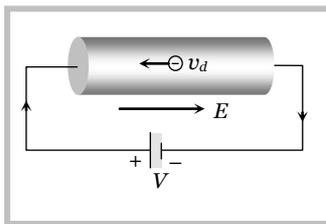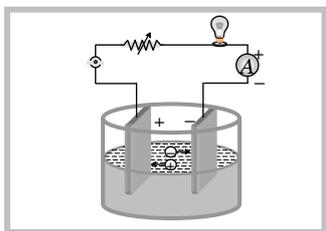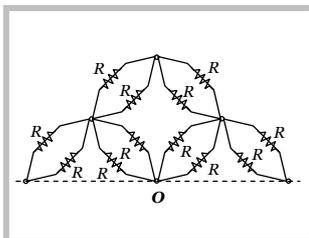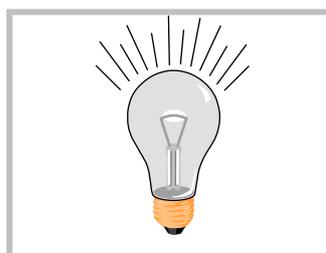genius PHYSICS

Current Electricity

Notes by Pradeep Kshetrapal

Electric current and Resistance

Cell, Kirchoff's law and Measuring instruments

Determination of resistance

Heating effect of current

Formulas in current electricity (Direct Current)

1	Electric Current	$i = q/t$	"q" is charge passing in normal direction through a cross section of conductor in time "t"
2	Drift velocity V_d with Electric field	$V_d = \dfrac{-e\overline{E}t}{m}$	e is charge and m is mass on electron, E is electric field, τ is relaxation time.
3	Current I with Drift velocity V_d	$I = n\,e\,A\,V_d$	n is number density with of free electrons, A is area of cross section.
4	Mobility of charge "μ"	$\mu = V_d / E = \dfrac{q\tau}{m}$	
5	Mobility and drift velocity	$V_d = \mu_e E$	
6	Current and Mobility	$I = Ane \times \mu_e E$	
7	Resistance, P.D., and Current	$R = V / I$	V Potential Difference, I Current .
8	Resistance R with specific Res.	$R = \rho\dfrac{l}{A}$	l is length of conductor and A is area of cross section
9	Specific Resistance, ρ	$\rho = R\dfrac{A}{l}$	
10	Resistivity with electrons	$\rho = \dfrac{m}{ne^2\tau}$	
11	Current density J	$\vec{J} = I / \vec{A}$	I is current, J current density, A is area of cross section
12	Current density magnitude	$J\,A\,\cos\theta = I$	θ is angle between $\vec{J}$ and $\vec{A}$
13	Conductance G	$G = 1/R$	
14	Conductivity σ	$\sigma = 1/\rho$	ρ is specific resistance
15	Microscopic form of Ohms Law	$J = \sigma E$	E is electric field
16	Temperature coefficient of Resistance α	$\alpha = \dfrac{Rt - R_0}{R_0 \times t}$	R_o is resistance at 0_o C. R_t is resistance at t^o and "t" is temperature difference.
17	Resistances in series	$R = R_1 + R_2 + R_3$	Same current through all resistances (circuit Current
	Resistances in parallel	$1/R_e = 1/R_1 + 1/R_2 + 1/R_3$	Same P.D. across each resistance (V of cell)
18	In a cell, emf and internal resistance	$I = \dfrac{E}{R + r}$	I is current, E is emf, R is external resistance, r is internal resistance.
19	In a circuit with a cell	$V = E - Ir$	V is terminal potential difference
20	n Cells of emf E in series	$Emf = nE$	
21	Resistance of n cells in series	$nr + R$	r is internal resistance of one cell, R external Resistance
22	Current in circuit with n cells in series	$I = \dfrac{nE}{R + nr}$	r is internal resistance of one cell, R external Resistance
23	n cells in parallel, then emf	$emf = E$	
24	n cells in parallel, resistance	$R + r/n$	R external resistance, r internal resistance
25	Cells in mixed group, condition for maximum current	$R = \dfrac{nr}{m}$	n is number of cells in one row, m is number of rows. r is internal resistance, R external resis.
26	Internal resistance of a cell	$r = \left(\dfrac{E - V}{V}\right) \times R$	E is emf, V is terminal Potential difference, R is external resistance.
27	Power of a circuit	$P = I.V = I^2R = V^2/R$	

28	Energy consumed	$E = I.V.\Delta T$	ΔT is time duration
29	Kirchoff Law (junction rule)	$\Sigma i = 0$	Sum of currents at junction is zero.
30	Kirchoff Law (Loop rule)	$\Sigma V = 0$	In a loop sum of all p.d.s is Zero
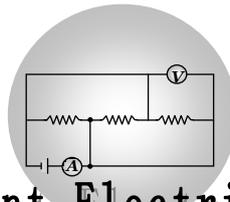
Current Electricity

Electric Current

(1) **Definition :** The time rate of flow of charge through any cross-section is called current. So if through a cross-section, ΔQ charge passes in time Δt then $i_{av} = \dfrac{\Delta Q}{\Delta t}$ and instantaneous current $i = \underset{\Delta t \to 0}{Lim} \dfrac{\Delta Q}{\Delta t} = \dfrac{dQ}{dt}$. If flow is uniform then $i = \dfrac{Q}{t}$. Current is a scalar quantity. It's S.I. unit is *ampere* (*A*) and C.G.S. unit is *emu* and is called *biot* (*Bi*), or *ab ampere*. $1A = (1/10)$ *Bi* (*ab amp.*)

(2) **The direction of current :** The conventional direction of current is taken to be the direction of flow of positive charge, *i.e.* field and is opposite to the direction of flow of negative charge as shown below.

Though conventionally a direction is associated with current (Opposite to the motion of electron), it is not a vector. It is because the current can be added algebraically. Only scalar quantities can be added algebraically not the vector quantities.

(3) **Charge on a current carrying conductor :** In conductor the current is caused by electron (free electron). The no. of electron (negative charge) and proton (positive charge) in a conductor is same. Hence the net charge in a current carrying conductor is zero.

(4) **Current through a conductor of non-uniform cross-section :** For a given conductor current does not change with change in cross-sectional area. In the following figure $i_1 = i_2 = i_3$
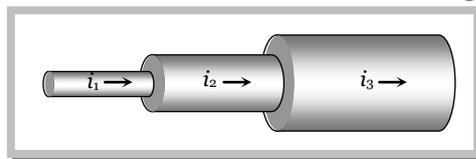

(5) **Types of current :** Electric current is of two type :

Alternating current (ac)	Direct current (dc)
(i) Magnitude and direction both varies with time ac $\to$ Rectifier $\to$ dc	(i) (Pulsating dc)　　　(Constant dc) dc $\to$ Inverter $\to$ ac
(ii)　Shows heating effect only	(ii) Shows heating effect, chemical effect and magnetic effect of current
(iii) It's symbol is　—⊙—	(iii) It's symbol is　⊣⊢

Note :❑ In our houses ac is supplied at 220*V*, 50*Hz*.

(6) **Current in difference situation :**

(i) **Due to translatory motion of charge**
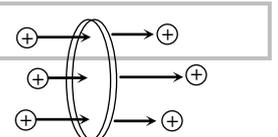
In n particle each having a charge q, pass through a given area in time t then $i = \dfrac{nq}{t}$

If n particles each having a charge q pass per second per unit area, the current associated with cross-sectional area A is $i = nqA$

If there are n particle per unit volume each having a charge q and moving with velocity v, the current thorough, cross section A is $i = nqvA$, for electrons i= **neav$_d$**
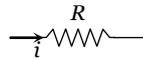
(ii) **Due to rotatory motion of charge**

If a point charge q is moving in a circle of radius r with speed v (frequency ν, angular speed ω and time

period T) then corresponding currents $i = q \, \nu = \dfrac{q}{T} = \dfrac{qv}{2\pi r} = \dfrac{q\,\omega}{2\pi}$
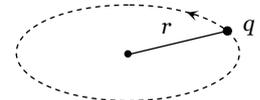
(iii) **When a voltage V applied across a resistance R** : Current flows through the conductor $i = \dfrac{V}{R}$

also by definition of power $i = \dfrac{P}{V}$

(7) **Current carriers :** The charged particles whose flow in a definite direction constitutes the electric current are called current carriers. In different situation current carriers are different.

(i) Solids : In solid conductors like metals current carriers are free electrons.

(ii) Liquids : In liquids current carriers are positive and negative ions.

(iii) Gases : In gases current carriers are positive ions and free electrons.

(iv) Semi conductor : In semi conductors current carriers are holes and free electrons.

Current density (J)

In case of flow of charge through a cross-section, current density is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point. Current density at point P is given by $\vec{J} = \dfrac{di}{dA}\,\vec{n}$
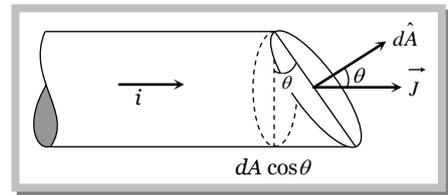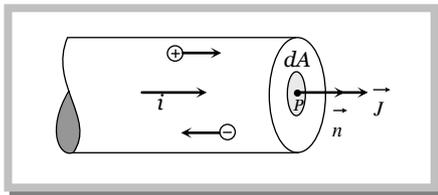
If the cross-sectional area is not normal to the current, the cross-sectional area normal to current in accordance with following figure will be $dA\cos\theta$ and so in this situation:

$J = \dfrac{di}{dA\cos\theta}$ i.e. $di = JdA\cos\theta$ or $di = \vec{J}.\overrightarrow{dA} \Rightarrow i = \int \vec{J}\cdot\overrightarrow{dA}$

i.e., in terms of current density, current is the flux of current density.

$\textit{Note}$: ❑   If current density $\vec{J}$ is uniform for a normal cross-section $\vec{A}$ then : $i = \int \vec{J}\cdot\overrightarrow{ds} = \vec{J}\cdot\int\overrightarrow{ds}$ [as

$\vec{J}$ = constant]

or $i = \vec{J}\cdot\vec{A} = JA\cos 0 = JA \Rightarrow J = \dfrac{i}{A}$ [as $\int\overrightarrow{dA} = \vec{A}$ and θ = 0º]

(1) **Unit and dimension :** Current density $\vec{J}$ is a vector quantity having S.I. unit Amp/m^2 and dimension.$[L^{-2}A]$

(2) **Current density in terms of velocity of charge :** In case of uniform flow of charge through a cross-section normal to it as $i = nqvA$ so, $\vec{J} = \dfrac{i}{A}\vec{n} = (nqv)\vec{n}$ or $\vec{J} = nq\vec{v} = \vec{v}\,(\rho)$ [With $\rho = \dfrac{\text{charge}}{\text{volume}} = nq$]

i.e., current density at a point is equal to the product of volume charge density with velocity of charge distribution at that point.

(3) **Current density in terms of electric field :** Current density relates with electric field as $J = \sigma E = \dfrac{E}{\rho}$; where σ = conductivity and ρ = resistivity or specific resistance of substance.

(i) Direction of current density $\vec{J}$ is same as that of electric field $\vec{E}$.

(ii) If electric field is uniform (*i.e.* $\vec{E}$ = constant) current density will be constant [as σ = constant]

(iii) If electric field is zero (as in electrostatics inside a conductor), current density and hence current will be zero.

Conduction of Current in Metals

According to modern views, a metal consists of a '*lattice*' of fixed positively charged ions in which billions and billions of free electrons are moving randomly at speed which at room temperature (*i.e.* 300 *K*) in accordance with kinetic theory of gases is given by $v_{rms} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3\times(1.38\times10^{-23})\times300}{9.1\times10^{-31}}} \simeq 10^5\,m/s$

The randomly moving free electrons inside the metal collide with the lattice and follow a zig-zag path as shown in figure (A).

(A)   (B)
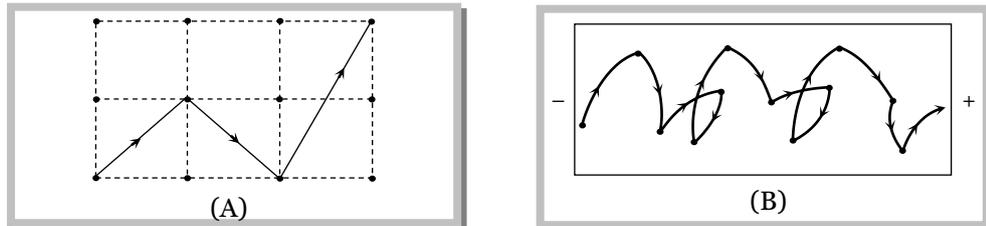
However, in absence of any electric field due to this random motion, the number of electrons crossing from left to right is equal to the number of electrons crossing from right to left (otherwise metal will not remain equipotential) so the net current through a cross-section is zero.

A motion of charge is possible by motion of electron or a current carrier.

Velocities of charged particle (electron) in a conductor
thermal velocity : All electrons in the atom are not capable of motion. Only a few which have little higher level of energy leave their orbit and are capable of moving around. These electrons are called "free electrons". These free electrons are in very large quantity $\approx 10^{29}$ m^{-3} in free metals. Due to temperature and thermal energy They have a *thermal velocity* $\approx 10^5$ ms^{-1} . This velocity is in all directions and of magnitudes varying from zero to maximum. Due to large number of electrons we can assume that vector sum of thermal velocities at any instant is zero.

i.e. $\vec{u_1} + \vec{u_2} + \vec{u_3} + \ldots\ldots + u_n = 0$

Mean Free path : The fast moving electrons keep striking other atoms/ions in the conductor. They are reflected and move in other direction. They keep moving till they strike another ion/atom.

The path between two consecutive collisions is called free path. The average length of these free paths is called "Mean Free Path".

8 Current Electricity

Relaxation Time :The time to travel mean free path is called Relaxation Period or Relaxation Time, denoted by Greek letter Tau "τ". If $t_1, t_2, ...t_n$ are the time periods for n collisions then Relaxation Time $\tau = \frac{1}{n}$ $(t_1+t_2+ ...t_n)$

Drift Velocity :When Electric Field is applied across a conductor, the free electrons experience a force in the direction opposite to field. Dur to this force they start drifting in the direction of force. The Velocity of this drift is called drift velocity "V_d". During the drift they maintain their thermal velocity.
The drift velocity can be calculated as averaged velocity of all the electrons drifting.

Relation between drift-velocity (V_d) and electric field applied.

When electric field is applied across a conductor each electron experience a Force $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. It acquires an acceleration $\alpha = \frac{e\vec{E}}{m}$ where e is charge on electron and m is its mass.

If n electrons are having initial speeds $u_1, u_2,...u_n$ and their time to travel free path is $t_1, t_2, ...t_n$ then final velocities are

$v_1 = u_1 + \alpha t_1,$

$V_2 = u_2 + \alpha t_2,$

$V_n = u_n + \alpha t_n$ and so on.

Drift velocity is average of these velocities of charged particles. Therefore

$$V_d = \frac{1}{n}(v_1 + V_2 + ... V_n)$$

$$= \frac{1}{n}(u_1 + \alpha t_1 + u_2 + \alpha t_2 + ... u_n + \alpha t_n)$$

$$= \frac{1}{n}(u_1 + u_2 + ... u_n + \alpha t_1 + \alpha t_2 + ... \alpha t_n)$$

$$= (u_1 + u_2 + ... u_n) + \frac{1}{n}(\alpha t_1 + \alpha t_2 + ... \alpha t_n)$$

$$= 0 + \alpha \frac{1}{n}(t_1 + t_2 + ...t_n)$$

$$= \alpha \tau$$

or $V_d = \frac{e\vec{E}\tau}{m}$ $(\alpha = \frac{e\vec{E}}{m})$ $\boxed{V_d = \frac{e\vec{E}\tau}{m}}$

    Relation of Current and Drift velocity : When an electric field is applied, inside the conductor due to electric force the path of electron in general becomes curved (parabolic) instead of straight lines and electrons drift opposite to the field figure (B). Due to this drift the random motion of electrons get modified and there is a net transfer of electrons across a cross-section resulting in current.

    Drift velocity is the average uniform velocity acquired by free electrons inside a metal by the application of an electric field which is responsible for current through it. Drift velocity is very small it is of the order of 10^{-4} m/s as compared to thermal speed ($\simeq 10^{5}$ m / s) of electrons at room temperature.

    If suppose for a conductor

n = Number of electron per unit volume of the conductor

A = Area of cross-section

V = potential difference across the conductor

E = electric field inside the conductor
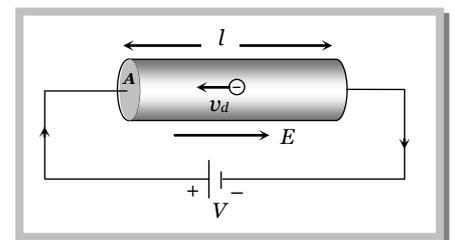
i = current, J = current density, ρ = specific resistance, σ = conductivity $\left(\sigma = \frac{1}{\rho}\right)$ then current relates

with drift velocity as $i = neAv_d$ we can also write $v_d = \frac{i}{neA} = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{V}{\rho l n e}$.

$\underline{Note}$: ❑ The direction of drift velocity for electron in a metal is opposite to that of applied electric field (*i.e.* current density $\vec{J}$).

❑ $v_d \propto E$ *i.e.*, greater the electric field, larger will be the drift velocity.

❑ When a steady current flows through a conductor of non-uniform cross-section drift velocity varies inversely with area of cross-section $\left(v_d \propto \dfrac{1}{A} \right)$

❑ If diameter of a conductor is doubled, then drift velocity of electrons inside it will not change.

(2) **Relaxation time (τ) :** The time interval between two successive collisions of electrons with the positive ions in the metallic lattice is defined as relaxation time $\tau = \dfrac{\text{mean free path}}{\text{r.m.s. velocity of electrons}} = \dfrac{\lambda}{v_{rms}}$ with rise in temperature v_{rms} increases consequently τ decreases.

(3) **Mobility :** Drift velocity per unit electric field is called mobility of electron *i.e.* $\mu = \dfrac{v_d}{E}$. It's unit is $\dfrac{m^2}{volt - \sec}$.

Concepts

☞ *Human body, though has a large resistance of the order of kΩ (say 10 kΩ), is very sensitive to minute currents even as low as a few mA. Electrocution, excites and disorders the nervous system of the body and hence one fails to control the activity of the body.*

☞ *1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross-section of the conductors.*

☞ *dc flows uniformly throughout the cross-section of conductor while ac mainly flows through the outer surface area of the conductor. This is known as skin effect.*

☞ *It is worth noting that electric field inside a charged conductor is zero, but it is non zero inside a current carrying conductor and is given by $E = \dfrac{V}{l}$ where V = potential difference across the conductor and l = length of the conductor. Electric field out side the current carrying is zero.*

☞
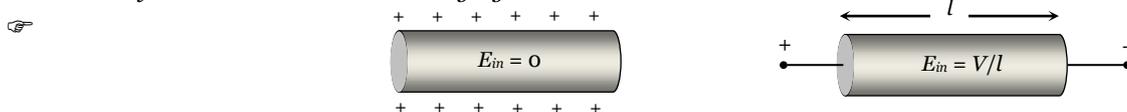
☞ *For a given conductor JA = i = constant so that $J \propto \dfrac{1}{A}$ i.e., $J_1 A_1 = J_2 A_2$; this is called equation of continuity*
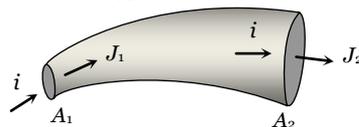
☞ *If cross-section is constant, $I \propto J$ i.e. for a given cross-sectional area, greater the current density, larger will be current.*

☞ *The drift velocity of electrons is small because of the frequent collisions suffered by electrons.*

☞ *The small value of drift velocity produces a large amount of electric current, due to the presence of extremely large number of free electrons in a conductor. The propagation of current is almost at the speed of light and involves electromagnetic process. It is due to this reason that the electric bulb glows immediately when switch is on.*

☞ *In the absence of electric field, the paths of electrons between successive collisions are straight line while in presence of electric field the paths are generally curved.*

☞ *Free electron density in a metal is given by $n = \dfrac{N_A x d}{A}$ where N_A = Avogrado number, x = number of free electrons per atom, d = density of metal and A = Atomic weight of metal.*

Example

Example: 1 The potential difference applied to an *X*-ray tube is 5 *KV* and the current through it is 3.2 *mA*. Then the number of electrons striking the target per second is

(a) 2×10^{16} (b) 5×10^6 (c) 1×10^{17} (d) 4×10^{15}

Solution : (a) $i = \dfrac{q}{t} = \dfrac{ne}{t}$ $\Rightarrow n = \dfrac{it}{e} = \dfrac{3.2 \times 10^{-3} \times 1}{1.6 \times 10^{-19}} = 2 \times 10^{16}$

Example: 2 A beam of electrons moving at a speed of 10^6 *m/s* along a line produces a current of 1.6×10^{-6} *A*. The number of electrons in the 1 *metre* of the beam is   **[CPMT 2000]**

(a) 10^6 (b) 10^7 (c) 10^{13} (d) 10^{19}

Solution : (b) $i = \dfrac{q}{t} = \dfrac{q}{(x/v)} = \dfrac{qv}{x} = \dfrac{nev}{x} \Rightarrow n = \dfrac{ix}{ev} = \dfrac{1.6 \times 10^{-6} \times 1}{1.6 \times 10^{-19} \times 10^6} = 10^7$

Example: 3 In the Bohr's model of hydrogen atom, the electrons moves around the nucleus in a circular orbit of a radius 5×10^{-11} *metre*. It's time period is 1.5×10^{-16} *sec*. The current associated is

(a) Zero   (b) $1.6 \times 10^{-19} A$ (c) $0.17 A$ (d) $1.07 \times 10^{-3} A$

Solution : (d) $i = \dfrac{q}{T} = \dfrac{1.6 \times 10^{-19}}{1.5 \times 10^{-16}} = 1.07 \times 10^{-3} A$

Example: 4 An electron is moving in a circular path of radius 5.1×10^{-11} *m* at a frequency of 6.8×10^{15} revolution/sec. The equivalent current is approximately

(a) $5.1 \times 10^{-3} A$ (b) $6.8 \times 10^{-3} A$ (c) $1.1 \times 10^{-3} A$ (d) $2.2 \times 10^{-3} A$

Solution : (c) $v = 6.8 \times 10^{15} \Rightarrow T = \dfrac{1}{6.8 \times 10^{15}} \sec \Rightarrow i = \dfrac{Q}{T} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15} = 1.1 \times 10^{-3} A$

Example: 5 A copper wire of length 1*m* and radius 1*mm* is joined in series with an iron wire of length 2*m* and radius 3*mm* and a current is passed through the wire. The ratio of current densities in the copper and iron wire is

[MP PMT 1994]

(a) $18:1$ (b) $9:1$ (c) $6:1$ (d) $2:3$

Solution : (b)   We know $J = \dfrac{i}{A}$ when i = constant $J \propto \dfrac{1}{A} \Rightarrow \dfrac{J_c}{J_i} = \dfrac{A_i}{A_c} = \left(\dfrac{r_i}{r_c}\right)^2 = \left(\dfrac{3}{1}\right)^2 = \dfrac{9}{1}$

Example: 6 A conducting wire of cross-sectional area 1 cm^2 has 3×10^{23} m^{-3} charge carriers. If wire carries a current of 24 *mA*, the drift speed of the carrier is   **[UPSEAT 2001]**

(a) $5 \times 10^{-6} \, m/s$ (b) $5 \times 10^{-3} \, m/s$ (c) $0.5 \, m/s$ (d) $5 \times 10^{-2} \, m/s$

Solution : (b) $v_d = \dfrac{i}{neA} = \dfrac{24 \times 10^{-3}}{3 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{-4}} = 5 \times 10^{-3} \, m/s$

Example: 7 A wire has a non-uniform cross-sectional area as shown in figure. A steady current *i* flows through it. Which one of the following statement is correct
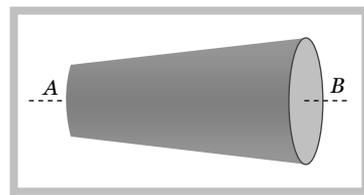
(a) The drift speed of electron is constant   (b) The drift speed increases on moving from
A to B

(c) The drift speed decreases on moving from A to B (d) The drift speed varies randomly

Solution : (c)   For a conductor of non-uniform cross-section $v_d \propto \dfrac{1}{\text{Area of cross - section}}$

Example: 8   In a wire of circular cross-section with radius r, free electrons travel with a drift velocity v, when a current i flows through the wire. What is the current in another wire of half the radius and of the some material when the drift velocity is $2v$

(a) $2i$ (b) i (c) $i/2$ (d) $i/4$

Solution : (c) $i = neAv_d = ne\pi r^2 v$ and $i' = ne\pi\left(\dfrac{r}{2}\right)^2 .2v = \dfrac{ne\,\pi r^2 v}{2} = \dfrac{i}{2}$

Example: 9   A potential difference of V is applied at the ends of a copper wire of length l and diameter d. On doubling only d, drift velocity

(a) Becomes two times   (b) Becomes half   (c) Does not change   (d) Becomes one fourth

Solution : (c)   Drift velocity doesn't depends upon diameter.

