245 × 3

(625 – 147)x = 735 – 625

478 x = 110

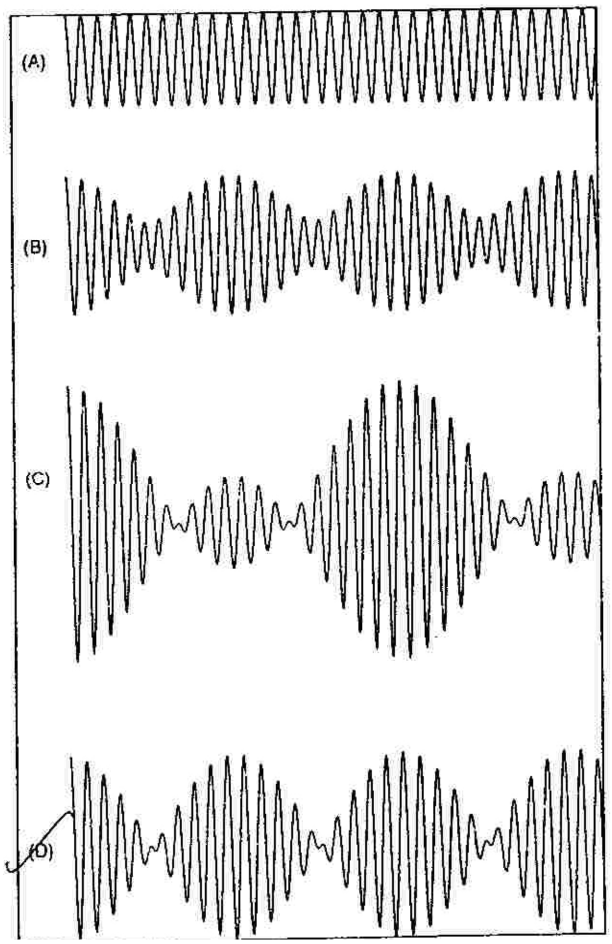
$$x = \frac{110}{478} \approx \frac{1}{4}$$

$$\therefore \quad \frac{v_1}{v_2} \approx \frac{1}{4}$$

Remark $\quad v_2 : v_1 \approx 4 : 1$

So no answer is correct

25. Three sinusoidal oscillations A sin (21t), Asin (20t), and A sin(19t) are superposed. Which of the following figures illustrates correctly the resultant displacement ?



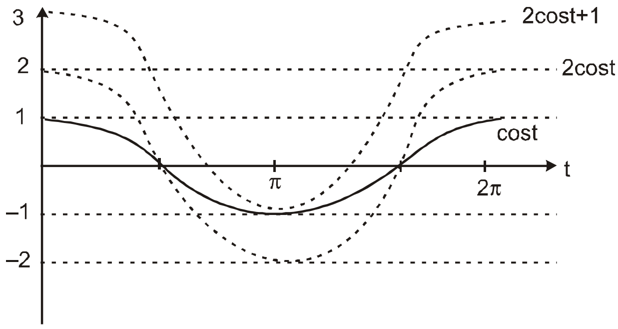
Ans. **(C)**

Sol.: A sin 21 t + A sin 20 t + A sin 19 t

$$2\,A \sin \frac{21t + 19t}{2} \cos \frac{21t - 19t}{2} + A \sin 20\,t$$

2 A sin 20 t cos t + A sin 20 t

A sin 20 t [2 cos t + 1]



2 cos t + 1 has extream values – 1 and 3 so one loop is small and one is big.

CHEMISTRY

26. In the oxidation of sulphite to sulphate using permanganate, the number of protons consumed by each maganese centre is,

(A) 5 (B) 2 (C) 6 (D) 3

Ans. **(D)**

Sol. $6H^+ + 2MnO_4^- + 5SO_3^{2-} \longrightarrow 2Mn^{2+} + 5SO_4^{2-} + 3H_2O$

H^+ consumed by each manganese = 3.

27. The symbols F, H, S, V_m and E° denote Helmboltz free energy, enthalpy, entropy, molar volume and standard electrode potential, respectively. The correct classification of the properties is

(A) F, H, S are intensive ; V_m and E° are extensive

(B) F, H, S extensive ; V_m and E° are intensive

(C) F, H, S and V_m are intensive ; E° are extensive

(D) F, H, S and E° are extensive ; V_m is intensive

Ans. **(B)**

Sol. F,H, S $\rightarrow$ Amount dependent (extensive).

V_m, E° $\rightarrow$ Amount independent (intensive).

28. For bromoalkanes

I $\rightarrow$ the boiling points decrease with increase in branching

II $\rightarrow$ S_N1 reaction rate decreases with increase in branching

III $\rightarrow$ S_N2 reaction rate decreases with increase in branching

IV $\rightarrow$ both S_N1 and S_N2 reaction rate decreases with increase in branching

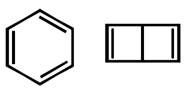
The correct statements are

(A) I and II (B) II and III (C) III and IV (D) I and III

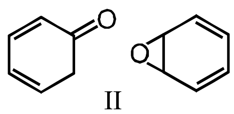
Ans. **(D)**

Sol. With increase in branching, vander walls attraction force decreases and hence the boiling point decreases. Rate of S_N2 reaction decreases with increases in branching.

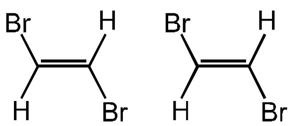
29. The relationship among the following pairs of isomers is



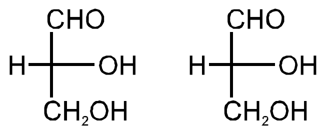
I



II



III



IV

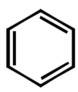
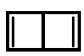
I	A : Constitutional
II	B :Configurational
III	C : Conformational
IV	D : Optical

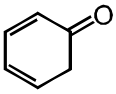
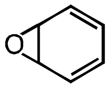
(A) I-A, II-B, III-B, IV-D (B) I-A, II-A, III-B, IV-B

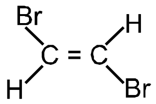
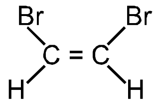
(C) I-B, II-A, III-B, IV-D (D) I-B, II-B, III-A, IV-B

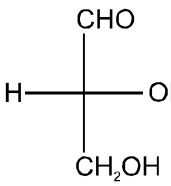
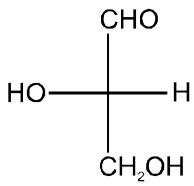
Ans. **(B)**

Sol. Most appropriate choice from given.

I  and  $\rightarrow$ A. Constitutinal

II  and  $\rightarrow$ A. Constitutinal

III  and  $\rightarrow$ B. Configurational.

IV  and  $\rightarrow$ B. Configurational as well as optical.

