Chapter 1

REPRESENTATION OF POWER SYSTEMS

- One Line Diagram
- Table of Symbols
- Sample Example System
- Impedance Diagram
- Reactance Diagram
- Examples

Chapter 2

SYMMETRICAL THREE PHASE FAULTS

- Transients on a Transmission line
- Short Circuit of Unloaded Syn. Machine
- Short Circuit Reactances
- Sort Circuit Current Oscillogram
- Short Circuit of a Loaded machine
- Examples

Chapter 3

SYMMETRICAL COMPONENTS

- What are Symmetrical Components?
- Resolution of components, Neutral Shift
- Phase shift in Y-Δ Transformer Banks
- Power in terms of Symmetrical Components
- Sequence Imps. & Sequence Networks
- Sequence Networks of system elements
- Examples

Chapter 4

UNSYMMETRICAL FAULTS

- Review of Symmetrical Components
- Preamble to Unsymmetrical Fault Analysis
- L-G, L-L, L-L-G & 3-Phase Faults
- Faults on Power Systems
- Effect of Fault Impedance
- Open Conductor Faults
- Examples

Chapter 5
STABILITY STUDIES

- Basic Stability terms
- Swing Equation & Swing Curve
- Power Angle Equation
- Equal Area Criterion
- Determination of Stability of a System
- Examples

EXPECTED PATTERN OF QUESTION PAPER

One question each on chapter 1 and 2 And **Two** questions each on chapters 3, 4 and 5.
Note: Five questions out of 8 are to be answered in full.

TEXTS/ REFERENCES:

1. **WD Stevenson**, Elements of Power System Analysis, MH.
2. **IJ Nagrath and DP Kothari,** modern Power System analysis, TMH..
3. **Hadi Sadat**: Power System Analysis, TMH
4. **GL Kusic**: Computer aided PSA, PHI

PREREQUISITE SUBJECTS:

1. DC and Synchronous Machines
2. Transmission and distribution
3. Transformers and induction machines

ELECTRICAL POWER SYSTEMS-

THE STATE-OF-THE-ART:

To Begin With ……

No.	TOPICS	SUB-TOPICS
1.	**Representation of Power systems**	- SLD/ OLD - React./ Imp. Diagram - per unit Systems
2.	**Electric Power System**	- Generation - Machines - Transmission Trans- - Distribution formers - Utilization – Tariffs
3.	**Fault studies**	- Sym. Faults - Sym. Components - Seq. Imps. / Networks - Unsymmetrical Faults
4.	**System Stability**	- SSS, TS, DS - Angle Stability - Solution of Equations - EAC, Clarke's Diagram
5.	**Linear Graph Theory (Linear Equations)**	- Incidence Matrices - Frames of Reference - Singular/ NS Transformations - Network Matrices - Node Elimination - Z_{BUS} Building
6.	**Power Flow Studies (NL Equations)**	- Buses, Y_{BUS} Advs., Loads flow equations - Iterative Methods - GS, NR, FDLF & DCLF

Present Scenario ……

No.	TOPICS	SUB-TOPICS
7.	**Reactive Power Management**	- Importance of VArs - Compensation Devices, Sizing, Placement, Design, Optimality, - VAr Dispatch - VAr Co-ordination
8.	**Gen. Expansion Planning**	- Optimality - Load Prediction: Short, Medium and Long Term Forecasting
9.	**Operation and Control**	-EMS: EMC, SLDC,RLDC -ALFC, Voltage Control -Tie-line Power Control
10.	**System Reliability**	- Requirements - Methods
11.	**Economic Operation**	- Unit Commitment - Parallel Operation - Optimal Load Dispatch - Constraints
12.	**Instrumentation**	- CTs, PTs
13.	**State Estimation**	- SCADA - Bad Data Elimination - Security/ Cont. Studies

Future Trends ……

No.	TOPICS	SUB-TOPICS
14.	**Voltage Stability**	- Importance - Angle/ Voltage stability - P-δ Vs. Q- $\lvert V \rvert$ Analysis - Proximity Indices - WBOV
15.	**Power System Simulators**	- Requirements - Control Blocks - Data-Base Definition
16.	**Energy Auditing**	- Deregulation
17.	**Demand Side Management**	- Time of Use Pricing
18.	**Renewable Energy**	- The Paradigm
19.	**Sparsity Oriented Programming**	- Sparsity: Y_{BUS} - Ordering Schemes - LU- Factorization: Fills - Pivoting - UD Table Storage
20.	**Recent Computer Applications**	- AI, Expert Systems - ANN, Genetic Algorithms - Fuzzy Logic

ELECTRICAL POWER SYSTEMS-

An Introduction

Energy in electrical form, apart from being clean, can be generated (converted from other natural forms) centrally in bulk; can be easily controlled; transmitted efficiently; and it is easily and efficiently adaptable to other forms of energy for various industrial and domestic applications. It is therefore a coveted form of energy and is an essential ingredient for the industrial and all-round development of any country.

The generation of electrical energy (by converting other naturally available forms of energy), controlling of electrical energy, transmission of energy over long distances to different load centers, and distribution and utilization of electrical energy together is called an electrical power system.

The subsystem that generates electrical energy is called generation subsystem or generating plants (stations). It consists of generating units (consisting of turbine-alternator sets) including the necessary accessories. Speed governors for the prime movers (turbines; exciters and voltage regulators for generators, and step-up transformers also form part of the generating plants.

The subsystem that transmits the electrical energy over long distances (from generating plants to main load centers) is called transmission subsystem. It consists of transmission lines, regulating transformers and static/rotating VAR units (which are used to control active/reactive powers).

The sub system that distributes of energy from load centers to individual consumer points along with end energy converting devices such as motors, resistances etc., is called distribution subsystems. It consists of feeders, step-down transformers, and individual consumer connections along with the terminal energy converting electrical equipment such as motors, resistors etc.

Electrical energy cannot be stored economically and the electric utility can exercise little control over the load demand (power) at any time. The power system must, therefore, be capable of matching the output from the generators to demand at any time at specified voltage and frequency.

With the constant increase in the electrical energy demand, more and more generating units, the transmission lines and distribution network along with the necessary controlling and protective circuits make the power system a large complex system. It is considered as one of the largest man-made systems. Hence highly trained engineers are needed to develop and implement the advances of technology for planning, operation and control of power systems.

The objective of the Course

The objective this course is to present methods of analysis with respect to the operation and control of power systems. Planning and expansion, operation and control of a power system require modeling (representation of the system suitable for analysis), load flow studies, fault calculations, protective schemes, and stability studies. In addition, there are more advanced issues such as economic operation which involve special algorithms for secured and economical operation of power systems.

Load flow analysis is the determination of the voltage, current, real and reactive powers at various points in the power network under normal operating conditions.

A fault in a power network is any failure which interferes with the normal operation of the system. Fault calculations or *Fault analysis* consist of determining the fault currents for various types of faults at various points of the network.

Faults can be very destructive to power systems. *System protection* schemes are therefore be evolved and implemented for the reliability and safety of power systems.

Stability analysis deals with the determination of the effects of disturbances on power systems. The disturbance may vary from be the usual fluctuation of the load to severe fault causing the loss of an important transmission line.

The *economic operation* requires power systems to be operated at such conditions which will ensure minimum cost of operation meeting all the conditions.

CHAPTER 1

REPRESENTATION OF POWER SYSTEMS

[CONTENTS: One line diagram, impedance diagram, reactance diagram, per unit quantities, per unit impedance diagram, formation of bus admittance & impedance matrices, examples]

1.1 One Line Diagram

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

Symbols used for SLD

Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.

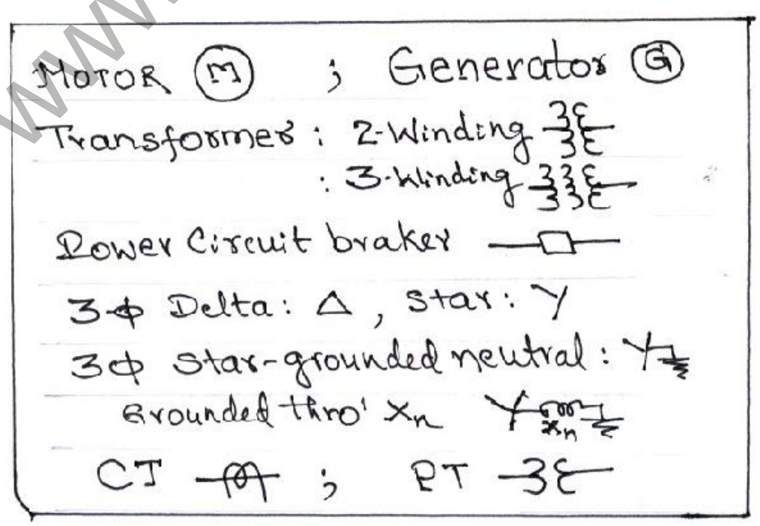
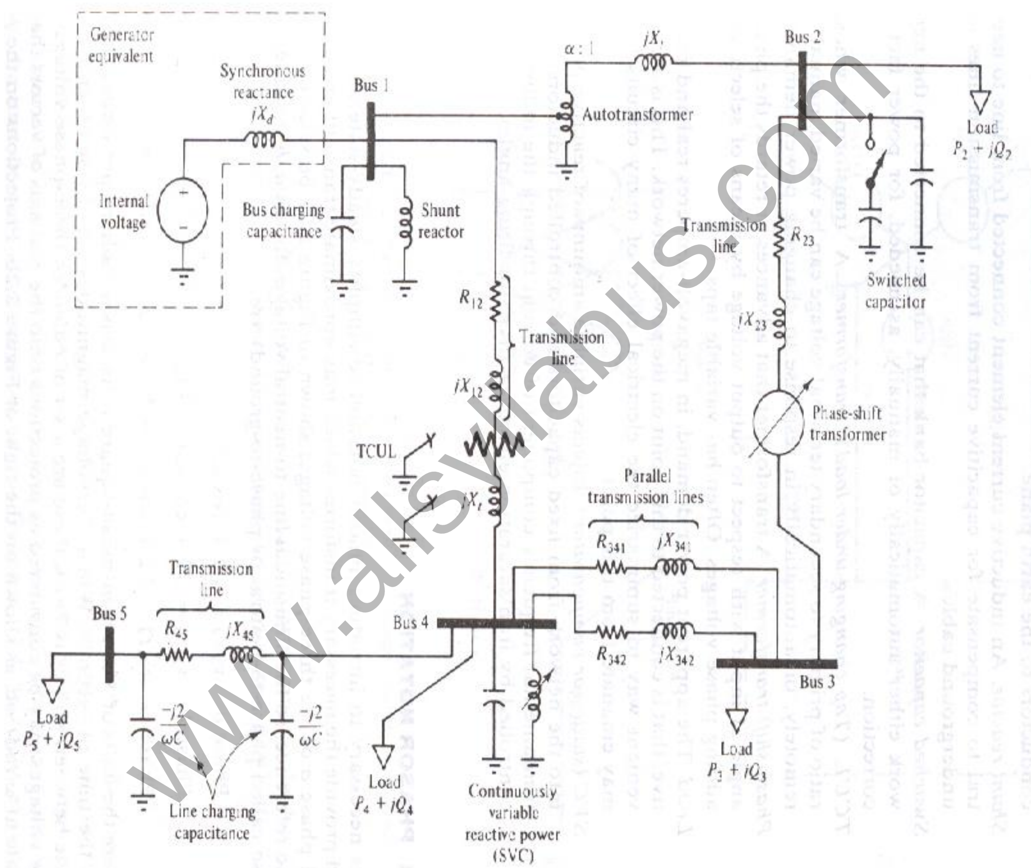


Figure 1. TABLE OF SYMBOLS FOR USE ON SLDS

Example system

Consider for illustration purpose, a sample example power system and data as under: Generator 1: 30 MVA, 10.5 KV, X"= 1.6 ohms, Generator 2: 15 MVA, 6.6 KV, X"= 1.2 ohms, Generator 3: 25 MVA, 6.6 KV, X"= 0.56 ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV, X=15.2 ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV, X=16.0 ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.



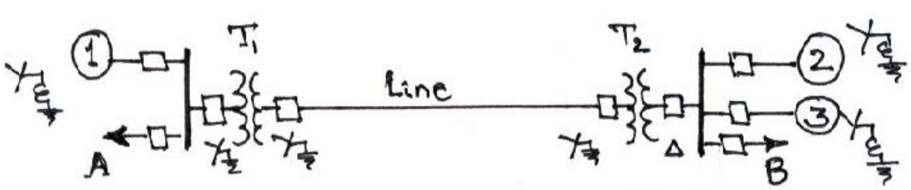


Figure 2. SAMPLE SYSTEM OLD

It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3-phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-L voltage and power factor.

1.2 Impedance Diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),
2. The magnetizing reactances of transformers are negligible,
3. The generators are represented as constant voltage sources with series resistance or reactance,
4. The transmission lines are approximated by their equivalent π-Models,
5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and
6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.

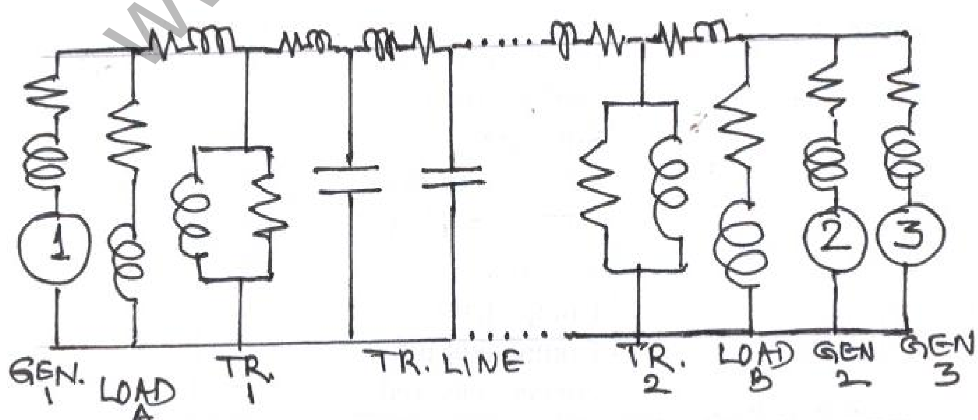


Figure 3. IMPEDANCE DIAGRAM

1.3 Reactance Diagram

With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Example system

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure 4.

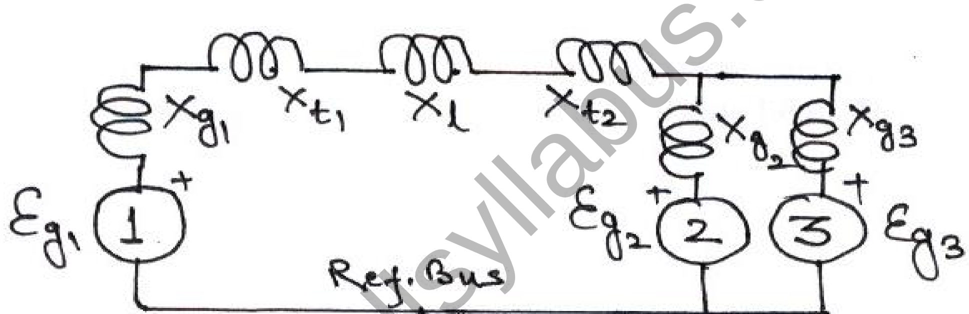


Figure 4. REACTANCE DIAGRAM

<u>Note</u>: These impedance & reactance diagrams are also refered as the *Positive Sequence Diagrams/ Networks.*

1.4 Per Unit Quantities

during the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Definition: Per Unit value of a given q*uantity is the ratio of the* ***actual value*** *in any given unit to the* ***base value*** *in the same unit.* The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit itself.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

If I_b is the base current in kilo amperes and V_b, the base voltage in kilovolts, then the base MVA is, $S_b = (V_b I_b)$. Then the base values of current & impedance are given by

$$\text{Base current (kA)}, \quad I_b = MVA_b/KV_b = S_b/V_b \qquad (1.1)$$

$$\text{Base impedance}, \quad Zb = (V_b/I_b) = (KV_b^2 / MVA_b) \qquad (1.2)$$

Hence the per unit impedance is given by

$$Z_{pu} = Z_{ohms}/Z_b = Z_{ohms}\,(MVA_b/KV_b^2) \qquad (1.3)$$

In 3-phase systems, KV_b is the line-to-line value & MVA_b is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

Changing the base of a given pu value:

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If Z_{pu}^{new} is the pu impedance required to be calculated on a new set of base values: MVA_b^{new} & KV_b^{new} from the already given per unit impedance Z_{pu}^{old}, specified on the old set of base values, MVA_b^{old} & KV_b^{old} , then we have

$$Z_{pu}^{new} = Z_{pu}^{old}\,(MVA_b^{new}/MVA_b^{old})\,(KV_b^{old}/KV_b^{new})^2 \qquad (1.4)$$

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

Merits and Demerits of pu System

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

Merits:

- The pu value is the same for both 1-phase and & 3-phase systems
- The pu value once expressed on a proper base, will be the same when refereed to either side of the transformer. Thus the presence of transformer is totally eliminated
- The variation of values is in a smaller range 9nearby unity). Hence the errors involved in pu computations are very less.
- Usually the nameplate ratings will be marked in pu on the base of the name plate ratings, etc.

Demerits:

- If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

1.5 pu Impedance / Reactance Diagram

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is refered as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

1. Obtain the one line diagram based on the given data
2. Choose a common base MVA for the system
3. Choose a base KV in any one section (Sections formed by transformers)
4. Find the base KV of all the sections present
5. Find pu values of all the parameters: R,X, Z, E, etc.
6. Draw the pu impedance/ reactance diagram.

1.6 Formation Of Y_{BUS} & Z_{BUS}

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS}\, I_{BUS}$$

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad (1.5)$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR}\, I_{BR}$$

$$I_{BR} = Y_{BR}\, E_{BR} \qquad (1.6)$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP}\, I_{LOOP}$$

$$I_{LOOP} = Y_{LOOP}\, E_{LOOP} \qquad (1.7)$$

Of the various network matrices refered above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3(V_1-V_3) + Y_6(V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5(V_2-V_3) + Y_6(V_2 - V_1)$

At node 3: $0 = Y_3(V_3-V_1) + Y_4V_3 + Y_5(V_3 - V_2) \qquad (1.8)$

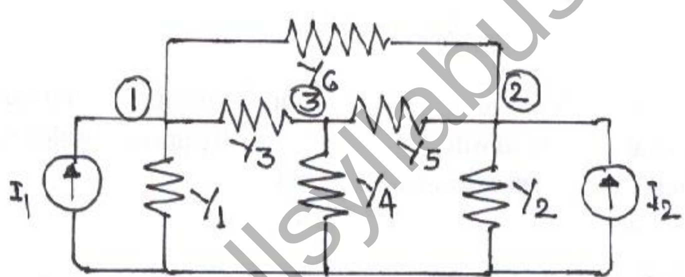


Figure 5. EXAMPLE SYSTEM FOR FINDING Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad (1.9)$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad (1.10)$$

Where, Y_{BUS} is the bus admittance matrix, I_{BUS} & E_{BUS} are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y_{BUS} of equation (9), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Y_{ii}) of the bus admittance matrix, Y_{BUS}, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Y_{ij}) of the bus admittance matrix, Y_{BUS}, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$

$$Y_{ij} = -\, y_{ij} \quad (j = 1,2,\ldots\ldots n) \qquad (1.11)$$

For i = 1,2,….n, n = no. of buses of the given system, y_{ij} is the admittance of element connected between buses i and j and y_{ii} is the admittance of element connected between bus i and ground (reference bus).

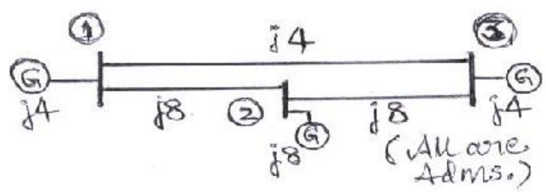
Bus impedance matrix

In cases where, the bus impedance matrix is also required, then it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible. **Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

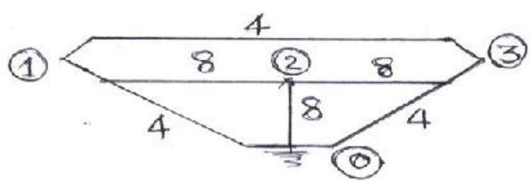
1.7 Examples

I EXAMPLES ON RULE OF INSPECTION:

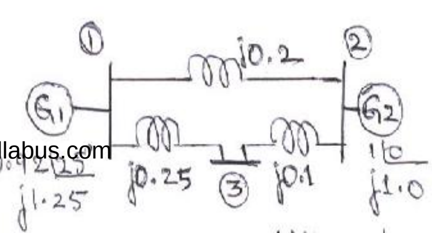
Problem #1: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection



$$\mathbf{Y_{BUS}} = \begin{vmatrix} \mathbf{16} & \mathbf{-8} & \mathbf{-4} \\ \mathbf{-8} & \mathbf{24} & \mathbf{-8} \\ \mathbf{-4} & \mathbf{-8} & \mathbf{16} \end{vmatrix}$$

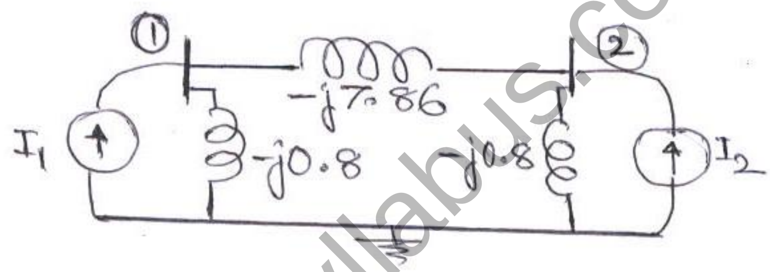


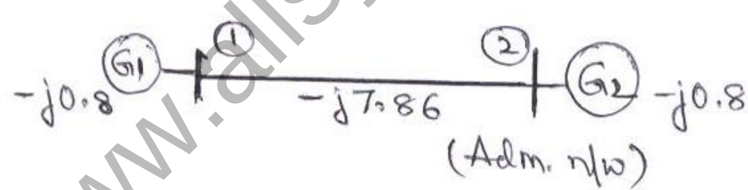
Problem #2: Obtain Y_{BUS} and Z_{BUS} matrices for the impedance network shown aside by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



$$Y_{BUS} = \begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1}$$





$$Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C$$

$$Y_{BUS} = \begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.86 \end{vmatrix}$$

II EXAMPLES ON PER UNIT ANALYSIS:

Problem #1:

Two generators rated 10 MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to 2 motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 KV. Assuming the base quantities as 50 MVA, 13.8 KV, draw the per unit reactance diagram. The percentage reactance for generators is 15% and that for motors is 20%.

Solution:

The one line diagram with the data is obtained as shown in figure P1(a).