Example: 10   A current flows in a wire of circular cross-section with the free electrons travelling with a mean drift velocity v. If an equal current flows in a wire of twice the radius new mean drift velocity is

(a) v (b) $\dfrac{v}{2}$ (c) $\dfrac{v}{4}$ (d) None of these

Solution : (c)   By using $v_d = \dfrac{i}{neA} \Rightarrow v_d \propto \dfrac{1}{A} \Rightarrow v' = \dfrac{v}{4}$

Example: 11   Two wires A and B of the same material, having radii in the ratio $1 : 2$ and carry currents in the ratio $4 : 1$. The ratio of drift speeds of electrons in A and B is

(a) $16 : 1$ (b) $1 : 16$ (c) $1 : 4$ (d) $4 : 1$

Solution : (a)   As $i = neA\,v_d \Rightarrow \dfrac{i_1}{i_2} = \dfrac{A_1}{A_2} \times \dfrac{v_{d_1}}{v_{d_2}} = \dfrac{r_1^2}{r_2^2}.\dfrac{v_{d_1}}{v_{d_2}} \Rightarrow \dfrac{v_{d_1}}{v_{d_2}} = \dfrac{16}{1}$

Tricky example: 1

In a neon discharge tube 2.9×10^{18} Ne^+ ions move to the right each second while 1.2×10^{18} electrons move to the left per second. Electron charge is 1.6×10^{-19} C. The current in the discharge tube   **[MP PET 1999]**

(a) $1\,A$ towards right   (b) $0.66\,A$ towards right   (c) $0.66\,A$ towards left   (d) Zero

Solution: (b)   Use following trick to solve such type of problem.

Trick : In a discharge tube positive ions carry q units of charge in t seconds from anode to cathode and negative carriers (electrons) carry the same amount of charge from cathode to anode in t′ second. The current in the tube is $i = \dfrac{q}{t} + \dfrac{q'}{t'}$.

Hence in this question current $i = \dfrac{2.9 \times 10^{18} \times e}{1} + \dfrac{1.2 \times 10^{18} \times e}{1} = 0.66A$ towards right.

Tricky example: 2

If the current flowing through copper wire of 1 mm diameter is 1.1 amp. The drift velocity of electron is (Given density of Cu is 9 gm/cm^3, atomic weight of Cu is 63 $grams$ and one free electron is contributed by each atom)

(a) $0.1\,mm/sec$ (b) $0.2\,mm/sec$ (c) $0.3\,mm/sec$ (d) $0.5\,mm/sec$

Solution: (a) 6.023×10^{23} atoms has mass $= 63 \times 10^{-3}$ kg

$$\text{So no. of atoms per } m^3 = n = \frac{6.023 \times 10^{23}}{63 \times 10^{-3}} \times 9 \times 10^3 = 8.5 \times 10^{28}$$

$$v_d = \frac{i}{neA} = \frac{1.1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times \pi \times (0.5 \times 10^{-3})^2} = 0.1 \times 10^{-3} \, m/\sec = 0.1 \, mm/\sec$$

Ohm's Law

If the physical circumstances of the conductor (length, temperature, mechanical strain etc.) remains constant, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends *i.e.* $i \propto V$

$\Rightarrow V = iR$ or $\dfrac{V}{i} = R$; where R is a proportionality constant, known as electric resistance.

(1) Ohm's law is not a universal law, the substance which obeys ohm's law are known as ohmic substance for such ohmic substances graph between V and i is a straight line as shown. At different temperatures *V-i* curves are different.

Slope of the line $= \tan\theta = \dfrac{V}{i} = R$

Here $\tan\theta_1 > \tan\theta_2$
So $R_1 > R_2$ *i.e.* $T_1 > T_2$
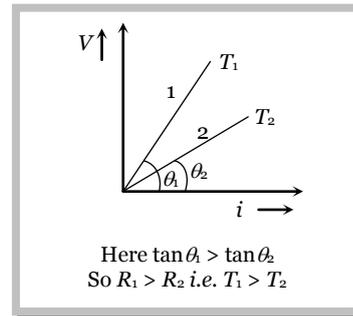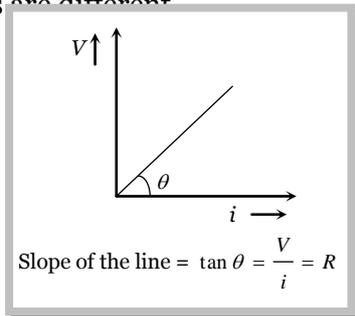
(2) The device or substances which doesn't obey ohm's law *e.g.* gases, crystal rectifiers, thermoionic valve, transistors etc. are known as non-ohmic or non-linear conductors. For these *V-i* curve is not linear. In these situation the ratio between voltage and current at a particular voltage is known as static resistance. While the rate of change of voltage to change in current is known as dynamic resistance.

$$R_{st} = \frac{V}{i} = \frac{1}{\tan\theta}$$

while $R_{dyn} = \dfrac{\Delta V}{\Delta I} = \dfrac{1}{\tan\phi}$
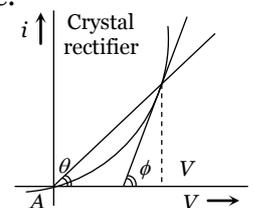
(3) Some other non-ohmic graphs are as follows :



(A)

(B)

(C)

(D)
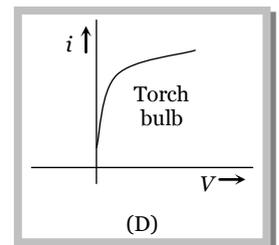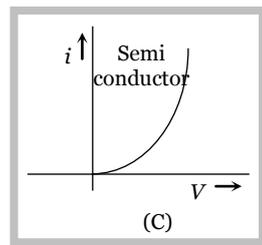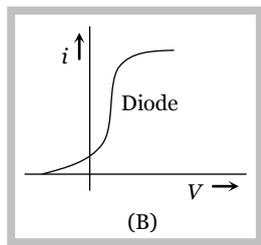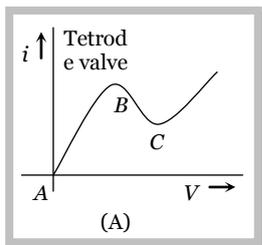
Resistance

(1) **Definition :** The property of substance by virtue of which it opposes the flow of current through it, is known as the resistance.

(2) **Cause of resistance of a conductor :** It is due to the collisions of free electrons with the ions or atoms of the conductor while drifting towards the positive end of the conductor.

(3) **Formula of resistance :** For a conductor if l = length of a conductor A = Area of cross-section of conductor, n = No. of free electrons per unit volume in conductor, τ = relaxation time then resistance of conductor $R = \rho \dfrac{l}{A} = \dfrac{m}{ne^2\tau} \cdot \dfrac{l}{A}$; where ρ = resistivity of the material of conductor

(4) **Unit and dimension :** It's S.I. unit is *Volt/Amp.* or *Ohm* (Ω). Also 1 *ohm*

$= \dfrac{1\,volt}{1\,Amp} = \dfrac{10^8\,emu\ \text{of potenti al}}{10^{-1}\,emu\ \text{of current}}$ = 10⁹ *emu* of resistance. It's dimension is $[ML^2T^{-3}A^{-2}]$.

(5) **Conductance (C) :** Reciprocal of resistance is known as conductance. $C = \dfrac{1}{R}$ It's unit is $\dfrac{1}{\Omega}$ or Ω^{-1} or "Siemen".
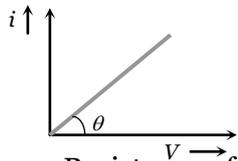
$$\text{Slope} = \tan\theta = \dfrac{i}{V} = \dfrac{1}{R} = C$$

(6) **Dependence of resistance :** Resistance of a conductor depends on the following factors.

(i) Length of the conductor : Resistance of a conductor is directly proportional to it's length *i.e.* $R \propto l$ *e.g.* a conducting wire having resistance R is cut in n equal parts. So resistance of each part will be $\dfrac{R}{n}$.

(ii) Area of cross-section of the conductor : Resistance of a conductor is inversely proportional to it's area of cross-section *i.e.* $R \propto \dfrac{1}{A}$

Less : Area of cross-section
More : Resistance

More : Area of cross-section
Less : Resistance
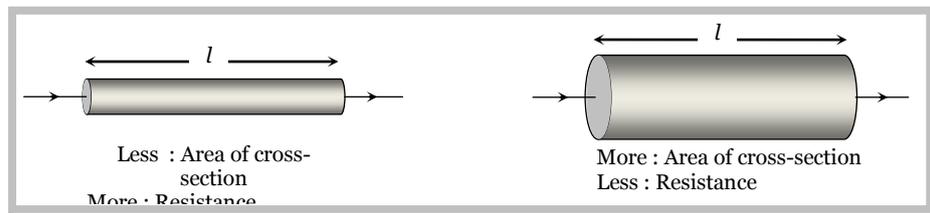
(iii) Material of the conductor : Resistance of conductor also depends upon the nature of material *i.e.* $R \propto \dfrac{1}{n}$, for different conductors n is different. Hence R is also different.

(iv) Temperature : We know that $R = \dfrac{m}{ne^2\tau} \cdot \dfrac{l}{A} \Rightarrow R \propto \dfrac{l}{\tau}$ when a metallic conductor is heated, the atom in the metal vibrate with greater amplitude and frequency about their mean positions. Consequently the number of collisions between free electrons and atoms increases. This reduces the relaxation time τ and increases the value of resistance R *i.e.* for a conductor **Resistance** $\propto$ **temperatur e** .

If $\quad R_0$ = resistance of conductor at 0ºC

$\quad\quad R_t$ = resistance of conductor at tºC

and α, β = temperature co-efficient of resistance (unit $\rightarrow$ perºC)

then $R_t = R_0(1 + \alpha t + \beta t^2)$ for $t > 300$ºC and $R_t = R_0(1 + \alpha t)$ for $t \le 300$ºC $\quad$ or $\quad \alpha = \dfrac{R_t - R_0}{R_0 \times t}$
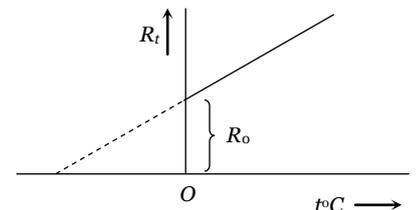
Note : ❑ $\quad$ If R_1 and R_2 are the resistances at t_1ºC and t_2ºC respectively then $\dfrac{R_1}{R_2} = \dfrac{1 + \alpha\,t_1}{1 + \alpha\,t_2}$.

❑ The value of α is different at different temperature. Temperature coefficient of resistance averaged over the temperature range $t_1°C$ to $t_2°C$ is given by $\alpha = \dfrac{R_2 - R_1}{R_1(t_2 - t_1)}$ which gives $R_2 = R_1$ $[1 + \alpha(t_2 - t_1)]$. This formula gives an approximate value.
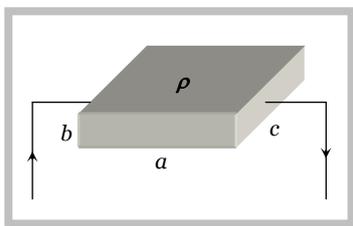
(v) **Resistance according to potential difference :** Resistance of a conducting body is not unique but depends on it's length and area of cross-section *i.e.* how the potential difference is applied. See the following figures
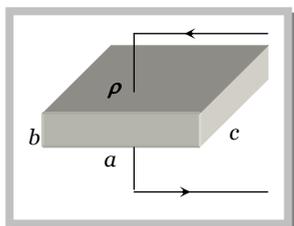
Length = b

Area of cross-section = $a \times c$

Resistance $R = \rho\left(\dfrac{b}{a \times c}\right)$

Length = a

Area of cross-section = $b \times c$

Resistance $R = \rho\left(\dfrac{a}{b \times c}\right)$
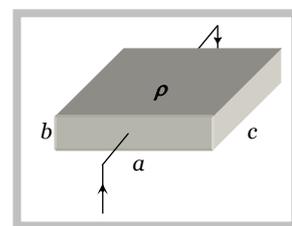
Length = c

Area of cross-section = $a \times b$

Resistance $R = \rho\left(\dfrac{c}{a \times b}\right)$

(7) **Variation of resistance of some electrical material with temperature :**

(i) Metals : For metals their temperature coefficient of resistance $\alpha > 0$. So resistance increases with temperature.

Physical explanation : Collision frequency of free electrons with the immobile positive ions increases

(ii) Solid non-metals : For these $\alpha = 0$. So resistance is independence of temperature.

Physical explanation : Complete absence of free electron.

(iii) Semi-conductors : For semi-conductor $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Covalent bonds breaks, liberating more free electron and conduction increases.

(iv) Electrolyte : For electrolyte $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : The degree of ionisation increases and solution becomes less viscous.

(v) Ionised gases : For ionised gases $\alpha < 0$ *i.e.* resistance decreases with temperature rise.

Physical explanation : Degree of ionisation increases.

(vi) Alloys : For alloys α has a small positive values. So with rise in temperature resistance of alloys is almost constant. Further alloy resistances are slightly higher than the pure metals resistance.

Alloys are used to made standard resistances, wires of resistance box, potentiometer wire, meter bridge wire *etc.*

Commonly used alloys are : Constantan, mangnin, Nichrome etc.

(vii) Super conductors : At low temperature, the resistance of certain substances becomes exactly zero. (*e.g. Hg* below 4.2 *K* or *Pb* below 7.2 *K*).

These substances are called super conductors and phenomenon super conductivity. The temperature at which resistance becomes zero is called critical temperature and depends upon the nature of substance.

Resistivity or Specific Resistance (ρ)

(1) **Definition :** From $R = \rho \dfrac{l}{A}$; If $l = 1m$, $A = 1\ m^2$ then $\boldsymbol{R = \rho}$ *i.e.* resistivity is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

(2) **Unit and dimension :** It's S.I. unit is $ohm \times m$ and dimension is $[ML^3T^{-3}A^{-2}]$

(3) **It's formula** : $\rho = \dfrac{m}{ne^2\tau}$

(4) **It's dependence :** Resistivity is the intrinsic property of the substance. It is independent of shape and size of the body (*i.e. l* and *A*). It depends on the followings :

(i) Nature of the body : For different substances their resistivity also different *e.g.* ρ_{silver} = minimum = 1.6×10^{-8} Ω-*m* and $\rho_{fused\ quartz}$ = maximum $\approx 10^{16}$ Ω-*m*
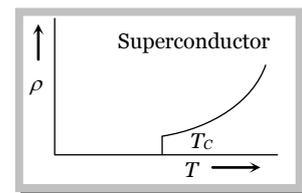
(ii) Temperature : Resistivity depends on the temperature. For metals $\rho_t = \rho_0\ (1 + \alpha\Delta t)$ *i.e.* resitivity increases with temperature.
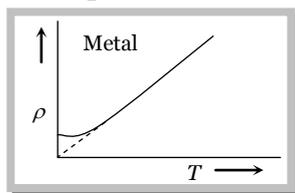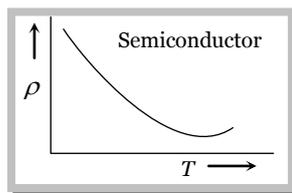
| ρ increases with temperature | ρ decreases with temperature | ρ decreases with temperature and becomes zero at a certain temperature |

(iii) Impurity and mechanical stress : Resistivity increases with impurity and mechanical stress.

(iv) Effect of magnetic field : Magnetic field increases the resistivity of all metals except iron, cobalt and nickel.

(v) Effect of light : Resistivity of certain substances like selenium, cadmium, sulphides is inversely proportional to intensity of light falling upon them.

(5) **Resistivity of some electrical material :** $\rho_{insulator} > \rho_{alloy} > \rho_{semi\text{-}conductor} > \rho_{conductor}$

Reciprocal of resistivity is called conductivity (σ) *i.e.* $\sigma = \dfrac{1}{\rho}$ with unit *mho/m* and dimensions $[M^{-1}L^{-3}T^3A^2]$.

| **Stretching of Wire** | If a conducting wire stretches, it's length increases, area of cross-section decreases so resistance increases but volume remain constant.

Suppose for a conducting wire before stretching it's length = l_1, area of cross-section = A_1, radius = r_1, diameter = d_1, and resistance $R_1 = \rho \dfrac{l_1}{A_1}$

Before stretching tretching

Volume remains constant *i.e.* $A_1 l_1 = A_2 l_2$
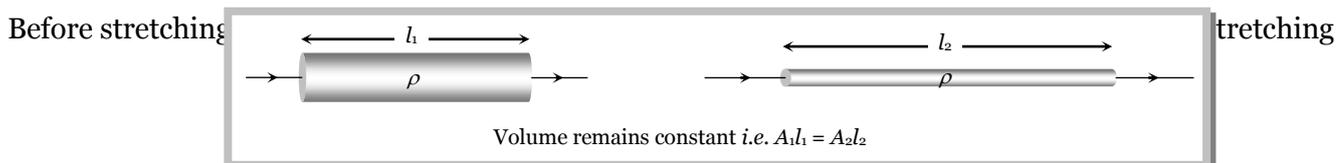
After stretching length = l_2, area of cross-section = A_2, radius = r_2, diameter = d_2 and resistance $= R_2 = \rho \dfrac{l_2}{A_2}$

Ratio of resistances $\quad \dfrac{R_1}{R_2} = \dfrac{l_1}{l_2} \times \dfrac{A_2}{A_1} = \left(\dfrac{l_1}{l_2}\right)^2 = \left(\dfrac{A_2}{A_1}\right)^2 = \left(\dfrac{r_2}{r_1}\right)^4 = \left(\dfrac{d_2}{d_1}\right)^4$

(1) If length is given then $R \propto l^2 \Rightarrow \dfrac{R_1}{R_2} = \left(\dfrac{l_1}{l_2}\right)^2$

(2) If radius is given then $R \propto \dfrac{1}{r^4} \Rightarrow \dfrac{R_1}{R_2} = \left(\dfrac{r_2}{r_1}\right)^4$

Note : ❑ After stretching if length increases by n times then resistance will increase by n^2 times

i.e. $R_2 = n^2 R_1$. Similarly if radius be reduced to $\dfrac{1}{n}$ times then area of cross-section decreases

$\dfrac{1}{n^2}$ times so the resistance becomes n^4 times i.e. $R_2 = n^4 R_1$.

❑ After stretching if length of a conductor increases by $x\%$ then resistance will increases by $2x \%$ (valid only if $x < 10\%$)

Various Electrical Conducting Material For Specific Use

(1) **Filament of electric bulb :** Is made up of tungsten which has high resistivity, high melting point.

(2) **Element of heating devices (such as heater, geyser or press) :** Is made up of nichrome which has high resistivity and high melting point.

(3) **Resistances of resistance boxes (standard resistances) :** Are made up of manganin, or constantan as these materials have moderate resistivity which is practically independent of temperature so that the specified value of resistance does not alter with minor changes in temperature.

(4) **Fuse-wire :** Is made up of tin-lead alloy (63% tin + 37% lead). It should have low melting point and high resistivity. It is used in series as a safety device in an electric circuit and is designed so as to melt and thereby open the circuit if the current exceeds a predetermined value due to some fault. The function of a fuse is independent of its length.

Safe current of fuse wire relates with it's radius as $i \propto r^{3/2}$.

(5) **Thermistors** : A thermistor is a heat sensitive resistor usually prepared from oxides of various metals such as nickel, copper, cobalt, iron etc. These compounds are also semi-conductor. For thermistors α is very high which may be positive or negative. The resistance of thermistors changes very rapidly with change of temperature.
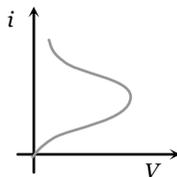
Thermistors are used to detect small temperature change and to measure very low temperature.

Concepts

☞ *In the absence of radiation loss, the time in which a fuse will melt does not depends on it's length but varies with radius as $t \propto r^4$.*

☞ *If length (l) and mass (m) of a conducting wire is given then $R \propto \dfrac{l^2}{m}$.*

☞ *Macroscopic form of Ohm's law is $R = \dfrac{V}{i}$, while it's microscopic form is $J = \sigma E$.*

Example

***Example*: 12**   Two wires of resistance R_1 and R_2 have temperature co-efficient of resistance α_1 and α_2 respectively. These are joined in series. The effective temperature co-efficient of resistance is

(a) $\dfrac{\alpha_1 + \alpha_2}{2}$ (b) $\sqrt{\alpha_1 \alpha_2}$ (c) $\dfrac{\alpha_1 R_1 + \alpha_2 R_2}{R_1 + R_2}$ (d) $\dfrac{\sqrt{R_1 R_2 \alpha_1 \alpha_2}}{\sqrt{R_1^2 + R_2^2}}$

Solution : (c)   Suppose at t^oC resistances of the two wires becomes R_{1t} and R_{2t} respectively and equivalent resistance becomes R_t. In series grouping $R_t = R_{1t} + R_{2t}$, also $R_{1t} = R_1(1 + \alpha_1 t)$ and $R_{2t} = R_2(1 + \alpha_2 t)$

$R_t = R_1(1 + \alpha_1 t) + R_2(1 + \alpha_2 t) = (R_1 + R_2) + (R_1\alpha_1 + R_2\alpha_2)t = (R_1 + R_2)\left[1 + \dfrac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2} t\right]$.