30. The temperature dependence of the e.m.f. of a standard electrochemical cell is given by

$$E = 1.02 - 4.0 \times 10^{-5}\,(T-20) - 9.0 \times 10^{-7}\,(T-20)^2$$

where, T is in ºC and E is in volts. The temperature coefficient of the e.m.f at 30ºC is

(A) -5.8×10^{-5} (B) -9.0×10^{-7} (C) $+6.3 \times 10^{-5}$ (D) $+9.5 \times 10^{-7}$

Ans. **(A)**

Sol. $E = 1.02 - 4 \times 10^{-5}\,(T-20) - 9 \times 10^{-7}\,(T-20)^2$

$$\frac{dE}{dT} = -4 \times 10^{-5} - 18 \times 10^{-7}\,T + 360 \times 10^{-7}$$

$$= -4 \times 10^{-5} - 18 \times 10^{-7}\,T + 360 \times 10^{-7}$$

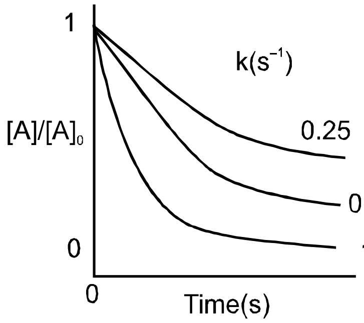
$$= (-400 - 540 + 360) \times 10^{-7}$$

$$= -580 \times 10^{-7} = -5.8 \times 10^{-5}.$$

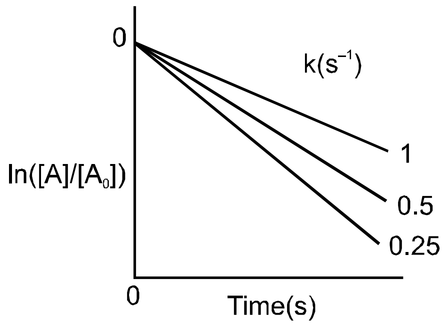
31. For a Ist order reaction of the form

$$A \xrightarrow{\Delta} B$$

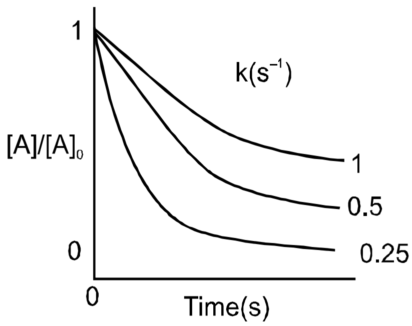
the correct representations are



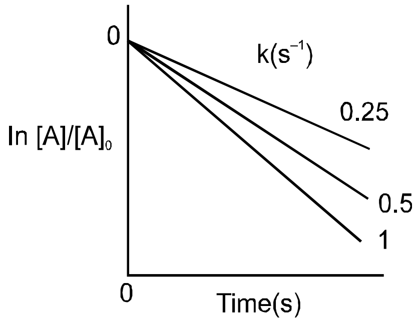
I



II



III



IV

(A) I and II (B) III and IV (C) I and IV (D) II and III

Ans. **(C)**

Sol. $K = \frac{1}{t} \ln \frac{A_0}{A}$

$Kt = \ln \frac{A_0}{A}$

$e^{Kt} = \frac{A_0}{A}$

$e^{-Kt} = \frac{A}{A_0}$

At any time (t), higher the value of K, lower will be the value of $\frac{A}{A_0}$.

32. Match each ore with the correct method used for the extraction of the metal

(a) Cr_2O_3	(i) Reduction with CO after roasting.
(b) Fe_2O_3	(ii) Reduction with Al
(c) Cu_2S	(iii) Self reduction after roasting
(d) ZnS	(iv) Reduction with CO

(A) (a) $\rightarrow$ (i) , (b) $\rightarrow$ (ii) , (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)
(B) (a) $\rightarrow$ (ii) , (b) $\rightarrow$ (iv) , (c) $\rightarrow$ (iii), (d) $\rightarrow$ (i)
(C) (a) $\rightarrow$ (iii) , (b) $\rightarrow$ (i) , (c) $\rightarrow$ (iv), (d) $\rightarrow$ (ii)
(D) (a) $\rightarrow$ (iv) , (b) $\rightarrow$ (ii) , (c) $\rightarrow$ (i), (d) $\rightarrow$ (iii)

Ans. **(B)**

Sol. Low electropositive metal oxides like Fe_2O_3 and ZnO can be reduced by CO, Cr_2O_3 cannot be reduced by CO and C because of there high melting point, some metal sulphides self reduced during roasting like Cu_2S, PbS and HgS.

33. Consider the reaction $2A \rightleftharpoons B$. The equilibrium constant is 1×10^2. If the initial concentration of 'A' is 0.12375 mol/L, the concentration of 'B' in mol/L at equilibrium is

(A) 0.10 (B)0.09 (C) 0.02 (D) 0.05

Ans. **(A)**

Sol.

	2A	$\rightleftharpoons$	B
t=0	0.12375		
t=eq	0.12375–x		$\frac{x}{2}$

$10^2 = \frac{x/2}{(0.12375 - x)^2}$

$10 = \frac{\sqrt{\frac{x}{2}}}{0.12375 - x}$

$1.2375 - 10x = \sqrt{\frac{x}{2}}$

$1.2375 = 10\,x + \sqrt{\frac{x}{2}}$

$x \approx 0.1$

34. The crystal field splitting energy (Δ_0), of
I = $[CoBr_6]^{3-}$, II =$[CoF_6]^{3-}$, III = $[Co(NCS)_6]^{3-}$ and IV = $[Co(CN)_6]^{3-}$
is in the order

(A) I < II < III < IV (B) II < IV < I < III (C) III < I < IV < II (D) IV < III < II < I

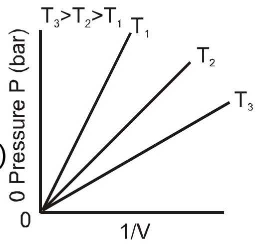
Ans. **(A)**

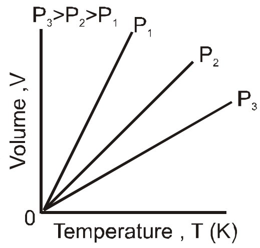
Sol. The crystal field splitting, Δ_0, depends upon the fields produced by the ligand and charge on the metal ion. Some ligands are able to produce strong fields in such a case, the splitting will be large whereas others produce weak fields and consequently result in small splitting of d orbitals. In general, ligands can be arranged in a series in the orders of increasing field strength as given below :

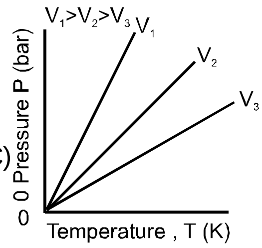
$I^- < Br^- < S^{2-} < Cl^- < NO_3^- < F^- < OH^- < EtOH < C_2O_4^{2-} < H_2O < NCS^- < EDTA < NH_3 < en < NO_2^- < CN^- < CO$

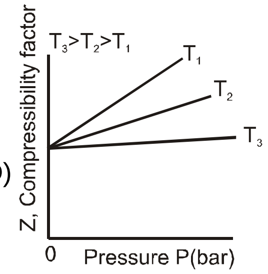
Such a series is termed as spectrochemical series.

35. For a fixed mass of an ideal gas the correct representation is

(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. From PV = nRT

$P \propto T$ at constant V.

36. The number of unpaired electrons present in $[Co(C_2O_4)_3]^{3-}$ and $[CoF_6]^{3-}$, respectively, are

(A) 4 and 4 (B) 4 and 0 (C) 0 and 4 (D) 0 and 0

Ans. **(C)**

Sol. Oxalate ion is strong field ligand with CO^{3+}, F^- is weak field ligand so $[Co(C_2O_4)_3]^{3-}$ is diamagnetic and numbe of unpaired electron is zero but $[CoF_6]^{3-}$ is paramagnetic and number of unpaired electron is four.

37. In the conversion of dinitrogen to hydrazine, the number of electrons and protons involved , respectively are

(A) 2 and 4 (B) 2 and 2 (C) 4 and 2 (D) 4 and 4

Ans. **(D)**

Sol. $N_2 \longrightarrow N_2H_4$.