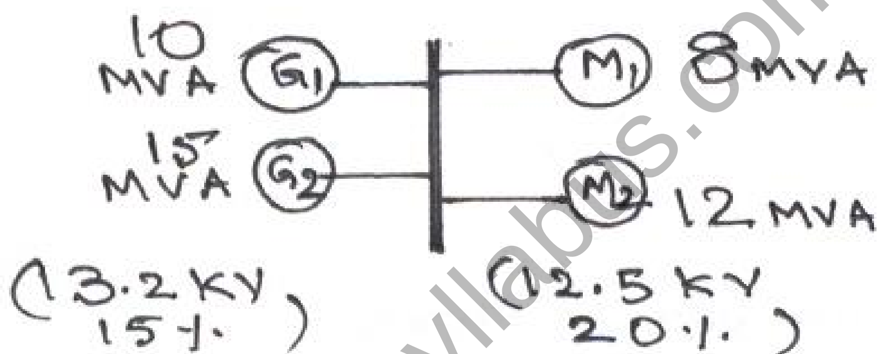


Figure P1(a). OLD of the given system

Selection of base quantities: **50** MVA, **13.8** KV (Given)

Calculation of pu values:

X_{G1} = j 0.15 (50/10) $(13.2/13.8)^2$ = j 0.6862 pu.

X_{G2} = j 0.15 (50/15) $(13.2/13.8)^2$ = j 0.4574 pu.

X_{m1} = j 0.2 (50/8) $(12.5/13.8)^2$ = j 1.0256 pu.

X_{m2} = j 0.2 (50/12) $(12.5/13.8)^2$ = j 0.6837 pu.

E_{g1} = E_{g2} = (13.2/13.8) = 0.9565 $\angle 0^0$ pu

E_{m1} = E_{m2} = (12.5/13.8) = 0.9058 $\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P1(b).

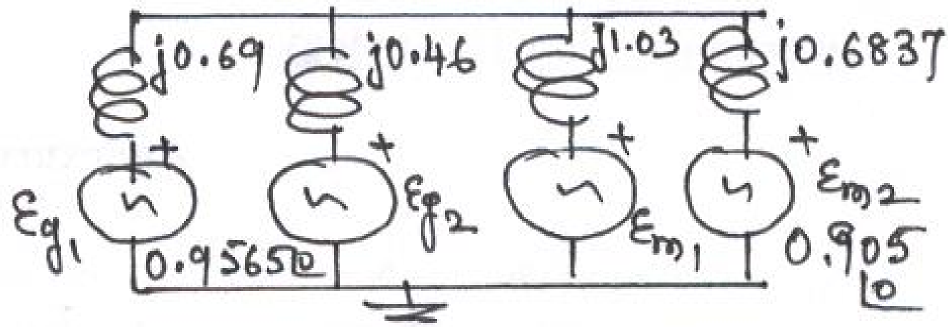


Figure P1(b).
Per Unit Reactance Diagram

Problem #2:

Draw the per unit reactance diagram for the system shown in figure below. Choose a base of 11 KV, 100 MVA in the generator circuit.

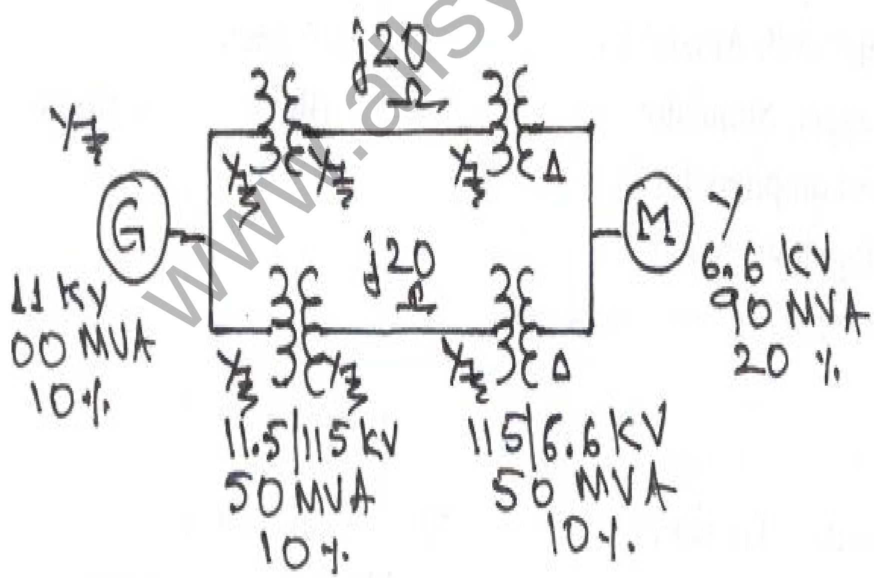


Figure P2(a). OLD of the given system

Solution:

The one line diagram with the data is considered as shown in figure.

Selection of base quantities:

100 MVA, **11** KV in the generator circuit(Given); the voltage bases in other sections are: 11 (115/11.5) = **110** KV in the transmission line circuit and 110 (6.6/11.5) = **6.31** KV in the motor circuit.

Calculation of pu values:

X_G = j 0.1 pu, X_m = j 0.2 (100/90) $(6.6/6.31)^2$ = j 0.243 pu.

X_{t1} = X_{t2} = j 0.1 (100/50) $(11.5/11)^2$ = j 0.2185 pu.

X_{t3} = X_{t4} = j 0.1 (100/50) $(6.6/6.31)^2$ = j 0.219 pu.

X_{lines} = j 20 $(100/110^2)$ = j 0.1652 pu.

$E_g = 1.0\angle 0^0$ pu, E_m = (6.6/6.31) = $1.045\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P2(b).

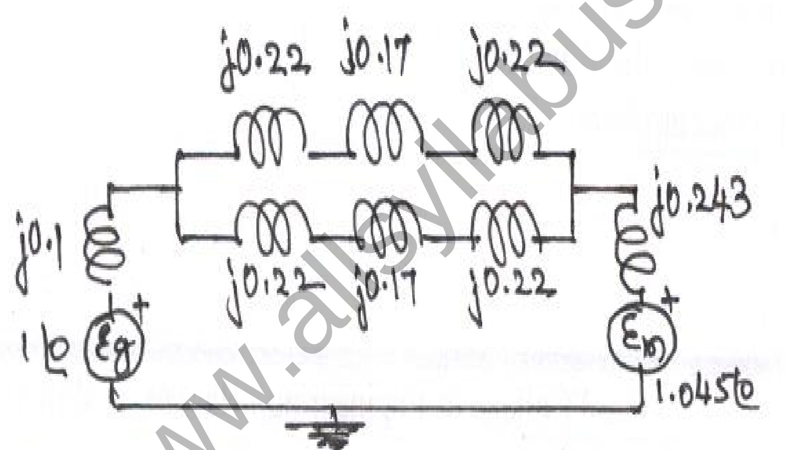


Figure P2(b). Per Unit Reactance Diagram

Problem #3:

A 30 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 2 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 20 MVA and 10 MVA at 12.8 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 35 MVA, 13.2 KV-Δ /115 KV-Y with 10 % leakage reactance. The line reactance is 80 ohms. Draw the equivalent per unit reactance diagram by selecting the generator ratings as base values in the generator circuit.

Solution:

The one line diagram with the data is obtained as shown in figure P3(a).

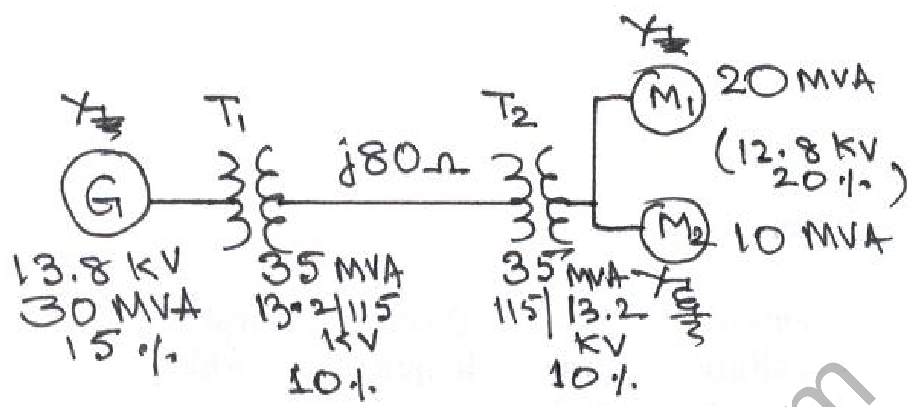


Figure P3(a). OLD of the given system

Selection of base quantities:

30 MVA, **13.8** KV in the generator circuit(Given);

The voltage bases in other sections are:

13.8 (115/13.2) = **120.23** KV in the transmission line circuit and

120.23 (13.26/115) = **13.8** KV in the motor circuit.

Calculation of pu values:

$X_G = j\ 0.15$ pu.

$X_{m1} = j\ 0.2\ (30/20)\ (12.8/13.8)^2 = j\ 0.516$ pu.

$X_{m2} = j\ 0.2\ (30/10)\ (12.8/13.8)^2 = j\ 0.2581$ pu.

$X_{t1} = X_{t2} = j\ 0.1\ (30/35)\ (13.2/13.8)^2 = j\ 0.0784$ pu.

$X_{line} = j\ 80\ (30/120.23^2) = j\ 0.17$ pu.

$E_g = 1.0\angle 0^0$ pu; $E_{m1} = E_{m2} = (6.6/6.31) = 0.93\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P3(b).

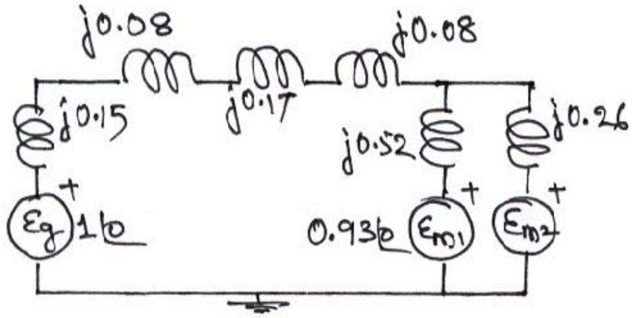


Figure P3(b). Per Unit Reactance Diagram

Problem #4:

A 33 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 0.5%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 25 MVA at 6.6 KV with 25% sub transient reactance. Draw the equivalent per unit impedance diagram by selecting 25 MVA (3ϕ), 6.6 KV (LL) as base values in the motor circuit, given the transformer and transmission line data as under:

Step up transformer bank: three single phase units, connected Δ–Y, each rated 10 MVA, 13.2/6.6 KV with 7.7 % leakage reactance and 0.5 % leakage resistance;

Transmission line: 75 KM long with a positive sequence reactance of 0.8 ohm/ KM and a resistance of 0.2 ohm/ KM; and

Step down transformer bank: three single phase units, connected Δ–Y, each rated 8.33 MVA, 110/3.98 KV with 8% leakage reactance and 0.8 % leakage resistance;

Solution:

The one line diagram with the data is obtained as shown in figure P4(a).

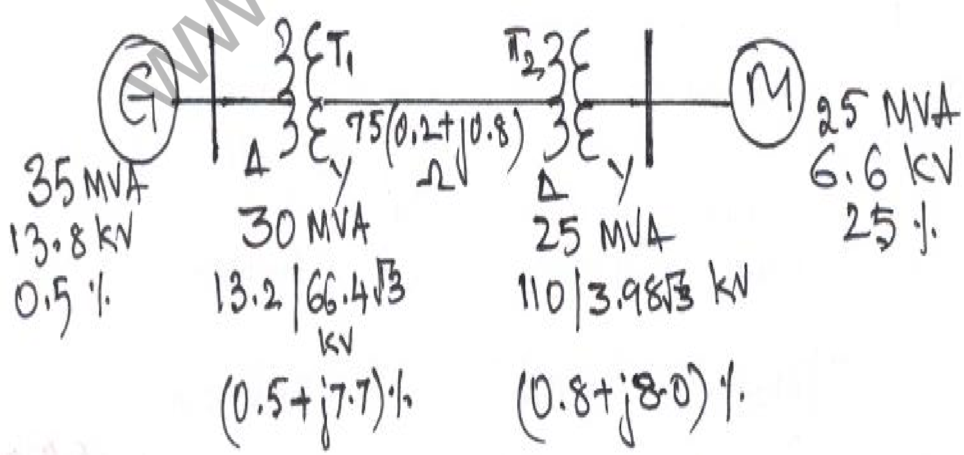


Figure P4(a). OLD of the given system

3-phase ratings of transformers:

T_1: 3(10) = 30 MVA, 13.2/ 66.4√3 KV = 13.2/ 115 KV, X = 0.077, R = 0.005 pu.
T_2: 3(8.33) = 25 MVA, 110/ 3.98√3 KV = 110/ 6.8936 KV, X = 0.08, R = 0.008 pu.

Selection of base quantities:

25 MVA, **6.6** KV in the motor circuit (Given); the voltage bases in other sections are: 6.6 (110/6.8936) = **105.316** KV in the transmission line circuit and 105.316 (13.2/115) = **12.09** KV in the generator circuit.

Calculation of pu values:

$X_m = j\,0.25$ pu; $E_m = 1.0\angle 0^0$ pu.

$X_G = j\,0.005\,(25/33)\,(13.8/12.09)^2 = j\,0.005$ pu; $E_g = 13.8/12.09 = 1.414\angle 0^0$ pu.

$Z_{t1} = 0.005 + j\,0.077\,(25/30)\,(13.2/12.09)^2 = 0.005 + j\,0.0765$ pu. (ref. to LV side)

$Z_{t2} = 0.008 + j\,0.08\,(25/25)\,(110/105.316)^2 = 0.0087 + j\,0.0873$ pu. (ref. to HV side)

$Z_{line} = 75\,(0.2+j\,0.8)\,(25/\,105.316^2) = 0.0338 + j\,0.1351$ pu.

Thus the pu reactance diagram can be drawn as shown in figure P4(b).

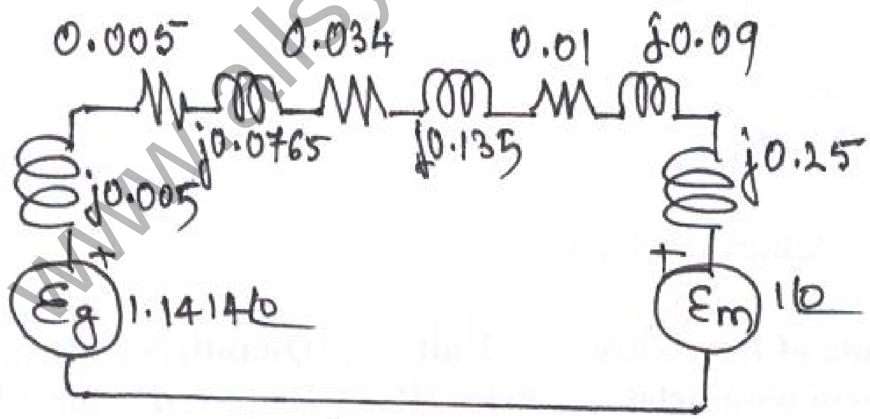


Figure P4(b).
Per Unit Reactance Diagram

1.8 Exercises for Practice

Problems

1. Determine the reactances of the three generators rated as follows on a common base of 200 MVA, 35 KV: Generator 1: 100 MVA, 33 KV, sub transient reactance of 10%; Generator 2: 150 MVA, 32 KV, sub transient reactance of 8% and Generator 3: 110 MVA, 30 KV, sub transient reactance of 12%.

[*Answers*: X_{G1} = j 0.1778, X_{g2} = j 0.089, X_{g3} = j 0.16 all in per unit]

2. A 100 MVA, 33 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 3 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 30 MVA, 20 MVA and 50 MVA, at 30 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 100 MVA, 32 KV-Δ /110 KV-Y with 8 % leakage reactance. The line has a reactance of 50 ohms. By selecting the generator ratings as base values in the generator circuit, determine the base values in all the other parts of the system. Hence evaluate the corresponding pu values and draw the equivalent per unit reactance diagram.

[*Answers*: X_G = j 0.15, X_{m1} = j 0.551, X_{m2} = j 0.826, X_{m3} = j 0.331, $E_{g1}=1.0\angle 0^0$, $E_{m1} = E_{m2} = E_{m3} = 0.91\angle 0^0$, $X_{t1} = X_{t2}$ = j 0.0775 and X_{line} = j 0.39 all in per unit]

3. A 80 MVA, 10 KV, 3-phase generator has a sub transient reactance of 10%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 95 MVA, 6.3 KV with 15% sub transient reactance. The step-up 3-phase transformer is rated at 90 MVA, 11 KV-Y /110 KV-Y with 10% leakage reactance. The 3-phase step-down transformer consists of three single phase Y-Δ connected transformers, each rated at 33.33 MVA, 68/6.6 KV with 10% leakage reactance. The line has a reactance of 20 ohms. By selecting the 11 KV, 100 MVA as base values in the generator circuit, determine the base values in all the other parts of the system. Hence evaluate the corresponding pu values and draw the equivalent per unit reactance diagram.

[*Answers*: X_G = j 1.103, X_m = j 0.165, $E_{g1}=0.91\angle 0^0$, $E_m= 1.022\angle 0^0$, X_{t1} = j 0.11, X_{t2} = j 0.114 and X_{line} = j 0.17 all in per unit]

4. For the three-phase system shown below, draw an impedance diagram expressing all impedances in per unit on a common base of 20 MVA, 2600 V on the HV side of the transformer. Using this impedance diagram, find the HV and LV currents.

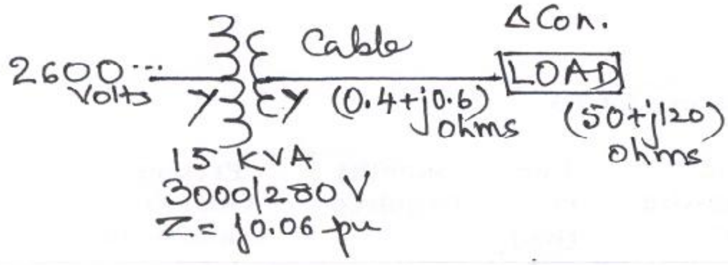


Figure E3. OLD of the Example system

[*Answers*: S_b = 20 MVA; V_b=2.6 KV (HV) and 0.2427 KV (LV); $V_t=1.0\angle 0^0$, X_t = j 0.107, Z_{cable} = 0.136 +j 0.204 and Z_{load} = 5.66 + j 2.26, I = 0.158 all in per unit, I (hv)= 0.7 A and I (lv) = 7.5 A]

Objective type questions

1. Under no load conditions the current in a transmission line is due to.
  a) Corona effects
  b) Capacitance of the line
  c) Back flow from earth
  d) None of the above

2. In the short transmission line which of the following is used?
  a) π - Model
  b) T – Model
  c) Both (a) and (b)
  d) None of the above

3. In the short transmission line which of the following is neglected?
  a) I^2 R loss
  b) Shunt admittance
  c) Series impedance
  d) All of the above

4. Which of the following loss in a transformer is zero even at full load?
  a) Eddy current
  b) Hysteresis
  c) Core loss
  d) Friction loss

5. The transmission line conductors are transposed to
  a) Balance the current
  b) Obtain different losses
  c) Obtain same line drops
  d) Balance the voltage

[Ans.: 1(b), 2(a), 3(b), 4(d), 5(c)]

CHAPTER 2

SYMMETRICAL THREE PHASE FAULTS

[CONTENTS: Preamble, transients on a transmission line, short circuit of an unloaded synchronous machine- short circuit currents and reactances, short circuit of a loaded machine, selection of circuit breaker ratings, examples]

2.1 Preamble

in practice, any disturbance in the normal working conditions is termed as a FAULT. The effect of fault is to load the device electrically by many times greater than its normal rating and thus damage the equipment involved. Hence all the equipment in the fault line should be protected from being overloaded. In general, overloading involves the increase of current up to 10-15 times the rated value. In a few cases, like the opening or closing of a circuit breaker, the transient voltages also may overload the equipment and damage them.

In order to protect the equipment during faults, fast acting circuit breakers are put in the lines. To design the rating of these circuit breakers or an auxiliary device, the fault current has to be predicted. By considering the equivalent per unit reactance diagrams, the various faults can be analyzed to determine the fault parameters. This helps in the protection and maintenance of the equipment.

Faults can be symmetrical or unsymmetrical faults. In symmetrical faults, the fault quantity rises to several times the rated value equally in all the three phases. For example, a 3-phase fault - a dead short circuit of all the three lines not involving the ground. On the other hand, the unsymmetrical faults may have the connected fault quantities in a random way. However, such unsymmetrical faults can be analyzed by using the *Symmetrical Components*. Further, the neutrals of the machines and equipment may or may not be grounded or the fault may occur through fault impedance. The three-phase fault involving ground is the most severe fault among the various faults encountered in electric power systems.

2.2 Transients on a transmission line

Now, let us Consider a transmission line of resistance R and inductance L supplied by an ac source of voltage v, such that $v = V_m \sin(\omega t+\alpha)$ as shown in figure 1. Consider the short circuit transient on this transmission line. In order to analyze this symmetrical 3-phase fault, the following assumptions are made:

- The supply is a constant voltage source,
- The short circuit occurs when the line is unloaded and

- The line capacitance is negligible.

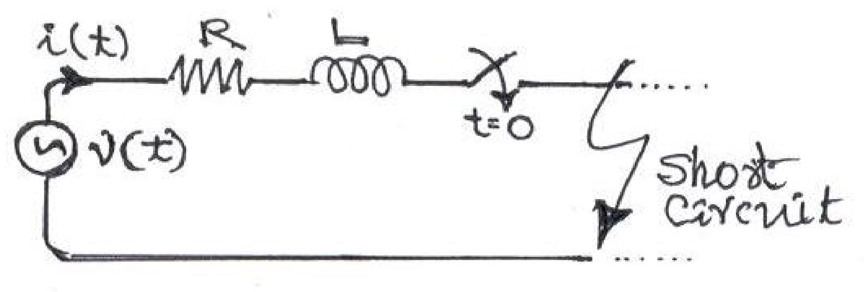


Figure 1. Short Circuit Transients on an Unloaded Line.

Thus the line can be modeled by a lumped R-L series circuit. Let the short circuit take place at t=0. The parameter, α controls the instant of short circuit on the voltage wave. From basic circuit theory, it is observed that the current after short circuit is composed of the two parts as under: $i = i_s + i_t$, Where, i_s is the steady state current and i_t is the transient current. These component currents are determined as follows.

Consider, $v = V_m \sin(\omega t+\alpha)$

$$= iR + L\,(di/dt) \qquad (2.1)$$

and $i = I_m \sin(\omega t+\alpha-\theta)$ (22.)

Where $V_m = \sqrt{2}V;\ I_m = \sqrt{2}I;\ Z_{mag} = \sqrt{[R^2+(\omega L)^2]} = \tan^{-1}(\omega L/R)$ (2.3)

Thus $i_s = [V_m/Z] \sin(\omega t+\alpha-\theta)$ (2.4)

Consider the performance equation of the circuit of figure 1 under circuit as:

$$iR + L\,(di/dt) = 0$$

i.e., $(R/L + d/dt)i = 0$ (2.5)

In order to solve the equation (5), consider the complementary function part of the solution as: $CF = C_1\, e^{(-t/\tau)}$ (2.6)

Where τ (= L/R) is the time constant and C_1 is a constant given by the value of steady state current at t = 0. Thus we have,

$$C_1 = -is(0)$$

$$= -[V_m/Z] \sin(\alpha-\theta)$$

$$= [V_m/Z] \sin(\theta-\alpha) \qquad (2.7)$$

Similarly the expression for the transient part is given by:

$$i_t = -is(0)\, e^{(-t/\tau)}$$

$$= [V_m/Z] \sin(\theta-\alpha)\, e^{(-R/L)t} \qquad (2.8)$$

Thus the total current under short circuit is given by the solution of equation (1) as [combining equations (4) and (8)],

$i = i_s + i_t$

$$= [\sqrt{2}V/Z] \sin(\omega t+\alpha-\theta) + [\sqrt{2}V/Z] \sin(\theta-\alpha)\, e^{(-R/L)t} \qquad (2.9)$$

Thus, i_s is the sinusoidal steady state current called as the *symmetrical short circuit current* and i_t is the unidirectional value called as the *DC off-set current*. This causes the total current to be unsymmetrical till the transient decays, as clearly shown in figure 2.