Hence effective temperature co-efficient is $\dfrac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$.

***Example*: 13**   From the graph between current i & voltage V shown, identity the portion corresponding to negative resistance

(a) *DE*

(b) *CD*

(c) *BC*

(d) *AB*
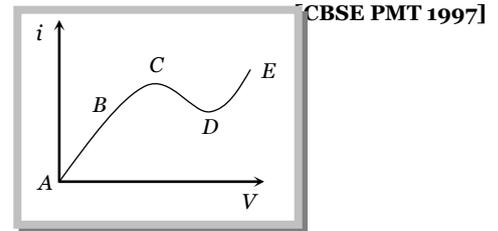
Solution : (b) $R = \dfrac{\Delta V}{\Delta I}$, in the graph *CD* has only negative slope. So in this portion R is negative.

***Example*: 14**   A wire of length L and resistance R is streched to get the radius of cross-section halfed. What is new resistance
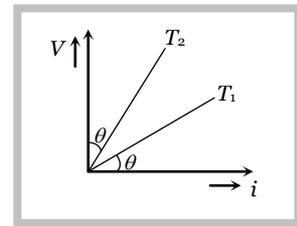
[NCERT 1974; CPMT 1994; AIIMS 1997; KCET 1999; Haryana PMT 2000; UPSEAT 2001]

(a) $5R$ (b) $8R$ (c) $4R$ (d) $16R$

Solution : (d)   By using $\dfrac{R_1}{R_2} = \left(\dfrac{r_2}{r_1}\right)^4$ $\Rightarrow$ $\dfrac{R}{R'} = \left(\dfrac{r/2}{r}\right)^4$ $\Rightarrow$ $R' = 16R$

***Example*: 15**   The V-i graph for a conductor at temperature T_1 and T_2 are as shown in the figure. $(T_2 - T_1)$ is proportional to

(a) $\cos 2\theta$

(b) $\sin \theta$

(c) $\cot 2\theta$

(d) $\tan \theta$

Solution : (c)   As we know, for conductors resistance $\propto$ Temperature.

From figure $R_1 \propto T_1 \Rightarrow \tan\theta \propto T_1 \Rightarrow \tan\theta = kT_1$ .... (i)   (k = constant)

and $R_2 \propto T_2 \Rightarrow \tan(90^o - \theta) \propto T_2 \Rightarrow \cot\theta = kT_2$ ....(ii)

From equation (i) and (ii) $k(T_2 - T_1) = (\cot\theta - \tan\theta)$

$(T_2 - T_1) = \left(\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}\right) = \dfrac{(\cos^2\theta - \sin^2\theta)}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = 2\cot 2\theta \Rightarrow (T_2 - T_1) \propto \cot 2\theta$

***Example*: 16**   The resistance of a wire at 20^oC is $20\ \Omega$ and at 500^oC is 60Ω. At which temperature resistance will be 25Ω

[UPSEAT 1999]

(a) 50^oC (b) 60^oC (c) 70^oC (d) 80^oC

Solution : (d)   By using $\dfrac{R_1}{R_2} = \dfrac{(1 + \alpha t_1)}{(1 + \alpha t_2)} \Rightarrow \dfrac{20}{60} = \dfrac{1 + 20\alpha}{1 + 500\alpha} \Rightarrow \alpha = \dfrac{1}{220}$

Again by using the same formula for 20Ω and $25\Omega \Rightarrow \dfrac{20}{25} = \dfrac{\left(1 + \dfrac{1}{220} \times 20\right)}{\left(1 + \dfrac{1}{220} \times t\right)} \Rightarrow t = 80^{o}C$

Example: 17 The specific resistance of manganin is 50×10^{-8} Ωm. The resistance of a manganin cube having length 50 cm is

(a) $10^{-6}\,\Omega$ (b) $2.5 \times 10^{-5}\,\Omega$ (c) $10^{-8}\,\Omega$ (d) $5 \times 10^{-4}\,\Omega$

Solution : (a) $R = \rho \dfrac{l}{A} = \dfrac{50 \times 10^{-8} \times 50 \times 10^{-2}}{(50 \times 10^{-2})^2} = 10^{-6}\,\Omega$

Example: 18 A rod of certain metal is 1 m long and 0.6 cm in diameter. It's resistance is $3 \times 10^{-3}\Omega$. A disc of the same metal is 1 mm thick and 2 cm in diameter, what is the resistance between it's circular faces.

(a) $1.35 \times 10^{-6}\Omega$ (b) $2.7 \times 10^{-7}\,\Omega$ (c) $4.05 \times 10^{-6}\Omega$ (d) $8.1 \times 10^{-6}\,\Omega$

Solution : (b)   By using $R = \rho . \dfrac{l}{A}; \dfrac{R_{\text{disc}}}{R_{\text{rod}}} = \dfrac{l_{\text{disc}}}{l_{\text{rod}}} \times \dfrac{A_{\text{rod}}}{A_{\text{disc}}} \Rightarrow \dfrac{R_{\text{disc}}}{3 \times 10^{-3}} = \dfrac{10^{-3}}{1} \times \dfrac{\pi(0.3 \times 10^{-2})^2}{\pi(10^{-2})^2} \Rightarrow R_{\text{disc}} = 2.7 \times 10^{-7}\Omega.$

Example: 19 An aluminium rod of length 3.14 m is of square cross-section 3.14×3.14 mm^2. What should be the radius of 1 m long another rod of same material to have equal resistance

(a) 2 mm (b) 4 mm (c) 1 mm (d) 6 mm

Solution : (c)   By using $R = \rho . \dfrac{l}{A} \Rightarrow l \propto A \Rightarrow \dfrac{3.14}{1} = \dfrac{3.14 \times 3.14 \times 10^{-6}}{\pi \times r^2} \Rightarrow r = 10^{-3}\,m = 1\,mm$

Example: 20 Length of a hollow tube is $5m$, it's outer diameter is 10 cm and thickness of it's wall is 5 mm. If resistivity of the material of the tube is 1.7×10^{-8} $\Omega \times m$ then resistance of tube will be
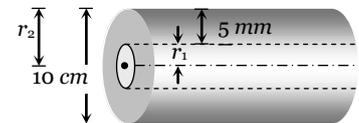
(a) $5.6 \times 10^{-5}\,\Omega$ (b) $2 \times 10^{-5}\,\Omega$ (c) $4 \times 10^{-5}\,\Omega$ (d) None of these

Solution : (a)   By using $R = \rho . \dfrac{l}{A}$; here $A = \pi(r_2^2 - r_1^2)$

Outer radius $r_2 = 5cm$

Inner radius $r_1 = 5 - 0.5 = 4.5$ cm

So $R = 1.7 \times 10^{-8} \times \dfrac{5}{\pi\{(5 \times 10^{-2})^2 - (4.5 \times 10^{-2})^2\}} = 5.6 \times 10^{-5}\,\Omega$

Example: 21 If a copper wire is stretched to make it 0.1% longer, the percentage increase in resistance will be

                                                  [MP PMT 1996, 2000; UPSEAT 1998; MNR 1990]

(a) 0.2 (b) 2 (c) 1 (d) 0.1

Solution : (a)   In case of streching $R \propto l^2$ So $\dfrac{\Delta R}{R} = 2\dfrac{\Delta l}{l} = 2 \times 0.1 = 0.2$

Example: 22 The temperature co-efficient of resistance of a wire is $0.00125/^{o}C$. At 300 K. It's resistance is 1Ω. The resistance of the wire will be 2Ω at   **[MP PMT 2001; IIT 1980]**

(a) 1154 K (b) 1127 K (c) 600 K (d) 1400 K

Solution: (b)   By using $R_t = R_0 (1 + \alpha \Delta t) \Rightarrow \dfrac{R_1}{R_2} = \dfrac{1 + \alpha t_1}{1 + \alpha t_2}$ So $\dfrac{1}{2} = \dfrac{1 + (300 - 273)\alpha}{1 + \alpha t_2} \Rightarrow t_2 = 854^{o}C = 1127$ K

***Example*: 23**   Equal potentials are applied on an iron and copper wire of same length. In order to have same current flow in the wire, the ratio $\left(\dfrac{r_{iron}}{r_{copper}}\right)$ of their radii must be [Given that specific resistance of iron = 1.0 × $10^{-7}\ \Omega m$ and that of copper = 1.7 × $10^{-8}\ \Omega m$]   **[MP PMT 2000]**

     (a) About 1.2   (b) About 2.4   (c) About 3.6   (d) About 4.8

Solution: (b) V = constant., i = constant.   So R = constant

$$\Rightarrow\ \frac{P_i l_i}{A_i} = \frac{\rho_{Cu} l_{Cu}}{A_{Cu}} \ \Rightarrow \frac{\rho_i l_i}{r_i^2} = \frac{\rho_{Cu} l_{Cu}}{r_{Cu}^2}$$

$$\Rightarrow \frac{r_i}{r_{Cu}} = \sqrt{\frac{\rho_i}{\rho_{Cu}}} = \sqrt{\frac{1.0\times 10^{-7}}{1.7\times 10^{-8}}} = \sqrt{\frac{100}{17}} \approx 2.4$$

***Example*: 24**   Masses of three wires are in the ratio 1 : 3 : 5 and their lengths are in the ratio 5 : 3 : 1. The ratio of their electrical resistance is

     (a) 1 : 3 : 5   (b) 5 : 3 : 1   (c) 1 : 15 : 125   (d) 125 : 15 : 1

Solution: (d) $R = \rho\dfrac{l}{A} = \rho\dfrac{l^2}{V} = \rho\dfrac{l^2}{m}\sigma \qquad \left(\because \sigma = \dfrac{m}{V}\right)$

$$R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = 25 : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

***Example*: 25**   Following figure shows cross-sections through three long conductors of the same length and material, with square cross-section of edge lengths as shown. Conductor *B* will fit snugly within conductor *A*, and conductor *C* will fit snugly within conductor *B*. Relationship between their end to end resistance is

     (a) $R_A = R_B = R_C$

     (b) $R_A > R_B > R_C$

     (c) $R_A < R_B < R$

     (d) Information is not sufficient
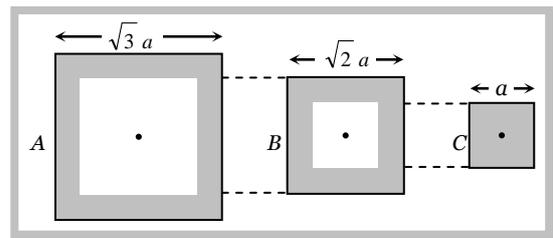
Solution : (a)   All the conductors have equal lengths. Area of cross-section of *A* is $\left\{(\sqrt{3}\,a)^2 - (\sqrt{2}\,a)^2\right\} = a^2$

     Similarly area of cross-section of *B* = Area of cross-section of *C* = a^2

     Hence according to formula $R = \rho\dfrac{l}{A}$; resistances of all the conductors are equal *i.e.* $R_A = R_B = R_C$

***Example*: 26**   Dimensions of a block are 1 *cm* × 1 *cm* × 100 *cm*. If specific resistance of its material is 3 × 10^{-7} *ohm-m*, then the resistance between it's opposite rectangular faces is

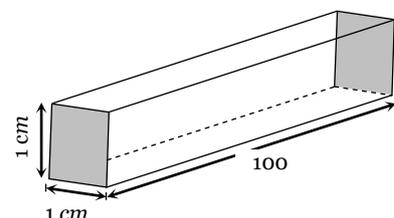   (a) 3 × 10^{-9} *ohm*   (b) 3 × 10^{-7} *ohm*   (c) 3 × 10^{-5} *ohm*   (d) 3 × 10^{-3} *ohm*

Solution: (b)   Length $l = 1\ cm = 10^{-2}\ m$

     Area of cross-section $A = 1\ cm \times 100\ cm$

                     $= 100\ cm^2 = 10^{-2}\ m^2$

     Resistance $R = 3\times 10^{-7} \times \dfrac{10^{-2}}{10^{-2}} = 3\times 10^{-7}\ \Omega$

Note :❏ In the above question for calculating equivalent resistance between two opposite square faces.

$$l = 100 \ cm = 1 \ m, A = 1 \ cm^2 = 10^{-4} \ m^2, \text{ so resistance } R = 3 \times 10^{-7} \times \frac{1}{10^{-4}} = 3 \times 10^{-3} \ \Omega$$

3. Two rods A and B of same material and length have their electric resistances are in ratio 1 : 2. When both the rods are dipped in water, the correct statement will be   **[RPMT 1997]**

    (a) A has more loss of weight   (b) B has more loss of weight

    (c) Both have same loss of weight   (d) Loss of weight will be in the ratio 1 : 2

Solution: (a) $R = \rho \dfrac{L}{A} \Rightarrow \dfrac{R_1}{R_2} = \dfrac{A_2}{A_1}$ (ρ, L constant) $\Rightarrow \dfrac{A_1}{A_2} = \dfrac{R_2}{R_1} = 2$

Now when a body dipped in water, loss of weight $= V\sigma_L g = AL\sigma_L g$

So $\dfrac{(\text{Loss of weight})_1}{(\text{Loss of weight})_2} = \dfrac{A_1}{A_2} = 2$; So A has more loss of weight

The *V-i* graph for a conductor makes an angle θ with *V*-axis. Here V denotes the voltage and i denotes current. The resistance of conductor is given by

    (a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$

Solution: (d)   At an instant approach the student will choose $\tan\theta$ will be the right answer. But it is to be seen here the curve makes the angle θ with the *V*-axis. So it makes an angle $(90 - \theta)$ with the *i*-axis. So resistance = slope = $\tan(90 - \theta) = \cot\theta$.
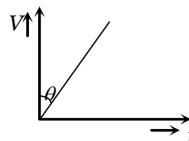
Colour Coding of Resistance

The resistance, having high values are used in different electrical and electronic circuits. They are generally made up of carbon, like 1 $k\Omega$, 2 $k\Omega$, 5 $k\Omega$ *etc*. To know the value of resistance colour code is used. These code are printed in form of set of rings or strips. By reading the values of colour bands, we can estimate the value of resistance.

The carbon resistance has normally four coloured rings or strips say A, B, C and D as shown in following figure.
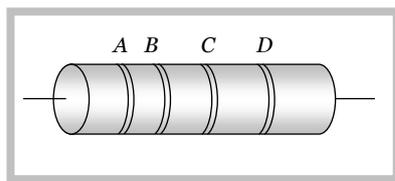
Colour band A and B indicate the first two significant figures of resistance in *ohm*, while the C band gives the decimal multiplier *i.e.* the number of zeros that follows the two significant figures A and B.

Last band (D band) indicates the tolerance in percent about the indicated value or in other ward it represents the percentage accuracy of the indicated value.

The tolerance in the case of gold is ± 5% and in silver is ± 10%. If only three bands are marked on carbon resistance, then it indicate a tolerance of 20%.

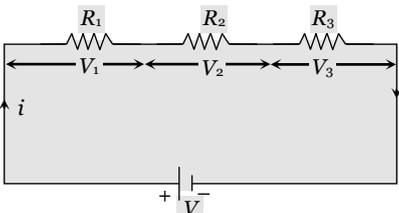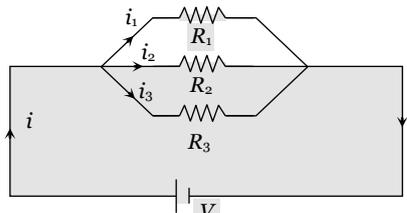The following table gives the colour code for carbon resistance.

Letters as an aid to memory	Colour	Figure (A, B)	Multiplier (C)
B	Black	0	10^0
B	Brown	1	10^1
R	Red	2	10^2
O	Orange	3	10^3
Y	Yellow	4	10^4
G	Green	5	10^5
B	Blue	6	10^6
V	Violet	7	10^7
G	Grey	8	10^8
W	White	9	10^9

Colour	Tolerance (D)
Gold	5%
Silver	10%
No-colour	20%

Note : ❑   To remember the sequence of colour code following sentence should kept in memory.

B B R O Y *Great* **B**ritain *Very* **G**ood **W**ife.

Grouping of Resistance

Series	Parallel
(1)	(1)
(2) Same current flows through each resistance but potential difference distributes in the ratio of resistance *i.e.* $V \propto R$ Power consumed are in the ratio of their resistance *i.e.* $P \propto R \Rightarrow P_1 : P_2 : P_3 = R_1 : R_2 : R_3$	(2) Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance *i.e.* $i \propto \dfrac{1}{R}$ Power consumed are in the reverse ratio of resistance *i.e.* $P \propto \dfrac{1}{R} \Rightarrow P_1 : P_2 : P_3 = \dfrac{1}{R_1} : \dfrac{1}{R_2} : \dfrac{1}{R_3}$
(3) $R_{eq} = R_1 + R_2 + R_3$ equivalent resistance is greater than the maximum value of resistance in the combination.	(3) $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ or $R_{eq} = (R_1^{-1} + R_2^{-1} + R_3^{-1})^{-1}$ or $\quad R_{eq} = \dfrac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_2 R_1}$ equivalent resistance is smaller than the minimum value of resistance in the combination.
(4) For two resistance in series $R_{eq} = R_1 + R_2$	(4) For two resistance in parallel $R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{\text{Multiplication}}{\text{Addition}}$

(5) Potential difference across any resistance $V' = \left(\dfrac{R'}{R_{eq}} \right) \cdot V$

Where R' = Resistance across which potential difference is to be calculated, R_{eq} = equivalent resistance of that line in which R' is connected, V = p.d. across that line in which R' is connected
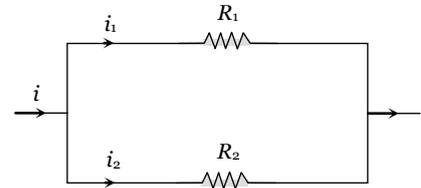
e.g.
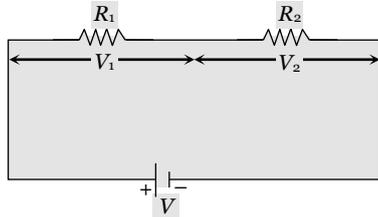
$V_1 = \left(\dfrac{R_1}{R_1 + R_2} \right) \cdot V$ and $V_2 = \left(\dfrac{R_2}{R_1 + R_2} \right) \cdot V$

(6) If n identical resistance are connected in series

$R_{eq} = nR$ and p.d. across each resistance $V' = \dfrac{V}{n}$

(5) Current through any resistance

$i' = i \times \left[\dfrac{\text{Resistance of opposite branch}}{\text{Total resistance}} \right]$

Where i' = required current (branch current)

i = main current

$i_1 = i \left(\dfrac{R_2}{R_1 + R_2} \right)$ and $i_2 = i \left(\dfrac{R_1}{R_1 + R_2} \right)$

(6) In n identical resistance are connected in parallel

$R_{eq} = \dfrac{R}{n}$ and current through each resistance $i' = \dfrac{i}{n}$

Note : ❑ In case of resistances in series, if one resistance gets open, the current in the whole circuit become zero and the circuit stops working. Which don't happen in case of parallel gouging.

❑ Decoration of lightning in festivals is an example of series grouping whereas all household appliances connected in parallel grouping.

❑ Using n conductors of equal resistance, the number of possible combinations is 2^{n-1}.

❑ If the resistance of n conductors are totally different, then the number of possible combinations will be 2^n.

Methods of Determining Equivalent Resistance For Some Difficult Networks

(1) **Method of successive reduction :** It is the most common technique to determine the equivalent resistance. So far, we have been using this method to find out the equivalent resistances. This method is applicable only when we are able to identify resistances in series or in parallel. The method is based on the simplification of the circuit by successive reduction of the series and parallel combinations. For example to calculate the equivalent resistance between the point A and B, the network shown below successively reduced.
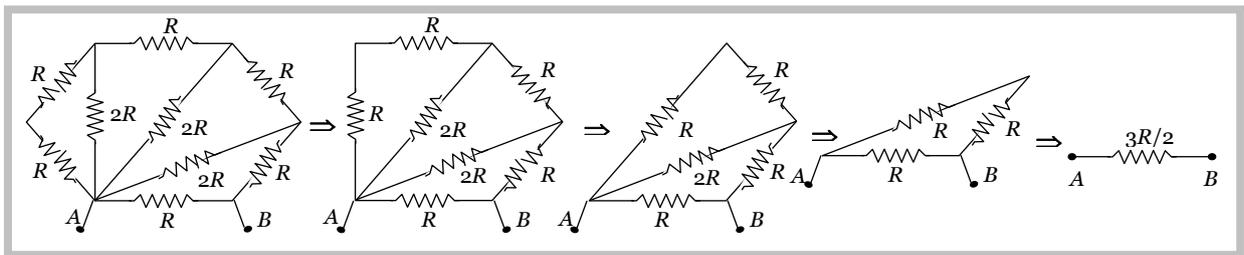

(2) **Method of equipotential points :** This method is based on identifying the points of same potential and joining them. The basic rule to identify the points of same potential is the symmetry of the network.

(i) In a given network there may be two axes of symmetry.

(a) Parallel axis of symmetry, that is, along the direction of current flow.

(b) Perpendicular axis of symmetry, that is perpendicular to the direction of flow of current.

For example in the network shown below the axis AA' is the parallel axis of symmetry, and the axis BB' is the perpendicular axis of symmetry.
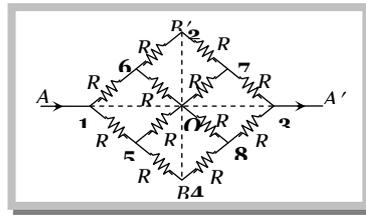
(ii) Points lying on the perpendicular axis of symmetry may have same potential. In the given network, point 2, 0 and 4 are at the same potential.
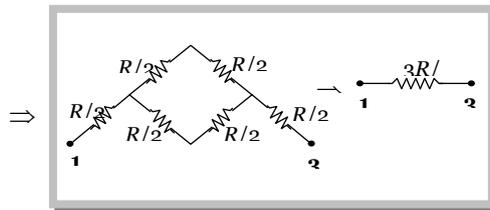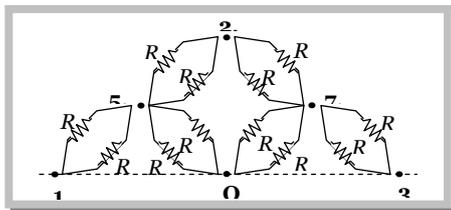
(iii) Points lying on the parallel axis of symmetry can never have same potential.

