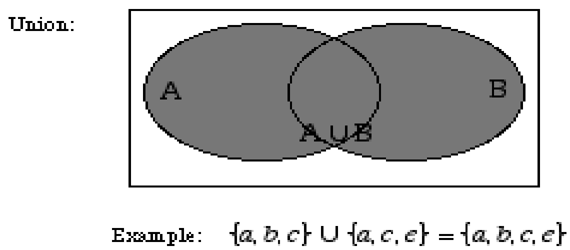
Long answers

Question 1: Write short notes of followings:

a. **Union of Sets**
b. **Intersection of Sets**
c. **Difference of Sets**
d. **Complements of Sets**

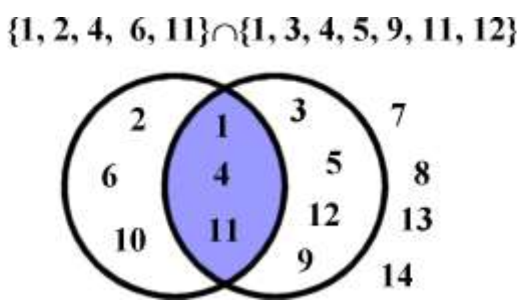
Answer

a. **Union:** The union or join of two sets A and B, written as A ∪ B, is the set of all elements, which are either in A, or in B, or in both. Example: Let A = {a, b, c} and B = {a, c, e}, Then A ∪ B = {a, b, c, e}



b. **Intersection:** The intersection or meet of two sets A and B written as A ∩ B, is the set of all elements that belongs to both A and B. For Example Let A = {1, 2, 4, 6, 11} and B = {1, 3, 4, 5, 9, 11, 12}

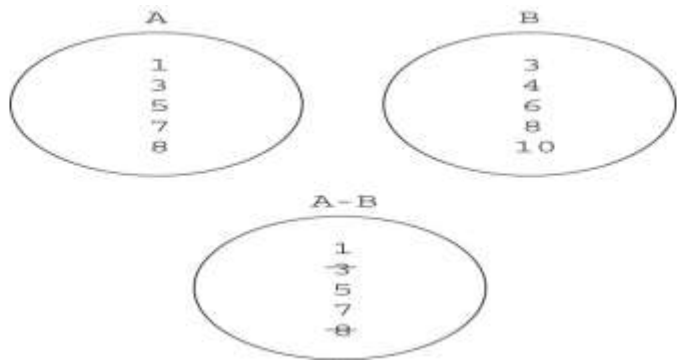
So A ∩ B = {1, 4, 11}



c. **Difference:** The difference of sets A and B is defined as the set A − B consisting of all elements of A that are not also in B. The difference of A and B is not the same as the difference of B and A. The definition of set difference does not imply that A and B have anything in common, nor does it say anything about their relative sizes:

For Example Let A = {1, 3, 5, 7, 8}, B = {3, 4, 6, 8, 10}, then

A − B = {1, 5, 7} and B − A = {4, 6, 10}



d. **Complement:** The complement (or absolute complement) of a set A is the set NOT(A) or A' consisting of all elements not in A. This definition requires the existence of a Universal set U:

For Example, Let A = {a, b, c} U = {a, b, c, ... , z}, then A' = {d, e, f, ..., z}

Questions 2: What are the Laws of Union of Sets?

Answer

1. **Idempotent law**

The literal meaning of the word "idempotent" is "unchanged when multiplied by itself". Following the clue, the union of a set with itself is the set itself. This is an equivalent statement conveying the meaning of "idempotent" in the context of union. Symbolically, $A \cup A = A$

The union set consists of distinct elements and common elements taken once. Between two sets here, all elements are common. The union set consists of all elements of either set.

2. **Identity law**

The algebraic operators like addition and multiplication have defined identities, which does not change the other operand of the operator. For example, if we add "0" to a number, it remains same. Hence, "0" is additive identity. Similarly, "1" is multiplicative identity.