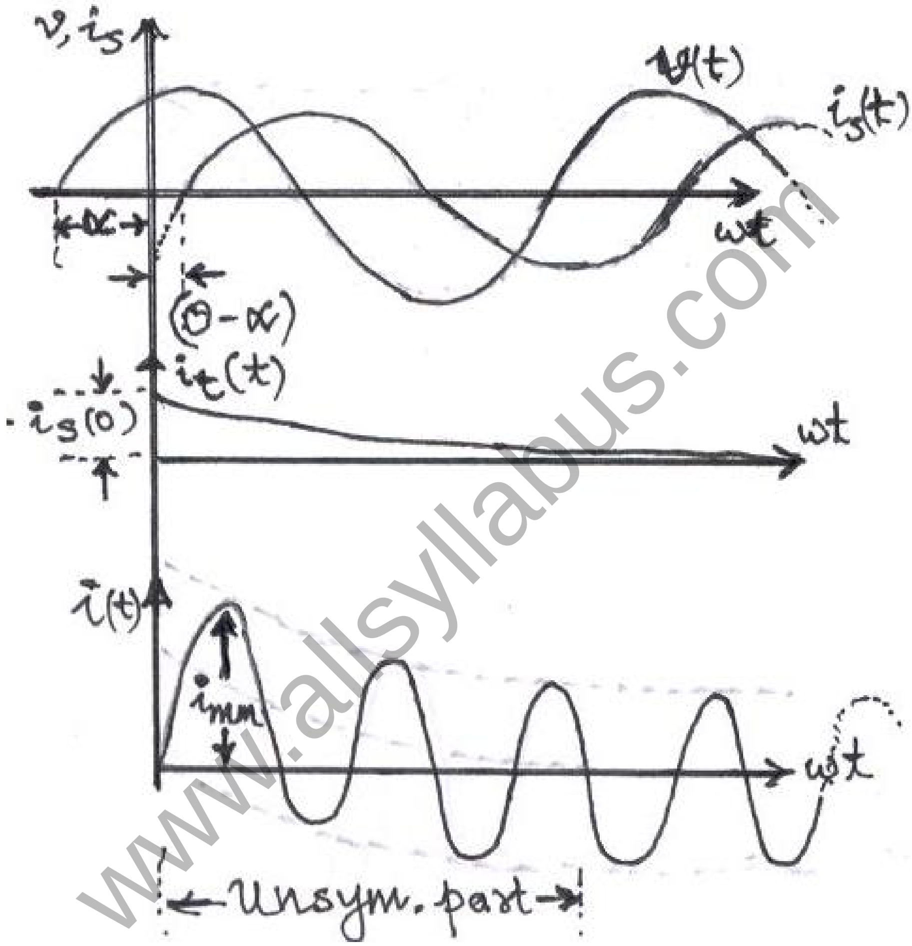


Figure 2. Plot of Symmetrical short circuit current, i(t).

The maximum momentary current, i_{mm} thus corresponds to the first peak. Hence, if the decay in the transient current during this short interval of time is neglected, then we have (sum of the two peak values);

$$i_{mm} = [\sqrt{2}V/Z] \sin(\theta-\alpha) + [\sqrt{2}V/Z] \qquad (2.10)$$

now, since the resistance of the transmission line is very small, the impedance angle θ, can be taken to be approximately equal to 90^0. Hence, we have

$$i_{mm} = [\sqrt{2}V/Z] \cos\alpha + [\sqrt{2}V/Z] \qquad (2.11)$$

This value is maximum when the value of α is equal to zero. This value corresponds to the short circuiting instant of the voltage wave when it is passing through zero. Thus the final expression for the maximum momentary current is obtained as:

$$i_{mm} = 2\,[\sqrt{2}V/Z] \qquad (2.12)$$

Thus it is observed that the maximum momentary current is twice the maximum value of symmetrical short circuit current. This is refered as the *doubling effect* of the short circuit current during the symmetrical fault on a transmission line.

2.3 Short circuit of an unloaded synchronous machine

2.3.1 Short Circuit Reactances

Under steady state short circuit conditions, the armature reaction in synchronous generator produces a demagnetizing effect. This effect can be modeled as a reactance, X_a in series with the induced emf and the leakage reactance, X_l of the machine as shown in figure 3. Thus the equivalent reactance is given by:

$$X_d = X_a + X_l \qquad (2.13)$$

Where Xd is called as the direct axis *synchronous reactance* of the synchronous machine. Consider now a sudden three-phase short circuit of the synchronous generator on no-load. The machine experiences a transient in all the 3 phases, finally ending up in steady state conditions.

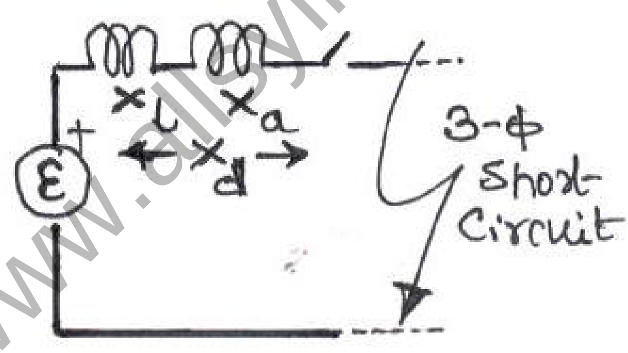


Figure 3. Steady State Short Circuit Model

Immediately after the short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. However, to encounter the demagnetization of the armature short circuit current, current appears in field and damper windings, assisting the rotor field winding to sustain the air-gap flux. Thus during the initial part of the short circuit, there is mutual coupling between stator, rotor and damper windings and hence the corresponding equivalent circuit would be as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d'' = X_l + [1/X_a + 1/X_f + 1/X_{dw}]^{-1} \qquad (2.14)$$

Where X_d" is called as the *sub-transient reactance* of the synchronous machine. Here, the equivalent resistance of the damper winding is more than that of the rotor field winding. Hence, the time constant of the damper field winding is smaller. Thus the damper field effects and the eddy currents disappear after a few cycles.

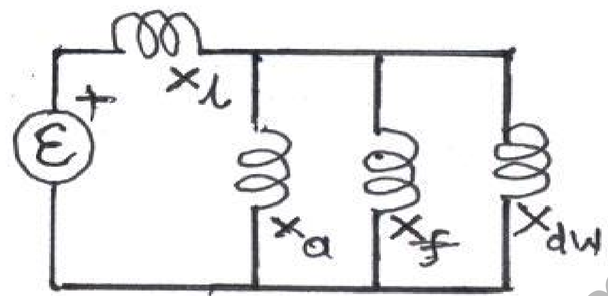


Figure 4. Model during Sub-transient Period of Short Circuit

In other words, X_{dw} gets open circuited from the model of Figure 5 to yield the model as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d' = X_l + [1/X_a + 1/X_f\,]^{-1} \tag{2.15}$$

Where X_d' is called as the *transient reactance* of the synchronous machine. Subsequently, X_f also gets open circuited depending on the field winding time constant and yields back the steady state model of figure 3.

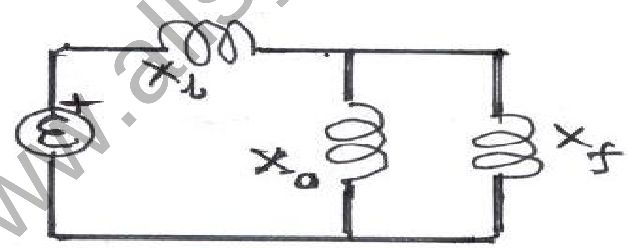


Figure 5. Model during transient Period of Short Circuit

Thus the machine offers a time varying reactance during short circuit and this value of reactance varies from initial stage to final one such that: Xd > Xd' > Xd'

2.3.2 Short Circuit Current Oscillogram

Consider the oscillogram of short circuit current of a synchronous machine upon the occurrence of a fault as shown in figure 6. The symmetrical short circuit current can be divided into three zones: the initial sub transient period, the middle transient period and finally the steady state period. The corresponding reactances, Xd," Xd' and Xd respectively, are offered by the synchronous machine during these time periods.

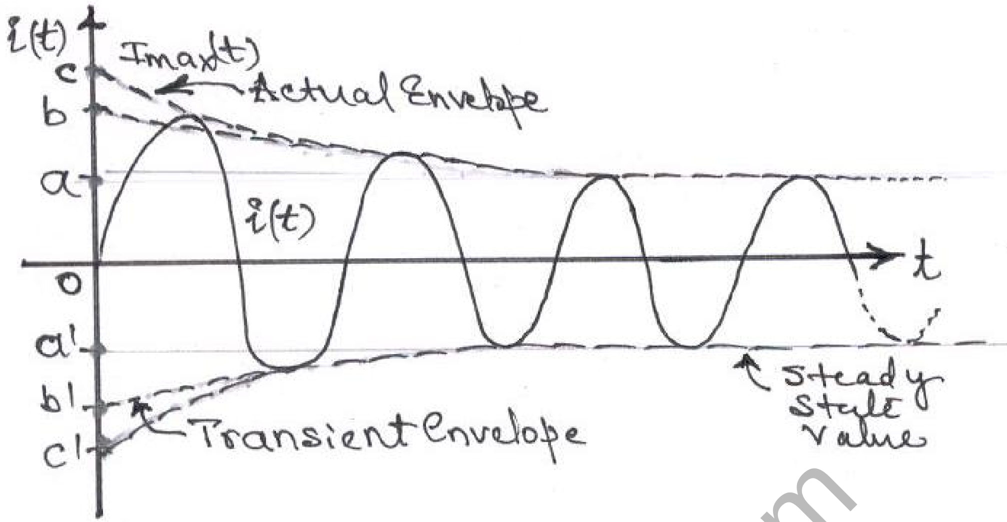


Figure 6. SC current Oscillogram of Armature Current.

The currents and reactances during the three zones of period are related as under in terms of the intercepts on the oscillogram (oa, ob and oc are the y-intercepts as indicated in figure 6):

RMS value of the steady state current = I = [oa/$\sqrt{2}$] = [E_g/X_d]
RMS value of the transient current = I' = [ob/$\sqrt{2}$] = [E_g/X_d']
RMS value of the sub transient current = I = [oc/$\sqrt{2}$] = [E_g/X_d''] (2.16)

2.4 short circuit of a loaded machine

In the analysis of section 2.3 above, it has been assumed that the machine operates at no load prior to the occurrence of the fault. On similar lines, the analysis of the fault occurring on a loaded machine can also be considered.

Figure 7 gives the circuit model of a synchronous generator operating under steady state conditions supplying a load current I_l to the bus at a terminal voltage V_t. E_g is the induced emf under the loaded conditions and X_d is the direct axis synchronous reactance of the generator.

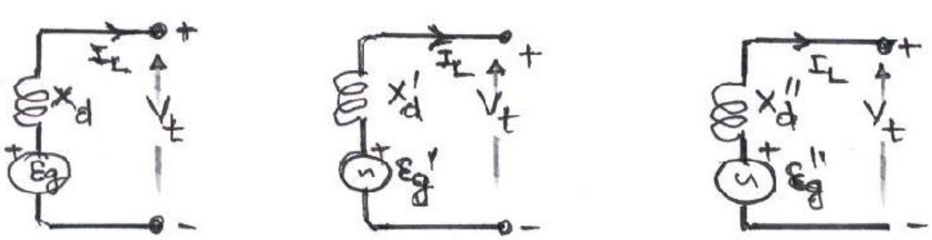


Figure 7. Circuit models for a fault on a loaded machine.

Also shown in figure 7, are the circuit models to be used for short circuit current calculations when a fault occurs at the terminals of the generator, for sub-transient current and transient current values. The induced emf values used in these models are given by the expressions as under:

$$E_g = V_t + j\, I_L X_d = \text{Voltage behind syn. reactance}$$
$$E_g' = V_t + j\, I_L X_d' = \text{Voltage behind transient reactance}$$
$$E_g'' = V_t + j\, I_L X_d'' = \text{Voltage behind subtr. Reactance} \qquad (2.17)$$

The synchronous motors will also have the terminal emf values and reactances. However, then the current direction is reversed. During short circuit studies, they can be replaced by circuit models similar to those shown in figure 7 above, except that the voltages are given by the relations as under:

$$E_m = V_t - j\, I_L X_d = \text{Voltage behind syn. reactance}$$
$$E_m' = V_t - j\, I_L X_d' = \text{Voltage behind transient reactance}$$
$$E_m'' = V_t - j\, I_L X_d'' = \text{Voltage behind subtr. Reactance} \qquad (2.18)$$

The circuit models shown above for the synchronous machines are also very useful while dealing with the short circuit of an interconnected system.

2.5 Selection of circuit breaker ratings

For *selection of circuit breakers*, the maximum momentary current is considered corresponding to its maximum possible value. Later, the current to be interrupted is usually taken as symmetrical short circuit current multiplied by an empirical factor in order to account for the DC off-set current. A value of 1.6 is usually selected as the multiplying factor.

Normally, both the generator and motor reactances are used to determine the momentary current flowing on occurrence of a short circuit. The interrupting capacity of a circuit breaker is decided by X_d'' for the generators and X_d' for the motors.

2.6 Examples

Problem #1: A transmission line of inductance 0.1 H and resistance 5 Ω is suddenly short circuited at t = 0, at the far end of a transmission line and is supplied by an ac source of voltage $v = 100 \sin(100\pi t + 15^0)$. Write the expression for the short circuit current, i(t). Find the approximate value of the first current maximum for the given values of α and θ. What is this value for α=0, and θ=90^0? What should be the instant of short circuit so that the DC offset current is (i)zero and (ii)maximum?

Solution:

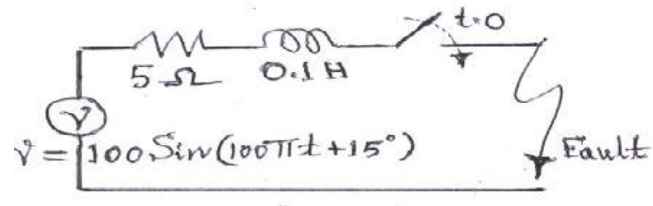


Figure P1.

Consider the expression for voltage applied to the transmission system given by

$$v = V_m \sin(\omega t+\alpha) = 100 \sin(100\pi t+15^0)$$

Thus we get: $V_m = 100$ volts; $f = 50$ Hz and $\alpha = 15^0$.

Consider the impedance of the circuit given by:

$$Z = R + j\omega L = 5 + j(100\pi)(0.1) = 5 + j\,31.416 \text{ ohms.}$$

Thus we have: Z_{mag}=31.8113 Ohms; θ=80.957^0 and τ=L/R=0.1/5=0.02 seconds.

The short circuit current is given by:

$$i(t) = [V_m/Z] \sin(\omega t+\alpha-\theta) + [V_m/Z] \sin(\theta-\alpha)\, e^{-(R/L)t}$$

$$= [100/31.8113] [\sin(100\pi t+15^0-80.957^0) + \sin(80.957^0-15^0)\, e^{-(t/0.02)}]$$

$$= 3.1435 \sin(314.16\,t - 65.96) + 2.871\, e^{-50t}$$

Thus we have:

i) $i_{mm} = 3.1435 + 2.871\, e^{-50t}$

where t is the time instant of maximum of symmetrical short circuit current. This instant occurs at $(314.16\, t^c - 65.96^0) = 90^0$; Solving we get, t = 0.00867 seconds so that i_{mm} = **5** Amps.

ii) $i_{mm} = 2V_m/Z$ = **6.287** A; for α=0, and θ=90^0 (Also, i_{mm} = 2 (3.1435) = 6.287 A)

iii) DC offset current = $[V_m/Z] \sin(\theta-\alpha)\, e^{-(R/L)t}$

= zero, if $(\theta-\alpha)$ = zero, i.e., $\theta = \alpha$, or α = **80.957^0**

= maximum if $(\theta-\alpha) = 90^0$, i.e., $\alpha = \theta - 90^0$, or α = **- 9.043^0.**

Problem #2: A 25 MVA, 11 KV, 20% generator is connected through a step-up transformer- T_1 (25 MVA, 11/66 KV, 10%), transmission line (15% reactance on a base of 25 MVA, 66 KV) and step-down transformer-T_2 (25 MVA, 66/6.6 KV, 10%) to a bus that supplies 3 identical motors in parallel (all motors rated: 5 MVA, 6.6 KV, 25%). A circuit breaker-A is used near the primary of the transformer T_1 and breaker-B is used near the motor M_3. Find the symmetrical currents to be interrupted by circuit breakers A and B for a fault at a point P, near the circuit breaker B.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

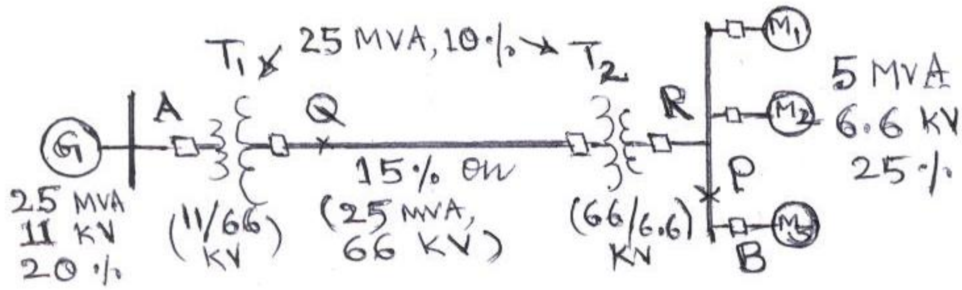


Figure P2(a)

Selection of bases:

S_b = 25 MVA (common); V_b = 11 KV (Gen. circuit)- chosen so that then V_b = 66 KV (line circuit) and V_b = 6.6 KV (Motor circuit).

Pu values:

X_g=j0.2 pu, X_{t1}=X_{t2}=j0.1 pu; X_{m1}=X_{m2}=X_{m3}=j0.25(25/5)=j1.25 pu; X_{line}=j0.15 pu.

Since the system is operating at no load, all the voltages before fault are 1 pu. Considering the pu reactance diagram with the faults at P, we have:

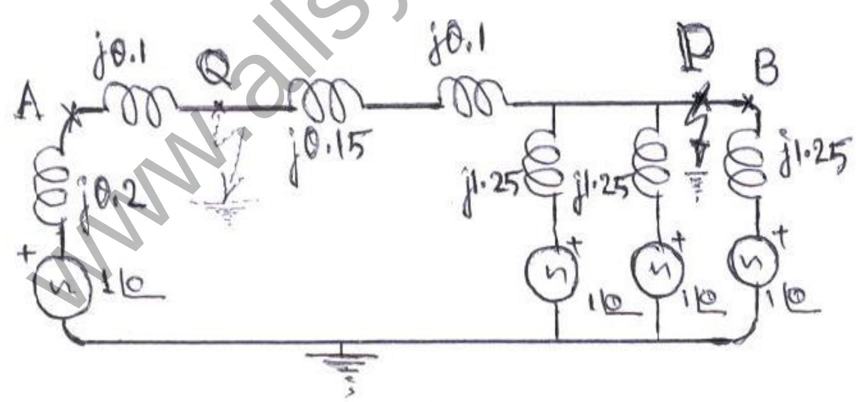


Figure P2(b)

Current to be interrupted by circuit breaker A = 1.0 /j[0.2+0.1+0.15+0.1]

= - j 1.818 pu = - j 1.818 (25/[√3(11)]) = - j 1.818 (1.312) KA = **2.386 KA**

And Current to be interrupted by breaker B = 1/j1.25 = - j 0.8 pu

= - j0.8 (25/[√3(6.6)]) = - j0.8 (2.187) KA = **1.75 KA.**

Problem #3: Two synchronous motors are connected to a large system bus through a short line. The ratings of the various components are: Motors(each)= 1 MVA, 440 volts, 0.1 pu reactance; line of 0.05 ohm reactance and the short circuit MVA at the bus of the large system is 8 at 440 volts. Calculate the symmetrical short circuit current fed into a three-phase fault at the motor bus when the motors are operating at 400 volts.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

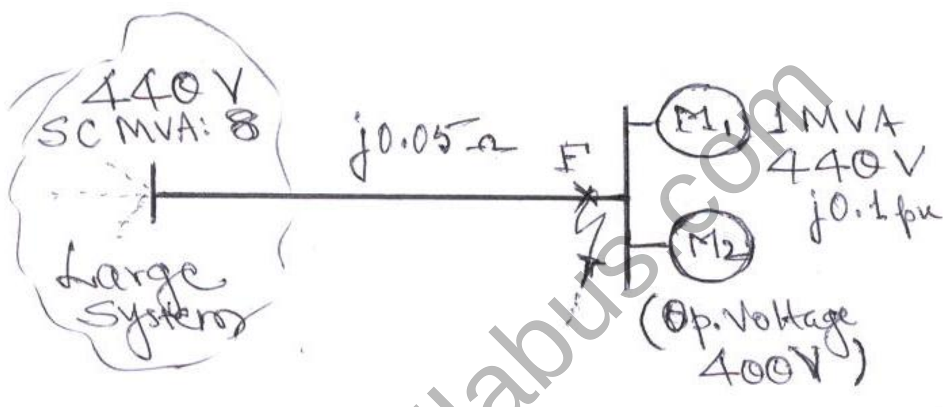


Figure P3.

S_b = 1 MVA; V_b = 0.44 KV (common)- chosen so that X_m(each)=j0.1 pu, Em = $1.0\angle 0^0$, X_{line}=j0.05 $(1/0.44^2)$ = j 0.258 pu and Xlarge-system -= (1/8) = j 0.125 pu.

Thus the prefault voltage at the motor bus; $V_t = 0.4/0.44 = 0.909\angle 0^0$,

Short circuit current fed to the fault at motor bus (I_f = YV);

$$I_f = [0.125 + 0.258]^{-1} + 2.0 \}0.909 = [20.55 \text{ pu}] [1000/(\surd 3(0.4))]$$

$$= 20.55 (1.312) \text{ KA} = \mathbf{26.966} \text{ KA}.$$

Problem #4: A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are: Gen.: 10 MVA, 6.6 KV, X_d" = 0.1 pu, X_d' = 0.2 pu and X_d = 0.8 pu; and Transformer: 10 MVA, 6.9/33 KV, X_l = 0.08 pu; The system is operating on no-load at a line voltage of 30 KV, when a three-phase fault occurs on the line just beyond the circuit breaker. Determine the following:

(i) Initial symmetrical RMS current in the breaker,
(ii) Maximum possible DC off-set current in the breaker,
(iii) Momentary current rating of the breaker,
(iv) Current to be interrupted by the breaker and the interrupting KVA and
(v) Sustained short circuit current in the breaker.