(iv) The network can be folded about the parallel axis of symmetry, and the overlapping nodes have same potential. Thus as shown in figure, the following points have same potential

(a) 5 and 6   (b) 2, 0 and 4   (c) 7 and 8

Note : ❑ Above network may be split up into two equal parts about the parallel axis of symmetry as

shown in figure each part has a resistance R', then the equivalent resistance of the network will be $R = \dfrac{R'}{2}$.
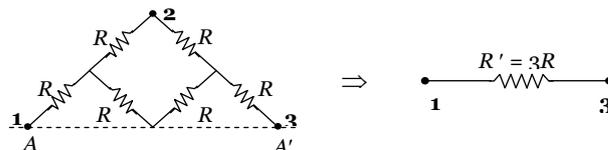
Some Standard Results for Equivalent Resistance

(1) **Equivalent resistance between points A and B in an unbalanced Wheatstone's bridge as shown in the diagram.**

(i)	(ii)
$R_{AB} = \dfrac{PQ(R+S)+(P+Q)RS+G(P+Q)(R+S)}{G(P+Q+R+S)+(P+R)(Q+S)}$	$R_{AB} = \dfrac{2PQ+G(P+Q)}{2G+P+Q}$

(2) **A cube each side have resistance R then equivalent resistance in different situations**

(i) Between E and C i.e. across the diagonal of the cube $R_{EC} = \dfrac{5}{6}R$
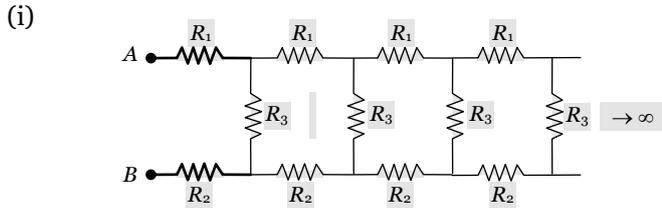
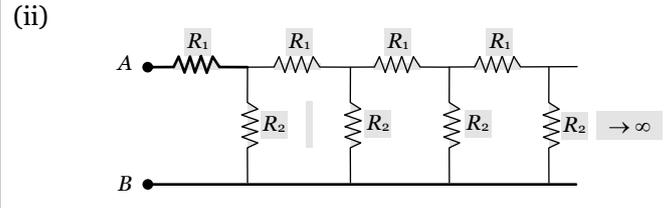(ii) Between A and B i.e. across one side of the cube $R_{AB} = \dfrac{7}{12}R$
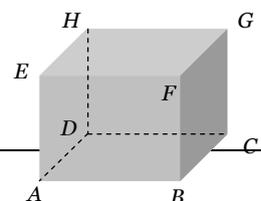
(iii) Between A and C *i.e.* across the diagonal of one face of the cube $R_{AC} = \dfrac{3}{4}R$

---

(3) The equivalent resistance of infinite network of resistances

(i)

$$R_{AB} = \frac{1}{2}(R_1 + R_2) + \frac{1}{2}\left[(R_1 + R_2)^2 + 4R_3(R_1 + R_2)\right]^{1/2}$$

(ii)

$$R_{AB} = \frac{1}{2}R_1\left[1 + \sqrt{1 + 4\left(\frac{R_2}{R_1}\right)}\right]$$

Concepts

☞ *If n identical resistances are first connected in series and then in parallel, the ratio of the equivalent resistance is given by*

$$\frac{R_p}{R_s} = \frac{n^2}{1}.$$

☞ *If equivalent resistance of R_1 and R_2 in series and parallel be R_s and R_p respectively then* $R_1 = \dfrac{1}{2}\left[R_s + \sqrt{R_s^2 - 4R_s R_p}\right]$

and $R_2 = \dfrac{1}{2}\left[R_s - \sqrt{R_s^2 - 4R_s R_p}\right].$

☞ *If a wire of resistance R, cut in n equal parts and then these parts are collected to form a bundle then equivalent resistance of combination will be* $\dfrac{R}{n^2}.$

Example

Example: 27   In the figure a carbon resistor has band of different colours on its body. The resistance of the following body is

(a) $2.2\ k\Omega$

(b) $3.3\ k\Omega$

(c) $5.6\ k\Omega$

(d) $9.1\ k\Omega$l
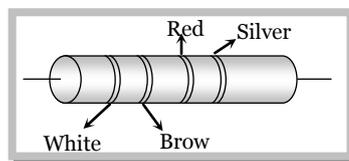
Solution : (d) $R = 91 \times 10^2 \pm 10\% \approx 9.1\ k\Omega$

Example: 28   What is the resistance of a carbon resistance which has bands of colours brown, black and brown **[DCE 1999]**
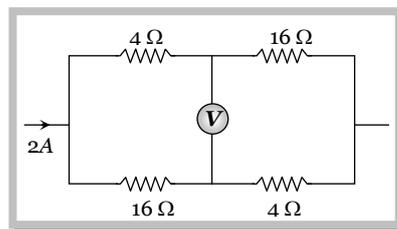
(a) $100\ \Omega$ (b) $1000\ \Omega$ (c) $10\ \Omega$ (d) $1\ \Omega$

Solution : (a) $R = 10 \times 10^1 \pm 20\% \approx 100\ \Omega$

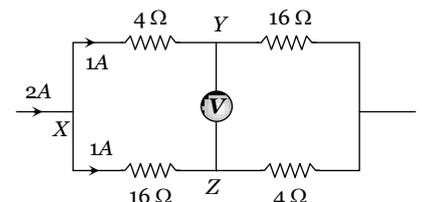**Example: 29**   In the following circuit reading of voltmeter V is   **[MP PET 2003]**

(a) $12\ V$

(b) $8\ V$

(c) $20\ V$

(d) $16\ V$

Solution : (a)   P.d. between X and Y is $V_{XY} = V_X - V_Y = 1 \times 4 = 4\ V$ .... (i)

and p.d. between X and Z is $V_{XZ} = V_X - V_Z = 1 \times 16 = 16\ V$ .... (ii)

On solving equations (i) and (ii) we get potential difference between Y and Z i.e., reading of voltmeter is $V_Y - V_Z = 12V$

Example: 30 An electric cable contains a single copper wire of radius 9 *mm*. It's resistance is 5 Ω. This cable is replaced by six insulated copper wires, each of radius 3 *mm*. The resultant resistance of cable will be **[CPMT 198**

(a) 7.5 Ω (b) 45 Ω (c) 90 Ω (d) 270 Ω

Solution : (a)   Initially : Resistance of given cable

$$R = \rho \frac{l}{\pi \times (9 \times 10^{-3})^2} \qquad ..... \text{(i)}$$

Finally : Resistance of each insulated copper wire is

$$R' = \rho \frac{l}{\pi \times (3 \times 10^{-3})^2}$$

Hence equivalent resistance of cable

$$R_{eq} = \frac{R'}{6} = \frac{1}{6} \times \left(\rho \frac{l}{\pi \times (3 \times 10^{-3})^2} \right) \quad ..... \text{(ii)}$$
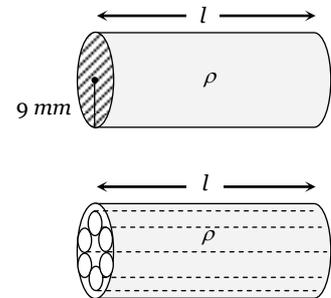
On solving equation (i) and (ii) we get R_{eq} = 7.5 Ω

Example: 31 Two resistance R_1 and R_2 provides series to parallel equivalents as $\frac{n}{1}$ then the correct relationship is

(a) $\left(\frac{R_1}{R_2} \right)^2 + \left(\frac{R_2}{R_1} \right)^2 = n^2$

(b) $\left(\frac{R_1}{R_2} \right)^{3/2} + \left(\frac{R_2}{R_1} \right)^{3/2} = n^{3/2}$

(c) $\left(\frac{R_1}{R_2} \right) + \left(\frac{R_2}{R_1} \right) = n$

(d) $\left(\frac{R_1}{R_2} \right)^{1/2} + \left(\frac{R_2}{R_1} \right)^{1/2} = n^{1/2}$
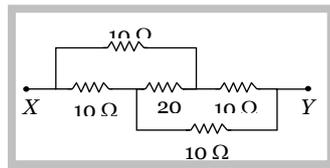
Solution : (d)   Series resistance $R_S = R_1 + R_2$ and parallel resistance $R_P = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow \frac{R_S}{R_P} = \frac{(R_1 + R_2)^2}{R_1 R_2} = n$

$$\Rightarrow \frac{R_1 + R_2}{\sqrt{R_1 R_2}} = \sqrt{n} \qquad \Rightarrow \frac{\sqrt{R_1^2}}{\sqrt{R_1 R_2}} + \frac{\sqrt{R_2^2}}{\sqrt{R_1 R_2}} = \sqrt{n} \Rightarrow \sqrt{\frac{R_1}{R_2}} + \sqrt{\frac{R_2}{R_1}} = \sqrt{n}$$

Example: 32 Five resistances are combined according to the figure. The equivalent resistance between the point X and Y will   be

(a) 10 Ω
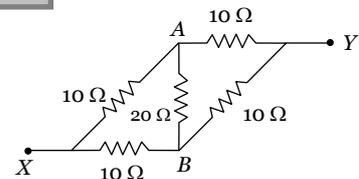
(b) 22 Ω

(c) 20 Ω

(d) 50 Ω

Solution : (a)   The equivalent circuit of above can be drawn as

Which is a balanced wheatstone bridge.
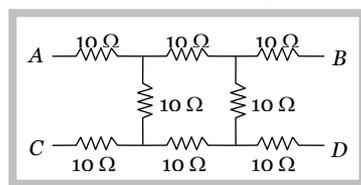
So current through AB is zero.

So $\frac{1}{R} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \Rightarrow R = 10$ Ω

Example: 33 What will be the equivalent resistance of circuit shown in figure between points A and D **[CBSE PMT 1996]**
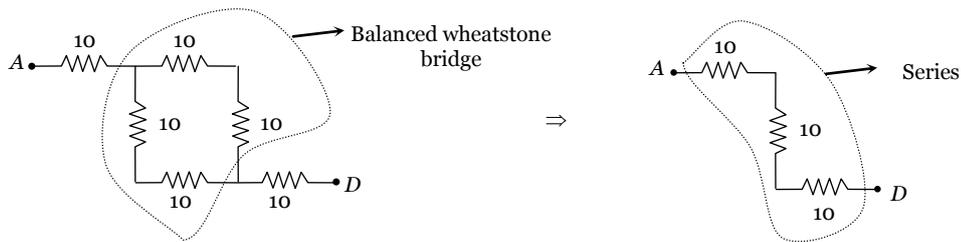
(a) 10 Ω

(b) 20 Ω

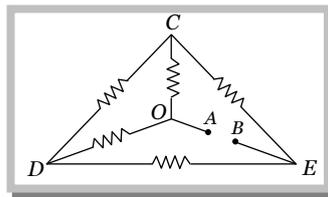(c) 30 Ω

(d) 40 Ω

*Solution* : (c)   The equivalent circuit of above fig between *A* and *D* can be drawn as

So $R_{eq} = 10 + 10 + 10 = 30\Omega$

Example: 34   In the network shown in the figure each of resistance is equal to 2Ω. The resistance between *A* and *B* is

[CBSE PMT 1995]

(a) 1 Ω
(b) 2 Ω
(c) 3 Ω
(d) 4 Ω

Solution : (b)   Taking the portion *COD* is figure to outside the triangle (left), the above circuit will be now as resistance of each is 2 Ω the circuit will behaves as a balanced wheatstone bridge and no current flows through *CD*. Hence $R_{AB} = 2\Omega$
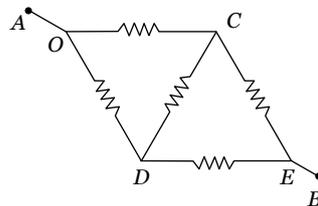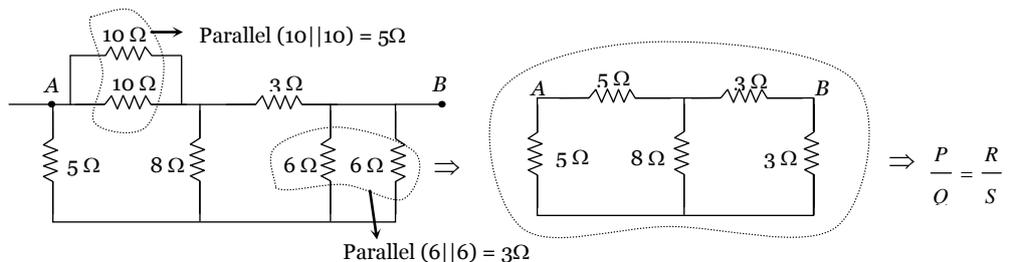

Example: 35   Seven resistances are connected as shown in figure. The equivalent resistance between *A* and *B* is   **[MP PET 2000]**
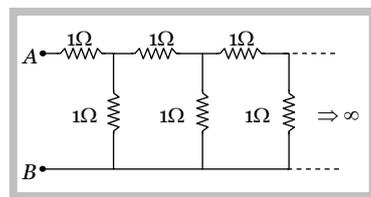
(a) 3 Ω
(b) 4 Ω
(c) 4.5 Ω
(d) 5 Ω
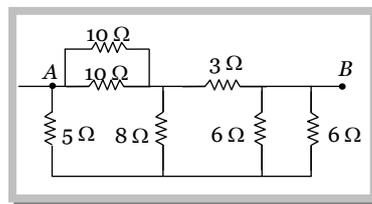
Solution : (b)

So the circuit is a balanced wheatstone bridge.
So current through 8Ω is zero $R_{eq} = (5 + 3) || (5 + 3) = 8 || 8 = 4\Omega$

Example: 36   The equivalent resistance between points *A* and *B* of an infinite network of resistance, each of 1 Ω, connected as shown is   **[CEE Haryana 1996]**
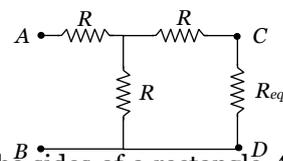
(a) Infinite
(b) 2 Ω

(c) $\dfrac{1+\sqrt{5}}{2}\,\Omega$

(d) Zero

Solution : (c)   Suppose the effective resistance between A and B is R_{eq}. Since the network consists of infinite cell. If we exclude one cell from the chain, remaining network have infinite cells *i.e.* effective resistance between C and D will also R_{eq}
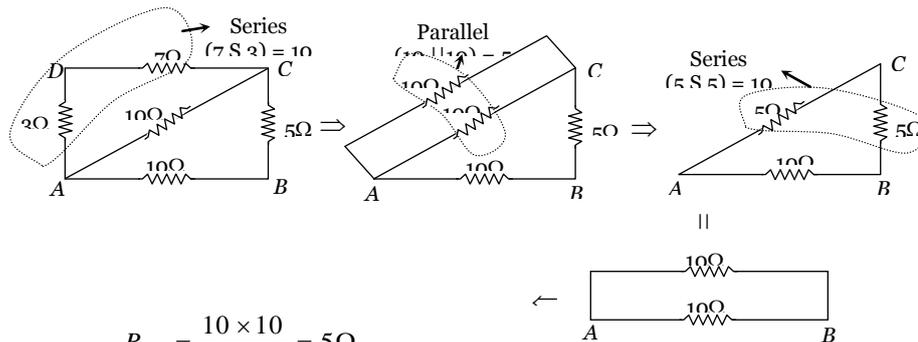
So now $R_{eq} = R_o + (R || R_{eq}) = R + \dfrac{R\,R_{eq}}{R + R_{eq}} \Rightarrow R_{eq} = \dfrac{1}{2}[1 + \sqrt{5}]$

Example: 37   Four resistances 10 Ω, 5 Ω, 7 Ω and 3 Ω are connected so that they form the sides of a rectangle AB, BC, CD and DA respectively. Another resistance of 10 Ω is connected across the diagonal AC. The equivalent resistance between A & B is
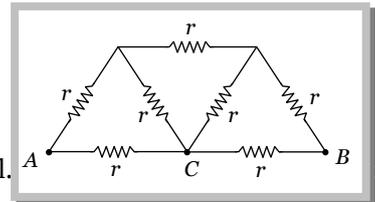
(a) 2 Ω   (b) 5 Ω   (c) 7 Ω   (d) 10 Ω
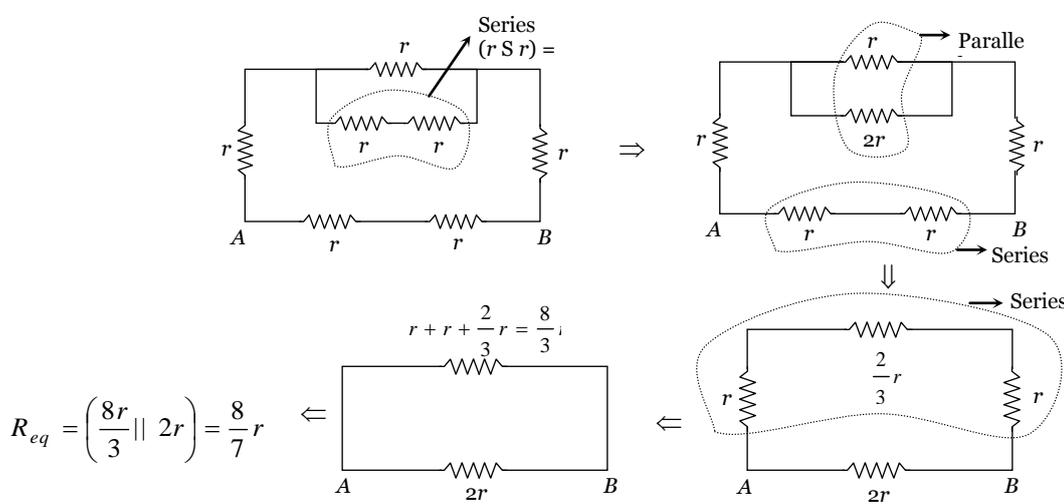
Solution : (b)

So $R_{eq} = \dfrac{10 \times 10}{10 + 10} = 5\Omega$

Example: 38   The equivalent resistance between A and B in the circuit will be

(a) $\dfrac{5}{4}r$ (b) $\dfrac{6}{5}r$ (c) $\dfrac{7}{6}r$ (d) $\dfrac{8}{7}r$

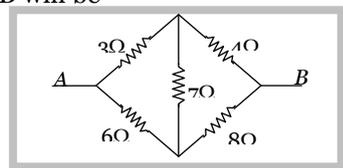*Solution* : (d)   In the circuit, by means of symmetry the point C is at zero potential. be drawn as

$R_{eq} = \left(\dfrac{8r}{3} || 2r\right) = \dfrac{8}{7}r$

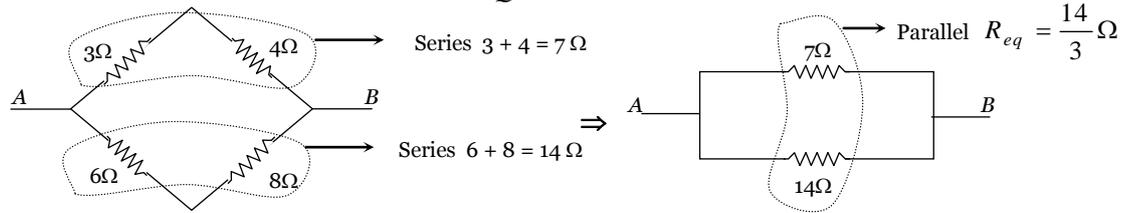**Example: 39**   In the given figure, equivalent resistance between A and B will be   **[CBSE PMT 2000]**

(a) $\dfrac{14}{3}\,\Omega$ (b) $\dfrac{3}{14}\,\Omega$

(c) $\dfrac{9}{14}\,\Omega$ (d) $\dfrac{14}{9}\,\Omega$

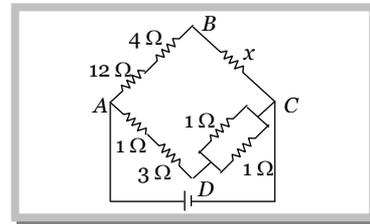*Solution* : (a) Given Wheatstone bridge is balanced because $\dfrac{P}{Q} = \dfrac{R}{S}$. Hence the circuit can be redrawn as follows

Series 3 + 4 = 7 Ω

Series 6 + 8 = 14 Ω

Parallel $R_{eq} = \dfrac{14}{3}\Omega$

Example: 40 In the combination of resistances shown in the figure the potential difference between *B* and *D* is zero, when unknown resistance (*x*) is
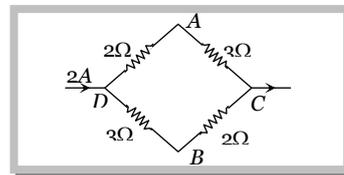
(a) 4 Ω

(b) 2 Ω

(c) 3 Ω

(d) The emf of the cell is required

Solution : (b) The potential difference across *B*, *D* will be zero, when the circuit will act as a balanced wheatstone bridge and $\dfrac{P}{Q} = \dfrac{R}{S} \Rightarrow \dfrac{12+4}{x} = \dfrac{1+3}{1/2} \Rightarrow x = 2\Omega$

Example: 41 A current of 2 *A* flows in a system of conductors as shown. The potential difference ($V_A - V_B$) will be

(a) + 2*V*

(b) + 1*V*

(c) − 1 *V*

(d) − 2 *V*

[CPMT 1975, 76]

Solution : (b) In the given circuit 2*A* current divides equally at junction *D* along the paths *DAC* and *DBC* (each path carry 1*A* current).
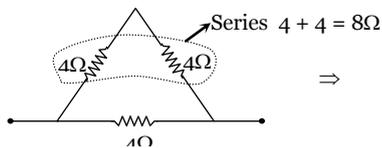
Potential difference between *D* and *A*, $\qquad V_D - V_A = 1 \times 2 = 2\ volt$ .... (i)

Potential difference between *D* and *B*, $\qquad V_D - V_B = 1 \times 3 = 3\ volt$ ..... (ii)

On solving (i) and (ii) $V_A - V_B = + 1\ volt$

Example: 42 Three resistances each of 4 Ω are connected in the form of an equilateral triangle. The effective resistance between two corners is

(a) 8 Ω (b) 12 Ω (c) $\dfrac{3}{8}\Omega$ (d) $\dfrac{8}{3}\Omega$

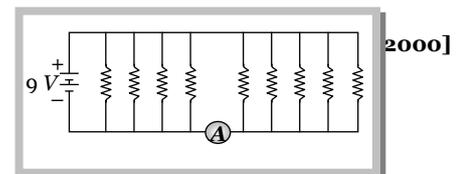*Solution* : (d)

Series 4 + 4 = 8Ω

$\Rightarrow$ On Solving further we get equivalent resistance is $\dfrac{8}{3}\Omega$

Example: 43 If each resistance in the figure is of 9 Ω then reading of ammeter is

(a) 5 *A* (b) 8 *A*

(c) 2 *A* (d) 9 *A*

2000]

Solution : (a) Main current through the battery $i = \dfrac{9}{1} = 9A$. Current through each resistance will be 1*A* and only 5 resistances on the right side of ammeter contributes for passing current through the ammeter. So reading of ammeter will be 5*A*.