In the case of union, we find that union of a set with empty set does not change the set. Hence, empty set is union identity. $A \cup \varphi = A$

As there is no element in empty set, union has same elements as that in "A".

3. **Law of U**

All sets are subsets of universal set for a given context. We have seen that union with subset results in the set itself. Clearly, union of universal set with its subset will result in the universal set itself.

Santoshsahni833@gmail.com

$A \cup U = U$

4. **Commutative law**

In order to assess whether commutative property holds or not, we consider the example, used earlier. Let the sets be :

A = {1, 2, 3, 4, 5, 6} and B = {4, 5, 6, 7, 8}

Then, $A \cup B$ = { 1,2,3,4,5,6,4,5,6,7,8 } = { 1,2,3,4,5,6,7,8 }

And $B \cup A$ = { 4,5,6,7,8,1,2,3,4,5,6 } = { 1,2,3,4,5,6,7,8 }

Thus, we see that order of operands with respect to the union operator is not differentiating. We can also appreciate this law on Venn diagram, which does not change by changing positions of sets across union operator.

5. **Associative law**

The associative property also holds with respect to union operator. We know that associative property is about changing the place of parentheses as here:

$(A \cup B) \cup C = A \cup (B \cup C)$

Question 3: What are the laws of Intersection of sets?

Answer

1. **Idempotent law:** The intersection of a set with itself is the set itself.

$A \cap A = A$

This is because intersection is a set of common elements. Here, all elements of a set is common with itself. The resulting intersection, therefore, is set itself.

2. **Identity law:** The intersection with universal set yields the set itself. Hence, universal set functions as the identity of the intersection operator.

$A \cap U = A$

It is easy to interpret this law. Only the elements in "A" are common to universal set. Hence, intersection, being the set of common elements, is set "A".

3. **Law of empty set:** Since empty set is element of all other sets, it emerges that intersection of an empty set with any set is an empty set (empty set is only common element between two sets).

$\varphi \cap A = \varphi$

4. **Commutative law:** The order of sets around intersection operator does not change the intersection. Hence, commutative property holds in the case of intersection operation.

A ∩ B = B ∩ A

5. **Associative law:** The associative property holds with respect to intersection operator.

(A ∩ B) ∩ C = A ∩ (B ∩ C)

6. **Distributive law:** The intersection operator(∩) is distributed over union operator (∪) :

A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)

Question 4: $1^2 + 2^2 + 3^2 + \cdots$...........................+ n^2 = 1/6 [n (n+1) (2n+1)]

Prove above statement by mathematical induction.

Solution

Let P (n): $1^2 + 2^2 + 3^2 + \cdots$...........................+ n^2 = 1/6 [n (n+1) (2n+1)]

Step 1: To prove that P (n) is true for smaller values:-

I.e. P (1): 1^2 = 1/6* (2) (3) =1 (True)

I.e. P (2): $1^2 + 2^2$ = 2/6 *(3) (5) =5 (True)

So we can say P (n) is true for smaller values.

Step 2: If P (n) is true, then proving P (m+1) be true.

By Replacing n by m+1:-

I.e. P (m+1): $1^2 + 2^2 + 3^2 + \cdots$...........................+ $m^2 + (m+1)^2$ = 1/6[(m+1)(m+2)(2m+3)]

L. H. S. = $1^2 + 2^2 + 3^2 + \cdots$...........................+ $m^2 + (m+1)^2$

= m/6[(m+1)(2m+1)] + $(m+1)^2$

= (m+1)/6[$2m^2 + m + 6m + 6$]

= 1/6[(m+1)(m+2)(2m+3)]

= R. H. S.

(Hence Proved)

So we can say P (n) is true for all values.

Question 5: Prove 1 + 2 + 3 + ……………………………….+ n = ½(n) (n+1) by Mathematical Induction.

Solution

Let P (n) : 1 + 2 + 3 + ……………………………..+ n = ½(n) (n+1)

Step 1: Prove be true for smaller values:-

P (1): 1 = ½ (1+1) = 1 (True)

P (2): 1 + 2 = (2 * 3) /2 =3 (True)

P (2): 1 + 2 +3 = (3 * 4) /2 =6 (True)

So we can say that P (n) is true for smaller values.