Solution:

Consider the base values selected as **10** MVA, **6.6** KV (in the generator circuit) and 6.6(33/6.9) = **31.56** KV(in the transformer circuit). Thus the base current is:

$I_b = 10 / [\sqrt{3}(31.56)] =$ **0.183** KA

The pu values are: $X_d'' = 0.1$ pu, $X_d' = 0.2$ pu and $X_d = 0.8$ pu; and $X_{Tr} = 0.08\,(6.9/6.6)^2 = 0.0874$ pu; $V_t = (30/31.6) = 0.95\angle 0^0$ pu.
Initial symmetrical RMS current = $0.95\angle 0^0 / [0.1 + 0.0874] = 5.069$ pu = **0.9277** KA;
Maximum possible DC off-set current = 2 (0.9277) = **1.312** KA;
Momentary current rating = 1.6(0.9277) = **1.4843** KA; (assuming 60% allowance)
Current to be interrupted by the breaker (5 Cycles) = 1.1(0.9277) = **1.0205** KA;
Interrupting MVA = 3(30) (1.0205) = **53.03** MVA;
Sustained short circuit current in the breaker = $0.95\angle 0^0\,(0.183) / [0.8 + 0.0874]$
= **0.1959** KA.

2.7 Exercises for Practice

PROBLEMS

1. The one line diagram for a radial system network consists of two generators, rated 10 MVA, 15% and 10 MVA, 12.5 % respectively and connected in parallel to a bus bar A at 11 KV. Supply from bus A is fed to bus B (at 33 KV) through a transformer T_1 (rated: 10 MVA, 10%) and OH line (30 KM long). A transformer T_2 (rated: 5 MVA, 8%) is used in between bus B (at 33 KV) and bus C (at 6.6 KV). The length of cable running from the bus C up to the point of fault, F is 3 KM. Determine the current and line voltage at 11 kV bus A under fault conditions, when a fault occurs at the point F, given that $Z_{cable} = 0.135 + j\,0.08$ ohm/ kM and $Z_{OH\text{-}line} = 0.27 + j\,0.36$ ohm/kM. [*Answer*: 9.62 kV at the 11 kV bus]

2. A generator (rated: 25MVA, 12. KV, 10%) supplies power to a motor (rated: 20 MVA, 3.8 KV, 10%) through a step-up transformer (rated:25 MVA, 11/33 KV, 8%), transmission line (of reactance 20 ohms) and a step-down transformer (rated:20 MVA, 33/3.3 KV, 10%). Write the pu reactance diagram. The system is loaded such that the motor is drawing 15 MW at 0.9 leading power factor, the motor terminal voltage being 3.1 KV. Find the sub-transient current in the generator and motor for a fault at the generator bus. [*Answer*: $I_g'' = 9.337$ KA; $I_m'' = 6.9$ KA]

3. A synchronous generator feeds bus 1 and a power network feed bus 2 of a system. Buses 1 and 2 are connected through a transformer and a line. Per unit reactances of the components are: Generator(bus-1):0.25; Transformer:0.12 and Line:0.28. The power network is represented by a generator with an unknown reactance in series. With the generator on no-load and with 1.0 pu voltage at each bus, a three phase fault occurring on bus-1 causes a current of 5 pu to flow into the fault. Determine the equivalent reactance of the power network. [*Answer*: X = 0.6 pu]

4. A synchronous generateor, rated 500 KVA, 440 Volts, 0.1 pu sub-transient reactance is supplying a passive load of 400 KW, at 0.8 power factor (lag). Calculate the initial symmetrical RMS current for a three-phase fault at the generator terminals.
[Answer: S_b=0.5 MVA; V_b=0.44 KV; load=$0.8\angle{-36.9^0}$; I_b=0.656 KA; I_f=6.97 KA]

OBJECTIVE TYPE QUESTIONS

1. When a 1-phase supply is across a 1-phase winding, the nature of the magnetic field produced is

a) Constant in magnitude and direction

b) Constant in magnitude and rotating at synchronous speed

c) Pulsating in nature

d) Rotating in nature

2. The damper windings are used in alternators to

a) Reduce eddy current loss

b) Reduce hunting

c) Make rotor dynamically balanced

d) Reduce armature reaction

3. The neutral path impedance Zn is used in the equivalent sequence network models as

a) Zn2

b) Zn

c) 3 Zn

d) An ineffective value

4. An infinite bus-bar should maintain

a) Constant frequency and Constant voltage

b) Infinite frequency and Infinite voltage

c) Constant frequency and Variable voltage

d) Variable frequency and Variable voltage

5. Voltages under extra high voltage are

a) 1KV & above

b) 11KV & above

c) 132 KV & above

d) 330 KV & above

[Ans.: 1(c), 2(b), 3(c), 4(a), 5(d)]

CHAPTER 3: SYMMETRICAL COMPONENTS

[CONTENTS: Introduction, The a operator, Power in terms of symmetrical components, Phase shift in Y-Δ transformer banks, Unsymmetrical series impedances, Sequence impedances, Sequence networks, Sequence networks of an unloaded generator, Sequence networks of elements, Sequence networks of power system]

3.1 INTRODUCTION

Power systems are large and complex three-phase systems. In the normal operating conditions, these systems are in balanced condition and hence can be represented as an equivalent single phase system. However, a fault can cause the system to become unbalanced. Specifically, the unsymmetrical faults: open circuit, LG, LL, and LLG faults cause the system to become unsymmetrical. The single-phase equivalent system method of analysis (using SLD and the reactance diagram) cannot be applied to such unsymmetrical systems. Now the question is how to analyze power systems under unsymmetrical conditions? There are two methods available for such an analysis: Kirchhoff's laws method and Symmetrical components method.

The method of symmetrical components developed by C.L. Fortescue in 1918 is a powerful technique for analyzing unbalanced three phase systems. Fortescue defined a linear transformation from phase components to a new set of components called symmetrical components. This transformation represents an unbalanced three-phase system by a set of three balanced three-phase systems. The symmetrical component method is a modeling technique that permits systematic analysis and design of three-phase systems. Decoupling a complex three-phase network into three simpler networks reveals complicated phenomena in more simplistic terms.

Consider a set of three-phase unbalanced voltages designated as V_a, V_b, and V_c. According to Fortescue theorem, these phase voltages can be resolved into following three sets of components.

1. Positive-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120^0 in phase, and having the same phase sequence as the original phasors, designated as V_{a1}, V_{b1}, and V_{c1}
2. Negative-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120^0 in phase, and having the phase sequence opposite to that of the original phasors, designated as V_{a2}, V_{b2}, and V_{c2}
3. Zero-sequence components, consisting of three phasors equal in magnitude, and with zero phase displacement from each other, designated as V_{a0}, V_{b0}, and V_{c0}
   Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terns of their components are

$$
\begin{aligned}
V_a &= V_{a1} + V_{a2} + V_{a0}\\
V_b &= V_{b1} + V_{b2} + V_{b0}\\
V_c &= V_{c1} + V_{c2} + V_{c0} \qquad (3.1)
\end{aligned}
$$

The synthesis of a set of three unbalanced phasors from the three sets of symmetrical components is shown in Figure1.

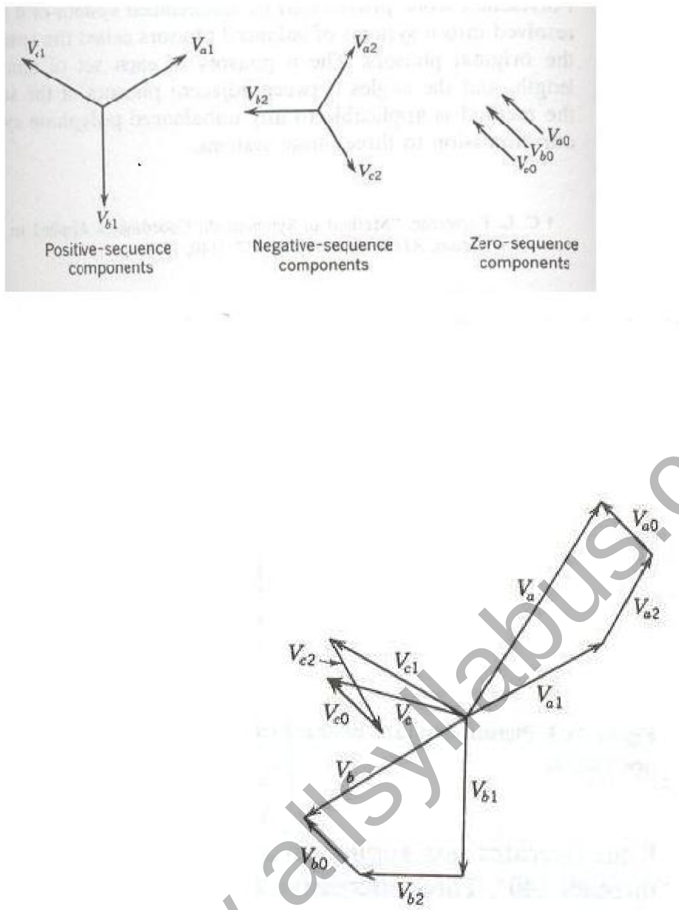


Figure 3.1 Graphical addition of symmetrical components To obtain unbalanced phasors.

3.2 THE OPERATOR 'a'

The relation between the symmetrical components reveals that the phase displacement among them is either 120^0 **or** 0^0**.** Using this relationship, only three independent components is sufficient to determine all the nine components. For this purpose an operator which rotates a given phasor by 120^0 in the positive direction (counterclockwise) is very useful. The letter 'a' is used to designate such a complex operator of unit magnitude with an angle of 120^0. It is defined by

$$a = 1\angle 120^0 = -0.5 + j\,0.866 \qquad (3.2)$$

If the operator 'a' is applied to a phasor twice in succession, the phasor is rotated through 240^0. Similarly, three successive applications of 'a' rotate the phasor through 360^0.

To reduce the number of unknown quantities, let the symmetrical components of V_b and V_c can be expressed as product of some function of the operator a and a component of V_a. Thus,

$$V_{b1} = a^2 V_{a1} \qquad V_{b2} = a V_{a2} \qquad V_{b0} = V_{a0}$$
$$V_{c1} = a V_{a1} \qquad V_{c2} = a^2 V_{a2} \qquad V_{c0} = V_{a0}$$

Using these relations the unbalanced phasors can be written as

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} \qquad (3.3)$$

In matrix form,

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix} \qquad (3.4)$$

$$\text{Let} \quad Vp = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}; \quad Vs = \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \qquad (3.5)$$

The inverse of A matrix is

$$A^{-1} = \tfrac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \qquad (3.6)$$

With these definitions, the above relations can be written as

$$\boldsymbol{V_p = A\, V_s; \qquad V_s = A^{-1} V_p} \qquad (3.7)$$

Thus the symmetrical components of V_a, V_b and V_c are given by

$$V_{a0} = 1/3\,(V_a + V_b + V_c)$$
$$V_{a1} = 1/3\,(V_a + a V_b + a^2 V_c)$$
$$V_{a2} = 1/3\,(V_a + a^2 V_b + a V_c) \qquad (3.8)$$

Since the sum of three balanced voltages is zero, the zero-sequence component voltage in a balanced three-phase system is always zero. Further, the sum of line voltages of even an unbalanced three-phase system is zero and hence the corresponding zero-sequence component of line voltages.

NUMERICAL EXAMPLES

Example 1 : The line currents in a 3-ph 4 –wire system are $Ia = 100<30^0$; $Ib = 50<300^0$; $Ic = 30<180^0$. Find the symmetrical components and the neutral current.

Solution:

$$Ia0 = 1/3(Ia + Ib + Ic) = 27.29 < 4.7^0\ A$$
$$Ia1 = 1/3(Ia + a\,Ib + a2Ic) = 57.98 < 43.3^0\ A$$
$$Ia2 = 1/3(Ia + a2\,Ib + a\,Ic) = 18.96 < 24.9^0\ A$$
$$In = Ia + Ib + Ic = 3\,Ia0 = 81.87 < 4.7^0\ A$$

Example 2: The sequence component voltages of phase voltages of a 3-ph system are: Va0 = 100 <00 V; Va1 = 223.6 < -26.60 V ; Va2 = 100 <1800 V. Determine the phase voltages.

Solution:

$$Va = Va0 + Va1 + Va2 = 223.6 < -26.60\ V$$
$$Vb = Va0 + a2Va1 + a\,Va2 = 213 < -99.90\ V$$
$$Vc = Va0 + a\,Va1 + a2\,Va2 = 338.6 < 66.20\ V$$

Example 3: The two seq. components and the corresponding phase voltage of a 3-ph system are $Va0 = 1<-60^0$ V; $Va1 = 2<0^0$ V ; & $Va = 3 <0^0$ V. Determine the other phase voltages.

Solution:

$$Va = Va0 + Va1 + Va2$$
$$Va2 = Va - Va0 - Va1 = 1 < 60^0\ V$$
$$Vb = Va0 + a2Va1 + a\,Va2 = 3 < -120^0\ V$$
$$Vc = Va0 + a\,Va1 + a2\,Va2 = 0\ V$$

Example 4: Determine the sequence components if $Ia = 10<60^0$ A; $Ib = 10<-60^0$ A; $Ic = 10 <180^0$ A.

Solution:

$$Ia0 = 1/3(Ia + Ib + Ic) = 0\ A$$
$$Ia1 = 1/3(Ia + a\,Ib + a2Ic) = 10<60^0\ A$$
$$Ia2 = 1/3(Ia + a2\,Ib + a\,Ic) = 0\ A$$

Observation: If the phasors are balanced, two sequence components will be zero.

Example 5: Determine the sequence components if $Va = 100 <30^0$ V; $Vb = 100 <150^0$ V & $Vc = 100 <-90^0$ V.

Solution:

$$Va0 = 1/3(Va + Vb + Vc) = 0\ V$$
$$Va1 = 1/3(Va + a\,Vb + a2Vc) = 0\ V$$
$$Va2 = 1/3(Va + a2\,Vb + a\,Vc) = 100<30^0\ V$$

Observation: If the phasors are balanced, two sequence components will be zero.

Example 6: The line b of a 3-ph line feeding a balanced Y-load with neutral grounded is open resulting in line currents: Ia = $10<0^0$ A & Ic = $10<120^0$ A. Determine the sequence current components.

Solution:

Ib = 0 A.
Ia0 = 1/3(Ia + Ib + Ic) = $3.33<60^0$ A
Ia1 = 1/3(Ia + a Ib + a2Ic) = $6.66<0^0$ A
Ia2 = 1/3(Ia + a2 Ib + a Ic) = $3.33<-60^0$ A

Example 7: One conductor of a 3-ph line feeding a balanced delta-load is open. Assuming that line c is open, if current in line a is 10<00 A , determine the sequence components of the line currents.

Solution:

Ic = 0 A; Ia = $10<0^0$ A. → Ib = $10<120^0$ A
Ia0 = 1/3(Ia + Ib + Ic) = 0 A
Ia1 = 1/3(Ia + a Ib + a2Ic) = $5.78<-30^0$ A
Ia2 = 1/3(Ia + a2 Ib + a Ic) = $5.78< 30^0$ A

Note: The zero-sequence components of line currents of a delta load (3-ph 3-wire) system are zero.

3.3 POWER IN TERMS OF SYMMETRICAL COMPONENTS

The power in a three-phase system can be expressed in terms of symmetrical components of the associated voltages and currents. The power flowing into a three-phase system through three lines a, b and c is

$$S = P + j\,Q = V_a I_a^* + V_b I_b^* + V_c I_c^* \qquad (3.9)$$

where V_a , V_b and V_c are voltages to neutral at the terminals and I_a , I_b, and I_c are the currents flowing into the system in the three lines. In matrix form

$$S = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Thus

$$S = [A\,V]^T\,[AI]^*$$

Using the reversal rule of the matrix algebra

$$S = V^T A^T A^* I^*$$

Noting that $A^T = A$ and a and a^2 are conjugates,

$$S = \begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

or, since $A^T A^*$ is equal to 3U where U is 3x3 unit matrix

$$S = 3 \begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Thus the complex three-phase power is given by

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3 V_{a0} I_{a0} + 3 V_{a1} I_{a1} + 3 V_{a2} I_{a2} \qquad (3.10)$$

Here, $3V_{a0}I_{a0}$, $3V_{a1}I_{a1}$ and $3V_{a2}I_{a2}$ correspond to the three-phase power delivered to the zero-sequence system, positive-sequence system, and negative-sequence system, respectively. Thus, the total three-phase power in the unbalanced system is equal to the sum of the power delivered to the three sequence systems representing the three-phase system.

3.4 PHASE SHIFT OF COMPONENTS IN Y-Δ TRANSFORMER BANKS

The dot convention is used to designate the terminals of transformers. The dots are placed at one end of each of the winding on the same iron core of a transformer to indicate that the currents flowing from the dotted terminal to the unmarked terminal of each winding produces an mmf acting in the same direction in the magnetic circuit. In that case, the voltage drops from dotted terminal to unmarked terminal in each side of the windings are in phase.

The HT terminals of three-phase transformers are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3. In Y-Y or Δ-Δ transformers, the markings are such that voltages to neutral from terminals H1, H2, and H3 are in phase with the voltages to neutral from terminals X1, X2, and X3, respectively. But, there will be a phase shift (of 30^0) between the corresponding quantities of the primary and secondary sides of a star-delta (or delta-star) transformer. The standard for connection and designation of transformer banks is as follows:

1. The HT side terminals are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3.
2. The phases in the HT side are marked in uppercase letters as A, B, and C. Thus for the sequence abc, A is connected to H1, B to H2 and C to H3. Similarly, the phases in the LT side are marked in lowercase letters as a, b and c.
3. The standard for designating the terminals H1 and X1 on transformer banks requires that the positive-sequence voltage drop from H1 to neutral lead the positive sequence voltage drop from X1 to neutral by 30^0 regardless of the type of connection in the HT

and LT sides. Similarly, the voltage drops from H2 to neutral and H3 to neutral lead their corresponding values, X2 to neutral and X3 to neutral by 30^0.

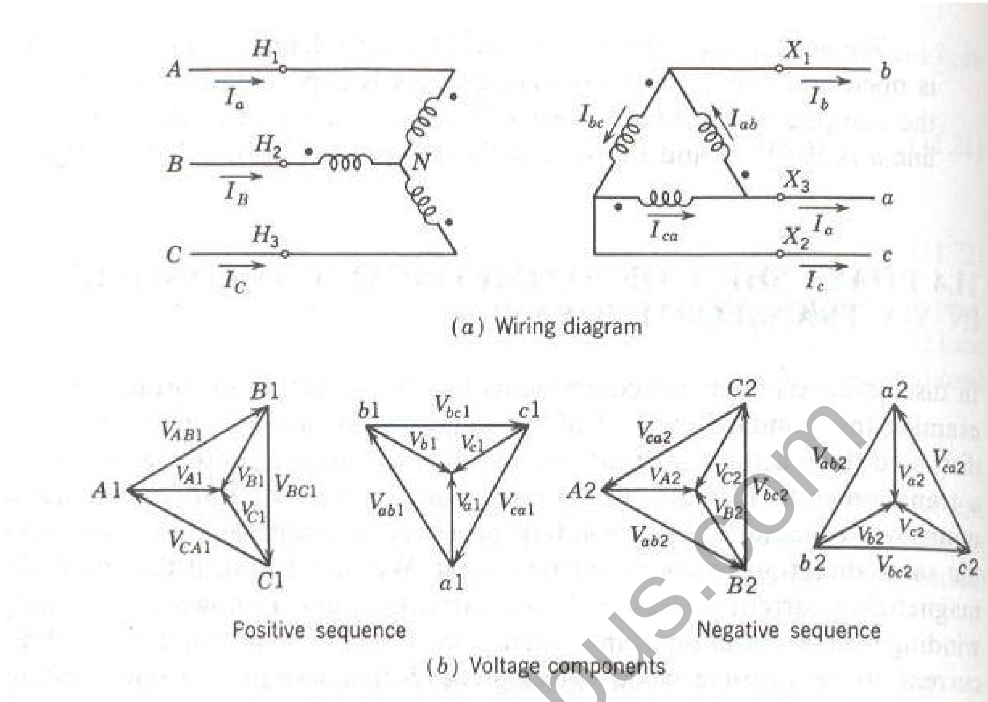


Figure 3.2 Wiring diagram and voltage phasors of a Y-Δ transformer With Y connection on HT side.

Consider a Y- Δ transformer as shown in Figure a. The HT side terminals H1, H2, and H3 are connected to phases A, B, and C, respectively and the phase sequence is ABC. The windings that are drawn in parallel directions are those linked magnetically (by being wound on the same core). In Figure a winding AN is the phase on the Y-side which is linked magnetically with the phase winding bc on the Δ side. For the location of the dots on the windings V_{AN} is in phase with V_{bc}. Following the standards for the phase shift, the phasor diagrams for the sequence components of voltages are shown in Figure b. The sequence component of V_{AN1} is represented as V_{A1} (leaving subscript '$_N$' for convenience and all other voltages to neutral are similarly represented. The phasor diagram reveals that V_{A1} leads V_{b1} by 30^0. This will enable to designate the terminal to which b is connected as X1. Inspection of the positive-sequence and negative-sequence phasor diagrams revels that V_{a1} leads V_{A1} by 90^0 and V_{a2} lags V_{A2} by 90^0.

From the dot convention and the current directions assumed in Figure a, the phasor diagram for the sequence components of currents can be drawn as shown in Figure c. Since the direction specified for I_A in Figure a is away from the dot in the winding and the direction of I_{bc} is also away from the dot in its winding, I_A and I_{bc} are 180^0 out of phase. Hence the phase relation between the Y and Δ currents is as shown in Figure c. From this diagram, it can be seen that I_{a1} leads I_{A1} by 90^0 and I_{a2} lags I_{A2} by 90^0. Summarizing these relations between the symmetrical components on the two sides of the transformer gives:

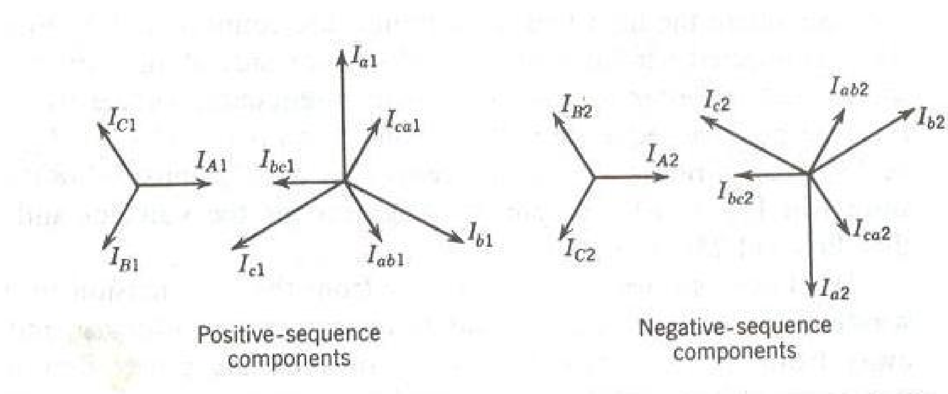


Figure 3.3 Current phasors of Y-Δ transformer with Y connection on HT side.

$V_{a1} = +j\, V_{A1}$ $\qquad$ $I_{a1} = +j\, I_{A1}$

$V_{a2} = -j\, V_{A2}$ $\qquad$ $I_{a1} = -j\, I_{A2}$ $\qquad$ (3.11)

Where each voltage and current is expressed in per unit. Although, these relations are obtained for Y- Δ transformer with Y connection in the HT side, they are valid even when the HT side is connected in Δ and the LT side in Y.

NUMERICAL EXAMPLES

Example 8: Three identical resistors are Y-connected to the LT Y-side of a delta-star transformer. The voltages at the resistor loads are |Vab| = 0.8 pu., |Vbc|=1.2 pu., and |Vca|=1.0 pu. Assume that the neutral of the load is not connected to the neutral of the transformer secondary. Find the line voltages on the HT side of the transformer.