Example: 44 A wire has resistance 12 Ω. It is bent in the form of a circle. The effective resistance between the two points on any diameter is equal to

(a) 12 Ω (b) 6 Ω (c) 3 Ω (d) 24 Ω

Solution : (c) Equivalent resistance of the following circuit will be

6Ω

6Ω
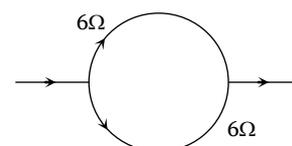
$$R_{eq} = \frac{6}{2} = 3\Omega$$

Example: 45 A wire of resistance 0.5 Ωm^{-1} is bent into a circle of radius 1 m. The same wire is connected across a diameter AB as shown in fig. The equivalent resistance is
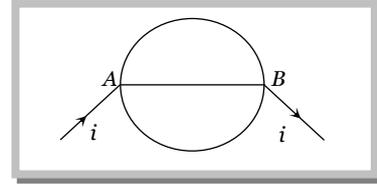
(a) $\pi\, ohm$

(b) $\pi(\pi+2)\, ohm$
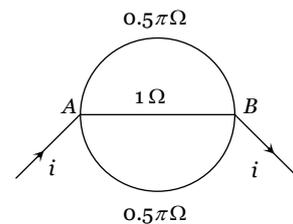
(c) $\pi/(\pi+4)\, ohm$

(d) $(\pi+1)\, ohm$

Solution : (c) Resistance of upper semicircle = Resistance of lower semicircle

$$= 0.5 \times (\pi R) = 0.5\ \pi\Omega$$
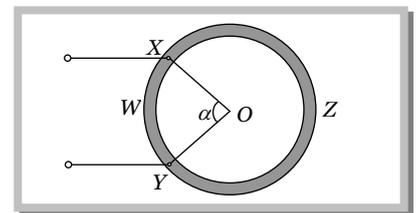
Resistance of wire AB $= 0.5 \times 2 = 1\ \Omega$

Hence equivalent resistance between A and B

$$\frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{1} + \frac{1}{0.5\pi} \Rightarrow R_{AB} = \frac{\pi}{(\pi+4)}\Omega$$

Example: 46 A wire of resistor R is bent into a circular ring of radius r. Equivalent resistance between two points X and Y on its circumference, when angle XOY is α, can be given by

(a) $\frac{R\alpha}{4\pi^2}(2\pi-\alpha)$ (b) $\frac{R}{2\pi}(2\pi-\alpha)$

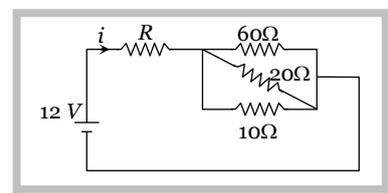(c) $R(2\pi-\alpha)$ (d) $\frac{4\pi}{R\alpha}(2\pi-\alpha)$

Solution : (a) Here $R_{XWY} = \frac{R}{2\pi r}\times(r\alpha) = \frac{R\alpha}{2\pi}$ $\left(\because \alpha = \frac{l}{r}\right)$ and $R_{XZY} = \frac{R}{2\pi r}\times r(2\pi-\alpha) = \frac{R}{2\pi}(2\pi-\alpha)$

$$R_{eq} = \frac{R_{XWY}R_{XZY}}{R_{XWY}+R_{XZY}} = \frac{\frac{R\alpha}{2\pi}\times\frac{R}{2\pi}(2\pi-\alpha)}{\frac{R\alpha}{2\pi}+\frac{R(2\pi-\alpha)}{2\pi}} = \frac{R\alpha}{4\pi^2}(2\pi-\alpha)$$

Example: 47 If in the given figure $i = 0.25\ amp$, then the value R will be **[RPET 2000]**

(a) $48\ \Omega$

(b) $12\ \Omega$

(c) $120\ \Omega$

(d) $42\ \Omega$

Solution : (d) $i = 0.25\ amp\ V = 12\ V$ $R_{eq} = \frac{V}{i} = \frac{12}{0.25} = 48\ \Omega$

Now from the circuit $R_{eq} = R + (60\,||\,20\,||\,10)$

$$= R + 6$$

$$\Rightarrow R = R_{eq} - 6 = 48 - 6 = 42\ \Omega$$
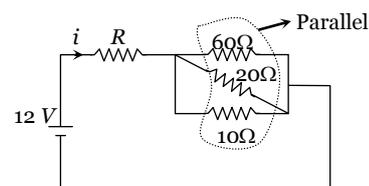
Example: 48 Two uniform wires A and B are of the same metal and have equal masses. The radius of wire A is twice that of wire B. The total resistance of A and B when connected in parallel is **[MNR 1994]**

(a) $4\ \Omega$ when the resistance of wire A is $4.25\ \Omega$ (b) $5\ \Omega$ when the resistance of wire A is $4\ \Omega$

(c) $4\ \Omega$ when the resistance of wire B is $4.25\ \Omega$ (d) $5\ \Omega$ when the resistance of wire B is $4\ \Omega$

Solution : (a) Density and masses of wire are same so their volumes are same *i.e.* $A_1 l_1 = A_2 l_2$

Ratio of resistances of wires A and B $\dfrac{R_A}{R_B} = \dfrac{l_1}{l_2} \times \dfrac{A_2}{A_1} = \left(\dfrac{A_2}{A_1}\right)^2 = \left(\dfrac{r_2}{r_1}\right)^4$

Since $r_1 = 2r_2$ so $\dfrac{R_A}{R_B} = \dfrac{1}{16} \Rightarrow R_B = 16\,R_A$

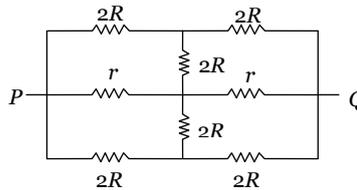Resistance R_A and R_B are connected in parallel so equivalent resistance $R = \dfrac{R_A R_B}{R_A + R_B} = \dfrac{16 R_A}{17}$, By checking correctness of equivalent resistance from options, only option (a) is correct.

Tricky Example: 5

The effective resistance between point P and Q of the electrical circuit shown in the figure is

[IIT-JEE 1991]

(a) $\dfrac{2Rr}{R+r}$ (b) $\dfrac{8R(R+r)}{3R+r}$ (c) $2r + 4R$ (d) $\dfrac{5R}{2} + 2r$

Solution : (a)   The points A, O, B are at same potential. So the figure can be redrawn as follows

$R_{eq} = \dfrac{2Rr}{R+r} \quad \Leftarrow \quad 4R \parallel 2r \parallel 4R \quad \Leftarrow$
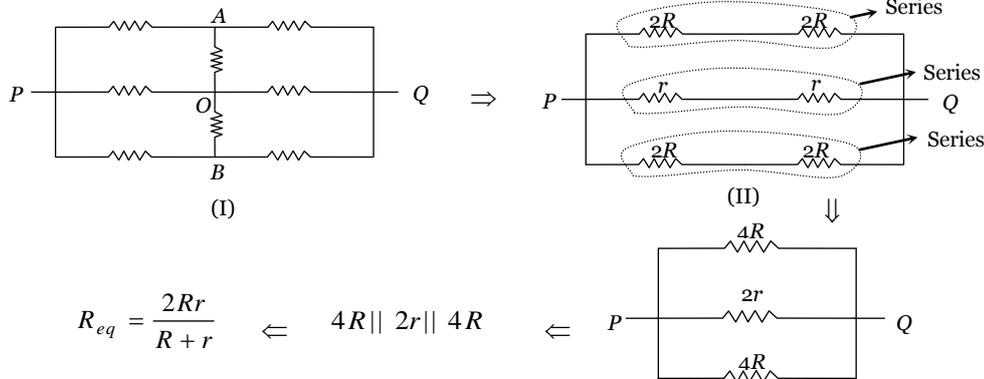
Tricky Example: 6

In the following circuit if key K is pressed then the galvanometer reading becomes half. The resistance of galvanometer is

$S = 40\ \Omega$

(a) $20\ \Omega$ (b) $30\ \Omega$ (c) $40\ \Omega$ (d) $50\ \Omega$
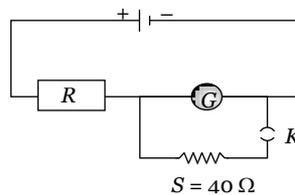
Solution : (c)   Galvanometer reading becomes half means current distributes equally between galvanometer and resistance of 40 Ω. Hence galvanometer resistance must be 40 Ω.

Cell

The device which converts chemical energy into electrical energy is known as electric cell.
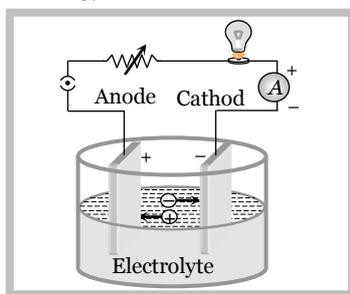
(1) A cell neither creates nor destroys charge but maintains the flow of charge present at various parts of the circuit by supplying energy needed for their organised motion.

(2) Cell is a source of constant emf but not constant current.

(3) Mainly cells are of two types :

(i) Primary cell : Cannot be recharged

(ii) Secondary cell : Can be recharged

(4) The direction of flow of current inside the cell is from negative to positive electrode while outside the cell is form positive to negative electrode.

(5) A cell is said to be ideal, if it has zero internal resistance.

(6) **Emf of cell (E) :** The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called it's electromotive force (emf) *i.e.* emf of cell $E = \dfrac{W}{q}$, It's unit is *volt*   or

The potential difference across the terminals of a cell when it is not given any current is called it's emf.

(7) **Potential difference (V) :** The energy given by the cell in the flow of unit charge in a specific part of electrical circuit (external part) is called potential difference. It's unit is also *volt*   or

The voltage across the terminals of a cell when it is supplying current to external resistance is called potential difference or terminal voltage. Potential difference is equal to the product of current and resistance of that given part *i.e.* $V = iR$.
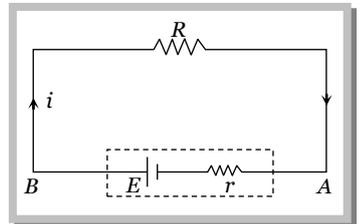
(8) **Internal resistance (r) :** In case of a cell the opposition of electrolyte to the flow of current through it is called internal resistance of the cell. The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes [$r \propto (1/A)$] and nature, concentration ($r \propto C$) and temperature of electrolyte [$r \propto (1/\text{temp.})$]. Internal resistance is different for different types of cells and even for a given type of cell it varies from to cell.

Cell in Various Position

(1) **Closed circuit (when the cell is discharging)**

(i) Current given by the cell $i = \dfrac{E}{R + r}$

(ii) Potential difference across the resistance $V = iR$

(iii) Potential drop inside the cell $= ir$

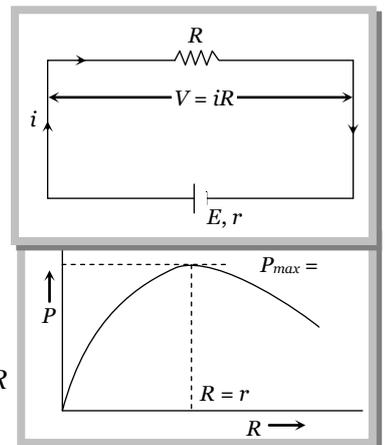(iv) Equation of cell $E = V + ir \ \ (E > V)$

(v) Internal resistance of the cell $r = \left(\dfrac{E}{V} - 1 \right) \cdot R$

(vi) Power dissipated in external resistance (load) $P = Vi = i^2 R = \dfrac{V^2}{R} = \left(\dfrac{E}{R + r} \right)^2 . R$

Power delivered will be maximum when $R = r$ so $P_{\max} = \dfrac{E^2}{4r}$.

This statement in generalised from is called "*maximum power transfer theorem*".

(vii) **Short trick to calculate E and r :** In the closed circuit of a cell having emf E and internal resistance r. If external resistance changes from R_1 to R_2 then current changes from i_1 to i_2 and potential difference changes from V_1 to V_2. By using following relations we can find the value of E and r.

$$E = \frac{i_1 i_2}{i_2 - i_1}(R_1 - R_2) \quad r = \left(\frac{i_2 R_2 - i_1 R_1}{i_1 - i_2}\right) = \frac{V_2 - V_1}{i_1 - i_2}$$

Note : ❑   When the cell is charging *i.e.* current is given to the cell then $E = V - ir$ and $E < V$.
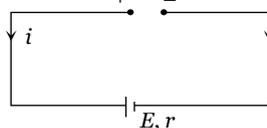
(2) **Open circuit and short circuit**

Open circuit	**Short circuit**
(i) Current through the circuit $i = 0$	(i) Maximum current (called short circuit current) flows momentarily $i_{sc} = \dfrac{E}{r}$
(ii) Potential difference between A and B, $V_{AB} = E$	(ii) Potential difference $V = 0$
(iii) Potential difference between C and D, $V_{CD} = 0$	

Note : ❑   Above information's can be summarized by the following graph

---
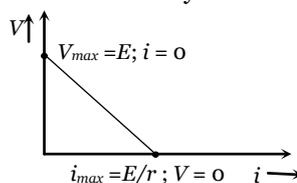
Concepts

☞   *It is a common misconception that "current in the circuit will be maximum when power consumed by the load is maximum."*

Actually current $i = E/(R+r)$ is maximum (= E/r) when R = min = 0 with $P_L = (E/r)^2 \times 0 = 0$ min. while power consumed by the load $E^2R/(R + r)^2$ is maximum (= $E^2/4r$) when $R = r$ and $i = (E/2r) \neq \max(= E/r)$.

☞   *Emf is independent of the resistance of the circuit and depends upon the nature of electrolyte of the cell while potential difference depends upon the resistance between the two points of the circuit and current flowing through the circuit.*

☞   *Emf is a cause and potential difference is an effect.*

☞   *Whenever a cell or battery is present in a branch there must be some resistance (internal or external or both) present in that branch. In practical situation it always happen because we can never have an ideal cell or battery with zero resistance.*

Example

***Example*: 49**   A new flashlight cell of emf 1.5 *volts* gives a current of 15 *amps*, when connected directly to an ammeter of resistance 0.04 Ω. The internal resistance of cell is

             (a) 0.04 Ω   (b) 0.06 Ω   (c) 0.10 Ω   (d) 10 Ω

Solution : (b)   By using $i = \dfrac{E}{R+r} \Rightarrow 15 = \dfrac{1.5}{0.04+r} \Rightarrow r = 0.06\ \Omega$

***Example*: 50**   For a cell, the terminal potential difference is 2.2 *V* when the circuit is open and reduces to 1.8 *V*, when the cell is connected across a resistance, $R = 5\Omega$. The internal resistance of the cell is

             (a) $\dfrac{10}{9}\Omega$ (b) $\dfrac{9}{10}\Omega$ (c) $\dfrac{11}{9}\Omega$ (d) $\dfrac{5}{9}\Omega$

Solution : (a)   In open circuit, $E = V = 2.2\ V,$ In close circuit, $V = 1.8\ V, R = 5\Omega$

             So internal resistance, $r = \left(\dfrac{E}{V} - 1\right)R = \left(\dfrac{2.2}{1.8} - 1\right)\times 5 \quad\Rightarrow\quad r = \dfrac{10}{9}\Omega$

***Example*: 51**   The internal resistance of a cell of emf 2*V* is 0.1 Ω. It's connected to a resistance of 3.9 Ω. The voltage across the cell will be   **[CBSE PMT 1999; AFMC 1999; MP PET 1993; CPMT 1990]**

             (a) 0.5 *volt*   (b) 1.9 *volt*   (c) 1.95 *volt*   (d) 2 *volt*

Solution : (c)   By using $r = \left(\dfrac{E}{V} - 1\right)R \Rightarrow 0.1 = \left(\dfrac{2}{V} - 1\right)\times 3.9 \Rightarrow V = 1.95\ volt$

***Example*: 52**   When the resistance of 2 Ω is connected across the terminal of the cell, the current is 0.5 *amp*. When the resistance is increased to 5 Ω, the current is 0.25 *amp*. The emf of the cell is

             (a) 1.0 *volt*   (b) 1.5 *volt*   (c) 2.0 *volt*   (d) 2.5 *volt*

Solution : (b)   By using $E = \dfrac{i_1 i_2}{(i_2 - i_1)}(R_1 - R_2) = \dfrac{0.5 \times 0.25}{(0.25 - 0.5)}(2 - 5) = 1.5\ volt$

***Example*: 53**   A primary cell has an emf of 1.5 *volts*, when short-circuited it gives a current of 3 *amperes*. The internal resistance of the cell is

             (a) 4.5 *ohm*   (b) 2 *ohm*   (c) 0.5 *ohm*   (d) 1/4.5 *ohm*

Solution : (c) $i_{sc} = \dfrac{E}{r} \Rightarrow 3 = \dfrac{1.5}{r} \Rightarrow r = 0.5\ \Omega$

***Example*: 54**   A battery of internal resistance 4 Ω is connected to the network of resistances as shown. In order to give the maximum power to the network, the value of *R* (in Ω) should be   **[IIT-JEE 1995]**

             (a) 4/9

             (b) 8/9

             (c) 2

             (d) 18
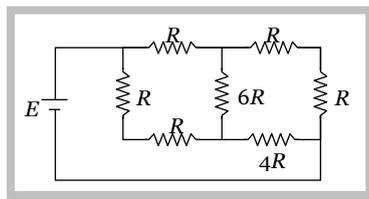
Solution : (c)   The equivalent circuit becomes a balanced wheatstone bridge
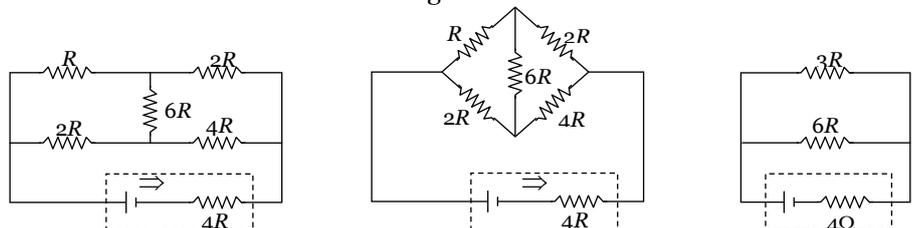
             For maximum power transfer, external resistance should be equal to internal resistance of source

$$\Rightarrow \frac{(R+2R)(2R+4R)}{(R+2R)+(2R+4R)} = 4 \;\; i.e. \;\; \frac{3R \times 6R}{3R+6R} = 4 \;\; or \; R = 2\Omega$$

Example: 55 A torch bulb rated as 4.5 W, 1.5 V is connected as shown in the figure. The emf of the cell needed to make the bulb glow at full intensity is
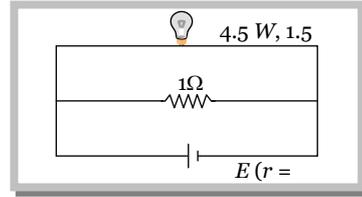
(a) 4.5 V

(b) 1.5 V

(c) 2.67 V

(d) 13.5 V

Solution : (d) When bulb glows with full intensity, potential difference across it is 1.5 V. So current through the bulb and resistance of 1Ω are 3 A and 1.5 A respectively. So main current from the cell $i = 3 + 1.5 = 4.5\ A$. By using $E = V + iR \Rightarrow E = 1.5 + 4.5 \times 2.67 = 13.5\ V$.

Tricky Example: 7

Potential difference across the terminals of the battery shown in figure is (r = internal resistance of battery)

(a) 8 V (b) 10 V

(c) 6 V (d) Zero
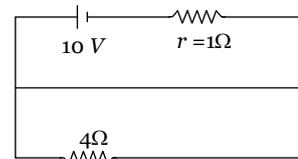
Solution : (d) Battery is short circuited so potential difference is zero.

Grouping of cell

Group of cell is called a battery.

(1) **Series grouping :** In series grouping anode of one cell is connected to cathode of other cell and so on.

(i) *n identical cells are connected in series*

(a) Equivalent emf of the combination $E_{eq} = nE$

(b) Equivalent internal resistance $r_{eq} = nr$

(c) Main current = Current from each cell $= i = \dfrac{nE}{R+nr}$

(d) Potential difference across external resistance $V = iR$

(e) Potential difference across each cell $V' = \dfrac{V}{n}$

(f) Power dissipated in the circuit $P = \left(\dfrac{nE}{R+nr}\right)^2 . R$

(g) Condition for maximum power $R = nr$ and $P_{max} = n\left(\dfrac{E^2}{4r}\right)$
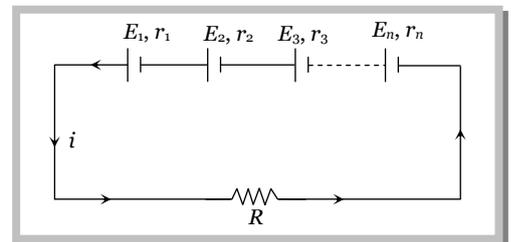
(h) This type of combination is used when $nr << R$.

(ii) **If non-identical cell are connected in series**

Cells are connected in right order	Cells are wrongly connected	
E_1, r_1　　E_2, r_2	E_1, r_1　　E_2, r_2	$(E_1 > E_2)$

(a) Equivalent emf $E_{eq} = E_1 + E_2$

(b) Current $i = \dfrac{E_{eq}}{R + r_{eq}}$

(c) Potential difference across each cell $V_1 = E_1 - ir_1$ and $V_2 = E_2 - ir_2$

(a) Equivalent emf $E_{eq} = E_1 - E_2$

(b) Current $i = \dfrac{E_1 - E_2}{R + r_{eq}}$

(c) in the above circuit cell 1 is discharging so it's equation is $E_1 = V_1 + ir_1 \Rightarrow \boldsymbol{V_1 = E_1 - ir_1}$ and cell 2 is charging so it's equation
$$E_2 = V_2 - ir_2 \Rightarrow \boldsymbol{V_2 = E_2 + ir_2}$$

(2) **Parallel grouping :** In parallel grouping all anodes are connected at one point and all cathode are connected together at other point.