Step 2: If P (m) is true, prove P (m+1) is also true, then we can say P (n) is always true.

Let P (m) be true for all 0 < m <= n

I.e. P (m): 1 +2 +3 + …………………………+ m =1/2 [(m)(m+1)] --- EQ (1)

To prove P (m+1) is also true:-

I.e. P (m+1): 1 + 2 + 3 + …………………………+ m + (m+1) = ½ [(m+1) (m+2)]

L. H. S. = 1 + 2 + 3 + ………………………….. + m + (m+1)

= ½ [m (m+1)] + (m+1) (From EQ 1)

= ½ (m+1)(m+2)

= R. H. S.

(Hence Proved)

Now we can say that P (n) is true for all values.

Question 6: By the help of mathematical induction prove that $2^{3n} - 1$ is divisible by 7.

Solution

Let P (n): $2^{3n} - 1$ is divisible by 7.

Santoshsahni833@gmail.com

Step 1: To Prove P (n) be true for smaller values.

P (1): $2^3 - 1$ = 7 = 7k (True)

P (2): $2^6 - 1$ = 63 = 7k (True)

P (3): $2^9 - 1$ = 511 = 7k (True)

So we can say that, P (n) is true for smaller values.

Step 2: Now P (n) is true, then we will prove P (m+1) be true.

P (m): $2^{3m} - 1$ is divisible by 7.

P (m+1): $2^{3(m+1)} - 1$ is divisible by 7.

$2^{3(m+1)} - 1 = 2^{3m+3} - 1$

$= 2^{3m} * 2^3 - 1$

$= 2^{3m} * 2^3 - 2^3 + 2^3 - 1$

$= 2^3 (2^{3m} - 1) + 7$

= 7k*8 + 7

= 7 (8k + 1)

=7h (Where h= 8k+1)

So P (m+1) is always divisible by 7.

(Hence Proved)

Question 7: A series comprises two numbers. Its mean is 25 G. M. is 20. Find out those two numbers.

Solution:

Let those two numbers be a and b.
So series will be a, b,
Now arithmetic mean of these numbers will be ½ (a + b).
i.e. ½(a + b) =25
Or a + b = 50 ………. EQ (1)
Now geometric mean of these numbers will be $\sqrt{ab}$
i .e. $\sqrt{a * b} = 20$
Or a * b =400 ………. EQ (2)

Since $(a-b)^2 = a^2 + b^2 - 2ab$
Or $(a-b)^2 = (a+b)^2 - 4ab$

On replacing the value from EQ (1) and EQ (2):

$(a-b)^2 = (50)^2 - 4 * 400$
Or $(a-b)^2 = 2500 - 1600$
Or $(a-b)^2 = 900$
a - b = ± 30 ………. EQ (3)

By solving EQ (1) and EQ (3):

Either a = 40 and b = 10 (By Taking a- b =30)
Or b = 40 and a = 10 (By Taking a- b =-30)

So numbers will be 10, 40 in any case.

Questions 8: The sum of three consecutive terms in A.P. is 15 and their product is 105. Find those numbers?

Solution

Let three terms of A. P. be a - d, a, a + d.

So as per question, (a – d) + (a) + (a + d) = 15

So 3a = 15

Or a = 5

Now product of terms, (a - d) * a * (a + d) =105

Or, $a(a^2 - d^2) = 105$

On Putting a = 5

$5(25 - d^2) = 105$

$So,\ d^2 = 4$

$So, d = \pm 2$

So Terms of series will be $5 \pm 2, 5, 5 \mp 2$

So series will be 3, 5, 7 or 7, 5, 3

(Answer)

Question 9: List out all the six properties of Log

Answer

Rule 1: (Log of a Product)

$\log uv = \log u + \log v$

Example: $\log e^3 p^4 = \log e^3 + \log p^4$

Rule 2(Log of a Quotient)