Solution:

Assuming an angle of 180^0 for Vca, find the angles of other voltages

Vab = $0.8<82.8^0$ pu
Vbc = $1.2<-41.4^0$ pu
Vca = $1.0<180^0$ pu

The symmetrical components of line voltages are

Vab0 = 1/3 (Vab +Vbc + Vca) = 0
Vab1 = 1/3 (Vab +aVbc + a2Vca) = $0.985<73.6^0$ V
Vab1 = 1/3 (Vab +a2Vbc + aVca) = $0.235<220.3^0$ V

Since Van1 = Vab1$<-30^0$ and Van2 = Vab2$<30^0$
Van1 = $0.985<73.6^0-30^0$
= $0.985<43.6^0$ pu (L-L base)
Van2 = $0.235<220.3^0+30^0$
= $0.235<250.3^0$ pu(L-L base)

Since each resistor is of 1.0<0 pu. Impedance,
Ian1 = (Van1/Z) = $0.985<43.6^0$ pu.

Ian2 = (Van2/Z) = 0.235<250.3^0 pu.

The directions are +ve for currents from supply toward the delta primary and away from the Y-side toward the load. The HT side line to neutral voltages are

VA1 = - j Va1 = 0.985<-46.4^0
VA2 = +j Va2 = 0.235<-19.7^0
VA = VA1 +VA2 = 1.2<-41.3^0 pu.
VB1 = a2VA1 and VB2 = a VA2
VB = VB1 + VB2 = 1<180^0 pu.
VC1 = a VA1 and VC2 = a2VA2
VC = VC1 + VC2 = 0.8<82.9^0 pu.

The HT side line voltages are

VAB = VA – VB = 2.06<-22.6^0 pu. (L-N base)
= (1/3) VAB = 1.19<-22.6^0 pu. (L-L base)
VBC = VB – Vc = 1.355<215.8^0 pu. (L-N base)
= (1/3) VBC = 0.782<215.8^0 pu. (L-L base)
VCA = VC – VA = 1.78<116.9^0 pu. (L-N base)
= (1/3) VCA = 1.028<116.9^0 pu. (L-L base)

3.5 UNSYMMETRICAL IMPEDANCES

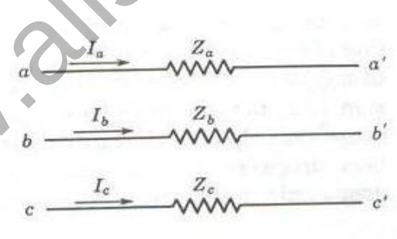


Figure 3.4 Portion of three-phase system representing three unequal series impedances.

Consider the network shown in Figure. Assuming that there is no mutual impedance between the impedances Za, Zb, and Zc, the voltage drops Vaa', vbb', and Vcc' can be expressed in matrix form as

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{3.12}$$

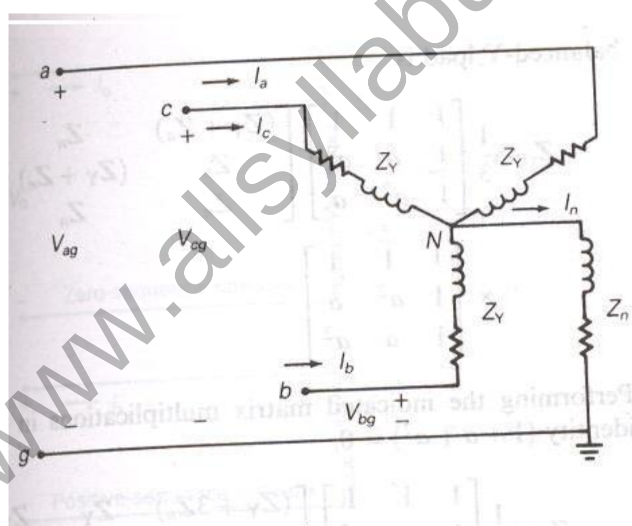
And in terms of symmetrical components of voltage and current as

$$A \begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} A \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{3.13}$$

If the three impedances are equal (i.e., if Za = Zb = Zc), Eq reduces to

$V_{aa'1} = Z_a I_{a1}$; $V_{aa'2} = Z_a I_{a2}$; $V_{aa'0} = Z_a I_{a0}$ (3.14)

Thus, the symmetrical components of unbalanced currents flowing in balanced series impedances (or in a balanced Y load) produce voltage drops of like sequence only. However, if the impedances are unequal or if there exists mutual coupling, then voltage drop of any one sequence is dependent on the currents of all the sequences.



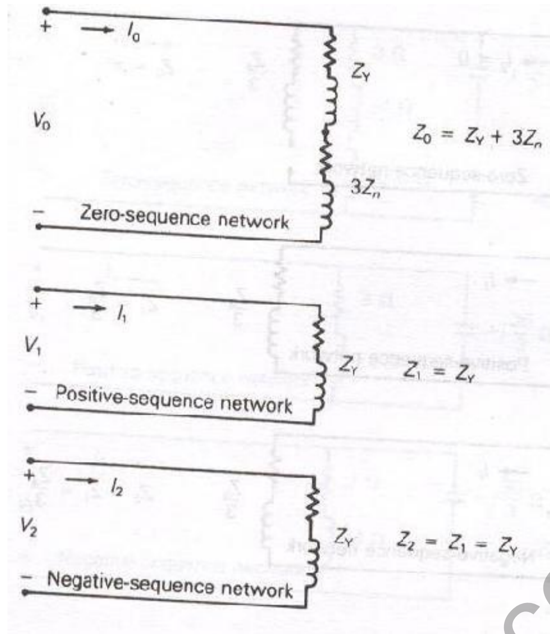


Figure 3.5 Sequence impedances of a Y-connected load.

NUMERICAL EXAMPLES

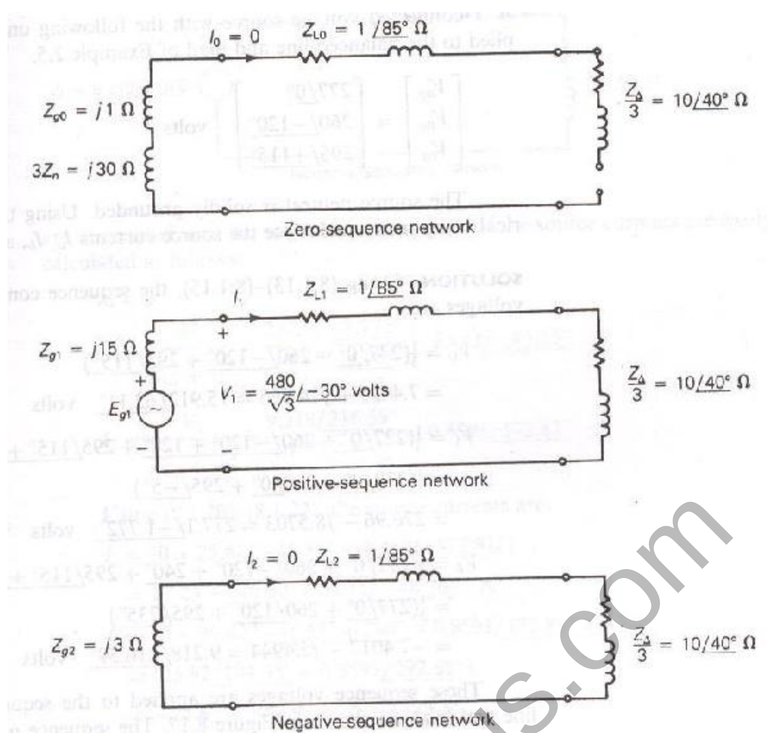
Example 9: A Y-connected source with phase voltages Vag = 277<0^0, Vbg = 260<-120^0 and Vcg = 295<115^0 is applied to a balanced Δ load of 30<40^0 Ω/phase through a line of impedance 1<85^0 Ω. The neutral of the source is solidly grounded. Draw the sequence networks of the system and find source currents.

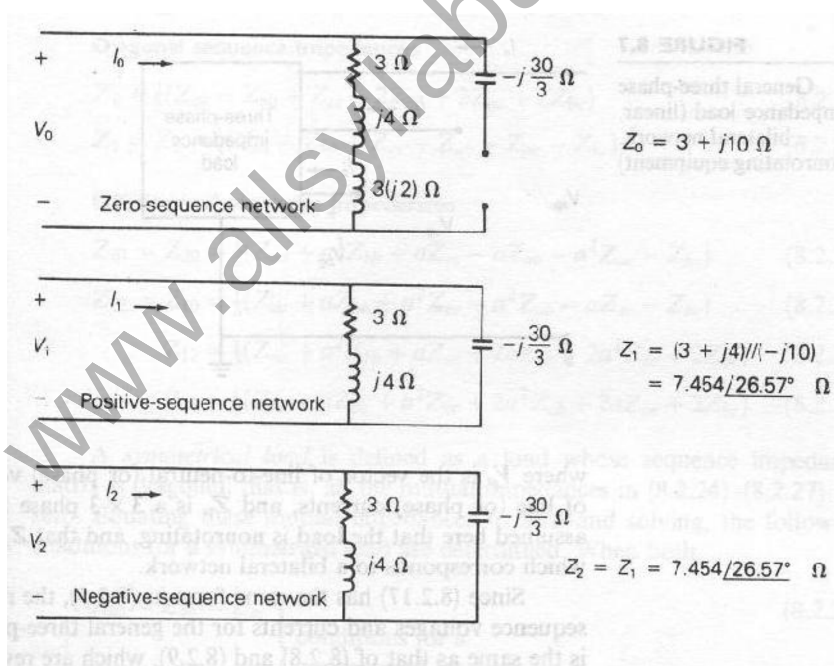
Solution:

Va0 = 15.91<62.110 V
Va1 = 277.1<-1.70 V
Va2 = 9.22<216.70 V
Y eq. of Δ load = 10<400 Ω/phase
Zline = 1<850 Ω.
Zneutral = 0

Ia0 = 0<00 A
Ia1 = 25.82<-45.60 A
Ia2 = 0.86<172.80 A

Ia = 25.15<-46.80 A
Ib = 25.71<196.40 A
Ic = 26.62<73.80 A





3.6 SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

The impedance of a circuit to positive-sequence currents alone is called the impedance to positive-sequence current or simply *positive-sequence impedance*, which is generally denoted as Z_1. Similarly, the impedance of a circuit to negative-sequence currents alone is called the impedance to negative-sequence current or simply *negative-sequence* *impedance*, which is generally denoted as Z_2. The impedance of a circuit to zero-sequence currents alone is called the impedance to zero-sequence current or simply *zero-sequence impedance*, which is generally denoted as Z_0. In the analysis of an unsymmetrical fault on a symmetrical system, the symmetrical components of the unbalanced currents that are flowing are determined. Since in a balanced system, the components currents of one sequence cause voltage drops of like sequence only and are independent of currents of other sequences, currents of any one sequence may be considered to flow in an independent network composed of the generated voltages, if any, and impedances to the current of that sequence only.

The single-phase equivalent circuit consisting of the impedances to currents of any one sequence only is called the *sequence network* of that particular sequence. Thus, the sequence network corresponding to positive-sequence current is called the *positive-sequence network.* Similarly, the sequence network corresponding to negative-sequence current is called *negative-sequence network*, and that corresponding to zero-sequence current is called *zero-sequence network.* The sequence networks are interconnected in a particular way to represent various unsymmetrical fault conditions. Therefore, to calculate the effect of a fault by the method of symmetrical components, it is required to determine the sequence networks.

3.7 SEQUENCE NETWORKS OF UNLOADED GENERATOR

Consider an unloaded generator which is grounded through a reactor as shown in Figure. When a fault occurs, unbalanced currents depending on the type of fault will flow through the lines. These currents can be resolved into their symmetrical components. To draw the sequence networks of this generator, the component voltages/currents, component impedances are to be determined. The generated voltages are of positive-sequence only as the generators are designed to supply balanced three-phase voltages. Hence, positive-sequence network is composed of an emf in series with the positive-sequence impedance. The generated emf in this network is the no-load terminal voltage to neutral, which is also equal to the transient and subtransient voltages as the generator is not loaded. The reactance in this network is the subtransient, transient, or synchronous reactance, depending on the condition of study.

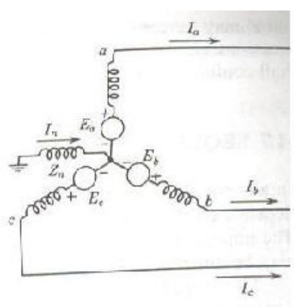


Figure 3.6 Circuit of an unloaded generator grounded through reactance.

The negative- and zero-sequence networks are composed of only the respective sequence impedances as there is no corresponding sequence emf. The reference bus for the positive- and negative-sequence networks is the neutral of the generator.

The current flowing in the impedance Zn between neutral and ground is $3I_{a0}$ as shown in Fig. 3.6. Thus the zero-sequence voltage drop from point a to the ground, is given by: $(-I_{a0}Z_{g0} - 3I_{a0}Zn)$, where Z_{g0} is the zero-sequence impedance of the generator. Thus the zero-sequence network, which is single-phase equivalent circuit assumed to carry only one phase, must have an zero-sequence impedance of $Zo = (Z_{g0} + 3Zn)$.

From the sequence networks, the voltage drops from point a to reference bus (or ground) are given by

$$
\begin{aligned}
V_{a1} &= Ea - I_{a1}Z_1 \\
V_{a2} &= - I_{a2}Z_2 \\
V_{a0} &= - I_{a0}\, Z_0
\end{aligned}
\tag{3.15}
$$

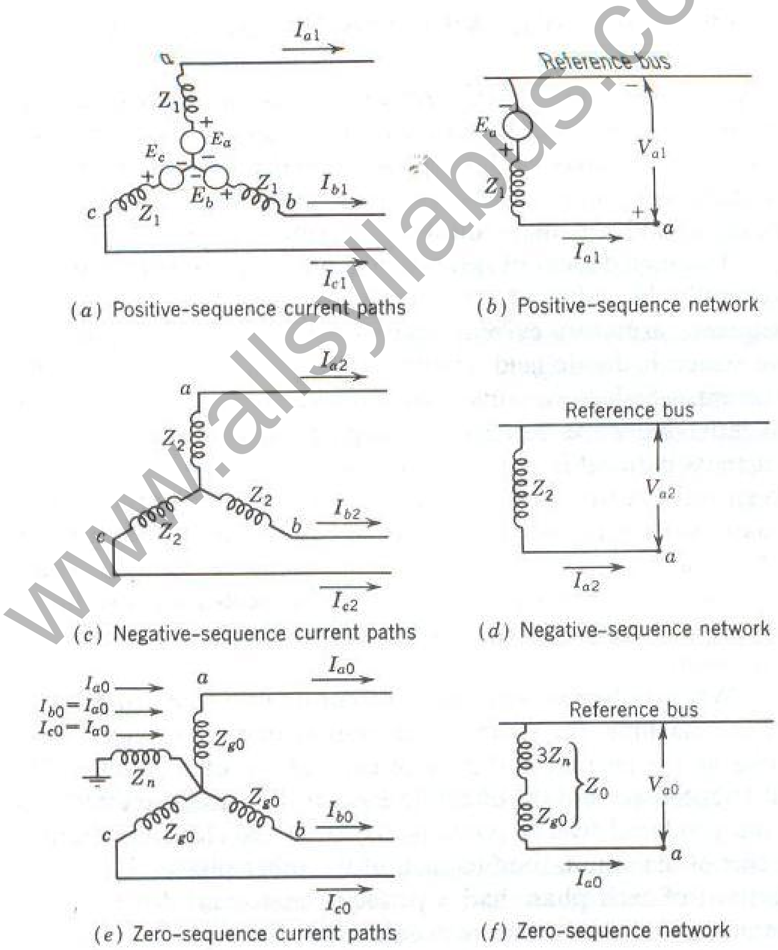


Figure 3.7 Sequence current paths in a generator and The corresponding sequence networks.

Eq. 3.15 applicable to any unloaded generator are valid for loaded generator under steady state conditions. These relations are also applicable for transient or subtransient conditions of a loaded generator if Eg' or Eg" is substituted for Ea.

3.8 SEQUENCE IMPEDANCE OF CIRCUIT ELEMENTS

For obtaining the sequence networks, the component voltages/ currents and the component impedances of all the elements of the network are to be determined. The usual elements of a power system are: passive loads, rotating machines (generators/ motors), transmission lines and transformers. The positive- and negative-sequence impedances of linear, symmetrical, static circuits are identical (because the impedance of such circuits is independent of phase order provided the applied voltages are balanced).

The sequence impedances of rotating machines will generally differ from one another. This is due to the different conditions that exists when the sequence currents flows. The flux due to negative-sequence currents rotates at double the speed of rotor while that the positive-sequence currents is stationary with respect to the rotor. The resultant flux due to zero-sequence currents is ideally zero as these flux components adds up to zero, and hence the zero-sequence reactance is only due to the leakage flux. Thus, the zero-sequence impedance of these machines is smaller than positive- and negative-sequence impedances.

The positive- and negative-sequence impedances of a transmission line are identical, while the zero-sequence impedance differs from these. The positive- and negative-sequence impedances are identical as the transposed transmission lines are balanced linear circuits. The zero-sequence impedance is higher due to magnetic field set up by the zero-sequence currents is very different from that of the positive- or negative-sequence currents (because of no phase difference). The zero-sequence reactance is generally 2 to 3.5 times greater than the positive- sequence reactance. It is customary to take all the sequence impedances of a transformer to be identical, although the zero-sequence impedance slightly differs with respect to the other two.

3.9 SEQUENCE NETWORKS OF POWER SYSTEMS

In the method of symmetrical components, to calculate the effect of a fault on a power system, the sequence networks are developed corresponding to the fault condition. These networks are then interconnected depending on the type of fault. The resulting network is then analyzed to find the fault current and other parameters.

Positive- and Negative-Sequence Networks: The positive-sequence network is obtained by determining all the positive-sequence voltages and positive-sequence impedances of individual elements, and connecting them according to the SLD. All the generated emfs are positive-sequence voltages. Hence all the per unit reactance/impedance diagrams obtained in the earlier chapters are positive-sequence networks. The negative-sequence generated emfs are not present. Hence, the negative-sequence network for a power system is obtained by omitting all the generated emfs (short circuiting emf sources) and replacing all impedances by negative-sequence impedances from the positive-sequence networks.

Since all the neutral points of a symmetrical three-phase system are at the same potential when balanced currents are flowing, the neutral of a symmetrical three-phase system is the logical reference point. It is therefore taken as the reference bus for the positive- and negative-sequence networks. Impedances connected between the neutral of the machine and ground is not a part of either the positive- or negative- sequence networks because neither positive- nor negative-sequence currents can flow in such impedances.

Zero-Sequence Networks: The zero-sequence components are the same both in magnitude and in phase. Thus, it is equivalent to a single-phase system and hence, zero-sequence currents will flow only if a return path exists. The reference point for this network is the ground (Since zero-sequence currents are flowing, the ground is not necessarily at the same point at all points and the reference bus of zero-sequence network does not represent a ground of uniform potential. The return path is conductor of zero impedance, which is the reference bus of the zero-sequence network.).

If a circuit is Y-connected, with no connection from the neutral to ground or to another neutral point in the circuit, no zero-sequence currents can flow, and hence the impedance to zero-sequence current is infinite. This is represented by an open circuit between the neutral of the Y-connected circuit and the reference bus, as shown in Fig. 3.8a. If the neutral of the Y-connected circuit is grounded through zero impedance, a zero-impedance path (short circuit) is connected between the neutral point and the reference bus, as shown in Fig. 3.8b. If an impedance Zn is connected between the neutral and the ground of a Y-connected circuit, an impedance of 3Zn must be connected between the neutral and the reference bus (because, all the three zero-sequence currents ($3I_{a0}$) flows through this impedance to cause a voltage drop of $3I_{a0}\, Z_0$), as shown in Fig. 3.8c.

A Δ-connected circuit can provide no return path; its impedance to zero-sequence line currents is therefore infinite. Thus, the zero-sequence network is open at the Δ-connected circuit, as shown in Fig.3.9 However zero-sequence currents can circulate inside the Δ-connected circuit.

The zero-sequence equivalent circuits of three-phase transformers deserve special attention. The different possible combinations of the primary and the secondary windings in Y and Δ alter the zero-sequence network. The five possible connections of two-winding transformers and their equivalent zero-sequence networks are shown in Fig.3.10. The networks are drawn remembering that there will be no primary current when there is no secondary current, neglecting the no-load component. The arrows on the connection diagram show the possible paths for the zero-sequence current. Absence of an arrow indicates that the connection is such that zero-sequence currents cannot flow. The letters P and Q identify the corresponding points on the connection diagram and equivalent circuit:

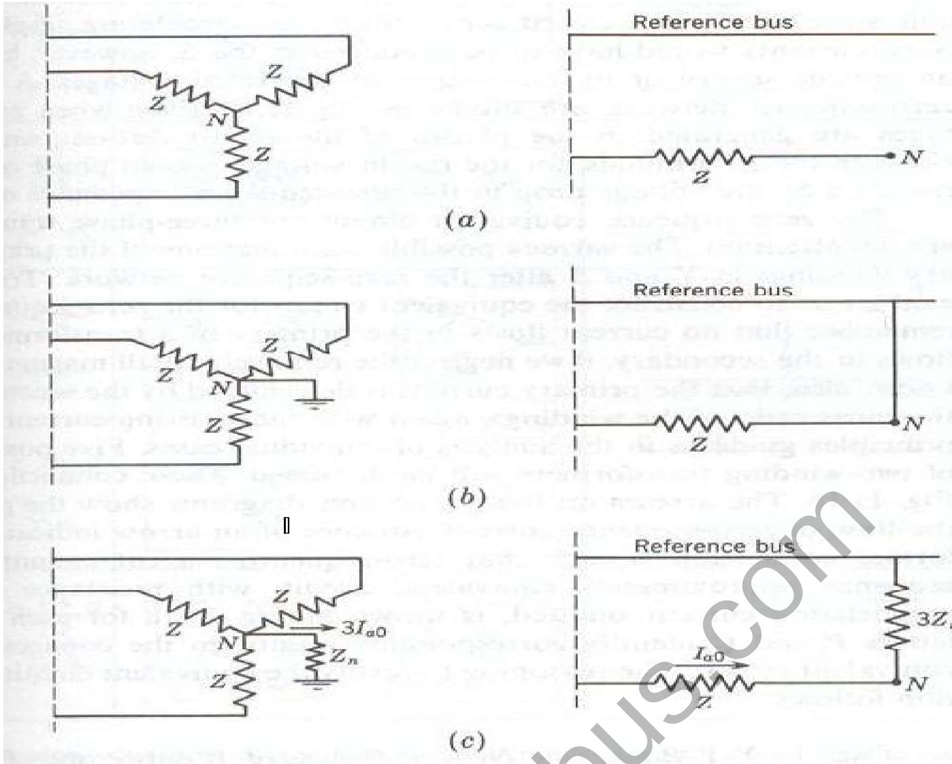


Figure 3.8 Zero-sequence equivalent networks of Y-connected load

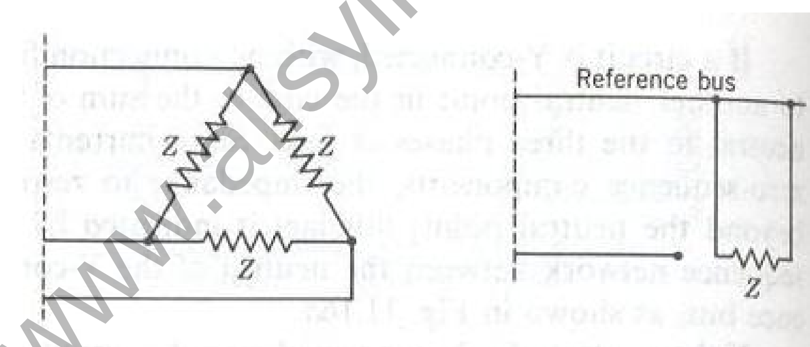


Figure 3.9 Zero-sequence equivalent networks of Δ -connected load

1. *Case 1: Y-Y Bank with one neutral grounded:* If either one of the neutrals of a Y-Y bank is ungrounded, zero-sequence current cannot flow in either winding (as the absence of a path through one winding prevents current in the other). An open circuit exists for zero-sequence current between two parts of the system connected by the transformer bank.
2. *Case 2: Y-Y Bank with both neutral grounded*: In this case, a path through transformer exists for the zero-sequence current. Hence zero-sequence current can flow in both sides of the transformer provided there is complete outside closed path for it to flow. Hence the points on the two sides of the transformer are connected by the zer0-sequence impedance of the transformer.