(i) **If n identical cells are connected in parallel**
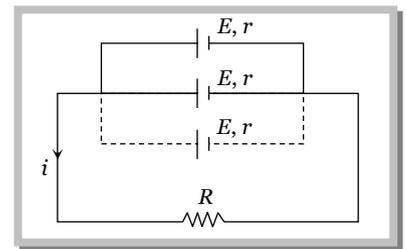
(a) Equivalent emf $E_{eq} = E$

(b) Equivalent internal resistance $R_{eq} = r/n$

(c) Main current $i = \dfrac{E}{R + r/n}$

(d) P.d. across external resistance = p.d. across each cell = $V = iR$

(e) Current from each cell $i' = \dfrac{i}{n}$ (f) Power dissipated in the circuit $P = \left(\dfrac{E}{R + r/n}\right)^2 . R$

(g) Condition for max power $R = r/n$ and $P_{\max} = n\left(\dfrac{E^2}{4r}\right)$ (h) This type of combination is used when nr $>> R$

(ii) **If non-identical cells are connected in parallel :** If cells are connected with right polarity as shown below then

(a) Equivalent emf $E_{eq} = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(b) Main current $i = \dfrac{E_{eq}}{r + R_{eq}}$

(c) Current from each cell $i_1 = \dfrac{E_1 - iR}{r_1}$ and $i_2 = \dfrac{E_2 - iR}{r_2}$
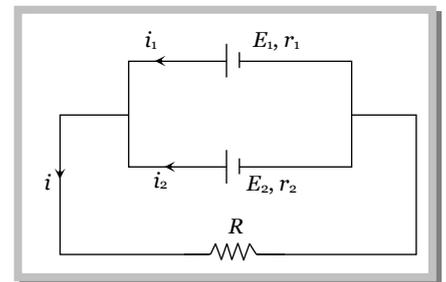
Note : ❑   In this combination if cell's are connected with reversed polarity as shown in figure then :

Equivalent emf $\boldsymbol{E_{eq}} = \dfrac{\boldsymbol{E_1 r_2 - E_2 r_1}}{\boldsymbol{r_1 + r_2}}$
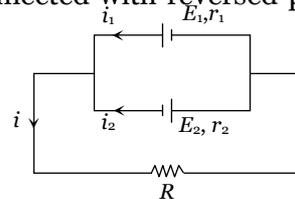
(3) **Mixed Grouping :** If n identical cell's are connected in a row and such m row's are connected in parallel as shown.

(i) Equivalent emf of the combination $E_{eq} = nE$

(ii) Equivalent internal resistance of the combination $r_{eq} = \dfrac{nr}{m}$
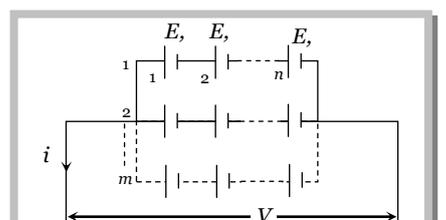
(iii) Main current flowing through the load $i = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{mnE}{mR + nr}$

(iv) Potential difference across load $V = iR$

(v) Potential difference across each cell $V' = \dfrac{V}{n}$

(vi) Current from each cell $i' = \dfrac{i}{n}$

(vii) Condition for maximum power $R = \dfrac{nr}{m}$ and $P_{max} = (mn)\dfrac{E^2}{4r}$

(viii) Total number of cell $= mn$

Concepts

☞ *In series grouping of cell's their emf's are additive or subtractive while their internal resistances are always additive. If dissimilar plates of cells are connected together their emf's are added to each other while if their similar plates are connected together their emf's are subtractive.*
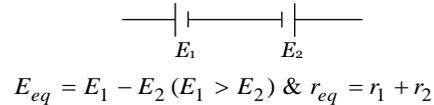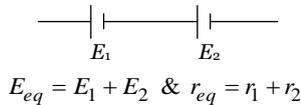
$E_{eq} = E_1 + E_2$ & $r_{eq} = r_1 + r_2$ $\qquad\qquad$ $E_{eq} = E_1 - E_2\ (E_1 > E_2)$ & $r_{eq} = r_1 + r_2$

☞ *In series grouping of identical cells. If one cell is wrongly connected then it will cancel out the effect of two cells e.g. If in the combination of n identical cells (each having emf E and internal resistance r) if x cell are wrongly connected then equivalent emf $E_{eq} = (n - 2x)E$ and equivalent internal resistance $r_{eq} = nr$.*

☞ *In parallel grouping of two identical cell having no internal resistance*

$E_{eq} = E$ $\qquad\qquad\qquad\qquad$ $E_{eq} = 0$
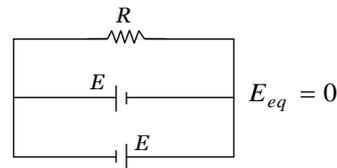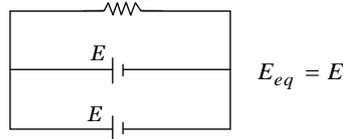
☞ *When two cell's of different emf and no internal resistance are connected in parallel then equivalent emf is indeterminate, note that connecting a wire with a cell but with no resistance is equivalent to short circuiting. Therefore the total current that will be flowing will be infinity.*
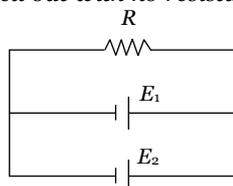
Example

Example: **56** A group of N cells whose emf varies directly with the internal resistance as per the equation $E_N = 1.5\ r_N$ are connected as shown in the following figure. The current i in the circuit is **[KCET 2003]**

(a) 0.51 *amp*

(b) 5.1 *amp*

(c) 0.15 *amp*

(d) 1.5 *amp*
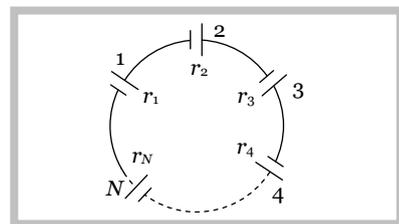
Solution : (d) $\quad i = \dfrac{E_{eq}}{r_{eq}} = \dfrac{1.5r_1 + 1.5r_2 + 1.5r_3 + \ldots\ldots}{r_1 + r_2 + r_3 + \ldots\ldots} = 1.5\ amp$

Example: 57 Two batteries A and B each of emf 2 *volt* are connected in series to external resistance $R = 1\ \Omega$. Internal resistance of A is 1.9 Ω and that of B is 0.9 Ω, what is the potential difference between the terminals of battery A **[MP PET 2001]**

(a) 2 V

(b) 3.8 V

(c) 0

(d) None of these
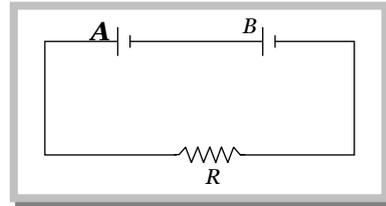
Solution : (c) $i = \dfrac{E_1 + E_2}{R + r_1 + r_2} = \dfrac{2 + 2}{1 + 1.9 + 0.9} = \dfrac{4}{3.8}$. Hence $V_A = E_A - ir_A = 2 - \dfrac{4}{3.8}1.9 = 0$

Example: 58 In a mixed grouping of identical cells 5 rows are connected in parallel by each row contains 10 cell. This combination send a current i through an external resistance of 20 Ω. If the emf and internal resistance of each cell is 1.5 *volt* and 1 Ω respectively then the value of i is

(a) 0.14   (b) 0.25   (c) 0.75   (d) 0.68

Solution : (d) No. of cells in a row $n = 10$;   No. of such rows $m = 5$

$$i = \dfrac{nE}{\left(R + \dfrac{nr}{m}\right)} = \dfrac{10 \times 1.5}{\left(20 + \dfrac{10 \times 1}{5}\right)} = \dfrac{15}{22} = 0.68\ amp$$

Example: 59 To get maximum current in a resistance of 3 Ω one can use n rows of m cells connected in parallel. If the total no. of cells is 24 and the internal resistance of a cell is 0.5 then

(a) $m = 12, n = 2$ (b) $m = 8, n = 4$ (c) $m = 2, n = 12$ (d) $m = 6, n = 4$

Solution : (a) In this question $R = 3\Omega$, $mn = 24$, $r = 0.5\Omega$ and $R = \dfrac{mr}{n}$. On putting the values we get $n = 2$ and $m = 12$.

Example: 60 100 cells each of emf $5V$ and internal resistance 1 Ω are to be arranged so as to produce maximum current in a 25 Ω resistance. Each row contains equal number of cells. The number of rows should be**[MP PMT 1**

(a) 2   (b) 4   (c) 5   (d) 100

Solution : (a) Total no. of cells, $= mn = 100$ .... (i)

Current will be maximum when $R = \dfrac{nr}{m}$; $25 = \dfrac{n \times 1}{m} \Rightarrow n = 25\ m$ .... (ii)

From equation (i) and (ii) $n = 50$ and $m = 2$

Example: 61 In the adjoining circuit, the battery E_1 has as emf of 12 *volt* and zero internal resistance, while the battery E has an emf of 2 *volt*. If the galvanometer reads zero, then the value of resistance $X\ ohm$ is **[NCERT 199**

(a) 10   (b) 100

(c) 500   (d) 200
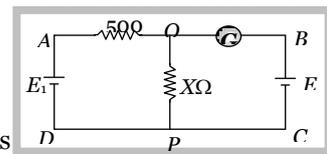
Solution : (b) For zero deflection in galvanometer the potential different across

In this condition $\dfrac{12X}{500 + X} = 2$

$\therefore\ X = 100\ \Omega$

Example: 62 In the circuit shown here $E_1 = E_2 = E_3 = 2V$ and $R_1 = R_2 = 4\ \Omega$. The current flowing between point A and B through battery E_2 is

(a) Zero

(b) 2 A from A to B
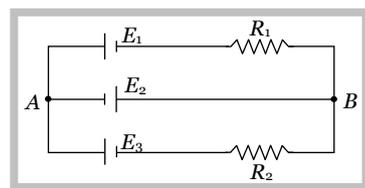
(c) $2\,A$ from B to A

(d) None of these

Solution : (b)   The equivalent circuit can be drawn as since E_1 & E_3 are parallely connected

So current $i = \dfrac{2+2}{2} = 2\,Amp$ from A to B.

Example: 63   The magnitude and direction of the current in the circuit shown will be   **[CPMT 1986, 88]**

(a) $\dfrac{7}{3}\,A$ from a to b through e (b) $\dfrac{7}{3}\,A$ from b and a through e

(c) $1.0\,A$ from b to a through e (d) $1.0\,A$ from a to b through e

Solution : (d)   Current $i = \dfrac{10-4}{3+2+1} = 1A$ from a to b via e

Example: 64   Figure represents a part of the closed circuit. The potential difference between points A and B $(V_A - V_B)$ is

(a) $+9\,V$ (b) $-9\,V$

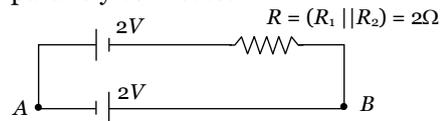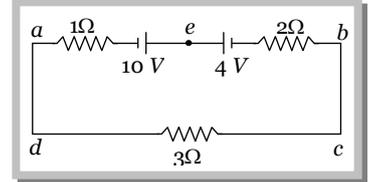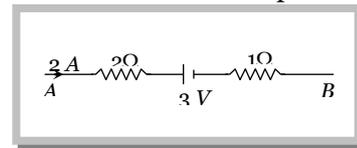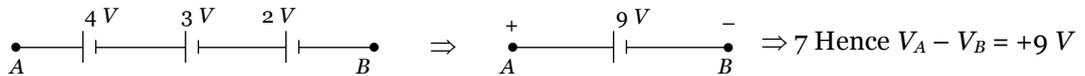(c) $+3\,V$ (d) $+6\,V$

Solution : (a)   The given part of a closed circuit can be redrawn as follows. It should be remember that product of current and resistance can be treated as an imaginary cell having emf = iR.

$\Rightarrow 7$ Hence $V_A - V_B = +9\,V$

Example: 65   In the circuit shown below the cells E_1 and E_2 have emf's $4\,V$ and $8\,V$ and internal resistance $0.5\,ohm$ and $1\,ohm$ respectively. Then the potential difference across cell E_1 and E_2 will be

(a) $3.75\,V, 7.5\,V$

(b) $4.25\,V, 7.5\,V$

(c) $3.75\,V, 3.5\,V$

(d) $4.25\,V, 4.25\,V$
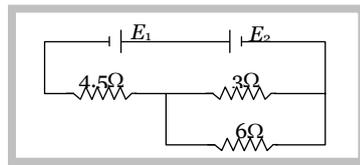
Solution : (b)   In the given circuit diagram external resistance $R = \dfrac{3 \times 6}{3+6} + 4.5 = 6.5\Omega$. Hence main current through

the circuit $i = \dfrac{E_2 - E_1}{R + r_{eq}} = \dfrac{8-4}{6.5 + 0.5 + 0.5} = \dfrac{1}{2}\,amp.$

Cell 1 is charging so from it's emf equation $E_1 = V_1 - ir_1 \Rightarrow 4 = V_1 - \dfrac{1}{2} \times 0.5 \Rightarrow V_1 = 4.25\,volt$

Cell 2 is discharging so from it's emf equation $E_2 = V_2 + ir_2 \Rightarrow 8 = V_2 + \dfrac{1}{2} \times 1 \Rightarrow V_2 = 7.5\,volt$

Example: 66   A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to this current, the temperature of the wire is raised by ΔT in time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but of length $2L$. The temperature of wire is raised by same amount ΔT in the same time t. The value of N is

(a) 4   (b) 6   (c) 8   (d) 9

Solution : (b)   Heat = $mS\Delta T = i^2 Rt$
Case I : Length $(L) \Rightarrow$ Resistance = R and mass = m
Case II : Length $(2L) \Rightarrow$ Resistance = $2R$ and mass = $2m$

So $\dfrac{m_1 S_1 \Delta T_1}{m_2 S_2 \Delta T_2} = \dfrac{i_1^2 R_1 t_1}{i_2^2 r_2 t_2}$ $\Rightarrow \dfrac{mS\Delta T}{2mS\Delta T} = \dfrac{i_1^2 Rt}{i_2^2 2Rt}$ $\Rightarrow i_1 = i_2 \Rightarrow \dfrac{(3E)^2}{12} = \dfrac{(NE)^2}{2R} \Rightarrow N = 6$

Tricky Example: 8

n identical cells, each of emf E and internal resistance r, are joined in series to form a closed

circuit. The potential difference across any one cell is

(a) Zero   (b) E (c) $\dfrac{E}{n}$ (d) $\left(\dfrac{n-1}{n}\right)E$

Solution: (a)   Current in the circuit $i = \dfrac{nE}{nr} = \dfrac{E}{r}$

The equivalent circuit of one cell is shown in the figure. Potential difference across the cell

$= V_A - V_B = -E + ir = -E + \dfrac{E}{r}.r = 0$

Kirchoff's Laws

(1) **Kirchoff's first law :** This law is also known as junction rule or current law (*KCL*). According to it the algebraic sum of currents meeting at a junction is zero *i.e.* $\Sigma i = \mathbf{0}$.

In a circuit, at any junction the sum of the currents entering the junction must equal the sum of the currents leaving the junction. $i_1 + i_3 = i_2 + i_4$

Here it is worthy to note that :

(i) If a current comes out to be negative, actual direction of current at the junction is opposite to that assumed, $i + i_1 + i_2 = 0$ can be satisfied only if at least one current is negative, *i.e.* leaving the junction.

(ii) This law is simply a statement of "*conservation of charge*" as if current reaching a junction is not equal to the current leaving the junction, charge will not be conserved.
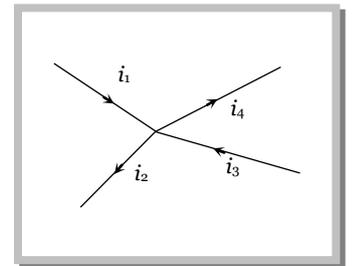
~~*Note*~~ : ❑   This law is also applicable to a capacitor through the concept of displacement current

treating the resistance of capacitor to be zero during charging or discharging and infinite in steady state as shown in figure.
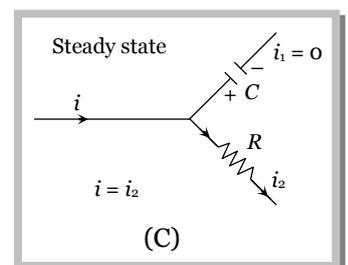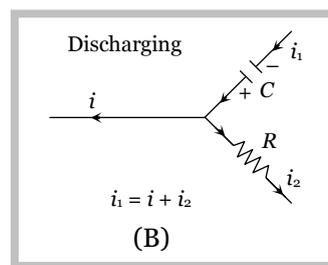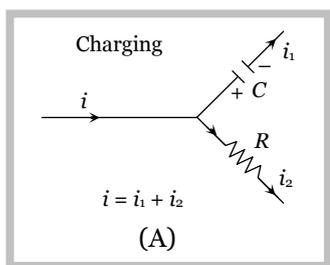
(2) **Kirchoff's second law :** This law is also known as loop rule or voltage law (KVL) and according to it "the algebraic sum of the changes in potential in complete traversal of a mesh (closed loop) is zero", *i.e.* $\Sigma V = \mathbf{0}$

e.g. In the following closed loop.

$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$
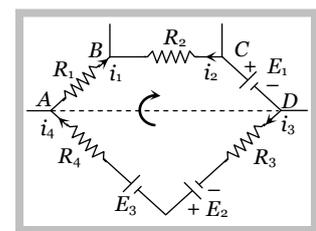
Here it is worthy to note that :

(i) This law represents "*conservation of energy*" as if the sum of potential changes around a closed loop is not zero, unlimited energy could be gained by repeatedly carrying a charge around a loop.

(ii) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be $(n-1)$.

(3) **Sign convention for the application of Kirchoff's law :** For the application of Kirchoff's laws following sign convention are to be considered

(i) The change in potential in traversing a resistance in the direction of current is $- iR$ while in the opposite direction $+iR$

(ii) The change in potential in traversing an emf source from negative to positive terminal is $+E$ while in the opposite direction $- E$ irrespective of the direction of current in the circuit.

(iii) The change in potential in traversing a capacitor from the negative terminal to the positive terminal is $+\dfrac{q}{C}$ while in opposite direction $-\dfrac{q}{C}$.

(iv) The change in voltage in traversing an inductor in the direction of current is $- L\dfrac{di}{dt}$ while in opposite direction it is $+ L\dfrac{di}{dt}$.

(4) **Guidelines to apply Kirchoff's law**

(i) Starting from the positive terminal of the battery having highest emf, distribute current at various junctions in the circuit in accordance with '*junction rule*'. It is not always easy to correctly guess the direction of current, no problem if one assumes the wrong direction.

(ii) After assuming current in each branch, we pick a point and begin to walk (mentally) around a closed loop. As we traverse each resistor, capacitor, inductor or battery we must write down, the voltage change for that element according to the above sign convention.

(iii) By applying KVL we get one equation but in order to solve the circuit we require as many equations as there are unknowns. So we select the required number of loops and apply Kirchhoff's voltage law across each such loop.

(iv) After solving the set of simultaneous equations, we obtain the numerical values of the assumed currents. If any of these values come out to be negative, it indicates that particular current is in the opposite direction from the assumed one.

Note : ❑   The number of loops must be selected so that every element of the circuit must be included in at least one of the loops.

❑ While traversing through a capacitor or battery we do not consider the direction of current.

❑ While considering the voltage drop or gain across as inductor we always assume current to be in increasing function.

(5) **Determination of equivalent resistance by Kirchoff's method :** This method is useful when we are not able to identify any two resistances in series or in parallel. It is based on the two Kirchhoff's laws. The method may be described in the following guideline.

(i) Assume an imaginary battery of emf E connected between the two terminals across which we have to calculate the equivalent resistance.

(ii) Assume some value of current, say i, coming out of the battery and distribute it among each branch by applying Kirchhoff's current law.

(iii) Apply Kirchhoff's voltage law to formulate as many equations as there are unknowns. It should be noted that at least one of the equations must include the assumed battery.

(iv) Solve the equations to determine $\dfrac{E}{i}$ ratio which is the equivalent resistance of the network.

e.g. Suppose in the following network of 12 identical resistances, equivalent resistance between point A and C is to be calculated.
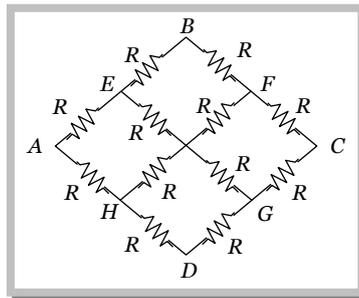
According to the above guidelines we can solve this problem as follows

Step (1)   **Step** (2)
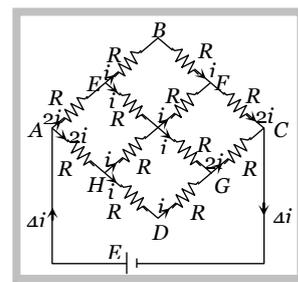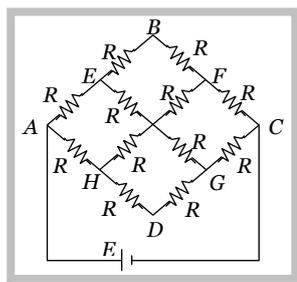
An imaginary battery of emf E is assumed across the terminals A and C

The current in each branch is distributed by assuming $4i$ current coming out of the battery.

Step (3) Applying KVL along the loop including the nodes A, B, C and the battery E. Voltage equation is $-2iR - iR - iR - 2iR + E = 0$

Step (4) After solving the above equation, we get $6iR = E \Rightarrow$ equivalent resistance between A and C is

$$R = \frac{E}{4i} = \frac{6iR}{4i} = \frac{3}{2}R$$

Concepts

☞ *Using Kirchoff's law while dividing the current having a junction through different arms of a network, it will be same through different arms of same resistance if the end points of these arms are equilocated w.r.t. exit point for current in network and will be different through different arms if the end point of these arms are not equilocated w.r.t. exit point for current of the network.*

e.g. In the following figure the current going in arms AB, AD and AL will be same because the location of end points B, D
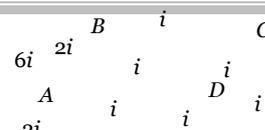
and L of these arms are symmetrically located w.r.t. exit point N of the network.