So, the number of proton and electrons involved 4 and 4.

38. The reduction potentials of M^{2+} /M follow the trend

(A) V < Fe < Ni < Cu (B) Fe < Cu < V < Ni

(C) Cu < V < Ni < Fe (D) Ni < Fe < Cu < V

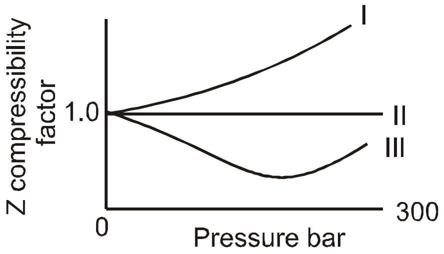
Ans. **(A)**

Sol. In 3d series $E^0_{M^{2+}/M}$ is positive only for Cu (+ 0.34 V). So(A) is correct. Otherwise SRP is negative for remaining all 3d elements.

39. The Boyle temperatures of three gas are given in the table.

Gas	Boyle Temperature (K)
Ethene	735
Oxygen	400
Hydrogen	110

If the compressibility factor was measure at 400 K, the gases are



(A) I-ethene, II-oxygen, III- hydrogen
(B) I-hydrogen, II- ethene, III- oxygen
(C) I - hydrogen, II- oxygen, III- ethene
(D) I- oxygen, II- ethene, III- hydrogen

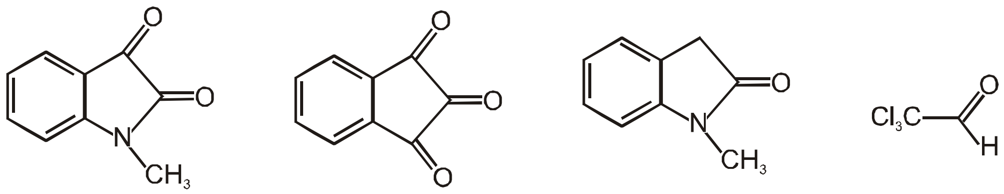
Ans. **(C)**

Sol. If $T_{gas} > T_B$ then gas will show + ve deviation under low pressure region.

T_B $\quad H_2 < O_2 < C_2H_4$

So, $\quad I \rightarrow H_2$, $II \rightarrow O_2$, $III \rightarrow C_2H_4$.

40. The compounds that form stable hydrates are

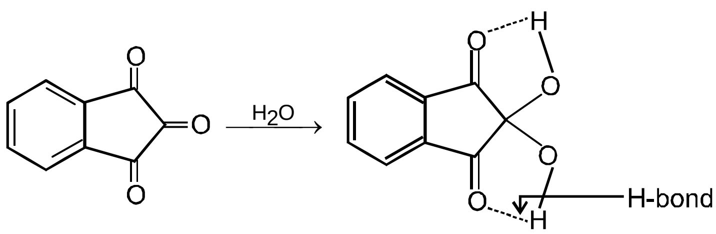


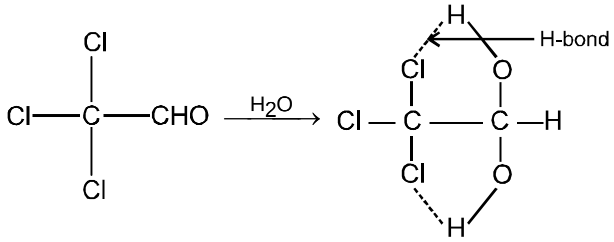
(I) (II) (III) (IV)

(A) I and IV
(B) II and IV
(C) II and III
(D) III and IV

Ans. **(B)**

Sol. Only II and IV form stable hydrates.





41. The major product of the following reaction -

$C_6H_5CH{=}CHCHO + H_2NCONHNH_2 \xrightarrow{H_3O^+}$

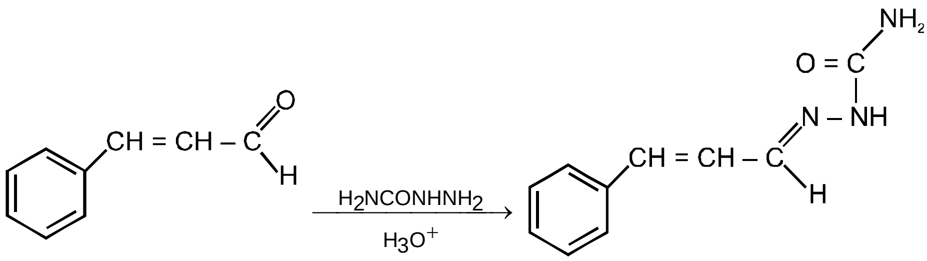
(A) $C_6H_5CH{=}CHCH{=}NNHCONH_2$

(B) $\begin{array}{l} NHCONHNH_2 \\ | \\ C_6H_5CHCH_2CHO \end{array}$

(C) $C_6H_5CH{=}CHCH{=}NCONHNH_2$

(D) $\begin{array}{l} NHNHCONH_2 \\ | \\ C_6H_5CHCH_2CHO \end{array}$

Ans. **(A)**

Sol. 

42. At 100 K, a reaction is 30% complete in 10 minutes, while at 200 K, 30% is complete in 5 minute. The activation energy of the reaction is :

(A) 2050 J (B) 4000 J (C) 3000 J (D) 1150 J

Ans. **(D)**

Sol. $K_{100} = \frac{1}{10} \ln \frac{100}{70}$

$K_{200} = \frac{1}{5} \ln \frac{100}{70}$

$\ln \frac{K_{200}}{K_{100}} = \frac{E_a}{R} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\ln 2 = \frac{E_a}{R} \left(\frac{1}{100} - \frac{1}{200}\right)$

$0.69 = \frac{E_a}{R} \frac{1}{200}$

$E_a = 200 \times 0.69R = 200 \times 0.69 \times \frac{25}{3}\ J = 200 \times 0.23 \times 25\ J = 1150\ J.$

43. The approximate standard enthalpies of formation of methanol and octane are determined to be – 1.5 kJ/mol and – 10.9 kJ/mol respectively. The standard enthalpies of combustion of octane is denoted as ΔH (octan) and that of methanol as ΔH (methanol). The correct statement is :

(A) ΔH (octane) is more negative than ΔH (methanol)

(B) ΔH (octane) is less negative than ΔH (methanol)

(C) ΔH (octane) is equal to ΔH (methanol)

(D) ΔH (octane) + ΔH (methanol) = 0

Ans. **(A)**

Sol. Enthalpy of combustion of C & H is very high & it will dominate ΔH_f^o difference

Datawise :

ΔH (octane) = – 5471 KJ/mol.

ΔH (methanol) = – 726 KJ/mol.

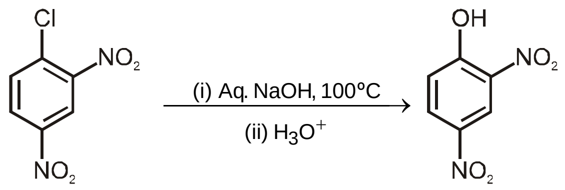
(Refer : Atkins Physical Chemistry).

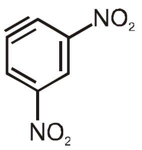
44. The correct statement regarding to functioning of a catalyst is that it
(I) alters the energy levels of the reactants and products
(II) provides an alternate path for climbing the activation energy barrier
(III) makes the reaction thermodynamically feasible
(IV) provides a different mechanism for the reaction
(A) I and II (B) I and III (C) II and IV (D) III and IV

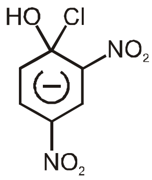
Ans. **(C)**

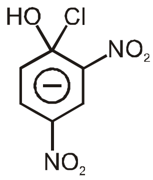
Sol. Catalyst does not affect ΔH of reaction.