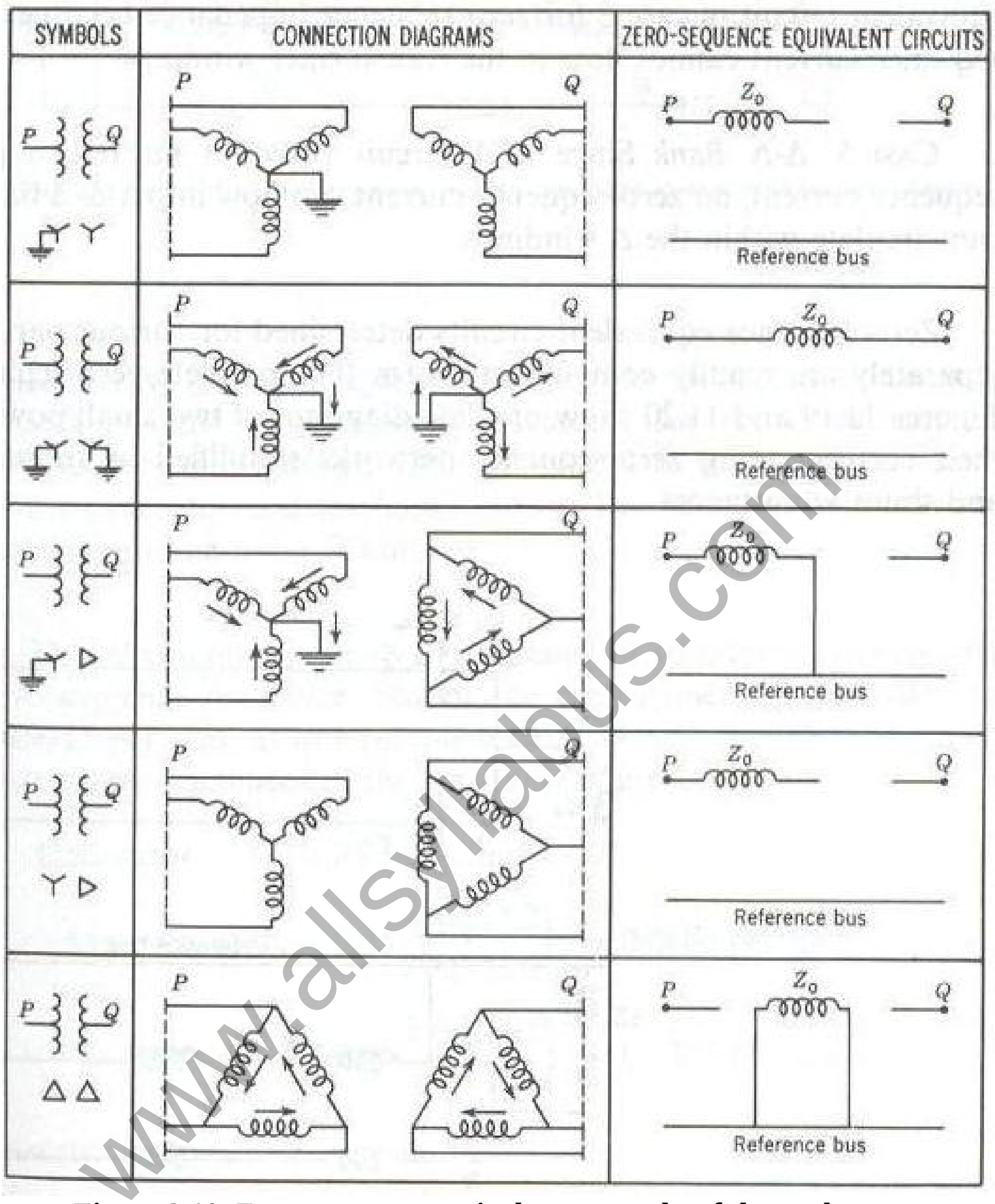


Figure 3.10 Zero-sequence equivalent networks of three-phase transformer banks for various combinations.

3. *Case 3: Y- Δ Bank with grounded Y:* In this case, there is path for zero-sequence current to ground through the Y as the corresponding induced current can circulate in the Δ. The equivalent circuit must provide for a path from lines on the Y side through zero-sequence impedance of the transformer to the reference bus. However, an open circuit must exist between line and the reference bus on the Δ side. If there is an impedance Zn between neutral and ground, then the zero-sequence impedance must include 3Zn along with zero-sequence impedance of the transformer.

4. *Case 4: Y- Δ Bank with ungrounded Y:* In this case, there is no path for zero-sequence current. The zero-sequence impedance is infinite and is shown by an open circuit.
5. *Case 5: Δ-Δ Bank:* In this case, there is no return path for zero-sequence current. The zero-sequence current cannot flow in lines although it can circulate in the Δ windings.
6. The zero-sequence equivalent circuits determined for the individual parts separately are connected according to the SLD to form the complete zero-sequence network.

Procedure to draw the sequence networks

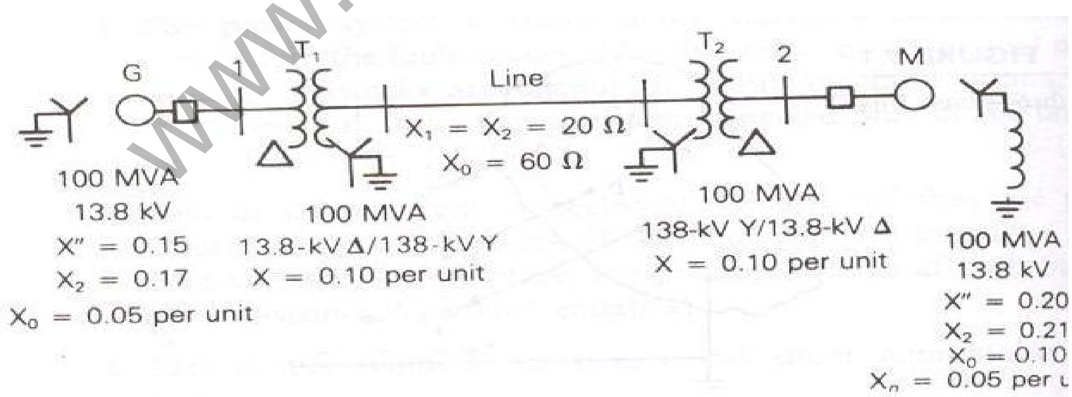
The sequence networks are three separate networks which are the single-phase equivalent of the corresponding symmetrical sequence systems. These networks can be drawn as follows:

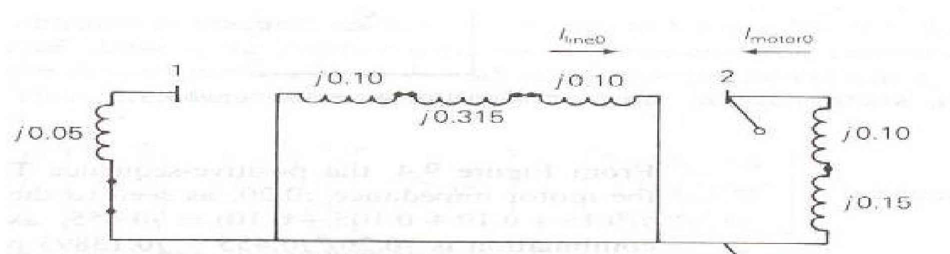
1. For the given condition (steady state, transient, or subtransient), draw the reactance diagram (selecting proper base values and converting all the per unit values to the selected base, if necessary). This will correspond to the positive-sequence network.

2. Determine the per unit negative-sequence impedances of all elements (if the values of negative sequence is not given to any element, it can approximately be taken as equal to the positive-sequence impedance). Draw the negative-sequence network by replacing all emf sources by short circuit and all impedances by corresponding negative-sequence impedances in the positive-sequence network.
3. Determine the per unit zero-sequence impedances of all the elements and draw the zero-sequence network corresponding to the grounding conditions of different elements.

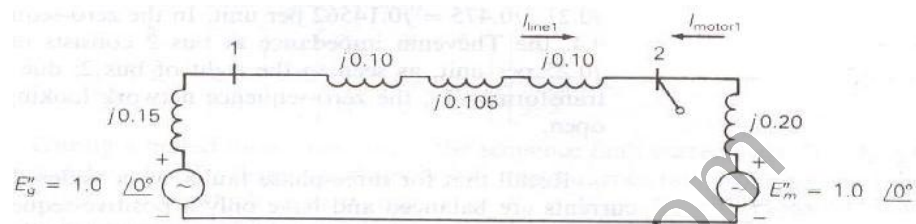
NUMERICAL EXAMPLES

Example 10: For the power system shown in the SLD, draw the sequence networks.

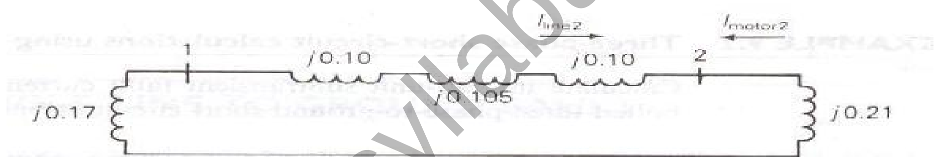




(a) Zero-sequence network

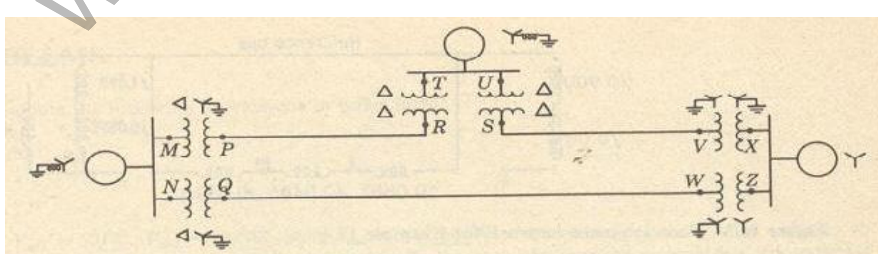


(b) Positive-sequence network



(c) Negative-sequence network

EXERCISE PROBLEM: For the power system shown in the SLD, draw the sequence networks.



CHAPTER 4: UNSYMMETRICAL FAULTS

[CONTENTS: Preamble, L-G, L-L, L-L-G and 3-phase faults on an unloaded alternator without and with fault impedance, faults on a power system without and with fault impedance, open conductor faults in power systems, examples]

4.1 PREAMBLE

The unsymmetrical faults will have faulty parameters at random. They can be analyzed by using the symmetrical components. The standard types of unsymmetrical faults considered for analysis include the following (in the order of their severity):

- Line–to–Ground (L-G) Fault
- Line–to–Line (L-L) Fault
- Double Line–to–Ground (L-L-G)Fault and
- Three-Phase–to–Ground (LLL-G) Fault.

Further the neutrals of various equipment may be grounded or isolated, the faults can occur at any general point F of the given system, the faults can be through a fault impedance, etc. Of the various types of faults as above, the 3-ϕ fault involving the ground is the most severe one. Here the analysis is considered in two stages as under: (i) Fault at the terminals of a Conventional (Unloaded) Generator and (ii) Faults at any point F, of a given Electric Power System (EPS).

Consider now the symmetrical component relational equations derived from the three sequence networks corresponding to a given unsymmetrical system as a function of sequence impedances and the positive sequence voltage source in the form as under:

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned} \qquad (4.1)$$

These equations are refered as the *sequence equations*. In matrix Form the sequence equations can be considered as:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.2)$$

This equation is used along with the equations i.e., conditions under fault (c.u.f.), derived to describe the fault under consideration, to determine the sequence current I_{a1} and hence the fault current I_f, in terms of E_a and the sequence impedances, Z_1, Z_2 and Z_0. Thus during unsymmetrical fault analysis of any given type of fault, two sets of equations as follows are considered for solving them simultaneously to get the required fault parameters:

- Equations for the conditions under fault (c.u.f.)

- Equations for the sequence components (sequence equations) as per (4.2) above.

4.2 SINGLE LINE TO GROUND FAULT ON A CONVENTIONAL (UNLOADED) GENERATOR

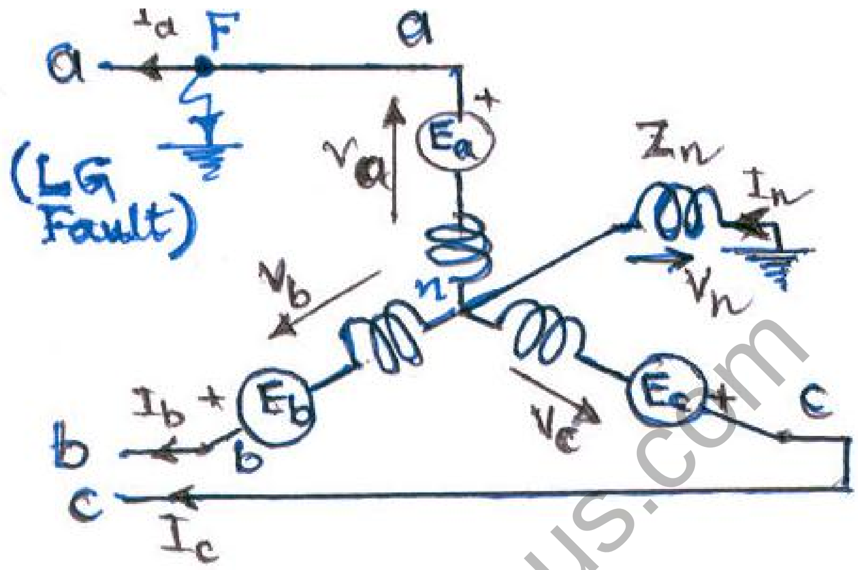


Figure 4.1 LG Fault on a Conventional Generator

A conventional generator is one that produces only the balanced voltages. Let Ea, nd Ec be the internally generated voltages and Zn be the neutral impedance. The fault is assumed to be on the phase'a' as shown in figure 4.1. Consider now the conditions under fault as under:

c.u.f.:

$$I_b = 0; \quad I_c = 0; \quad \text{and} \quad V_a = 0. \tag{4.3}$$

Now consider the symmetrical components of the current I_a with I_b=I_c=0, given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} I_a \\ 0 \\ 0 \end{vmatrix} \tag{4.4}$$

Solving (4.4) we get,

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3) \tag{4.5}$$

Further, using equation (4.5) in (4.2), we get,

$$\begin{vmatrix} V_{a0} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \end{vmatrix} \begin{vmatrix} I_{a1} \\ I_{a1} \end{vmatrix}$$

$V_{a2} \quad 0 \quad\quad 0 \quad 0 \quad Z_2 \quad I_{a1}$ (4.6)

Pre-multiplying equation (4.6) throughout by [1 1 1], we get,

$V_{a1}+V_{a2}+V_{a0} = - I_{a1}Z_0 + E_a – I_{a1}Z_1 – I_{a2}Z_2$

i.e., $Va = E_a – I_{a1} (Z_1 + Z_2 + Z_0) =$ zero,

Or in other words,

$\mathbf{I_{a1} = [E_a/(Z_1 + Z_2 + Z_0)]}$ (4.7)

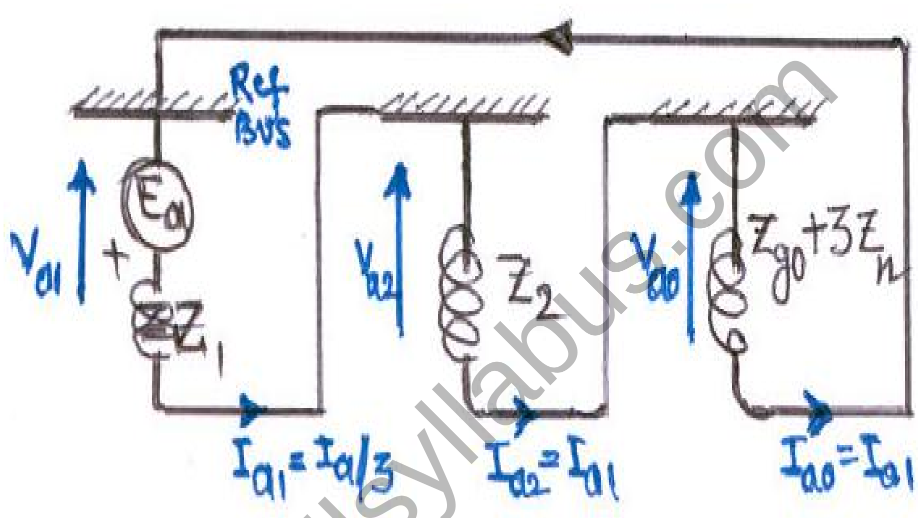


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Figure 4.2 Connection of sequence networks for LG Fault on phase a of a Conventional Generator

The equation (4.7) derived as above implies that the three sequence networks are connected in series to simulate a LG fault, as shown in figure 4.2. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = I_{a2} = I_{a0} = (I_a/3) = [E_a/(Z_1 + Z_2 + Z_0)]$
2. **Fault current $\mathbf{I_f = I_a = 3I_{a1} = [3E_a/(Z_1 + Z_2 + Z_0)]}$**
3. $V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0)$
4. $V_{a2} = - E_aZ_2/(Z_1+Z_2+Z_0)$
5. $V_{a0} = - E_aZ_0/(Z_1+Z_2+Z_0)$
6. Fault phase voltage $V_a = 0$,
7. Sound phase voltages $V_b = a^2V_{a1}+aV_{a2}+V_{a0}$; $V_c = aV_{a1}+a^2V_{a2}+V_{a0}$
8. Fault phase power: $V_aI_a^* = 0$, Sound pahse powers: $V_bI_b^* = 0$, and $V_cI_c^* = 0$,
9. If $Z_n = 0$, then $Z_0 = Z_{g0}$,

10. If $Z_n = \infty$, then $Z_0 = \infty$, i.e., the zero sequence network is open so that then, $I_f=I_a=0$.

4.3 LINE TO LINE FAULT ON A CONVENTIONAL GENERATOR

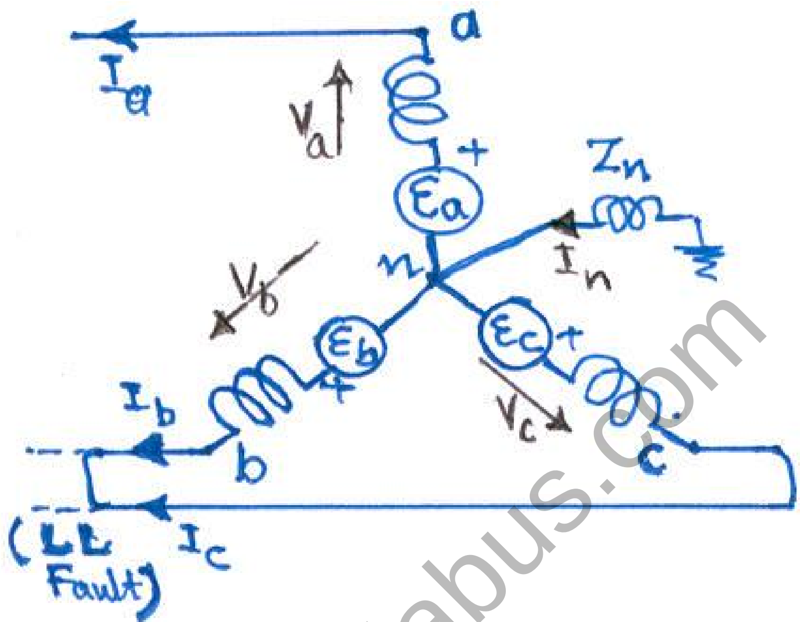


Figure 4.3 LL Fault on a Conventional Generator

Consider a line to line fault between phase ‘b’ and phase ‘c’ as shown in figure 4.3, at the terminals of a conventional generator, whose neutral is grounded through a reactance. Consider now the conditions under fault as under:

c.u.f.:

$I_a = 0$; $I_b = - I_c$; and $V_b = V_c$ (4.8)

Now consider the symmetrical components of the voltage V_a with $V_b=V_c$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_b \end{vmatrix} \qquad (4.9)$$

Solving (4.4) we get,

$V_{a1} = V_{a2}$ (4.10)

Further, consider the symmetrical components of current I_a with $I_b=-I_c$, and $I_a=0$; given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} 0 \\ I_b \\ -I_b \end{vmatrix} \qquad (4.11)$$

Solving (4.11) we get,

$$I_{a0} = 0;\ \text{and } I_{a2} = -I_{a1} \tag{4.12}$$

Using equation (4.10) and (4.12) in (4.2), and since $V_{a0} = 0$ (I_{a0} being 0), we get,

$$\begin{vmatrix} 0 \\ V_{a1} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{vmatrix} \tag{4.13}$$

Pre-multiplying equation (4.13) throughout by [0 1 -1], we get,

$$V_{a1}-V_{a1} = E_a - I_{a1}Z_1 - I_{a1}Z_2 = 0$$

Or in other words,

$$\mathbf{I_{a1} = [E_a/(Z_1 + Z_2)]} \tag{4.14}$$

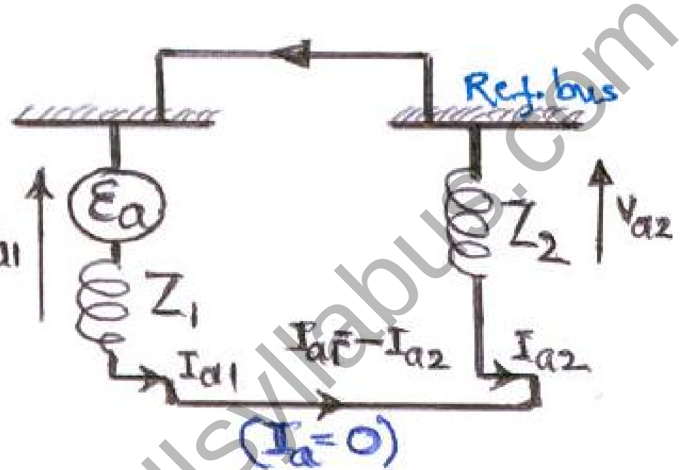


Figure 4.4 Connection of sequence networks for LL Fault on phases b & c of a Conventional Generator