Example

Example: 67   In the following circuit $E_1 = 4V$, $R_1 = 2\Omega$ **[MP PET 2003]**

$E_2 = 6V$, $R_2 = 2\Omega$ and $R_3 = 4\Omega$. The current i_1 is

(a) $1.6\,A$

(b) $1.8\,A$

(c) $2.25\,A$

(d) $1\,A$
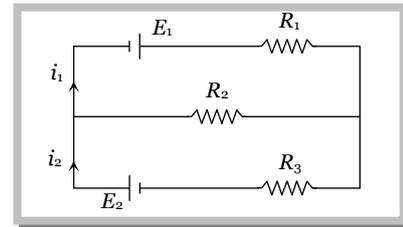
Solution : (b)   For loop (1) $-2i_1 - 2(i_1 - i_2) + 4 = 0 \Rightarrow 2i_1 - i_2 = 2$ ...... (i)

For loop (2) $-4i_2 + 2(i_1 - i_2) + 6 = 0 \Rightarrow 3i_2 - i_1 = 3$ ...... (ii)

After solving equation (i) and (ii) we get $i_1 = 1.8\,A$ and $i_2 = 1.6\,A$
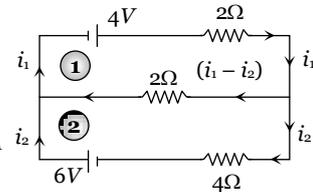
Example: 68   Determine the current in the following circuit

(a) $1\,A$

(b) $2.5\,A$

(c) $0.4\,A$

(d) $3\,A$
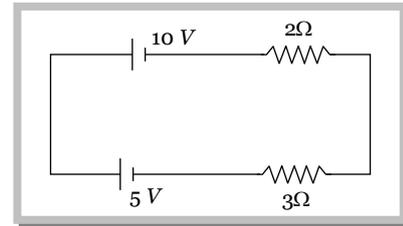
Solution : (a)   Applying KVL in the given circuit we get $-2i + 10 - 5 - 3i = 0 \Rightarrow i = 1A$

Second method : Similar plates of the two batteries are connected together, so the net emf = 10 − 5 = 5*V*

Total resistance in the circuit = 2 + 3 = 5Ω

$\therefore \quad i = \dfrac{\Sigma V}{\Sigma R} = \dfrac{5}{5} = 1A$

Example: 69   In the circuit shown in figure, find the current through the branch *BD*

(a) $5\,A$ (b) $0\,A$

(c) $3\,A$ (d) $4\,A$
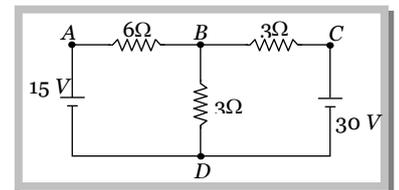
Solution : (a)   The current in the circuit are assumed as shown in the fig.

Applying KVL along the loop *ABDA*, we get

$-6i_1 - 3\,i_2 + 15 = 0$ or $2i_1 + i_2 = 5$ ...... (i)

Applying KVL along the loop *BCDB*, we get

$-3(i_1 - i_2) - 30 + 3i_2 = 0$ or $-i_1 + 2i_2 = 10$ ...... (ii)

Solving equation (i) and (ii) for i_2, we get $i_2 = 5\,A$
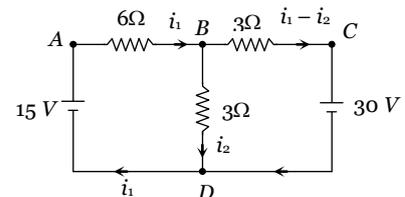
Example: 70   The figure shows a network of currents. The magnitude of current is shown here. The current i will be **[MP PMT 1995]**

(a) $3\,A$

(b) $13\,A$

(c) $23\,A$

(d) $-3\,A$
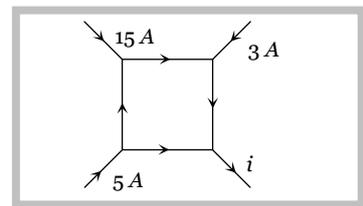
Solution : (c) $i = 15 + 3 + 5 = 23A$

Example: 71 Consider the circuit shown in the figure. The current i_3 is equal to **[AMU 1995]**

(a) 5 *amp*

(b) 3 *amp*

(c) -3 *amp*

(d) $-5/6$ *amp*
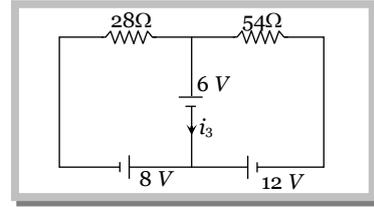
Solution : (d) Suppose current through different paths of the circuit is as follows.

After applying *KVL* for loop (1) and loop (2)

We get $28i_1 = -6 - 8 \Rightarrow i_1 = -\dfrac{1}{2}A$

and $54i_2 = -6 - 12 \Rightarrow i_2 = -\dfrac{1}{3}A$

Hence $i_3 = i_1 + i_2 = -\dfrac{5}{6}A$
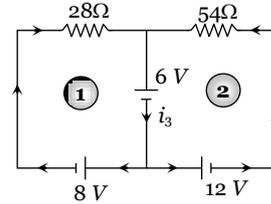
Example: 72 A part of a circuit in steady state along with the current flowing in the branches, with value of each resistance is shown in figure. What will be the energy stored in the capacitor C_0

(a) $6 \times 10^{-4}J$

(b) $8 \times 10^{-4}J$

(c) $16 \times 10^{-4}J$

(d) Zero
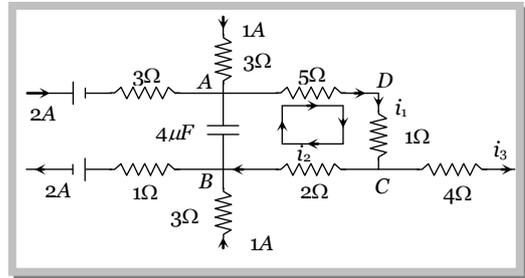
Solution : (b) Applying Kirchhoff's first law at junctions A and B respectively we have $2 + 1 - i_1 = 0$ *i.e.*, $i_1 = 3A$

and $i_2 + 1 - 2 - 0 = 0$ *i.e.*, $i_2 = 1A$

Now applying Kirchhoff's second law to the mesh *ADCBA* treating capacitor as a seat of emf *V* in open circuit

$-3 \times 5 - 3 \times 1 - 1 \times 2 + V = 0$ *i.e.* $V(= V_A - V_B) = 20$ V

So, energy stored in the capacitor $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(4 \times 10^{-6}) \times (20)^2 = 8 \times 10^{-4}J$

Example: 73 In the following circuit the potential difference between *P* and *Q* is

(a) 15 *V* (b) 10 *V*

(c) 5 *V* (d) 2.5 *V*

Solution : (c) By using KVL $-5 \times 2 - V_{PQ} + 15 = 0 \Rightarrow V_{PQ} = 5V$

Tricky Example: 9

As the switch *S* is closed in the circuit shown in figure, current passed through it is

(a) $4.5\,A$ (b) $6.0\,A$ (c) $3.0\,A$ (d) Zero

Solution : (a)   Let *V* be the potential of the junction as shown in figure. Applying junction law, we have

or $\dfrac{20-V}{2} + \dfrac{5-V}{4} = \dfrac{V-0}{2}$ or $40 - 2V + 5 - V = 2V$

or $5V = 45 \Rightarrow V = 9V$

$\therefore \quad i_3 = \dfrac{V}{2} = 4.5\,A$

Different Measuring Instruments

(1) **Galvanometer :** It is an instrument used to detect small current passing through it by showing deflection. Galvanometers are of different types *e.g.* moving coil galvanometer, moving magnet galvanometer, hot wire galvanometer. In dc circuit usually moving coil galvanometer are used.

(i) **It's symbol :**   —————G—————   ; where *G* is the total internal resistance of the galvanometer.

(ii) **Principle :** In case of moving coil galvanometer deflection is directly proportional to the current that passes through it *i.e.* $i \propto \theta$.

(iii) **Full scale deflection current :** The current required for full scale deflection in a galvanometer is called full scale deflection current and is represented by i_g.

(iv) **Shunt :** The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer is known as shunt.

Merits of shunt	Demerits of shunt
(a) To protect the galvanometer coil from burning	Shunt resistance decreases the sensitivity of galvanometer.
(b) It can be used to convert any galvanometer into ammeter of desired range.	

(2) **Ammeter :** It is a device used to measure current and is always connected in series with the 'element' through which current is to be measured.

(i) The reading of an ammeter is always lesser than actual current in the circuit.

(ii) Smaller the resistance of an ammeter more accurate will be its reading. An ammeter is said to be ideal if its resistance *r* is zero.

(iii) **Conversion of galvanometer into ammeter :** A galvanometer may be converted into an ammeter by connecting a low resistance (called shunt *S*) in parallel to the galvanometer *G* as shown in figure.

(a) Equivalent resistance of the combination $= \dfrac{GS}{G+S}$

(b) *G* and *S* are parallel to each other hence both will have equal potential difference *i.e.* $i_g G = (i - i_g)S$; which gives

Required shunt $S = \dfrac{i_g}{(i - i_g)} G$

(c) To pass nth part of main current (*i.e.* $i_g = \dfrac{i}{n}$) through the galvanometer, required shunt

$$S = \dfrac{G}{(n-1)}\,.$$

(3) **Voltmeter :** It is a device used to measure potential difference and is always put in parallel with the 'circuit element' across which potential difference is to be measured.

(i) The reading of a voltmeter is always lesser than true value.

(ii) Greater the resistance of voltmeter, more accurate will be its reading. A voltmeter is said to be ideal if its resistance is infinite, *i.e.*, it draws no current from the circuit element for its operation.

(iii) **Conversion of galvanometer into voltmeter :** A galvanometer may be converted into a voltmeter by connecting a large resistance R in series with the galvanometer as shown in the figure.
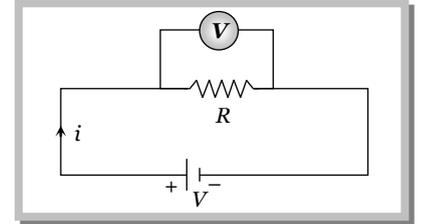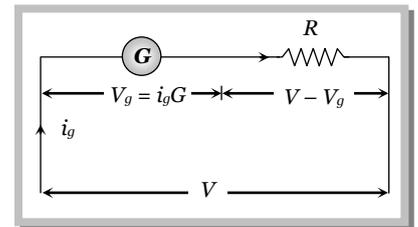
(a) Equivalent resistance of the combination $= G + R$

(b) According to ohm's law $V = i_g (G + R)$; which gives

Required series resistance $R = \dfrac{V}{i_g} - G = \left(\dfrac{V}{V_g} - 1\right) G$

(c) If n^{th} part of applied voltage appeared across galvanometer (*i.e.* $V_g = \dfrac{V}{n}$) then required series resistance $R = (n-1)\,G$.

(4) **Wheatstone bridge :** Wheatstone bridge is an arrangement of four resistance which can be used to measure one of them in terms of rest. Here arms AB and BC are called ratio arm and arms AC and BD are called conjugate arms

(i) **Balanced bridge :** The bridge is said to be balanced when deflection in galvanometer is zero *i.e.* no current flows through the galvanometer or in other words $V_B = V_D$. In the balanced condition $\dfrac{P}{Q} = \dfrac{R}{S}$, on mutually changing the position of cell and galvanometer this condition will not change.
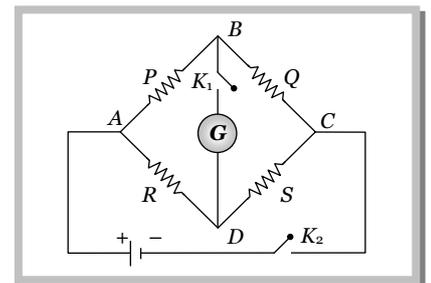
(ii) **Unbalanced bridge :** If the bridge is not balanced current will flow from D to B if $V_D > V_B$ *i.e.* $(V_A - V_D) < (V_A - V_B)$ which gives $PS > RQ$.
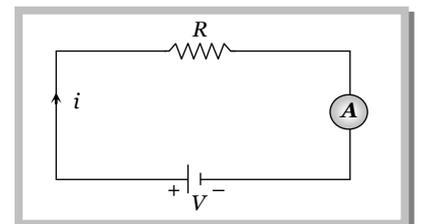
(iii) **Applications of wheatstone bridge :** Meter bridge, post office box and Carey Foster bridge are instruments based on the principle of wheatstone bridge and are used to measure unknown resistance.

(5) **Meter bridge :** In case of meter bridge, the resistance wire AC is 100 *cm* long. Varying the position of tapping point B, bridge is balanced. If in balanced position of bridge $AB = l$, $BC (100 - l)$ so that

$$\dfrac{Q}{P} = \dfrac{(100 - l)}{l}\,. \text{ Also } \dfrac{P}{Q} = \dfrac{R}{S} \Rightarrow S = \dfrac{(100 - l)}{l}\,R$$
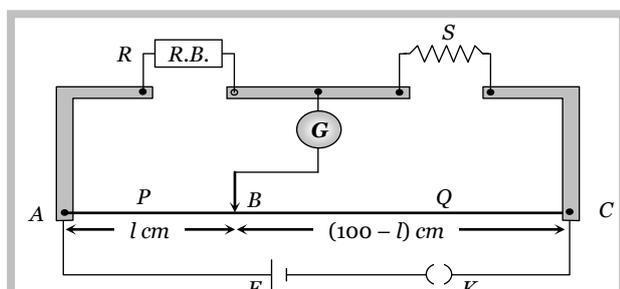
Concepts

☞ *Wheatstone bridge is most sensitive if all the arms of bridge have equal resistances i.e. P = Q = R = S*

☞ *If the temperature of the conductor placed in the right gap of metre bridge is increased, then the balancing length decreases and the jockey moves towards left.*

☞ *In Wheatstone bridge to avoid inductive effects the battery key should be pressed first and the galvanometer key afterwards.*

☞ *The measurement of resistance by Wheatstone bridge is not affected by the internal resistance of the cell.*

Example

Example: 74 The scale of a galvanometer of resistance 100 Ω contains 25 divisions. It gives a deflection of one division on passing a current of 4×10^{-4} *A*. The resistance in *ohms* to be added to it, so that it may become a voltmeter of range 2.5 *volt* is

(a) 100 (b) 150 (c) 250 (d) 300

Solution : (b) Current sensitivity of galvanometer = 4×10^{-4} *Amp/div.*

So full scale deflection current (i_g) = Current sensitivity × Total number of division = $4 \times 10^{-4} \times 25 = 10^{-2}$ *A*

To convert galvanometer in to voltmeter, resistance to be put in series is

$R = \dfrac{V}{i_g} - G = \dfrac{2.5}{10^{-2}} - 100 = 150 \ \Omega$

Example: 75 A galvanometer, having a resistance of 50 Ω gives a full scale deflection for a current of 0.05 *A*. the length in meter of a resistance wire of area of cross-section 2.97×10^{-2} *cm²* that can be used to convert the galvanometer into an ammeter which can read a maximum of 5*A* current is : (Specific resistance of the wire = 5×10^{-7} *Ωm*) **[EAMCET 2003]**

(a) 9 (b) 6 (c) 3 (d) 1.5

Solution : (c) Given $G = 50 \ \Omega$, $i_g = 0.05 \ Amp.$, $i = 5A$, $A = 2.97 \times 10^{-2} \ cm^2$ and $\rho = 5 \times 10^{-7} \Omega\text{-}m$

By using $\dfrac{i}{i_g} = 1 + \dfrac{G}{S} \Rightarrow S = \dfrac{G.i_g}{(i - i_g)} \Rightarrow \dfrac{\rho l}{A} = \dfrac{Gi_g}{(i - i_g)} \Rightarrow l = \dfrac{Gi_g A}{(i - i_g)\rho}$ on putting values $l = 3 \ m$.

Example: 76 100 *mA* current gives a full scale deflection in a galvanometer of resistance 2 Ω. The resistance connected with the galvanometer to convert it into a voltmeter of 5 *V* range is

[KCET 2002; UPSEAT 1998; MNR 1994 Similar to MP PMT 1999]

(a) 98 Ω (b) 52 Ω (c) 80 Ω (d) 48 Ω

Solution : (d) $R = \dfrac{V}{I_g} - G = \dfrac{5}{100 \times 10^{-3}} - 2 = 50 - 2 = 48 \ \Omega$.

Example: 77 A milliammeter of range 10 *mA* has a coil of resistance 1 Ω. To use it as voltmeter of range 10 *volt*, the resistance that must be connected in series with it will be

(a) 999 Ω (b) 99 Ω (c) 1000 Ω (d) None of these

Solution : (a) By using $R = \dfrac{V}{ig} - G \Rightarrow R = \dfrac{10}{10 \times 10^{-3}} - 1 = 999\Omega$

Example: 78 In the following figure ammeter and voltmeter reads 2 *amp* and 120 *volt* respectively. Resistance of voltmeter is

(a) 100 Ω

(b) 200 Ω

(c) 300 Ω

(d) 400 Ω
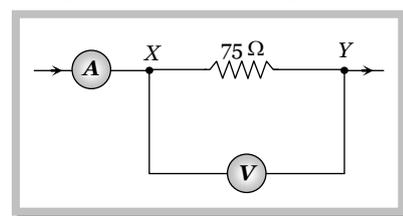
Solution : (c)   Let resistance of voltmeter be R_V. Equivalent resistance between X and Y is $R_{XY} = \dfrac{75\,R_V}{75 + R_V}$

Reading of voltmeter = potential difference across X and Y = 120 = $i \times R_{XY}$ = $2 \times \dfrac{75\,R_V}{75 + R_V} \Rightarrow R_V =$

300Ω

Example: 79   In the circuit shown in figure, the voltmeter reading would be

(a) Zero

(b) 0.5 *volt*

(c) 1 *volt*

(d) 2 *volt*

Solution : (a)   Ammeter has no resistance so there will be no potential difference across it, hence reading of voltmeter is zero.

Example: 80   Voltmeters V_1 and V_2 are connected in series across a d.c. line. V_1 reads 80 V and has a per *volt* resistance of 200 Ω, V_2 has a total resistance of 32 $k\Omega$. The line voltage is
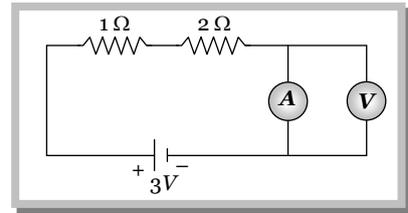
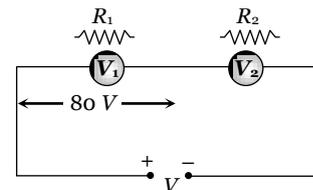(a) 120 V (b) 160 V (c) 220 V (d) 240 V

Solution : (d)   Resistance of voltmeter V_1 is $R_1 = 200 \times 80 = 16000$ Ω and resistance of voltmeter V_2 is $R_2 = 32000$ Ω

By using relation $V' = \left(\dfrac{R'}{R_{eq}}\right)V$;   where V' = potential difference across any resistance R' in a series grouping.

So for voltmeter V_1 potential difference across it is

$$80 = \left(\dfrac{R_1}{R_1 + R_2}\right).V \Rightarrow V = 240\ V$$

Example: 81   The resistance of 1 A ammeter is 0.018 Ω. To convert it into 10 A ammeter, the shunt resistance required will be

(a) 0.18 Ω   (b) 0.0018 Ω   (c) 0.002 Ω   (d) 0.12 Ω

Solution : (c)   By using $\dfrac{i}{i_g} = 1 + \dfrac{4}{S} \Rightarrow \dfrac{10}{1} = 1 + \dfrac{0.018}{S} \Rightarrow S = 0.002$ Ω

Example: 82   In meter bridge the balancing length from left and when standard resistance of 1 Ω is in right gas is found to be 20 *cm*. The value of unknown resistance is   **[CBSE PMT 1999]**

(a) 0.25 Ω   (b) 0.4 Ω   (c) 0.5 Ω   (d) 4 Ω

Solution: (a)   The condition of wheatstone bridge gives $\dfrac{X}{R} = \dfrac{20\,r}{80\,r}$, r- resistance of wire per *cm*, X- unknown resistance

$$\therefore\ \ X = \dfrac{20}{80} \times R = \dfrac{1}{4} \times 1 = 0.25\ \Omega$$
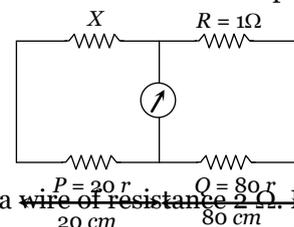
Example: 83   A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 *amp*, the part of it passing through the shunt will be   **[CBSE PMT 1998]**

(a) 0.25 *amp*   (b) 0.8 *amp*   (c) 0.2 *amp*   (d) 0.5 *amp*

Solution : (b)   Fraction of current passing through the galvanometer

$$\frac{i_g}{i} = \frac{S}{S+G} \quad \text{or} \quad \frac{i_g}{i} = \frac{2}{2+8} = 0.2$$

So fraction of current passing through the shunt

$$\frac{i_s}{i} = 1 - \frac{i_g}{i} = 1 - 0.2 = 0.8 \ amp$$

Example : **84**   A moving coil galvanometer is converted into an ammeter reading upto 0.03 *A* by connecting a shunt of resistance $4r$ across it and into an ammeter reading upto 0.06 *A* when a shunt of resistance r connected across it. What is the maximum current which can be through this galvanometer if no shunt is used

[MP PMT 1996]

(a) 0.01 *A*   (b) 0.02 *A*   (c) 0.03 *A*   (d) 0.04 *A*

Solution : (b)   For ammeter, $S = \dfrac{i_g}{(i - i_g)} G \quad \Rightarrow i_g G = (i - i_g)S$
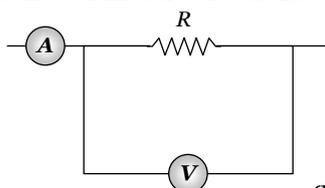
So $i_g G = (0.03 - i_g)4r$ ...... (i)   and $i_g G = (0.06 - i_g)r$ ...... (ii)

Dividing equation (i) by (ii) $1 = \dfrac{(0.03 - i_g)4}{0.06 - i_g} \Rightarrow 0.06 - i_g = 0.12 - 4i_g$

$\Rightarrow 3i_g = 0.06 \Rightarrow i_g = 0.02 \ A$

Tricky Example: 10

The ammeter *A* reads 2 *A* and the voltmeter *V* reads 20 *V*. The value of resistance *R* is

(a) Exactly 10 *ohm*   (b) Less than 10 *ohm*

(c) More than 10 *ohm*   (d) We cannot definitely say

Solution : (c)   If current goes through the resistance *R* is *i* then $iR = 20 \ volt \Rightarrow R = \dfrac{20}{i}$. Since $i < 2A$ so $R >$ 10Ω.

Potentiometer

Potentiometer is a device mainly used to measure emf of a given cell and to compare emf's of cells. It is also used to measure internal resistance of a given cell.

(1) **Superiority of potentiometer over voltmeter** : An ordinary voltmeter cannot measure the emf accurately because it does draw some current to show the deflection. As per definition of emf, it is the potential difference when a cell is in open circuit or no current through the cell. Therefore voltmeter can only measure terminal voltage of a give *n* cell.

Potentiometer is based on no deflection method. When the potentiometer gives zero deflection, it does not draw any current from the cell or the circuit *i.e.* potentiometer is effectively an ideal instrument of infinite resistance for measuring the potential difference.

(2) **Circuit diagram :** Potentiometer consists of a long resistive wire *AB* of length *L* (about 6*m* to 10 *m* long) made up of mangnine or constantan. A battery of known voltage *e* and internal resistance *r* called supplier battery or driver cell. Connection of these two forms primary circuit.