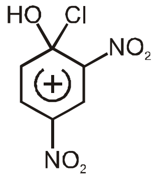
45. The intermediate formed in the following reaction is



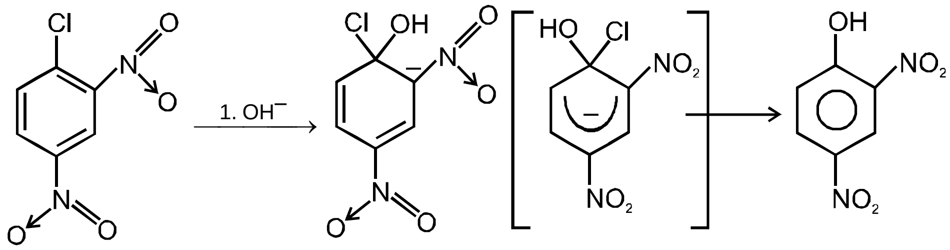
(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. 

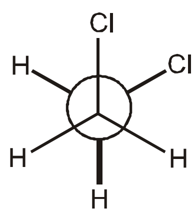
46. The toal number of isomers expected for $[Pt(NCS)_2(en)_2]^{2+}$ (where en = ethylenediamine)
(A) 9 (B) 8 (C) 6 (D) 3

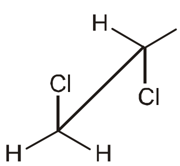
Ans. **(A)**

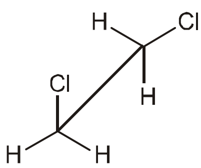
Sol. $\left.\begin{array}{l} Pt(NCS)_2(en)_2 \\ Pt(NCS)(SCN)(en)_2 \\ Pt(SCN)_2(en)_2 \end{array}\right\}$ 1trans, 1 cis and 1 mirror image of cis. So, 3 isomers for each.

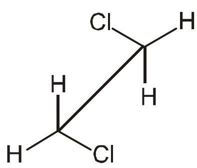
Total isomers = 3 × 3 = 9.

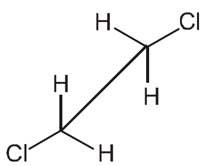
47. For the following Newman projection



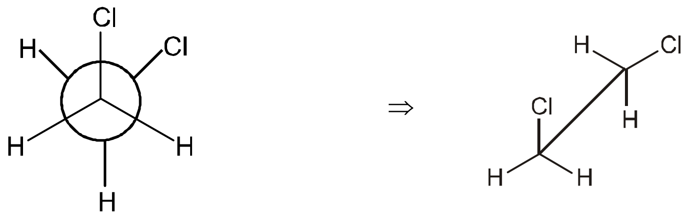
(A) 

(B) 

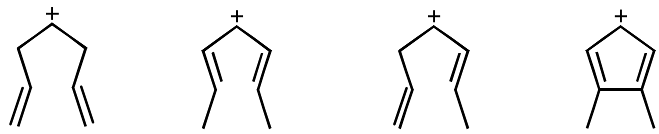
(C) 

(D) 

Ans. **(B)**

Sol. 

48. The stability order of the following carbocations is :



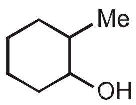
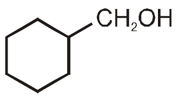
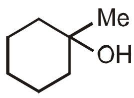
I II III IV

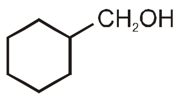
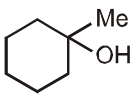
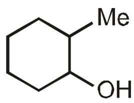
(A) II > IV > III > I (B) IV > II > III > I (C) II > III > I > IV (D) I > III > II > IV

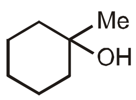
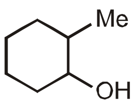
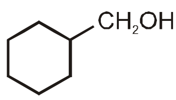
Ans. **(C)**

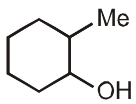
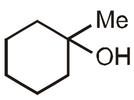
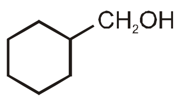
Sol. II is more and IV is antiaromatic.

49. Two isomerism alkenes A and B on hydrogenation in the presence of Pd/C gave methylcyclohexane. Hydroboration and oxidation of A and B formed two isomeric alcohols X and Y, respectively. Both A and B on oxymercuration followed by $NaBH_4$ reduction provided to alcohol Z which was isomeric with X and Y. The alcohols X, Y and Z are :

(A) X =  Y =  Z = 

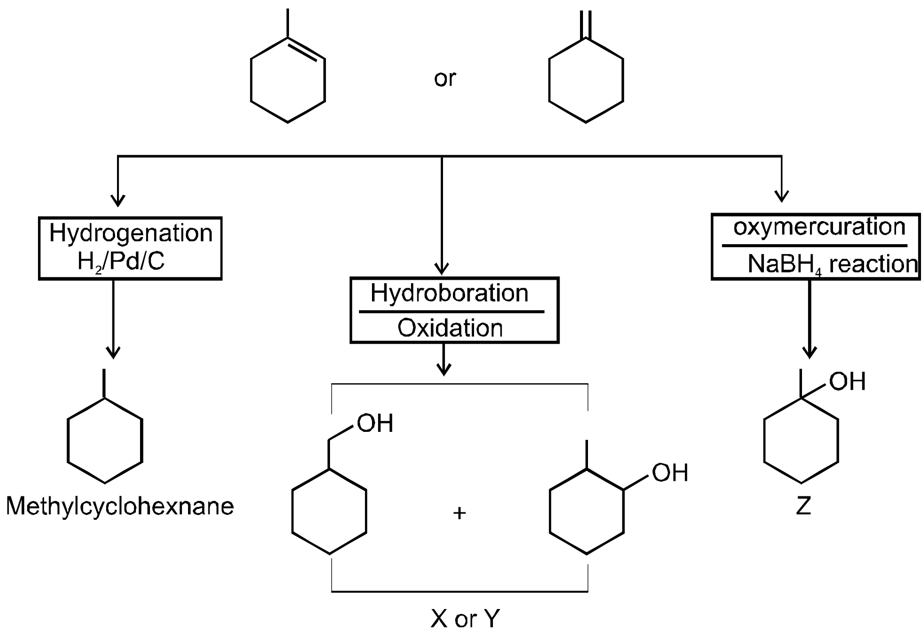
(B) X =  Y =  Z = 

(C) X =  Y =  Z = 

(D) X =  Y =  Z = 

Ans. **(A)**

Sol. Two isomeric alkenes may be



50. Lithium nitrate when heated gives

(A) $LiNO_2$ and O_2 (B) Li_2O, N_2 and O_2 (C) Li_2O, NO_2 and O_2 (D) Li_2O, NO and O_2

Ans. **(C)**

Sol. Lithium nitrate when heated gives lithium oxide, Li_2O, whereas other alkali metal nitrates decompose to give the corresponding nitrite. (M is alkali metals other than Li).

