M.Sc. (Previous) – Mathematics

Paper – I : Advanced Algebra

Unit – 1. Direct products of groups (external and internal).

Unit – 2. Isomorphism theorems; Conjugacy and the class equation of a group.

Unit – 3. Commutators, Derived subgroups, Solvable groups, Subnormal series and Refinement theorem, Composition series and Jordan-Holder Theorem.

Unit – 4. Euclidean rings : Division in commutative rings, Units, Associates and Prime elements, Unique factorization domain.

Unit – 5. Modules, Submodules, Quotient modules, Direct sums, Module homomorphisms, Generation of modules, Cyclic modules.

Unit – 6. Linear transformation of vector spaces, Dual spaces, Dual basis and their properties, Dual maps.

Unit – 7. Basic theory of field extensions, Simple field extension, Algebraic and Transcendental extensions.

Unit – 8. Splitting fields, Normal extension, Separable and Inseparable extensions, Automorphism of extensions.

Unit – 9. Galois thoery : Galois extension and Galois group, Fundamental theorem of Galois theory, Extensions by radicals and solvability, Insolvability of the quintic.

Unit – 10. Matrices of linear maps of composite maps and of dual maps.

Unit – 11. Rank and Nullity of linear maps and matrices, Invertible matrices, Eigen values and Eigen vectors, Change of basis and similar matrices.

Unit – 12. Determinants of matrices and their properties, Existence and Uniqueness of determinants, Characteristic polynomial and Eigen values.

Unit – 13. Real Inner product space, Schwartz's inequality, Orthogonality, Pythagoras theorem, Gram-Schmidt orthogonalization.

Unit – 14. Bessel's inequality, Parseval's identity, Direct Sum, Adjoint of a linear map, Self-adjoint linear maps and matrices.

Unit – 15. Orthogonal linear transformation and matrices, Principal axis theorem.

Paper – II: Real Analysis and Topology

Unit 1: Algebra and algebras of sets, Algebras generated by a class of subsets, Borel sets.

Unit 2: Lebesgue measure of sets of real numbers. Measurability and Measure of a set, Existence of Non-measurable sets.

Unit 3: Measurable functions, Realization of non-negative measurable function as limit of an increasing sequence of simple functions, Structure of measurable functions. Convergence in measure, Egoroff's theorem.

Unit 4: Weierstrass's theorem on the approximation of continuous function by polynomials, Lebesgue integral of bounded measurable functions, Lebesgue theorem on the passage to the limit under the integral sign for bounded measurable functions.

Unit 5: Summable functions, Space of square summable functions.

Unit 6: Fourier series and coefficients, Parseval's identity, Riesz-Fisher Theorem.

Unit 7: L^p-spaces, Holder-Minkowski inequalities. Completeness of L^p-spaces.

Unit 8: Topological spaces, Subspaces, Open sets, Closed sets, Neighbourhood system.

Unit 9: Bases and sub-bases, Continuous mapping and Homeomorphism.

Unit 10: Separation axioms (To, T_1, T_2, T_3, T_4).

Unit 11: Compact and locally compact spaces.

Unit 12: Tychonoff's one point compactification.

Unit 13: Connected and Locally connected spaces.

Unit 14: Product and Quotient spaces.

Unit 15: Nets, Filters.

Paper – III: Differential Equations, Calculus of Variations & Special Functions:

Unit 1: Non-linear ordinary differential equations of particular forms, Riccati's equation –General solution and the solution when one, two or three particular solutions are known.

Unit 2: Total Differential equations.

Unit 3: Partial differential equations of second order with variable co-efficients-Monge's method.

Unit 4: Classification of linear partial differential equation of second order, Cauchy's problem, Method of separation of variables.

Unit 5: Laplace, Wave and Diffusion equations, Canonical forms.

Unit 6: Linear homogeneous boundary value problems. Eigen values and eigen functions. Sturm-Liouville boundary value problems, Orthogonality of eigen functions, Reality of eigen values.

Unit 7: Calculus of variation – Functionals, Variation of a functional and its properties, Variational problems with fixed boundaries, Euler's equation, Extremals, Functional dependent on several unknown functions and their first order derivatives.

Unit 8: Functionals dependent on higher order derivatives, Functionals dependent on the function of more than one independent variable, Variational problems in parametric form.

Unit9: Series solution of a second order linear differential equation near a regular/singular point (Method of Frobenius) with special reference to Gauss hypergeometric equation and Legendre's equation.

Unit 10: Gauss hypergeometric function and its properties, Integral representation.