The equation (4.14) derived as above implies that the three sequence networks are connected such that the zero sequence network is absent and only the positive and negative sequence networks are connected in series-opposition to simulate the LL fault, as shown in figure 4.4. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = - I_{a2} = [E_a/(Z_1 + Z_2)]$ and $I_{a0} = 0$,
2. Fault current **$I_f = I_b = - I_c$ = [Ě3$E_a/(Z_1 + Z_2)$]** (since $I_b = (a^2-a)I_{a1} = \sqrt{3}I_{a1}$)
3. $V_{a1} = E_a - I_{a1}Z_1 = E_aZ_2/(Z_1+Z_2)$
4. $V_{a2} = V_{a1} = E_aZ_2/(Z_1+Z_2)$
5. $V_{a0} = 0$,
6. Fault phase voltages; $V_b = V_c = aV_{a1}+a^2V_{a2}+V_{a0} = (a+a^2)V_{a1} = - V_{a1}$
7. Sound phase voltage; $Va = V_{a1}+V_{a2}+V_{a0} = 2V_{a1}$;
8. Fault phase powers are $V_bI_b^*$ and $V_cI_c^*$,
9. Sound phase power: $V_aI_a^* = 0$,

10. Since $I_{a0}=0$, the presence of absence of neutral impedance does not make any difference in the analysis.

4.4 DOUBLE LINE TO GROUND FAULT ON A CONVENTIONAL GENERATOR

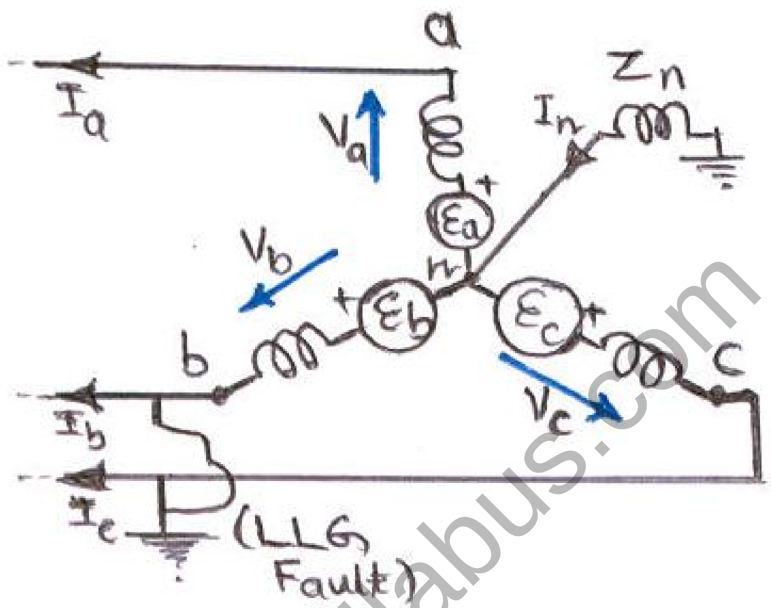


Figure 4.5 LLG Fault on a Conventional Generator

Consider a double-line to ground fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between phase ‘b’ and phase ‘c’ as shown in figure 4.5, Consider now the conditions under fault as under:

c.u.f.:

$$I_a = 0 \text{ and } V_b = V_c = 0 \qquad (4.15)$$

Now consider the symmetrical components of the voltage with $V_b=V_c=0$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ 0 \\ 0 \end{vmatrix} \qquad (4.16)$$

Solving (4) we get,

$$V_{a1} = V_{a2} = V_{a0} = V_a/3 \qquad (4.17)$$

Consider now the sequence equations (4.2) as under,

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.18)$$

Pre-multiplying equation (4.18) throughout by

$$Z^{-1} = \begin{matrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{matrix} \qquad (4.19)$$

We get,

$$Z^{-1}\begin{vmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{vmatrix} = Z^{-1}\begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - Z^{-1}\begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.20)$$

Using the identity: $V_{a1}= (E_a – I_{a1}Z_1)$ in equation (4.19), pre-multiplying throughout by [1 1 1] and finally adding, we get,

$$E_a/Z_0 - I_{a1}(Z1/Z_0) + (E_a/Z_1) - I_{a1} + E_a/Z_2 - I_{a1}(Z1/Z_2) = (E_a/Z_1) – (I_{a0}+I_{a1}+I_{a2})$$

$$= (E_a/Z_1) - I_a \quad = (E_a/Z_1) \qquad (4.21)$$

Since $I_a = 0$, solving the equation (4.21), we get,

$$\mathbf{I_{a1} = \{ E_a/ [Z_1 + Z_2Z_0/(Z_2+Z_0)] \}} \qquad (4.22)$$

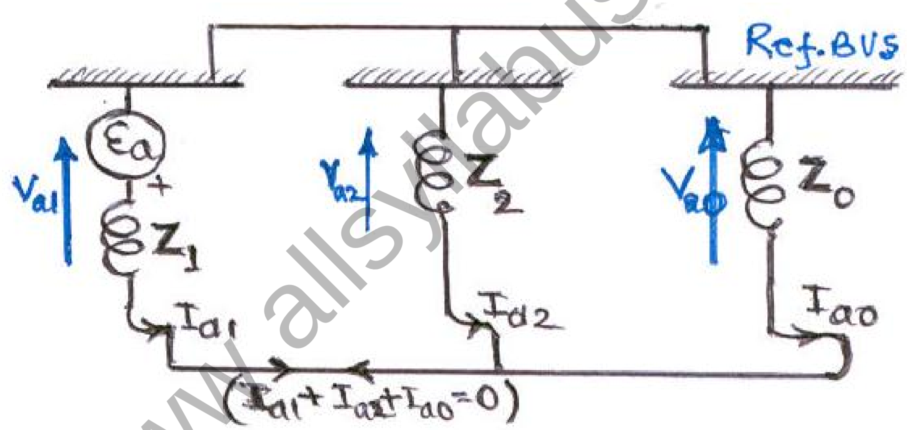


Figure4.6 Connection of sequence networks for LLG Fault on phases b and c of a Conventional Generator

The equation (4.22) derived as above implies that, to simulate the LLG fault, the three sequence networks are connected such that the positive network is connected in series with the parallel combination of the negative and zero sequence networks, as shown in figure 4.6. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = \{E_a/ [Z_1+Z_2Z_0/(Z_2+Z_0)]\}$; $I_{a2}= -I_{a1}Z_0/(Z_2 + Z_0)$ and $I_{a0} = -I_{a1}Z_2/(Z_2 + Z_0)$,
2. Fault current I_f: $I_{a0}=(1/3)(I_a+I_b+I_c) = (1/3)(I_b+I_c) = I_f/3$, Hence $\mathbf{I_f = 3I_{a0}}$
3. $I_a = 0$, $V_b=V_c=0$ and hence $V_{a1}=V_{a2}=V_{a0}=V_a/3$
4. Fault phase voltages; $V_b = V_c = 0$
5. Sound phase voltage; $Va = V_{a1}+V_{a2}+V_{a0} = 3V_{a1}$;
6. Fault phase powers are $V_bI_b^* = 0$, and $V_cI_c^* = 0$, since $V_b=V_c=0$

7. Healthy phase power: $V_a I_a^* = 0$, since $I_a=0$
8. If $Z_0=\infty$, (i.e., the ground is isolated), then $I_{a0}=0$, and hence the result is the same as that of the LL fault [with $Z_0=\infty$, equation (4.22) yields equation (4.14)].

4.5 THREE PHASE TO GROUND FAULT ON A CONVENTIONAL GENERATOR

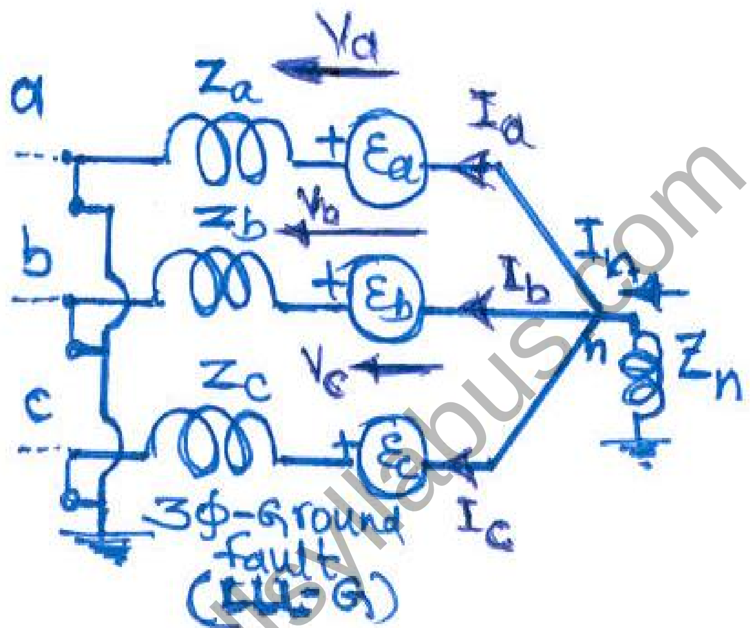


Figure 4.7 Three phase ground Fault on a Conventional Generator

Consider a three phase to ground (LLLG) fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between all its three phases a, b and c, as shown in figure 4.7, Consider now the conditions under fault as under:

c.u.f.:

$$V_a = V_b = V_c = 0, \quad I_a + I_b + I_c = 0 \qquad (4.23)$$

Now consider the symmetrical components of the voltage with $V_a=V_b=V_c= 0$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \qquad (4.24)$$

Solving (4.24) we get,

$$V_{a1} = V_{a2} = V_{a0} = 0 \tag{4.25}$$

Thus we have

$$V_{a1} = E_{a1} - I_{a1}Z_1 \tag{4.26}$$

So that after solving for I_{a1} we, get,

$$\mathbf{I_{a1} = [\ E_a / Z_1\ ]} \tag{4.27}$$

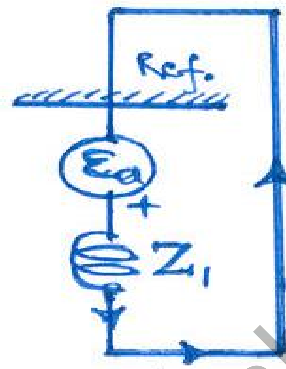


Figure 4.8 Connection of sequence networks for 3-phase ground Fault on phases b and c of a Conventional Generator

The equation (4.26) derived as above implies that, to simulate the 3-phase ground fault, the three sequence networks are connected such that the negative and zero sequence networks are absent and only the positive sequence network is present, as shown in figure 4.8. Further the fault current, I_f in case of a 3-phase ground fault is given by

$$\mathbf{I_f = I_{a1} = I_a = (E_a/Z_1)} \tag{4.28}$$

It is to be noted that the presence of a neutral connection without or with a neutral impedance, Z_n will not alter the simulated conditions in case of a three phase to ground fault.

4.6 UNSYMMETRICAL FAULTS ON POWER SYSTEMS

In all the analysis so far, only the fault at the terminals of an unloaded generator have been considered. However, faults can also occur at any part of the system and hence the power system fault at any general point is also quite important. The analysis of unsymmetrical fault on power systems is done in a similar way as that followed thus far for the case of a fault at the terminals of a generator. Here, instead of the sequence impedances of the generator, each and every element is to be replaced by their corresponding sequence impedances and the fault is analyzed by suitably connecting them together to arrive at the Thevenin equivalent impedance if that given sequence. Also, the internal voltage of the generators of the equivalent circuit for the positive sequence network is now V_f (and not E_a), the pre-fault voltage to neutral at the point of fault (PoF) (ref. Figure 4.9).

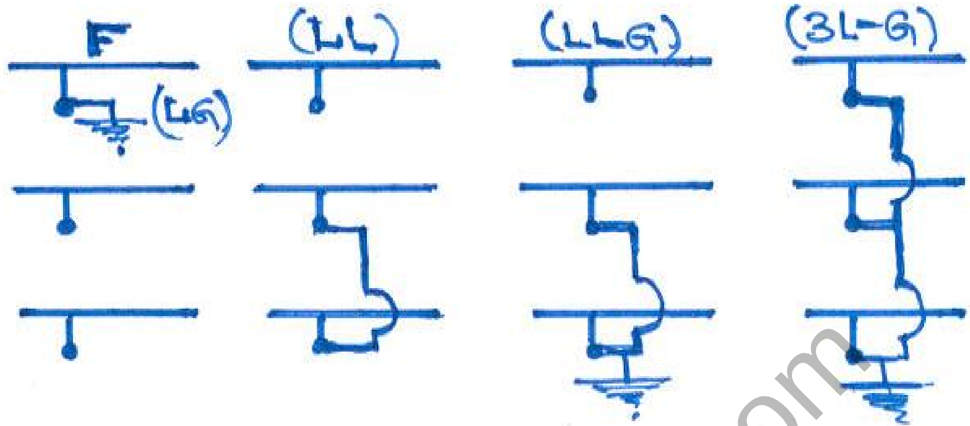


Figure 4.9 Unsymmetrical faults in Power Systems

Thus, for all the cases of unsymmetrical fault analysis considered above, the sequence equations are to be changed as under so as to account for these changes:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ V_f \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \quad (4.29)$$

(i) LG Fault at any point F of a given Power system

Let phase 'a' be on fault at F so that then, the c.u.f. would be:

$I_b = 0$; $I_c = 0$; and $V_a = 0$.

Hence the derived conditions under fault would be:

$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$

$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0)]$ and

$I_f = 3I_{a1}$ (4.30)

(ii) LL Fault at any point F of a given Power system

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$I_a = 0$; $I_b = - I_c$; and $V_b = V_c$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2}$; $I_{a0} = 0$; $I_{a2} = -I_{a1}$

$I_{a1} = [V_f / (Z_1 + Z_2)]$ and

$I_f = I_b = - I_c = [\sqrt{3}\, V_f / (Z_1 + Z_2)]$ (4.31)

(ii) LLG Fault at any point F of a given Power system

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$I_a = 0$ and $V_b = V_c = 0$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$

$I_{a1} = \{V_f / [Z_1+Z_2Z_0/(Z_2+Z_0)]\}$

$I_{a2}= -I_{a1}Z_0/(Z_2 + Z_2)$; $I_{a0} = -I_{a1}Z_2/(Z_2 + Z_2)$ and

$\mathbf{I_f = 3I_{a0}}$ (4.32)

(ii) Three Phase Fault at any point F of a given Power system

Let all the 3 phases a, b and c be on fault at F so that then, the c.u.f. would be:

$V_a = V_b = V_c = 0$, $I_a + I_b + I_c = 0$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2} = V_{a0} = V_a/3$

$V_{a0} = V_{a1} = V_{a2} = 0$; $I_{a0} = I_{a2} = 0$,

$I_{a1} = [V_f /Z_1]$ and $I_f = I_{a1}=I_a$ (4.33)

4.7 OPEN CONDUCTOR FAULTS

Various types of power system faults occur in power systems such as the *shunt type faults* (LG, LL, LLLG faults) and *series type faults* (open conductor and cross country faults). While the symmetrical fault analysis is useful in determination of the rupturing capacity of a given protective circuit breaker, the unsymmetrical fault analysis is useful in the determination of relay setting, single phase switching and system stability studies.

When one or two of a three-phase circuit is open due to accidents, storms, etc., then unbalance is created and the asymmetrical currents flow. Such types of faults that come in series with the lines are refered as the *open conductor faults*. The open conductor faults can be analyzed by using the sequence networks drawn for the system under consideration as seen from the point of fault, F. These networks are then suitably connected to simulate the given type of fault. The following are the cases required to be analyzed (ref. fig.4.10).

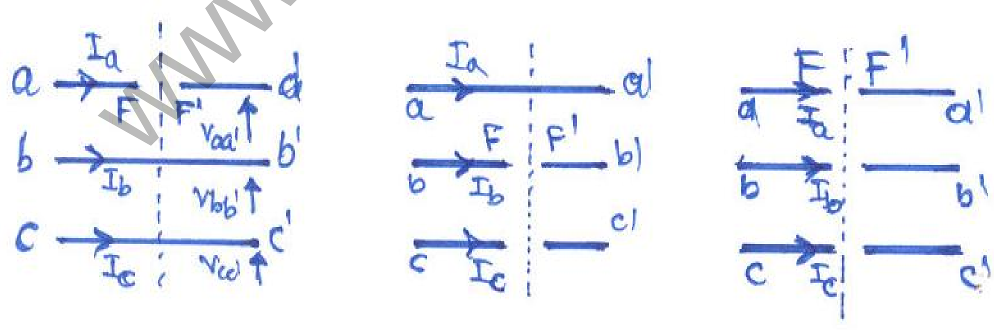


Figure 4.10 Open conductor faults.

(i) Single Conductor Open Fault: consider the phase 'a' conductor open so that then the conditions under fault are:

$I_a = 0$; $V_{bb'} = V_{cc'} = 0$

The derived conditions are:

$$I_{a1} + I_{a2} + I_{a0} = 0 \text{ and}$$
$$V_{aa1'} = V_{aa2'} = V_{aa0'} = (V_{aa'}/3) \qquad (4.34)$$

These relations suggest a parallel combination of the three sequence networks as shown in fig. 4.11.

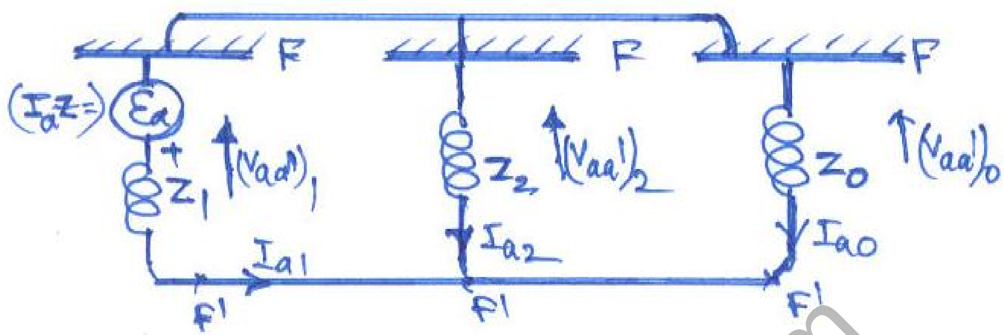


Figure 4.11 Sequence network connection for 1-conductor open fault

It is observed that a single conductor fault is similar to a LLG fault at the fault point F of the system considered.

(ii) Two Conductor Open Fault: consider the phases 'b' and 'c' under open condition so that then the conditions under fault are:

$$I_b = I_c = 0; \; V_{aa'} = 0$$

The derived conditions are:

$$I_{a1} = I_{a2} = I_{a0} = I_a/3 \text{ and}$$
$$V_{aa1'} = V_{aa2'} = V_{aa0'} = 0 \qquad (4.35)$$

These relations suggest a series combination of the three sequence networks as shown in fig. 4.12. It is observed that a double conductor fault is similar to a LG fault at the fault point F of the system considered.

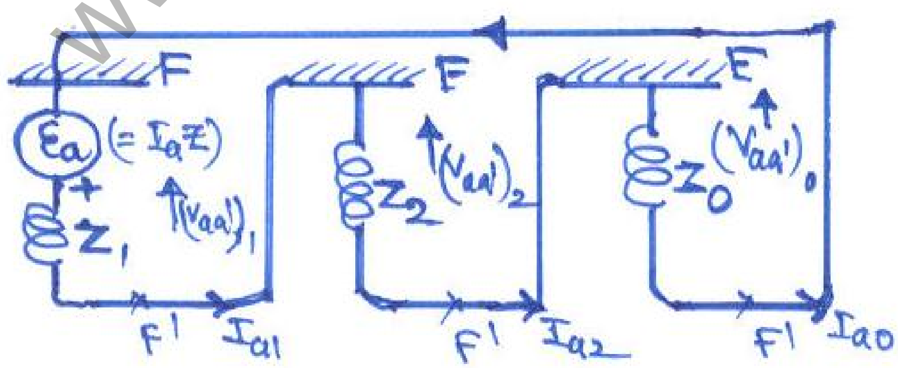


Figure 4.12 Sequence network connection for 2-conductor open fault.

(iii) Three Conductor Open Fault: consider all the three phases a, b and c, of a 3-phase system conductors be open. The conditions under fault are:

$I_a + I_b + I_c = 0$

The derived conditions are:

$I_{a1} = I_{a2} = I_{a0} = 0$ and

$V_{a0} = V_{a2} = 0$ and $V_{a1} = V_f$ (4.36)

These relations imply that the sequence networks are all open circuited. Hence, in a strict analystical sense, this is not a fault at all!

4.8 FAULTS THROUGH IMPEDANCE

All the faults considered so far have comprised of a direct short circuit from one or two lines to ground. The effect of impedance in the fault is found out by deriving equations similar to those for faults through zero valued neutral impedance. The connections of the hypothetical stubs for consideration of faults through fault impedance Z_f are as shown in figure 4.13.

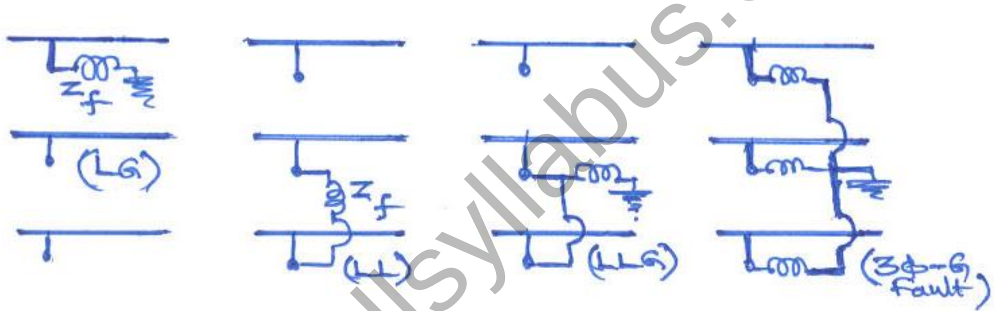


Figure 4.13 Stubs Connections for faults through fault impedance Z_f.

(i) LG Fault at any point F of a given Power system through Z_f

Let phase 'a' be on fault at F *through Z_f,* so that then, the c.u.f. would be:

$I_b = 0$; $I_c = 0$; and $V_a = 0$.