(a) $4LiNO_3 \longrightarrow 2\,Li_2O + 4\,NO_2 + O_2$

(b) $2MNO_3 \xrightleftharpoons{500°C} 2MNO_2 + O_2$

(c) $4MNO_3 \xrightleftharpoons{800°C} 2M_2O + 5O_2 + 2N_2$

(d) $2NO + O_2 \longrightarrow 2NO_2$

MATHEMATICS

51. If $y^x - x^y = 1$, then the value of $\frac{dy}{dx}$ at x = 1

(A) 2(1 – log2) (B) 2(1 + log2) (C) 2 – log 2 (D) 2 + log 2

Sol. **(A)**

$y^x - x^y = 1$

when x = 1, y = 2

Differentiating w.r.t. x

$$y^x\left(\ell n\, y + \frac{x}{y}\frac{dy}{dx}\right) - x^y\left(\frac{dy}{dx}\ell n\, x + \frac{y}{x}\right) = 0$$

Putting x = 1 ; y = 2 in above equation, we get

$$\left.\frac{dy}{dx}\right]_{(1,\,2)} = 2(1 - \log 2)$$

52. The number of distinct real value of λ for which the vector $\lambda^3\vec{i} + \vec{k}$, $\vec{i} - \lambda^3\hat{j}$ and $\vec{i}$ + $(2\lambda - \sin\lambda)$ $\vec{j} - \lambda\vec{k}$ are coplanar is

(A) 0 (B) 1 (C) 2 (D) 3

Sol. **(B)**

For given three vectors to be coplanar

$$\begin{vmatrix} \lambda^3 & 0 & 1 \\ 1 & -\lambda^3 & 0 \\ 1 & 2\lambda - \sin\lambda & -\lambda \end{vmatrix} = 0$$

$\lambda^7 + \lambda^3 + 2\lambda - \sin\lambda = 0$

Clearly $\lambda = 0$, is a solution of above equation

If $f(\lambda) = \lambda^7 + \lambda^3 + 2\lambda - \sin\lambda$

then $f'(\lambda) = 7\lambda^6 + 3\lambda^2 + 2 - \cos\lambda > 0$

i.e. $f(\lambda)$ is strictly increasing.

Hence $\lambda = 0$ is the only value for which the three vectors are coplanar.

53. The set of all 2 × 2 matrices which commute with the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ with respect to matrix multiplication is

(A) $\left\{\begin{bmatrix} p & q \\ r & r \end{bmatrix} : p, q, r \in R\right\}$ (B) $\left\{\begin{bmatrix} p & q \\ q & r \end{bmatrix} : p, q, r \in R\right\}$

(C) $\left\{\begin{bmatrix} p-q & p \\ q & r \end{bmatrix} : p, q, r \in R\right\}$ (D) $\left\{\begin{bmatrix} p & q \\ q & p-q \end{bmatrix} : p, q \in R\right\}$

Sol. **(D)**

For A and B to commute AB = BA

By hit and trial clearly (D) is the solution

54. Let $f : [0, 4] \to R$ be a continuous function such that $|f(x)| \le 2$ for all $x \in [0, 4]$ and $\int_0^4 f(t)\,dt = 2$. Then, for all $x \in [0, 4]$, the value of $\int_0^x f(t)\,dt$ lies in the interval

(A) $[-6 + 2x, 10 - 2x]$
(B) $[-12 + 2x, -7 + 2x]$
(C) $[11 - 2x, 17 + 2x]$
(D) $[-8 - 2x, 6 - 2x]$

Sol. **(A)**

Let $F(x) = \int_0^x f(t)\,dt$ i.e. $F'(x) = f(x)$

Using LMVT on F(x) over the interval [x, 4]

$$F'(c) = \frac{F(4) - F(x)}{4 - x}$$

$$f(c) = \frac{F(4) - F(x)}{4 - x} = \frac{2 - F(x)}{4 - x}$$

Now $|f(x)| \le 2$

$\Rightarrow$ $-2 \le f(x) \le 2$ for all $x \in [0, 4]$

$\Rightarrow$ $-2 \le f(x) \le 2$

$\Rightarrow$ $-2 \le \dfrac{2 - F(x)}{4 - x} \le 2$

$\Rightarrow$ $-8 + 2x \le 2 - F(x) \le 8 - 2x$

$\Rightarrow$ $-10 + 2x \le -F(x) \le 6 - 2x$

$\Rightarrow$ $2x - 6 \le F(x) \le 10 - 2x$

$\Rightarrow$ $2x - 6 \le \int_0^x f(t)\,dt \le 10 - x$

55. Let the line segment joining the centers of the circles $x^2 - 2x + y^2 = 0$ and $x^2 + 4x + y^2 + 8y + 16 = 0$ intersect the circles at points P and Q respectively. Then the equation of the circle with PQ as its diameter is

(A) $5x^2 + 5y^2 - 2x - 16y + 8 = 0$
(B) $5x^2 + 5y^2 - 8x - 24y + 27 = 0$
(C) $5x^2 + 5y^2 + 8x + 24y + 27 = 0$
(D) $5x^2 + 5y^2 + 2x + 16y + 8 = 0$

Sol. **(D)**

Equation of PQ is same as equation of line joining the two centres

Let $C_1 \equiv (1, 0)$; $r_1 = 1$

$C_2 \equiv (-2, -4)$; $r_2 = 2$

$m_{PQ} = m_{C_1C_2} = \dfrac{4}{3} = \tan\theta$ $\Rightarrow$ $\sin\theta = \dfrac{4}{5}$; $\cos\theta = \dfrac{3}{5}$

Equation of PQ in parametric form $\dfrac{x - 1}{3/5} = \dfrac{y - 0}{4/5} = r$

Put r = – 1 and r = – 3 to obtain points P and Q which are $\left(\dfrac{2}{5}, -\dfrac{4}{5}\right)$ and $\left(-\dfrac{4}{5}, -\dfrac{12}{5}\right)$

Using diameter form of circle we get

$5x^2 + 5y^2 + 2x + 16y + 8 = 0$

56. The probability that a randomly selected calculator from a store is of brand r is proportional to r, r = 1, 2,....6. Further, the probability of a calculator of brand r being defective is $\frac{7-r}{21}$, r = 1, 2,......, 6. Then the probability that a calculator randomly selected from the store being defective is

(A) $\frac{8}{63}$ (B) $\frac{13}{63}$ (C) $\frac{55}{63}$ (D) $\frac{50}{63}$

Sol. (A)

Probability that calculator of brand 'r' is selected and is defective = (kr) $\left(\frac{7-r}{21}\right)$ = $\frac{k}{21}$ $(7r - r^2)$

$\Rightarrow$ probability that calculator is defective = $\sum_{r=1}^{6} \frac{k}{21}(7r-r^2) = \frac{8k}{3}$

Let E_r denote the event that calculator of brand r is selected
$P(E_r) = kr$
Since E_r (r = 1, 2, ..., 6) are mutually exclusive and exhaustive events we must have

$$\sum_{r=1}^{6} P(E_r) = 1 \quad \Rightarrow \quad \sum_{r=1}^{6} kr = 1 \quad \Rightarrow \quad k(21) = 1 \quad \Rightarrow \quad k = \frac{1}{21}$$

$\Rightarrow$ Required probability = $\frac{8k}{3} = \frac{8}{63}$

57. Let $f(\theta) = \frac{1}{\tan^9\theta}$ $((1 + \tan\theta)^{10} + (2 + \tan\theta)^{10} + ... + (20 + \tan)^{10}) - 20 \tan\theta$.