Unit 11: Linear transformation formulas, Contiguous function relations, Differentiation formulae, Linear relation between the solutions of Gauss hypergeometric equation, Kummer's confluent hypergeometric function and its properties, Integral representation, Kummer's first transformation.

Unit 12: Legendre polynomials and functions $P_n(x)$ and $Q_n(x)$.

Unit 13: Bessel functions $J_n(x)$.

Unit 14: Hermite polynomials $H_n(x)$.

Unit 15: Laguerre and Associated Laguerre polynomials.

Paper- IV: Differential Geometry and Tensors

Unit 1: Space curves, Tangent, Contact of curve and surface, Osculating plane.

Unit 2: Principal normal and Binormal, Curvature, Torsion, Serret-Frenet's formulae, Osculating circle and Osculating sphere.

Unit 3: Existence and Uniqueness theorems, Bertrand curves, Involute, Evolutes, Conoids, Inflexional tangents, Singular points, Indicatrix.

Unit 4: Envelope, Edge of regression, Ruled surface, Developable surface, Tangent plane to a ruled surface.

Unit 5: Necessary and sufficient condition that a surface $\zeta = f(\xi,\eta)$ should represent a developable surface, Metric of a surface.

Unit 6: First, second and third fundamental forms, Fundamental magnitudes of some important surfaces, Orthogonal trajectories, Normal curvature.

Unit 7: Meunier's theorem, Principal directions and Principal curvatures, First curvature, Mean curvature, Gaussion curvature, Umbilics. Radius of curvature of any normal section at an umbilic on $z = f(x,y)$, Radius of curvature of a given section through any point on $z = f(x,y)$, Lines of curvature.

Unit 8: Principal radii, Relation between fundamental forms, Asymptotic lines, Differential equation of an asymptotic line, Curvature and Torsion of an asymptotic line.

Unit 9: Geodesics, Differential equation of a geodesic, Single differential equation of a geodesic, Geodesic on a surface of revolution, Geodesic Curvature and Torsion, Gauss-Bonnet Theorem.

Unit 10: Gauss's formulae, Gauss's characteristic equation, Weingarten equations, Mainardi-Codazzi equations. Fundamental existence theorem for surfaces, Parallel surfaces, Gaussian and mean curvature for a parallel surface, Bonnet's theorem on parallel surfaces.

Unit 11: Tensor Analysis. Kronecker delta, Contravariant and Covariant tensors, Symmetric tensors, Quotient law of tensors, Relative tensor.

Unit 12: Riemannian space. Metric tensor, Indicator, Permutation symbols and Permutation tensors, Christoffel symbols and their properties.

Unit 13: Covariant differentiation of tensors, Ricci's theorem, Intrinsic derivative.

Unit 14: Geodesics, Differential equation of geodesic, Geodesic coordinates, Field of parallel vectors.

Unit 15: Reimann-Christoffel tensor and its properties, Covariant curvature tensor, Einstein space, Bianchi's identity, Einstein tensor, Flate space, Isotropic point, Schur's theorem.

Paper – V: Mechanics

Unit – 1. D'Alembert's principle, General equation of motion of rigid body, Motion of centre of inertia, Motion relative to centre of inertia.

Unit – 2. Motion about a fixed axis, Compound pendulum, Centre of percussion.

Unit – 3. Motion of a rigid body in two dimension under finite forces, Motion under impulsive forces.

Unit – 4. Motion in three dimension with reference to Euler's dynamical and geometrical equations, Motion under finite forces

Unit – 5. Motion under no forces, Motion under impulsive forces.

Unit – 6. Conservation of linear and angular momentums, Conservation of energy for finite and impulsive forces.

Unit – 7. Lagrange's equation, Energy equation for conservative field, Small oscillations, Motion under impulsive forces (Lagrange's equations for blows)

Unit – 8 Motion of a top: Equation of motion of a top, Steady motion of a top, Stability conditions.

Unit – 9. Hamilton's principle, Principle of least action

Unit – 10. Kinematics of ideal fluid, Lagrange's and Euler's methods, Streamlines, Path lines, Stream function in two dimensions.

Unit – 11. Velocity potential, Rotational and Irrotational motion in two dimensions.

Equation of Continuity, Lagranges approach. Eulerian approach, Equivalence of these two approaches.

Unit – 12. Equation of Continuity: Cartesian, Cylindrical and Spherical polar coordinates, Boundary surfaces.

Unit – 13. Euler's hydrodynamical equations, Bernoulli's theorem, Helmholtz equations.

Unit – 14 Cauchy's integrals, Motion due to impulsive forces.

Unit – 15. Motion in two dimensions: Complex potential, Cauchy-Riemann equations, Two dimensional Sources, Sinks, Doublets and their images.