$\log\left(\frac{u}{v}\right) = \log u - \log v$ (Given u > v)

Example: $\log\frac{p^2}{c} = \log p^2 - \log c$

Rule 3 (Log of a power)

$\log u^v = v \log u$

Example: $\log 5^2 = 2\log 5$

Rule 4 (Conversion of Log base)

$\log_b u = (\log_b e)\ (\log_e u)$ (Given u > 0)

Example: $\log_{10} 3 = (\log_{10} 5)\ (\log_5 3)$

Rule 5 (Inversion of base)

$$\log_b u = \frac{1}{\log_u b}$$

Example: $\log_5 10 = 1/\log_{10} 5$

Question 10: If P = {1, 2, 3, 4, 5} and Q= {1, 3, 5, 7, 9}, then find P ∩ Q and $(\boldsymbol{P} - \boldsymbol{Q}) \cup (\mathbf{Q} - \mathbf{P})$

Solution:

Given P = {1, 2, 3, 4, 5}

And Q= {1, 3, 5, 7, 9}

As P ∩ Q contains all elements which are either in P or in Q,

So, P ∩ Q = {1, 3, 5}

(Answer 1)

$P - Q$ Contain all elements which are available in P but not available in Q and similarly $Q - P$ contain all elements which are available in Q but not available in P.

So $P - Q = \{2, 4\}$

And $Q - P = \{7, 9\}$

So $(P - Q) \cup (\mathrm{Q} - \mathrm{P}) = \{2, 4, 7, 9\}$

(Answer 2)

Santoshsahni833@gmail.com

Question 11: Find the sum of the series $2 + 6 + 18 + \ldots\ldots\ldots\ldots.. + 4374$.

Solution:

As $2nd\ term/1st\ term = 3rd\ /2nd\ term = \cdots \ldots\ldots\ldots\ldots$=3

So the given series is in G. P.

So $a\ = 2\ and\ r = 3\ and\ l = 4374$

Since we know that $l = ar^{n-1}$

So, $4374 = 2 * 3^{n-1}$

Or $3^{n-1} = 2187$

Or $3^{n-1} = 3^7$

So $n - 1 = 7$

So $n = 8$

Now $S\ =\ a\ (r^n - 1)/(r - 1)$

Or $S = \frac{2(3^8-1)}{3-1}$

$S = 6551$

(Answer)

Question 12: In a group of athletic team in a certain institute, 21 are in the basketball team, 26 are in hockey team and 29 are in football team. If 14 play hockey and basketball, 12 play football and basketball, 15 play and 8 play all the three games.

i. How many players are there in total?
ii. How many play only football?

Solution:

Given

No. of basketball players (B) = 21,

No. of hockey players (H) = 26,

No. of football players (F) = 29,

No. of basketball and hockey players (H ∩ B) = 14,

No. of basketball and football players (F ∩ B) = 12,

No. of football and hockey players (H ∩ F) = 15,

No. of hockey, football and basketball players (H ∩ F ∩ B) = 8.

Therefore,

i. The no. of total players = $n(B \cup \mathrm{H} \cup \mathrm{F})$
$= n(B) + n(H) + n(F) - n(\mathrm{H} \cap \mathrm{B}) - \mathrm{n}(\mathrm{H} \cap \mathrm{F}) - \mathrm{n}(\mathrm{F} \cap \mathrm{B}) + \mathrm{n}(\mathrm{H} \cap \mathrm{F} \cap \mathrm{B})$

$= 21 + 26 + 29 - 14 - 12 - 15 + 8$

$= 8$

$(Answer)$

ii. No. of Playing Football but not hockey=$12 - 8 = 4$
No. of Playing Football, hockey but not basketball=$15 - 8 = 7$

$No. of\ Playing\ Football$
$= No. of\ Players\ playing\ football - (No. of\ Playing\ Football\ and\ hockey$
$+ No. of\ Playing\ Football, basketball\ and\ hockey)$

$= 29 - (7 + 4 + 8)$

$= 10$

(Answer)

Question 13:

(a) To solve $\log_4 64 - \log_{64} 4 = X$
(b)Can You Prove $2^{log5} - 5^{log2} = 0$

Solution:

(a)$\log_4 64 - \log_{64} 4 = X$

$\log_4 4^3 - \log_{64} 64^{\frac{1}{3}} = X$

$3\ \log_4 4 - \frac{1}{3}\log_{64} 64 = X$

$3 - \frac{1}{3} = X$

$X = \frac{8}{3}$

(Answer)

(b)$2^{log\,5} - 5^{log\,2} = 0$

Or $2^{log\,5} = 5^{log\,2}$

Taking log both sides

$log\ 2^{log\ 5} = log\ 5^{log\ 2}$

$\log 5 * \log 2 = log2 * log5$

L. H. S. = R. H. S.

(Hence Proved)

Question 14: Write Short notes on following:

- **a. Null or Zero Matrix**
- **b. Row Matrix and Column Matrix**
- **c. Square Matrix**
- **d. Unit Matrix**

Solution:

a. Null Matrix: A matrix who's all the elements are zero is called null matrix or zero matrix.

For example: A= $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b. Row Matrix: A matrix that has a single row is called a row matrix. It is the same as a row vector. The order of a row matrix is 1 × m.

For Example: A= $[3 \quad 4]$

Column Matrix: A matrix that has a single column is called a column matrix. It is the same as a column vector. The order of a row matrix is m × 1.

For Example: $A = \begin{matrix} 4 \\ 4 \\ 7 \end{matrix}$

c. Square Matrix: A matrix that has equal numbers of rows and column is called square matrix. Thus every matrix of the order n*n is a square matrix.

For Example: $A = \begin{matrix} 8 & 11 & 8 \\ 3 & -5 & 5 \\ 4 & 5 & 4 \end{matrix}$

d. Unit Matrix: A Square matrix having all its diagonal elements equal to 1 and rest of the elements zero is called a unit matrix. For Example:

For Example: $A = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} or\ B = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$

Question 15: Write Short notes on following:

a. **Diagonal Matrix**
b. **Scalar Matrix**
c. **Triangular Matrix**

Solution:

Diagonal Matrix: A matrix what has all the non-diagonal elements are zero is called a diagonal matrix.

For Example: $A = \begin{matrix} 34 & 0 & 0 \\ 0 & 23 & 0 \\ 0 & 0 & 45 \end{matrix}$

Scalar Matrix: A Diagonal square matrix that has all the diagonal elements are equal is called scalar matrix.

For Example: $A = \begin{matrix} 5 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 25 \end{matrix}$

Triangular Matrix: A Square matrix that has all the elements below its diagonal are zero is called an upper triangular matrix. And Id all the elements below its diagonal are zero it is called a lower triangular matrix. In either case it is simply called a triangular matrix.

For Example: $A = \begin{matrix} 34 & 11 & 43 \\ 0 & 23 & 23 \\ 0 & 0 & 45 \end{matrix}$ Here A is upper triangular matrix.

$B = \begin{matrix} 34 & 0 & 0 \\ 14 & 23 & 0 \\ 45 & 35 & 45 \end{matrix}$ Here B is lower triangular matrix.

Question 16: For What Value of m the equation

(m+1) x^2 + 2 (m+3) x + (m+8) = 0 has equal roots.

Solution:

(m+1) x^2 + 2 (m+3) x + (m+8) = 0

By comparing this equation by $Ax^2 + Bx + C = 0$

$A = (m + 1)$

$B = 2(m + 3)$

$C = (m + 8)$

Let roots of these equations be x and y.

If roots are equal then $B^2 = 4AC$

Or $[2(m+3)]^2 = 4(m+1) * (m+8)$

Or $4(m^2+9+6m) = 4(m+1) * (m+8)$

Or $(m^2+9+6m) = (m+1) * (m+8)$

Or $(m^2+9+6m) = (m^2+9m+8)$

Or $3m = 1$

Or $m = \frac{1}{3}$

So we can say that for $m = 1/3$ above equation has equal roots.