One terminal of another cell (whose emf E is to be measured) is connected at one end of the main circuit and the other terminal at any point on the resistive wire through a galvanometer G. This forms the secondary circuit. Other details are as follows
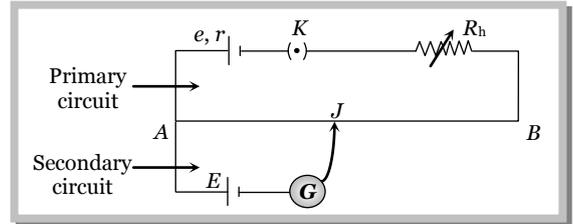
J = Jockey

K = Key

R = Resistance of potentiometer wire,

ρ = Specific resistance of potentiometer wire.

R_h = Variable resistance which controls the current through the wire AB

(3) **Points to be remember**

(i) The specific resistance (ρ) of potentiometer wire must be high but its temperature coefficient of resistance (α) must be low.

(ii) All higher potential points (terminals) of primary and secondary circuits must be connected together at point A and all lower potential points must be connected to point B or jockey.

(iii) The value of known potential difference must be greater than the value of unknown potential difference to be measured.

(iv) The potential gradient must remain constant. For this the current in the primary circuit must remain constant and the jockey must not be slided in contact with the wire.

(v) The diameter of potentiometer wire must be uniform everywhere.

(4) **Potential gradient (x) :** Potential difference (or fall in potential) per unit length of wire is called

potential gradient *i.e.* $x = \dfrac{V}{L} \dfrac{volt}{m}$ where $V = iR = \left(\dfrac{e}{R + R_h + r} \right).R$. So $x = \dfrac{V}{L} = \dfrac{iR}{L} = \dfrac{i\rho}{A} = \dfrac{e}{(R + R_h + r)} . \dfrac{R}{L}$

(i) Potential gradient directly depends upon

(a) The resistance per unit length (R/L) of potentiometer wire.

(b) The radius of potentiometer wire (*i.e.* Area of cross-section)

(c) The specific resistance of the material of potentiometer wire (*i.e.* ρ)

(d) The current flowing through potentiometer wire (i)

(ii) x indirectly depends upon

(a) The emf of battery in the primary circuit (*i.e.* e)

(b) The resistance of rheostat in the primary circuit (*i.e.* R_h)

Note: ❑   When potential difference V is constant then $\dfrac{x_1}{x_2} = \dfrac{L_2}{L_1}$

❑ Two different wire are connected in series to form a potentiometer wire then $\dfrac{x_1}{x_2} = \dfrac{R_1}{R_2} . \dfrac{L_2}{L_1}$

❑ If the length of a potentiometer wire and potential difference across it's ends are kept constant and if it's diameter is changed from $d_1 \to d_2$ then potential gradient remains unchanged.

❑ The value of x does not change with any change effected in the secondary circuit.

(5) **Working :** Suppose jocky is made to touch a point J on wire then potential difference between A and J will be $V = xl$

At this length (l) two potential difference are obtained
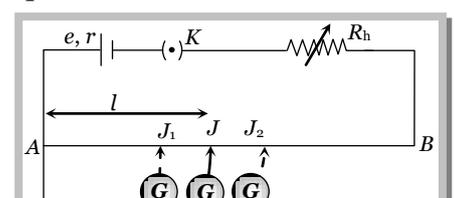
(i) V due to battery e and

(ii) E due to unknown cell

If $V > E$ then current will flow in galvanometer circuit in one direction

If $V < E$ then current will flow in galvanometer circuit in opposite direction

If $V = E$ then no current will flow in galvanometer circuit this condition to known as null deflection position, length l is known as balancing length.

In balanced condition $E = xl$ or $E = xl = \dfrac{V}{L}l = \dfrac{iR}{L}l = \left(\dfrac{e}{R + R_h + r}\right).\dfrac{R}{L} \times l$

Note : ❏ If V is constant then $L \propto l \quad \Rightarrow \dfrac{L_1}{L_2} = \dfrac{l_1}{l_2}$

(6) **Standardization of potentiometer :** The process of determining potential gradient experimentally is known as standardization of potentiometer.

Let the balancing length for the standard emf E_0 is l_0 then by the principle of potentiometer $E_0 = xl_0 \Rightarrow x = \dfrac{E_0}{l_0}$


(7) **Sensitivity of potentiometer :** A potentiometer is said to be more sensitive, if it measures a small potential difference more accurately.

(i) The sensitivity of potentiometer is assessed by its potential gradient. The sensitivity is inversely proportional to the potential gradient.

(ii) In order to increase the sensitivity of potentiometer

(a) The resistance in primary circuit will have to be decreased.

(b) The length of potentiometer wire will have to be increased so that the length may be measured more accuracy.
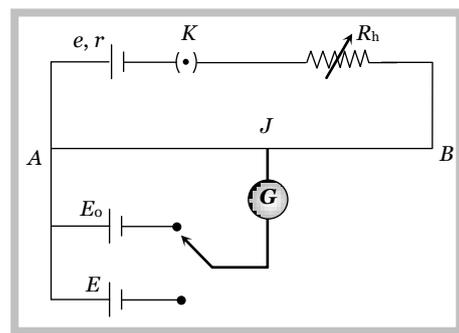
(8) **Difference between voltmeter and potentiometer**

|	Voltmeter	Potentiometer
(i)	It's resistance is high but finite	Its resistance is high but infinite
(ii)	It draws some current from source of emf	It does not draw any current from the source of known emf
(iii)	The potential difference measured by it is lesser than the actual potential difference	The potential difference measured by it is equal to actual potential difference
(iv)	Its sensitivity is low	Its sensitivity is high
(v)	It is a versatile instrument	It measures only emf or potential difference
(vi)	It is based on deflection method	It is based on zero deflection method

Application of Potentiometer

(1) **To determine the internal resistance of a primary cell**

(i) Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1 *i.e.* $E = xl_1$ .... (i)

(ii) Now key K' is closed so cell E comes in closed circuit. If the process is repeated again then potential difference V balances on length l_2 i.e. $V = xl_2$ .... (ii)

(iii) By using formula internal resistance $r = \left(\dfrac{E}{V} - 1\right).R'$

$$r = \left(\dfrac{l_1 - l_2}{l_2}\right).R'$$
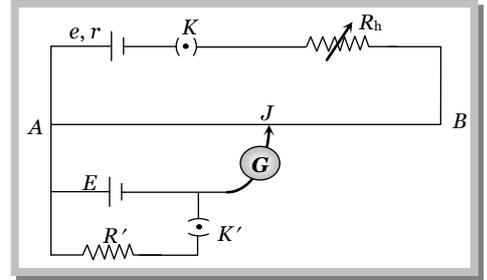
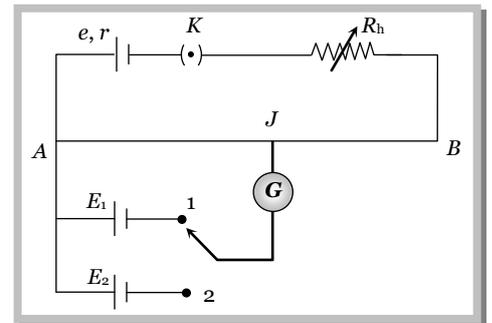(2) **Comparison of emf's of two cell :** Let l_1 and l_2 be the balancing lengths with the cells E_1 and E_2 respectively then $E_1 = xl_1$ and $E_2 = xl_2$ $\Rightarrow \dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$

Note : ☐ Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cell assist each other and it is l_2 when they oppose each other as shown then :
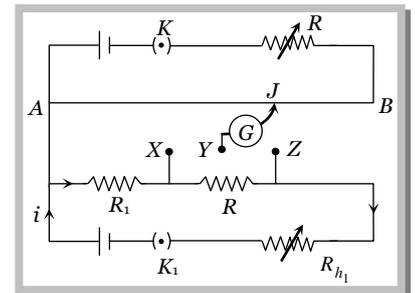
$(E_1 + E_2) = xl_1$ $\qquad$ $(E_1 - E_2) = xl_2$

$\Rightarrow \qquad \dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{l_1}{l_2}$ $\qquad$ or $\qquad \dfrac{E_1}{E_2} = \dfrac{l_1 + l_2}{l_1 - l_2}$

(3) **Comparison of resistances :** Let the balancing length for resistance R_1 (when XY is connected) is l_1 and let balancing length for resistance $R_1 + R_2$ (when YZ is connected) is l_2. $\qquad$ Then $\quad iR_1 = xl_1$ and $i(R_1 + R_2) = xl_2$

$\Rightarrow \qquad \dfrac{R_2}{R_1} = \dfrac{l_2 - l_1}{l_1}$

(4) **To determine thermo emf**

(i) The value of thermo-emf in a thermocouple for ordinary temperature difference is very low (10^{-6} *volt*). For this the potential gradient x must be also very low (10^{-4} *V/m*). Hence a high resistance (R) is connected in series with the potentiometer wire in order to reduce current.

(ii) The potential difference across R must be equal to the emf of standard cell *i.e.* $iR = E_0$ $\therefore$ $i = \dfrac{E_0}{R}$

(iii) The small thermo emf produced in the thermocouple $e = xl$

(iv) $x = i\rho = \dfrac{iR}{L}$ $\quad \therefore$ $e = \dfrac{iRl}{L}$ $\quad$ where L = length of potentiometer wire, ρ = resistance per unit length, l = balancing length for e
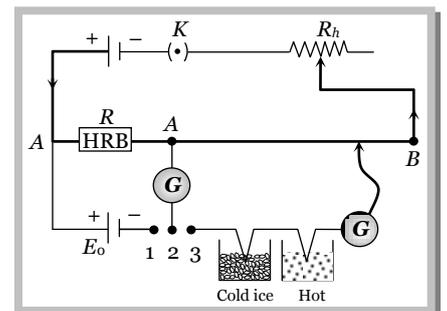
(5) **To calibrate ammeter and voltmeter**

Calibration of ammeter

(i) If p.d. across 1Ω resistance is measured by potentiometer, then current through this (indirectly measured) is thus known or if R is known then $i = V/R$ can be found.

(ii) **Circuit and method**

(a) Standardisation is required and per formed as already described earlier. ($x = E_0/l_0$)

(b) The current through R or 1Ω coil is measured by the connected ammeter and same is calculated by potentiometer by finding a balancing length as described blow.

Let i' current flows through 1Ω resistance giving p.d. as $V' = i'(1) = xl_1$ where l_1 is the balancing length. So error can be found as [i (measured by ammeter) $\Delta i' = i - i' \left[= xl_1 = \left(\dfrac{E_0}{l_0} \right) l_1 \right]$
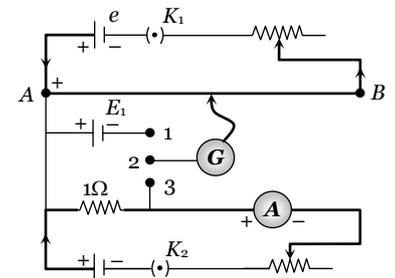

Calibration of voltmeter

(i) Practical voltmeters are not ideal, because these do not have infinite resistance. The error of such practical voltmeter can be found by comparing the voltmeter reading with calculated value of p.d. by potentiometer.

(ii) **Circuit and procedure**

(a) **Standardisation :** If l_0 is balancing length for E_0 the emf of standard cell by connecting 1 and 2 of bi-directional key, then $x = E_0/l_0$.

(b) The balancing length l_1 for unknown potential difference V' is given by (by closing 2 and 3) $V' = xl_1 = (E_0 / l_0)l_1$.

If the voltmeter reading is V then the error will be $(V - V')$ which may be $+ve, -ve$ or zero.
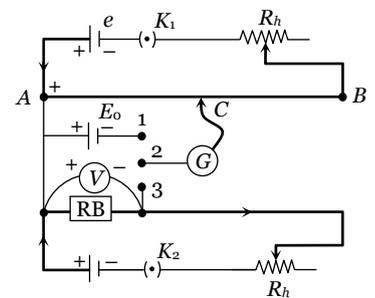
Concepts

☞ *In case of zero deflection in the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.*

☞ *A potentiometer can act as an ideal voltmeter.*

Example

Example: 85 A battery with negligible internal resistance is connected with 10m long wire. A standard cell gets balanced on 600 cm length of this wire. On increasing the length of potentiometer wire by 2m then the null point will be displaced by

(a) 200 cm (b) 120 cm (c) 720 cm (d) 600 cm

Solution : (b)   By using $\dfrac{L_1}{L_2} = \dfrac{l_1}{l_2} \Rightarrow \dfrac{10}{12} = \dfrac{600}{l_2} \Rightarrow l_2 = 720 \, cm$.

Hence displacement = 720 − 600 = 120 cm

Example: 86 In the following circuit a 10 m long potentiometer wire with resistance 1.2 ohm/m, a resistance R_1 and an accumulator of emf 2 V are connected in series. When the emf of thermocouple is 2.4 mV then the deflection in galvanometer is zero. The current supplied by the accumulator will be
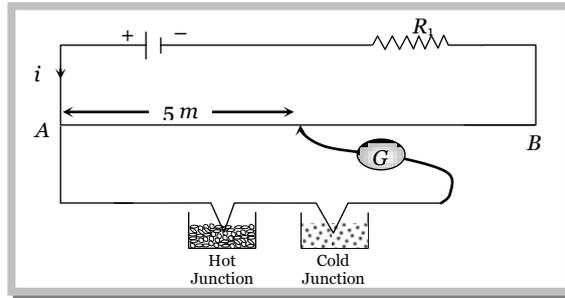
(a) $4 \times 10^{-4} A$ (b) $8 \times 10^{-4} A$ (c) $4 \times 10^{-3} A$ (d) $8 \times 10^{-3} A$

Solution : (a) $E = x \, l = i \rho l$ $\therefore \quad i = \dfrac{E}{\rho l} = \dfrac{E}{\rho l} = \dfrac{2.4 \times 10^{-3}}{1.2 \times 5} = 4 \times 10^{-4} A$.

***Example*: 87** The resistivity of a potentiometer wire is $40 \times 10^{-8} \ \Omega m$ and its area of cross section is $8 \times 10^{-6} \ m^2$. If $0.2 \ amp$. Current is flowing through the wire, the potential gradient will be

(a) $10^{-2} \ volt/m$ (b) $10^{-1} \ volt/m$ (c) $3.2 \times 10^{-2} \ volt/m$ (d) $1 \ volt/m$

Solution : (a)   Potential gradient $= \dfrac{V}{L} = \dfrac{iR}{L} = \dfrac{i\rho L}{AL} = \dfrac{i\rho}{A} = \dfrac{0.2 \times 40 \times 10^{-8}}{8 \times 10^{-6}} = 10^{-2} \ V/m$

***Example*: 88** A deniel cell is balanced on $125 \ cm$ length of a potentiometer wire. When the cell is short circuited with a $2 \ \Omega$ resistance the balancing length obtained is $100 \ cm$. Internal resistance of the cell will be **[RPMT 1998]**

(a) $1.5 \ \Omega$ (b) $0.5 \ \Omega$ (c) $1.25 \ \Omega$ (d) $4/5 \ \Omega$

Solution: (b)   By using $r = \dfrac{l_1 - l_2}{l_2} \times R' \ \Rightarrow r = \dfrac{125 - 100}{100} \times 2 = \dfrac{1}{2} = 0.5 \ \Omega$

***Example*: 89** A potentiometer wire of length $10 \ m$ and a resistance $30 \ \Omega$ is connected in series with a battery of emf $2.5 \ V$ and internal resistance $5 \ \Omega$ and an external resistance R. If the fall of potential along the potentiometer wire is $50 \ \mu V/mm$, the value of R is (in Ω)

(a) 115 (b) 80 (c) 50 (d) 100

Solution : (a)   By using $x = \dfrac{e}{(R + R_h + r)} \cdot \dfrac{R}{L}$

$\Rightarrow \dfrac{50 \times 10^{-6}}{10^{-3}} = \dfrac{2.5}{(30 + R + 5)} \times \dfrac{30}{10} \Rightarrow R = 115$

***Example*: 90** A $2 \ volt$ battery, a $15 \ \Omega$ resistor and a potentiometer of $100 \ cm$ length, all are connected in series. If the resistance of potentiometer wire is $5 \ \Omega$, then the potential gradient of the potentiometer wire is **[AIIMS 1982**

(a) $0.005 \ V/cm$ (b) $0.05 \ V/cm$ (c) $0.02 \ V/cm$ (d) $0.2 \ V/cm$

Solution : (a)   By using $x = \dfrac{e}{(R + R_h + r)} \cdot \dfrac{R}{L} \ \Rightarrow x = \dfrac{2}{(5 + 15 + 0)} \times \dfrac{5}{1} = 0.5 \ V/m = 0.005 \ V/cm$

***Example*: 91** In an experiment to measure the internal resistance of a cell by potentiometer, it is found that the balance point is at a length of $2 \ m$ when the cell is shunted by a $5 \ \Omega$ resistance; and is at a length of $3 \ m$ when the cell is shunted by a $10 \ \Omega$ resistance. The internal resistance of the cell is, then

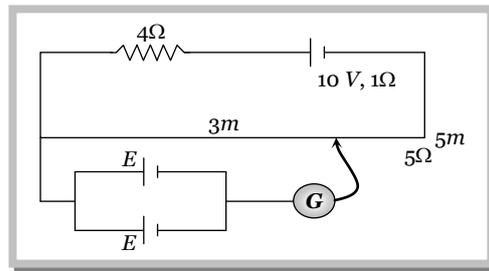(a) $1.5 \ \Omega$ (b) $10 \ \Omega$ (c) $15 \ \Omega$ (d) $1 \ \Omega$

Solution : (b)   By using $r = \left(\dfrac{l_1 - l_2}{l_2}\right) R' \Rightarrow r = \left(\dfrac{l_1 - 2}{2}\right) \times 5$ ...... (i)

and $r = \left(\dfrac{l_1 - 3}{3}\right) \times 10$ .... (ii)

On solving (i) and (ii) $r = 10\ \Omega$

Example: 92   A resistance of 4 Ω and a wire of length 5 *metres* and resistance 5 Ω are joined in series and connected to a cell of emf 10 *V* and internal resistance 1 Ω. A parallel combination of two identical cells is balanced across 300 *cm* of the wire. The emf *E* of each cell is

(a) 1.5 *V*   (b) 3.0 *V*   (c) 0.67 *V*   (d) 1.33 *V*

Solution : (b)   By using $E_{eq} = \dfrac{e}{(R + R_h + r)} \cdot \dfrac{R}{L} \times l \Rightarrow E = \dfrac{10}{(5 + 4 + 1)} \times \dfrac{5}{5} \times 3 \Rightarrow E = 3\ volt$

Example: 93   A potentiometer has uniform potential gradient across it. Two cells connected in series (i) to support each other and (ii) to oppose each other are balanced over 6 *m* and 2 *m* respectively on the potentiometer wire. The emf's of the cells are in the ratio of   **[MP PMT 2002; RPMT 2000]**

(a) 1 : 2   (b) 1 : 1   (c) 3 : 1   (d) 2 : 1

Solution : (d)   If suppose emf's of the cells are E_1 and E_2 respectively then

$E_1 + E_2 = x \times 6$ .... (i)   [x = potential gradient)]

and $E_1 - E_2 = x \times 2$ ....(ii)

$\Rightarrow$ $\dfrac{E_1 + E_2}{E_1 - E_2} = \dfrac{3}{1} \Rightarrow \dfrac{E_1}{E_2} = \dfrac{2}{1}$

Example: 94   In the following circuit the potential difference between the points *B* and *C* is balanced against 40 *cm* length of potentiometer wire. In order to balance the potential difference between the points *C* and *D*, where should jockey be pressed
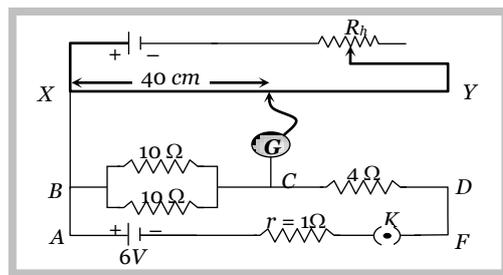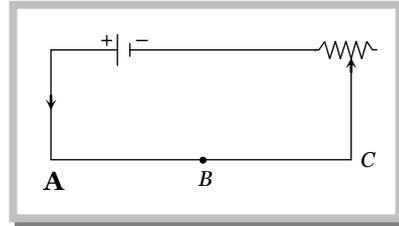

(a) 32 *cm*   (b) 16 *cm*   (c) 8 *cm*   (d) 4 *cm*

Solution : (a) $\dfrac{1}{R} = \dfrac{1}{10} + \dfrac{1}{10} = \dfrac{2}{10} = \dfrac{1}{5}$ or $R_1 = 5\ \Omega$

$R_2 = 4\Omega,\ l_1 = 40\ cm,\ l_2 = ?$ $l_2 = l_1 \dfrac{R_2}{R_1}$ or $l_2 = \dfrac{40 \times 4}{5} = 32\ cm$

***Example*: 95** In the following circuit diagram fig. the lengths of the wires *AB* and *BC* are same but the radius of *AB* is three times that of *BC*. The ratio of potential gradients at *AB* and *BC* will be

(a) 1 : 9

(b) 9 : 1

(c) 3 : 1

(d) 1 : 3

Solution : (a) $x \propto R_p \propto \dfrac{1}{r^2} \Rightarrow \dfrac{x_1}{x_2} = \dfrac{r_2^2}{r_1^2} = \left(\dfrac{r}{3r}\right)^2 = \dfrac{1}{9}$

***Example*: 96** With a certain cell the balance point is obtained at 0.60 *m* from one end of the potentiometer. With another cell whose emf differs from that of the first by 0.1 *V*, the balance point is obtained at 0.55 *m*. Then, the two emf's are

(a) 1.2 *V*, 1.1 *V*   (b) 1.2 *V*, 1.3 *V*   (c) − 1.1 *V*, − 1.0 *V*   (d) None of the above

Solution : (a) $E_1 = x\,(0.6)$ and $E_2 = E_1 - 0.1 = x\,(0.55) \Rightarrow \dfrac{E_1}{E_1 - 0.1} = \dfrac{0.6}{0.55}$

or $55\,E_1 = 60\,E_1 - 6 \Rightarrow E_1 = \dfrac{6}{5} = 1.2\,V$ thus $E_2 = 1.1\,V$

Tricky Example: 11

A cell of internal resistance 1.5Ω and of emf 1.5 *volt* balances 500 *cm* on a potentiometer wire. If a wire of 15Ω is connected between the balance point and the cell, then the balance point will shift

[MP PMT 1985]

(a) To zero   (b) By 500 *cm*

(c) By 750 *cm*   (d) None of the above

Solution : (d)   In balance condition no current flows in the galvanometer circuit. Hence there will be no shift in balance point after connecting a resistance between balance point and cell.

.