The left hand limit of $f(\theta)$ as $\theta \to \frac{\pi}{2}$ is

(A) 1900 (B) 2000 (C) 2100 (D) 2200

Sol. (C)

Put $\tan\theta = \frac{1}{y}$

Given limit reduces to $\lim_{y\to 0}\left(y^9\left[\left(1+\frac{1}{y}\right)^{10}+\left(2+\frac{1}{y}\right)^{10}+.....+\left(20+\frac{1}{y}\right)^{10}\right]-\frac{20}{y}\right)$

$$= \lim_{y\to 0}\frac{(1+y)^{10}+(1+2y)^{10}+(1+3y)^{10}+.....+(1+20y)^{10}-20}{y}$$

$= \lim_{y\to 0} [({}^{10}C_1 + {}^{10}C_1 . 2 + {}^{10}C_1 . 3 + ...+ {}^{10}C_1 . 20) + y (.....) + y^2 (....) + ...] = 10 [1 + 2 + ... + 20] = 2100$

58. Let $P = \begin{bmatrix} \cos\frac{\pi}{9} & \sin\frac{\pi}{9} \\ -\sin\frac{\pi}{9} & \cos\frac{\pi}{9} \end{bmatrix}$ and α, β, γ be nonzero real numbers such that $\alpha P^6 + \beta P^3 + \gamma I$ is the zero matrix.

Then $(\alpha^2 + \beta^2 + \gamma^2)^{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$ is -

(A) π (B) $\frac{\pi}{2}$ (C) 0 (D) 1

Sol. **(D)**

$$P^2 = \begin{bmatrix} \cos\frac{2\pi}{9} & \sin\frac{2\pi}{9} \\ -\sin\frac{2\pi}{9} & \cos\frac{2\pi}{9} \end{bmatrix}$$

$$P^3 = P^2 . P = \begin{bmatrix} \cos\frac{3\pi}{9} & \sin\frac{3\pi}{9} \\ -\sin\frac{3\pi}{9} & \cos\frac{3\pi}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^6 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$\alpha P^6 + \beta P^3 + \gamma I = 0$

i.e. $$\begin{bmatrix} -\frac{\alpha}{2}+\frac{\beta}{2}+\gamma & \frac{\sqrt{3}}{2}\alpha+\frac{\sqrt{3}}{2}\beta \\ -\frac{\sqrt{3}}{2}\alpha-\frac{\sqrt{3}}{2}\beta & -\frac{\alpha}{2}+\frac{\beta}{2}+\gamma \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\alpha = -\beta$ and $\beta = -\gamma \quad \Rightarrow \quad \alpha = \gamma$

$\Rightarrow \quad (\alpha^2 + \beta^2 + \gamma^2)^0 = 1$

59. Let r > 1 and n > 2 be integers. Suppose L and M are the coefficients of $(3r)^{th}$ and $(r + 2)^{th}$ terms respectively in the binomial expansion of $2n(1 + x)^{2n-1}$. If $(r + 2) L = (3r)M$, then n is

(A) 2r – 1 (B) 2r (C) 2r + 1 (D) 2r + 2

Sol. **(C)**

$L = 2n . {}^{2n-1}C_{3r-1}$

$M = 2n . {}^{2n-1}C_{r+1}$

$(r + 2) L = (3r)M$

$(r + 2) . {}^{2n-1}C_{3r-1} = (3r)\, {}^{2n-1}C_{r+1}$

$\frac{1}{3r}\, {}^{2n-1}C_{3r-1} = \frac{1}{r+2}\, {}^{2n-1}C_{r+1}$

${}^{2n}C_{3r} = {}^{2n}C_{r+2} \qquad \left(\text{Using } {}^{n}C_r = \frac{n}{r} . {}^{n-1}C_{r-1}\right)$

$\Rightarrow \quad 3r = 2n - (r + 2) \Rightarrow \quad n = 2r + 1$

60. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and the area of the triangle with adjacent sides parallel to $\vec{a}$ and $\vec{b}$ is 3, then $\vec{a}.\vec{b}$ is .

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$

Sol. **(B)**

$\frac{1}{2}\, |\vec{a}\times\vec{b}| = 3$

$|\vec{a}||\vec{b}| \sin\frac{\pi}{3} = 6 \quad \Rightarrow \quad |\vec{a}||\vec{b}| = \frac{12}{\sqrt{3}}$

$\Rightarrow \quad \vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\frac{\pi}{3} = \frac{12}{\sqrt{3}} . \frac{1}{2} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

61. In the interval $\left[0, \frac{\pi}{2}\right]$, the equation $\cos^2 x - \cos x - x = 0$ has

(A) No solution
(B) Exactly one solution
(C) Exactly two solutions
(D) More than two solutions

Sol. **(B)**

Let $f(x) = \cos^2 x - \cos x - x$

$f(0) = 1 - 1 - 0 = 0$

$$f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$$

$f'(x) = -\sin 2x + \sin x - 1 < 0$ for $x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow$ f(x) is strictly decreasing for $x \in \left(0, \frac{\pi}{2}\right)$

Hence x = 0 is the only solution for $x \in \left[0, \frac{\pi}{2}\right]$

62. For a real number x, let [x] denote the greatest integer less than or equal to x. Let $f : R \to R$ be defined by $f(x) = 2x + [x] + \sin x \cos x$. Then f is

(A) one-one but NOT onto
(B) Onto but NOT one-one
(C) both one-one and onto
(D) NEITHER one-one NOR onto

Sol. **(A)**

If x = a, where 'a' is an integer, then

$$f(a) = 2a + a + \frac{\sin 2a}{2}$$

But $\lim\limits_{h \to 0} f(a - h) = 2a + a - 1 + \frac{\sin 2a}{2}$

Therefore, values between $\lim\limits_{h \to 0} f(a - h)$ and f(a) are never achieved

Since between any two consecutive integers

$f'(x) = 2 + \cos 2x > 0$

i.e. f(x) is strictly increasing

Thus f(x) is NOT ONTO but is definitely ONE-ONE.

63. Let M be a 3 × 3 non-singular matrix with $\det(M) = \alpha$. If $M^{-1}\text{adj}(\text{adj } M) = kI$, then the value of k is

(A) 1
(B) α
(C) α^2
(D) α^3

Sol. **(B)**

M^{-1} adj (adj M) = kI, Pre-multiplying by M

$\Rightarrow$ adj (adj M) = kM

$\Rightarrow$ det (adj (adj M)) = det (kM)

$\Rightarrow$ $(\det M)^{(3-1)^2} = k^3 \det(M)$

$\Rightarrow$ $(\det(M))^4 = k^3 \det(M)$

$\Rightarrow$ $k^3 = (\det M)^3 = \alpha^3$

$\Rightarrow$ $k = \alpha$

64. There are 10 girls and 8 boys in a class room including Mr. Ravi, Ms. Rani and Ms. Radha. A list of speakers consisting of 8 girls and 6 boys has to be prepared. Mr. Ravi refuses to speak if Ms. Rani is a speaker. Ms. Rani refuse to speak if Ms. Radha is a speaker. The number of ways the list can be prepared is

(A) 202
(B) 308
(C) 567
(D) 952

Sol. **(B)**

Case-I Ms. Rani Speaks
Mr. Ravi cannot speak. Thus boys can be selected in ${}^7C_6$ = 7 ways
Als, Mr. Radha cannot speak. Thus girls can be selected in ${}^8C_7$ ways i.e. 8 ways
i.e. The list can be prepared in 7 × 8 = 56 ways

Case-II Ms. Rani does not speak
Boys can be selected in ${}^8C_6$ = 28 ways
Girls can be selected in ${}^9C_8$ = 9 ways
i.e. the list can be prepaped in 28 × 9 = 252 ways
Thus total number of ways = 56 + 252 = 308 ways

65. Let f(x) = log(sinx + cosx), $x \in \left(\frac{-\pi}{4}, \frac{3\pi}{4}\right)$. Then f is strictly increasing in the interval

(A) $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(0, \frac{3\pi}{8}\right)$ (C) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right]$ (D) $\left(\frac{\pi}{2}, \frac{3\pi}{4}\right]$

Sol. **(A)**
$f(x) = \log_e (\sin x + \cos x)$

$$f'(x) = \frac{1}{\sin x + \cos x} (\cos x - \sin x) = \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

for $f'(x) > 0$

$$0 < \frac{\pi}{4} - x < \frac{\pi}{2} \Rightarrow \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

66. The minimum value of $| z_1 - z_2 |$ as z_1 and z_2 vary over the curves $\left|\sqrt{3}(1-2z) + 2i\right| = 2\sqrt{7}$ and $\left|\sqrt{3}(-1-z) - 2i\right| = \left|\sqrt{3}(9-z) + 18i\right|$, respectively, is

(A) $\frac{7\sqrt{7}}{2\sqrt{3}}$ (B) $\frac{5\sqrt{7}}{2\sqrt{3}}$ (C) $\frac{14\sqrt{7}}{\sqrt{3}}$ (D) $\frac{7\sqrt{7}}{5\sqrt{3}}$

Sol. **(B)**

Converting to Cartesian coordinates we have z_1 lying an the circle $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = \frac{7}{3}$ and z_2 lying on the line $3x + 2\sqrt{3}y - 28 = 0$

Shortest distance between z_1 and z_2 will along line perpendicular to $3x + 2\sqrt{3}\,y - 28 = 0$ and passing through the centre $\left(\frac{1}{2}, \frac{1}{\sqrt{3}}\right)$ of the circle .

Distance of $\left(\frac{1}{2}, \frac{1}{\sqrt{3}}\right)$ from line = $\dfrac{\left|3\left(\frac{1}{2}\right) + 2\sqrt{3}\left(\frac{1}{\sqrt{3}}\right) - 28\right|}{\sqrt{3^2 + (2\sqrt{3})^2}} = \dfrac{7\sqrt{7}}{2\sqrt{3}}$

$$\Rightarrow \quad |z_1 - z_2|_{min} = \frac{7\sqrt{7}}{2\sqrt{3}} - (\text{radius}) = \frac{7}{2}\frac{\sqrt{7}}{\sqrt{3}} - \frac{\sqrt{7}}{\sqrt{3}} = \frac{5\sqrt{7}}{2\sqrt{3}}$$

67. The age distribution of 400 persons in a colony having median age 32 is given below:

age (in years)	20-25	25-30	30-35	35-40	40-45	45-50
number of persons	110	x	75	55	y	30

Then x - y is

(A) 10 (B) 20 (C) -10 (D) -20

Sol. **(C)**

Medium = L + $\left(\dfrac{\frac{N}{2}-f}{f}\right)h$

$\ell = 30$; $\dfrac{N}{2} = 135 + \dfrac{x+y}{2}$; f = 75 ; h = 5 ; median = 32

$\Rightarrow$ $x - y = -10$

68. The points with position vectors $\alpha\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}-\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$, $\hat{i}+\hat{j}+\beta\hat{k}$ are coplanar if

(A) $(1-\alpha)(1+\beta) = 0$ (B) $(1-\alpha)(1-\beta) = 0$ (C) $(1+\alpha)(1+\beta) = 0$ (D) $(1+\alpha)(1-\beta) = 0$

Sol. **(A)**

Let A, B, C, D have position vectors $\alpha\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\hat{j}-\hat{k}$, $\hat{i}+2\hat{j}-\hat{k}$, $\hat{i}+\hat{j}+\beta\hat{k}$ respectively.

If A, B, C, D are coplanar then $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ are coplanar

i.e. $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} 1-\alpha & -2 & -2 \\ 1-\alpha & 1 & -2 \\ 1-\alpha & 0 & \beta-1 \end{vmatrix} = 0$$

$(1-\alpha)(1+\beta) = 0$

69. Let $f : (0,1) \to (0,1)$ be a differential function such that $f'(x) \neq 0$ for all $x \in (0,1)$ and $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. Suppose for all x.

$$\lim_{t\to x}\frac{\int_0^t \sqrt{1-(f(s))^2}\,ds - \int_0^x \sqrt{1-(f(s)^2}\,ds}{f(t)-f(x)} = f(x)$$

Then the value of $f\left(\frac{1}{4}\right)$ belongs to

(A) $\{\sqrt{7}, \sqrt{15}\}$ (B) $\left\{\frac{\sqrt{7}}{2}, \frac{\sqrt{15}}{2}\right\}$ (C) $\left\{\frac{\sqrt{7}}{3}, \frac{\sqrt{15}}{3}\right\}$ (D) $\left\{\frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4}\right\}$

Sol. **(D)**

Using L.H. Rule we get

$$\frac{\sqrt{1-(f(x))^2} - 0}{f'(x) - 0} = f(x)$$

If y = f(x), then we have $\frac{\sqrt{1-y^2}}{y'} = y$

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$$

$$\frac{y}{\sqrt{1-y^2}}\, dy = dx$$

Integrating, we get $-\sqrt{1-y^2} = x + c$

Since $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, we get c = – 1

$$\sqrt{1-(f(x))^2} = x - 1$$

Put $x = \frac{1}{4}$, $f(x) = \frac{\sqrt{7}}{4}$

70. A random variable X takes values -1,0,1,2 with probabilities $\frac{1+3p}{4}$, $\frac{1-p}{4}$, $\frac{1+2p}{4}$, $\frac{1-4p}{4}$ respectively where p varies over R. Then the minimum and maximum value of the mean of X are respectively.

(A) $\frac{-7}{4}$ and $\frac{1}{2}$ (B) $\frac{-1}{16}$ and $\frac{5}{16}$ (C) $\frac{-7}{4}$ and $\frac{5}{16}$ (D) $\frac{-1}{16}$ and $\frac{5}{4}$

Sol. **(D)**

$$\left.\begin{array}{l} 0 \le \frac{1+3p}{4} \le 1 \Rightarrow -\frac{1}{3} \le p \le 1 \\ 0 \le \frac{1-p}{4} \le 1 \Rightarrow -3 \le p \le 1 \\ 0 \le \frac{1+2p}{4} \le 1 \Rightarrow -\frac{1}{2} \le p \le \frac{3}{2} \\ 0 \le \frac{1-4p}{4} \le 1 \Rightarrow -\frac{3}{4} \le p \le \frac{1}{4} \end{array}\right\} -\frac{1}{3} \le p \le \frac{1}{4}$$

Mean, $X = (-1)\left(\frac{1+3p}{4}\right) + (0)\left(\frac{1-p}{4}\right) + (1)\left(\frac{1+2p}{4}\right) + (2)\left(\frac{1-4p}{4}\right) = \frac{2-9p}{4}$

Since, $p \in \left[-\frac{1}{3}, \frac{1}{4}\right] \Rightarrow$ Mean $\in \left[-\frac{1}{16}, \frac{5}{4}\right]$

71. Suppose an ellipse and a hyperbola have the same pair of foci on the x- axis with centers at the origin, and that they interest at (2,2). If the eccentricity of the ellipse is $\frac{1}{2}$, then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{7}{4}}$ (B) $\sqrt{\frac{7}{3}}$ (C) $\sqrt{\frac{5}{4}}$ (D) $\sqrt{\frac{5}{3}}$

Sol. **(B)**

Let ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ i.e. $e = \frac{1}{2}$ $\Rightarrow$ $\frac{1}{4} = 1 - \frac{b^2}{a^2}$ $\Rightarrow$ $b^2 = \frac{3}{4}a^2$

$\frac{4}{a^2} + \frac{4}{b^2} = 1$ $\Rightarrow$ $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{4}$ $\Rightarrow$ $\frac{1}{a^2} + \frac{4}{3a^2} = \frac{1}{4}$ $\Rightarrow$ $a = 2\sqrt{\frac{7}{3}}$

$\Rightarrow$ focus $\equiv$ (ae, 0) $\equiv \left(\sqrt{\frac{7}{3}}, 0\right)$

which is focus of the hyperbola as well

Let hyperbola be $\frac{x^2}{A^2}$, $- \frac{y^2}{B^2} = 1$, $AE = \sqrt{\frac{7}{3}}$

$$\frac{1}{A^2} - \frac{1}{B^2} = \frac{1}{4}$$

$$\frac{1}{A^2} - \frac{1}{A^2(E^2-1)} = \frac{1}{4}$$

$$\frac{E^2-2}{E^2-1} = \frac{A^2}{4} = \frac{7}{12E^2} \Rightarrow 24E^4 - 31E^2 + 7 = 0$$

$\Rightarrow$ $E^2 = \frac{31 \pm \sqrt{961-336}}{24} = \frac{56}{24} = \frac{7}{3}$ (Neglecting – ve sign)

$\Rightarrow$ $E = \sqrt{\frac{7}{3}}$

72. The number of solutions of the equations

$$\cos^2(x + \frac{\pi}{6}) + \cos^2 x - 2\cos(x + \frac{\pi}{6})\cos\frac{\pi}{6} = \sin^2\frac{\pi}{6}$$

in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ is

(A) 0 (B) 1 (C) 2 (D) 3

Sol. **(C)**

$$\cos^2\left(x + \frac{\pi}{6}\right) + \cos^2\frac{\pi}{6} - 2\cos\left(x + \frac{\pi}{6}\right)\cos\frac{\pi}{6} + \cos^2 x = \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{6}$$

$$\left(\cos\left(x + \frac{\pi}{6}\right) - \cos\frac{\pi}{6}\right)^2 + \cos^2 x = 1$$

$$\left(\cos\left(x + \frac{\pi}{6}\right) - \cos\frac{\pi}{6}\right)^2 = \sin^2 x$$

Either $\cos\left(x + \frac{\pi}{6}\right) - \cos\frac{\pi}{6} = \sin x$ or $\cos\left(x + \frac{\pi}{6}\right) - \cos\frac{\pi}{6} = -\sin x$

$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2} = \sin x$ $\qquad$ $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x - \frac{\sqrt{3}}{2} = -\sin x$

$\frac{\sqrt{3}}{2}(\cos x - 1) = \frac{3}{2}\sin x$ $\qquad$ $\frac{\sqrt{3}}{2}(\cos x - 1) = -\frac{1}{2}\sin x$

$1 - \cos x + \sqrt{3}\ \sin x = 0$ $\qquad$ $\sqrt{3}\ (1 - \cos x) = \sin x$

$$\sqrt{3}\left(2\sin^2\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = 0$ $\qquad$ $\sin\frac{x}{2} = 0 \qquad \tan\frac{x}{2} = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6}$

$\cos\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$ $\qquad$ $x = 0,\ x = \frac{\pi}{3}$

$x + \frac{\pi}{3} = -\frac{\pi}{3}, \frac{\pi}{3}$

$x = 0$

73. The value of the integral $\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}dx$ is

(A) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} + \frac{5}{12}\log 2 - \frac{1}{4}\log 3$ $\qquad$ (B) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} - \frac{5}{12}\log 2 - \frac{1}{12}\log 3$

(C) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} + \frac{5}{12}\log 2 + \frac{1}{4}\log 3$ $\qquad$ (D) $\frac{\sqrt{2}}{3}\tan^{-1}\sqrt{2} - \frac{5}{12}\log 2 + \frac{1}{12}\log 3$

Sol. **(D)**

$$\int_0^2 \frac{\log(x^2+2)}{(x+2)^2}\,dx = \left|-\frac{1}{x+2}\log(x^2+2)\right|_0^2 + \int_0^2 \frac{2x}{(x+2)(x^2+2)}\,dx$$

$$= \frac{1}{4}\log\left(\frac{2}{3}\right) - \frac{2}{3}\int_0^2 \frac{1}{x+2}\,dx + \frac{1}{3}\int_0^2 \frac{2x}{x^2+2}\,dx + \frac{2}{3}\int_0^2 \frac{1}{x^2+2}\,dx$$

$$= \frac{\sqrt{2}}{3}\tan^{-1}(\sqrt{2}) - \frac{5}{12}\log 2 + \frac{1}{12}\log 3$$

74. Let a be non-zero real number and α, β be the roots of the equations $ax^2 + 5x + 2 = 0$. Then the absolute value of the difference of the roots of equations $a^3(x+5)^2 - 25a(x+5) + 50 = 0$ is

(A) $\left|\alpha^2 - \beta^2\right|$ $\qquad$ (B) $\left|\alpha\beta(\alpha^2 - \beta^2)\right|$ $\qquad$ (C) $\left|\frac{\alpha^2 - \beta^2}{\alpha\beta}\right|$ $\qquad$ (D) $\left|\frac{\alpha^2 - \beta^2}{\alpha^2\beta^2}\right|$

Sol. **(A)**

If α, β be the roots $ax^2 + 5x + 2 = 0$

then $\alpha + \beta = -\frac{5}{a}$; $\alpha\beta = \frac{2}{a}$; $|\alpha - \beta| = \left|\frac{\sqrt{25 - 8a}}{a}\right|$

Similarly, the non-negative difference of roots of the equation $a^3(x+5)^2 - 25a(x+5) + 50 = 0$ is given

by $\left|\frac{\sqrt{625a^2 - 200a^3}}{a^3}\right| = \left|\frac{5\sqrt{25 - 8a}}{a^2}\right| = 5\left|\frac{a(\alpha - \beta)}{a^2}\right| = 5\left|\frac{(\alpha - \beta)}{a}\right| = 5\left|\frac{(\alpha - \beta)(\alpha + \beta)}{-5}\right| = |\alpha^2 - \beta^2|$

75. The equations of the circles which cuts each of the three circles $x^2 + y^2 = 4$, $(x - 1)^2 + y^2 = 4$ and $x^2 + (y - 2)^2 = 4$ orthogonally is

(A) $x^2 + y^2 + x + 2y + 4 = 0$
(B) $x^2 + y^2 + x - 2y + 4 = 0$
(C) $x^2 + y^2 - x - 2y + 4 = 0$
(D) $x^2 + y^2 - x + 2y + 4 = 0$

Sol. **(D)**

Given circles are $x^2 + y^2 - 4 = 0$
$x^2 + y^2 - 2x - 3 = 0$
$x^2 + y^2 - 4y = 0$

Let the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

Using the condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$ with above three circles we get $g = -\frac{1}{2}$; $f = 1$; $c = 4$

$\Rightarrow$ The required circle is $x^2 + y^2 - x + 2y + 4 = 0$