Hence the derived conditions under fault would be:

$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$

$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0+3Z_f)]$ and

$I_f = 3I_{a1}$ (4.37)

(ii) LL Fault at any point F of a given Power system through Z_f

Let phases 'b' and 'c' be on fault at F *through Z_f,* so that then, the c.u.f. would be:

$I_a = 0$; $I_b = - I_c$; and $V_b = V_c$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2}$; $I_{a0} = 0$; $I_{a2} = -I_{a1}$

$I_{a1} = [V_f / (Z_1 + Z_2+Z_f)]$ and

$I_f = I_b = - I_c = [\sqrt{3}\, V_f / (Z_1 + Z_2+Z_f)]$ (4.38)

(iii) LLG Fault at any point F of a given Power system through Z_f

Let phases 'b' and 'c' be on fault at F *through* Z_f, so that then, the c.u.f. would be:

$I_a = 0$ and $V_b = V_c = 0$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$

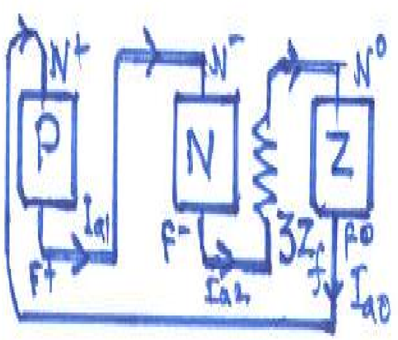
$I_{a1} = \{V_f / [Z_1+Z_2(Z_0+3Z_f)/(Z_2+Z_0+3Zf)]\}$

$I_{a2} = -I_{a1}(Z_0+3Z_f)/(Z_2+Z_0+3Z_f)$; $I_{a0} = -I_{a1}Z_2/(Z_2+(Z_0+3Z_f)$ and

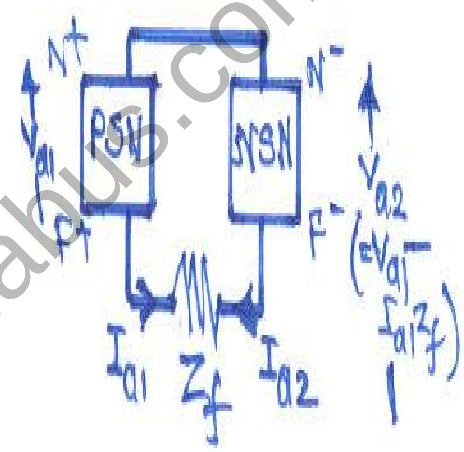
$\mathbf{I_f = 3I_{a0}}$ (4.39)

(iv) Three Phase Fault at any point F of a given Power system through Z_f

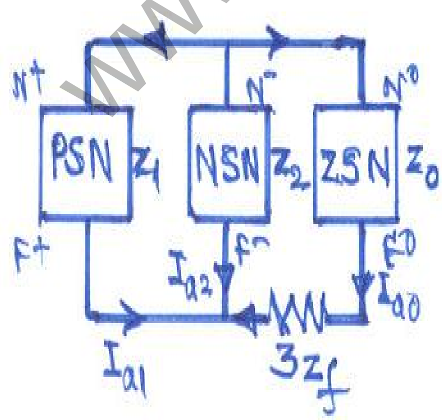
Let all the 3 phases a, b and c be on fault at F, through Z_f so that the c.u.f. would be: $V_a = I_aZ_f$; Hence the derived conditions under fault would be: $I_{a1} = [V_f /(Z_1+Z_f)$; The connections of the sequence networks for all the above types of faults through Z_f are as shown in figure 4.14.



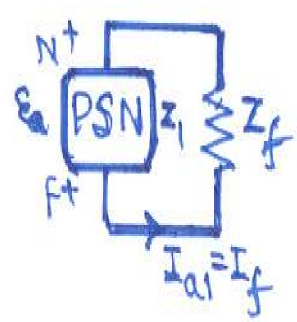
LG Fault



LL Fault



LLG Fault



3-Ph. Fault

Figure 4.15 Sequence network connections for faults through impedance

4.9 EXAMPLES

Example-1: A three phase generator with constant terminal voltages gives the following currents when under fault: 1400 A for a line-to-line fault and 2200 A for a line-to-ground fault. If the positive sequence generated voltage to neutral is 2 ohms, find the reactances of the negative and zero sequence currents.

Solution: Case a) Consider the conditions w.r.t. the LL fault:

$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$

$I_f = I_b = - I_c = \sqrt{3}\, I_{a1}$

$= \sqrt{3}\, E_{a1} / (Z_1 + Z_2)$ or

$(Z_1 + Z_2) = \sqrt{3}\, E_{a1} / I_f$

i.e., $2 + Z_2 = \sqrt{3}\, [2000/1400]$

Solving, we get, $Z_2 = 0.474$ ohms.

Case b) Consider the conditions w.r.t. a LG fault:

$I_{a1} = [E_{a1}/(Z_1 + Z_2+Z_0)]$

$I_f = 3\, I_{a1}$

$= 3\, E_{a1} / (Z_1 + Z_2+Z_0)$ or

$(Z_1 + Z_2+Z_0) = 3\, E_{a1} / I_f$

i.e., $2 + 0.474 + Z_0 = 3\, [2000/2200]$

Solving, we get, $Z_0 = 0.253$ ohms.

Example-2: A dead fault occurs on one conductor of a 3-conductor cable supplied y a 10 MVA alternator with earhed neutral. The alternator has +ve, -ve and 0-sequence components of impedances per phase respectively as: (0.5+j4.7), (0.2+j0.6) and (j0.43) ohms. The corresponding LN values for the cable up to the point of fault are: (0.36+j0.25), (0.36+j0.25) and (2.9+j0.95) ohms respectively. If the generator voltage at no load (E_{a1}) is 6600 volts between the lines, determine the (i)Fault current, (ii)Sequence components of currents in lines and (iii)Voltages of healthy phases.

Solution: There is LG fault on any one of the conductors. Consider the LG fault to be on conductor in phase a. Thus the fault current is given by:

(i) Fault current: $I_f = 3I_{a0} = [3E_a/(Z_1 + Z_2 + Z_0)]$

$= 3(6600/\sqrt{3})/ (4.32+j7.18)$

$= 1364.24 \angle 58.97^0$.

(ii) Sequence components of line currents:

$I_{a1} = I_{a2} = I_{a0} = I_a/3 = I_f/3 = 454.75 \angle 58.97^0$.

(iii) Sound phase voltages:

$V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0) = 1871.83 \angle -26.17^0$,
$V_{a2} = - E_aZ_2/(Z_1+Z_2+Z_0) = 462.91 \angle 177.6^0$,
$V_{a0} = - E_aZ_0/(Z_1+Z_2+Z_0) = 1460.54 \angle 146.5^0$,

Thus,

Sound phase voltages $V_b = a^2V_{a1}+aV_{a2}+V_{a0} = 2638.73 \angle -165.8^0$ Volts,
And $V_c = aV_{a1}+a^2V_{a2}+V_{a0} = 3236.35 \angle 110.8^0$ Volts.

Example-3: A generator rated 11 kV, 20 MVA has reactances of X_1=15%, X_2=10% and X_0=20%. Find the reactances in ohms that are required to limit the fault current to 2 p.u. when a a line to ground fault occurs. Repeat the analysis for a LLG fault also for a fault current of 2 pu.

Solution: **Case a:** Consider the fault current expression for LG fault given by:

$I_f = 3\ I_{a0}$

i.e., $2.0 = 3Ea\ /\ j[X_1+X_2+X_0]$

$= 3(1.0\angle 0^0)\ /\ j[0.15+0.1+0.2+3X_n]$

Solving we get

$3X_n = 2.1$ pu

$= 2.1\ (Z_b)$ ohms $= 2.1\ (11^2/20) = 2.1(6.05)$

$= 12.715$ ohms.

Thus $X_n = 4.235$ ohms.

Case b: Consider the fault current expression for LLG fault given by:

$\mathbf{I_f = 3I_{a0} = 3\ \{} -I_{a1}X_2/(X_2 + X_0+3X_n)\}= 2.0$,

where, $I_{a1} = \{E_a/ [X_1+X2(X0+3X_n)/(X_2+X_0+3X_n)]\}$

Substituting and solving for X_n we get,

Xn = 0.078 pu
= 0.47 ohms.

Example-4: A three phase 50 MVA, 11 kV generator is subjected to the various faults and the surrents so obtained in each fault are: 2000 A for a three phase fault; 1800 A for a line-to-line fault and 2200 A for a line-to-ground fault. Find the sequence impedances of the generator.

Solution: Case a) Consider the conditions w.r.t. the three phase fault:

$I_f = I_a = I_{a1} = E_{a1}/Z_1$

i.e., $2000 = 11000/\ (\sqrt{3}Z_1)$

Solving, we get, $Z_1 = 3.18$ ohms (1.3 pu, with $Z_b = (11^2/50) = 2.42$ ohms).

Case b) Consider the conditions w.r.t. the LL fault:

$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$

$I_f = I_b = - I_c = \sqrt{3}\, I_{a1}$

$= \sqrt{3}\, E_{a1} / (Z_1 + Z_2)$ or

$(Z_1 + Z_2) = \sqrt{3}\, E_{a1} / I_f$

i.e., $3.18 + Z_2 = \sqrt{3}\,(11000/\sqrt{3})/1800$

Solving, we get, $Z_2 = 2.936$ ohms $= 1.213$ pu.

Case c) Consider the conditions w.r.t. a LG fault:

$I_{a1} = [E_{a1}/(Z_1 + Z_2+Z_0)]$

$I_f = 3\, I_{a1}$

$= 3\, E_{a1} / (Z_1 + Z_2+Z_0)$ or

$(Z_1 + Z_2+Z_0) = 3\, E_{a1} / I_f$

i.e., $3.18+ 2.936 + Z_0 = 3\,(11000/\sqrt{3})/ 2200$

Solving, we get, $Z_0 = 2.55$ ohms $= 1.054$ pu.

CHAPTER 5: POWER SYSTEM STABILITY

5.1 INTRODUCTION

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability, inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact*".

The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements.

A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

5.2 CLASSIFICATION OF POWER SYSTEM STABILITY

The high complexity of stability problems has led to a meaningful classification of the power system stability into various categories. The classification takes into account the main system variable in which instability can be observed, the size of the disturbance and the time span to be considered for assessing stability.

5.2.1 ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

Small single (or small disturbance) rotor angle stability: It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

Large-signal rotor angle stability or transient stability: This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance.

The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

5.2.2 VOLTAGE STABILITY

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance. It depends on the ability of the system to maintain equilibrium between load demand and load supply. Instability results in a progressive fall or rise of voltages of some buses, which could lead to loss of load in an area or tripping of transmission lines, leading to cascading outages. This may eventually lead to loss of synchronism of some generators.

The cause of voltage instability is usually the loads. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network. Voltage stability is also threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources. Voltage stability is categorized into the following sub-categories:

Small – disturbance voltage stability: It refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in load. This is primarily influenced by the load characteristics and the controls at a given point of time.

Large disturbance voltage stability: It refers to the systems ability to maintain steady voltages following large disturbances; It requires computation of the non-linear response of the power system to include interaction between various devices like motors, transformer tap changers and field current limiters. Short term voltage stability involves dynamics of fast acting load components and period of interest is in the order of several seconds. Long term voltage stability involves slower acting equipment like tap-changing transformers and generator current limiters. Instability is due to loss of long-term equilibrium.

5.2.3 FREQUENCY STABILITY

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe disturbance, causing considerable imbalance between generation and load. Instability occurs in the form of sustained frequency swings leading to tripping of generating units or loads. During frequency swings, devices such as under frequency load shedding, generator controls and protection equipment get activated in a few seconds. However, devices such as prime mover energy supply systems and load voltage regulators respond after a few minutes. Hence, frequency stability could be a short-term or a long-term phenomenon.

5.3 MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle θ_m is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \qquad (5.1)$$

The unit is radian. Angular velocity ω_m is defined as

$$\omega_m = \frac{d\theta_m}{dt} \qquad (5.2)$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \qquad (5.3)$$

The torque on a body due to a tangential force F at a distance *r* from axis of rotation is given by $T = r\,F$ (5.4)

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\,dF \qquad (5.5)$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\alpha$, where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\,dm = r\,\alpha\,dm \qquad (5.6)$$

The torque required for the particle is

$$dT = r\,dF = r^2\,\alpha\,dm \qquad (5.7)$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I\,\alpha \qquad (5.8)$$

Here, $I = \int r^2 dm$ (5.9)

It is called the moment of inertia of the body. The unit is Kg – m^2. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\,F$$

Now each force acts through a distance, $ds = r\,d\theta_m$ and the work done is $\sum F \, . \, ds$ i.e.,

$$dW = \sum F\, r\, d\theta_m = d\theta_m\, T$$

$$W = \int T\, d\theta_m \tag{5.10}$$

and

$$T = \frac{dW}{d\theta_m} \tag{5.11}$$

Thus the unit of torque may also be Joule per radian. The power is defined as rate of doing work. Using (5.11)

$$P = \frac{dW}{dt} = \frac{T\, d\theta_m}{dt} = T\,\omega_m \tag{5.12}$$

The angular momentum M is defined as

$$M = I\,\omega_m \tag{5.13}$$

And the kinetic energy is given by

$$KE = \frac{1}{2} I\,\omega_m^{\,2} = \frac{1}{2} M\,\omega_m \tag{5.14}$$

From (5.14) we can see that the unit of M has to be J-sec/rad.

5.4 SWING EQUATION:

The laws of rotation developed in section.3 are applicable to the synchronous machine. From(.5.8)

$$I\alpha = T$$

or

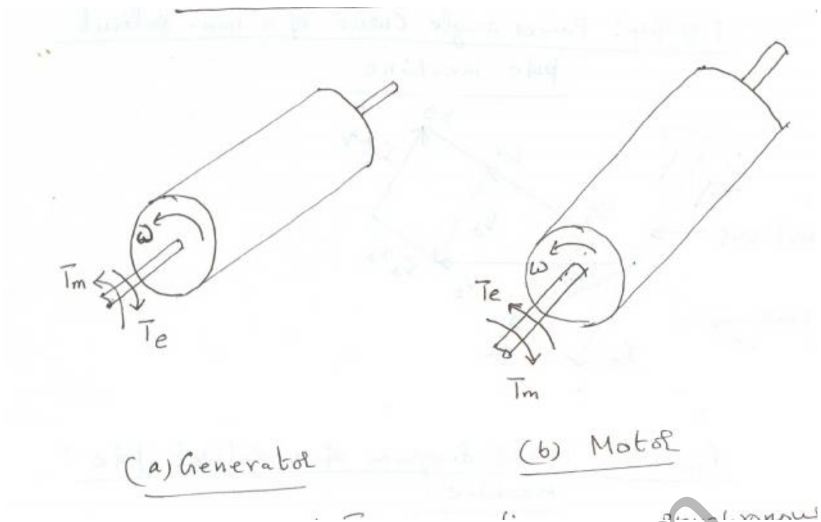
$$\frac{I\, d^2\theta_m}{dt^2} = T \tag{5.15}$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let T_m = shaft torque or mechanical torque corrected for rotational losses

T_e = Electromagnetic or electrical torque

For a generator T_m tends to accelerate the rotor in positive direction of rotation as shown in Fig 5.1. It also shows the corresponding torque for a motor with respect to the direction of rotation.



(a) Generator **(b) Motor**

Fig. 5.1 Torque acting on a synchronous machine

The accelerating torque for a generator is given by:

$$T_a = T_m \quad T_e \tag{5.16}$$

Under steady-state operation of the generator, T_m is equal to T_e and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, T_m is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (5.15) has to be solved to determine ¯$_m$ as a function of time. Since ¯$_m$ is measured with respect to a stationary reference axis on the stator, it is the measure of the absolute rotor angle and increases continuously with time even at constant synchronous speed. Since machine acceleration /deceleration is always measured relative to synchronous speed, the rotor angle is measured with respect to a synchronously rotating reference axis. Let

¢$_m$ = ¯$_m$ μ$_{sm}$ t (5.17)

where ω_{sm} is the synchronous speed in mechanical rad/s and θ_m is the angular displacement in mechanical radians. Taking the derivative of (5.17) we get

$$\frac{d\theta_m}{dt} = \frac{d\delta_m}{dt} \qquad \omega_{sm}$$

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{5.18}$$

Substituting (5.18) in (5.15) we get

$$I\,\frac{d^2\theta_m}{dt^2} = T_a = T_m \quad T_e \text{ N-m} \tag{5.19}$$

Multiplying by ω_m on both sides we get

$$\omega_m I\,\frac{d^2\theta_m}{dt^2} = \omega_m (T_m \quad T_e) \text{ N-m} \tag{5.20}$$

From (5.12) and (5.13), we can write

$$M\,\frac{d^2\theta_m}{dt^2} = P_m - P_a \qquad W \tag{5.21}$$

where M is the angular momentum, also called inertia constant, P_m is shaft power input less rotational losses, P_e is electrical power output corrected for losses and P_a is the acceleration power. M depends on the angular velocity ω_m, and hence is strictly not a constant, because ω_m deviates from the synchronous speed during and after a disturbance. However, under stable conditions ω_m does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\begin{array}{c}\text{stored kinetic energy in mega joules}\\ \text{at sychronous speed}\end{array}}{\text{Machine rating in MVA}}\ MJ/MVA \tag{5.22}$$

H falls within a narrow range and typical values are given in Table 5.1. If the rating of the machine is G MVA, from (5.22) the stored kinetic energy is GH Mega Joules. From (5.14)

$$GH = \frac{1}{2} M \omega_{sm} \quad \text{MJ} \tag{5.23}$$

or

$$M = \frac{2GH}{\omega_{sm}} \quad \text{MJ-s/mech rad} \tag{5.24}$$

The swing equation (5.21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{G} = \frac{P_{m} - P_e}{G} \tag{5.25}$$

In (5.25) δ_m is expressed in mechanical radians and ω_{sm} in mechanical radians per second (the subscript *m* indicates mechanical units). If δ and ω have consistent units (both are mechanical or electrical units) (5.25) can be written as

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.26}$$

Here ω_s is the synchronous speed in electrical rad/s ($\omega_s = \left(\frac{p}{2}\right)\omega_{sm}$) and P_a is acceleration power in per unit on same base as H. For a system with an electrical frequency f Hz, (5.26) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.27}$$

when δ is in electrical radians and

$$\frac{H}{180 f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.28}$$

when δ is in electrical degrees. Equations (5.27) and (5.28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = P_m - P_e \quad \text{pu} \tag{5.29}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{5.30}$$

In which ω, ω_s and δ are in electrical units. In deriving the swing equation, damping has been neglected.

Table 5.1 H constants of synchronous machines

Type of machine		H (MJ/MVA)
Turbine generator condensing	1800 rpm	9 – 6
	3600 rpm	7 – 4
Non condensing	3600 rpm	4 – 3
Water wheel generator		
	Slow speed < 200 rpm	2 – 3
	High speed > 200 rpm	2 – 4
Synchronous condenser	Large	1.25 } 25% less for hydrogen cooled
	Small	1.0 }
Synchronous motor with load varying from 1.0 to 5.0		2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let G_{mach} = Machine MVA rating (base)

G_{system} = System MVA base

In (5.25), H is computed on the machine rating $G = G_{mach}$

Multiplying (5.25) by $\dfrac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{μ_{sm}}\frac{d^2¢_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \tag{5.31}$$

$$\frac{2H_{system}}{μ_{sm}}\frac{d^2¢_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

where $H_{system} = H\dfrac{G_{mach}}{G_{system}}$ (5.32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant. The concept is best understood by considering a two machine system.

5.4.1 SWING EQUATION OF TWO COHERENT MACHINES

The swing equations for two machines on a common system base are:

$$\frac{2H_1}{\mu_s}\frac{d^2 \text{¢}_1}{dt^2} = P_{m1} - P_{e1} \text{ pu} \tag{5.33}$$

$$\frac{2H_2}{\mu_s}\frac{d^2 \text{¢}_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \tag{5.34}$$

Now $\text{¢}_1 = \text{¢}_2 = \text{¢}$ (since they swing together). Adding (5.33) and (5.34) we get

$$\frac{2H_{eq}}{\mu_s}\frac{d^2 \text{¢}}{dt^2} = P_m - P_e \text{ pu} \tag{5.35}$$

Where $H_{eq} = H_1 + H_2$

$P_m = P_{m1} + P_{m2}$

$P_e = P_{e1} + P_{e2}$

The relation (5.35) represents the dynamics of the single equivalent machine.

5.4.2 SWING EQUATION OF TWO NON – COHERENT MACHINES

For any two non – coherent machines also (5.33) and (5.34) are valid. Subtracting (5.34) from (33) we get

$$\frac{2}{\mu_s}\frac{d^2 \text{¢}_1}{dt^2} - \frac{2}{\mu_s}\frac{d^2 \text{¢}_2}{dt^2} = \frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} \tag{5.36}$$

Multiplying both sides by $\frac{H_1 H_2}{H_1 + H_2}$ we get

$$\frac{2}{\mu_s}\left(\frac{H_1 H_2}{H_1+H_2}\right)\frac{d^2\left(\text{¢}_1-\text{¢}_2\right)}{dt^2}=\frac{P_{m1}H_2-P_{m2}H_1}{H_1+H_2}-\frac{P_{e1}H_2-p_{e2}H_1}{H_1+H_2}$$

i.e
$$\frac{2}{\mu_s}H_{12}\frac{d^2\text{¢}_{12}}{dt^2}=P_{m12}-P_{e12} \qquad (5.37)$$

where $\text{¢}_{12}=\text{¢}_1-\text{¢}_2$, the relative angle of the two machines

$$H_{12}=\frac{H_1 H_2}{H_1+H_2}$$

$$P_{m12}=\frac{p_{m1}H_2-p_{m2}H_1}{H_1+H_2}$$

$$P_{e12}=\frac{p_{e1}H_2-p_{e2}H_1}{H_1+H_2}$$

From (5.37) it is obvious that the swing of a machine is associated with dynamics of other machines in the system. To be stable, the angular differences between all the machines must decrease after the disturbance. In many cases, when the system loses stability, the machines split into two coherent groups, swinging against each other. Each coherent group of machines can be replaced by a single equivalent machine and the relative swing of the two equivalent machines solved using an equation similar to (5.37), from which stability can be assessed.

The acceleration power is given by $P_a = P_m - P_e$. Hence, under the condition that P_m is a constant, an accelerating machine should have a power characteristic, which would increase P_e as δ increases.

This would reduce P_a and hence the acceleration and help in maintaining stability. If on the other hand, P_e decreases when δ increases, P_a would further increase which is detrimental to stability. Therefore, $\frac{\partial P}{\partial \text{¢}}$ must be positive for a stable system. Thus the power-angle relationship plays a crucial role in stability.

5.5 POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made

(i) Mechanical power input P_m is a constant during the period of interest, immediately after the disturbance

(ii) Rotor speed changes are insignificant.

(iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant.

As discussed in section 4, the power–angle relationship plays a vital role in the solution of the swing equation.

5.5.1 POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf. The phasor diagram of the voltages and currents at constant speed and excitation is shown in Fig. 5.2.

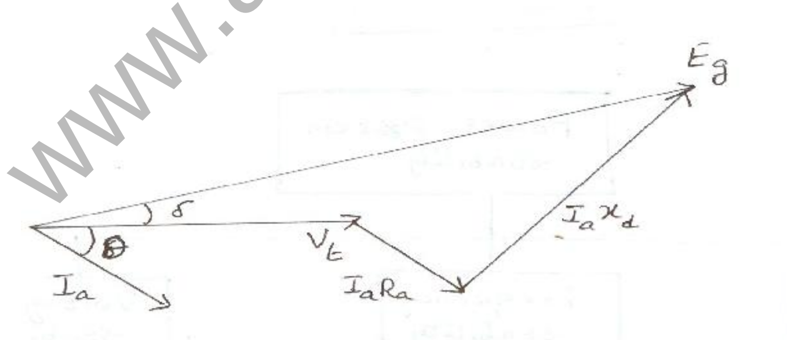


Fig 5.2 Phasor diagram of a non–salient pole synchronous generator

E_g = Generator internal emf.

V_t = Terminal voltage

θ = Power factor angle

I_a = Armature current

R_a = Armature resistance

x_d = Direct axis reactance

The power output of the generator is given by the real part of $E_g\, I_a^*$.

$$I_a = \frac{E_g \angle\delta - V_t \angle 0°}{R_a + jx_d} \tag{5.38}$$

Neglecting R_a, $$I_a = \frac{E_g \angle\delta - V_t \angle 0°}{j x_d}$$

$$P = \mathbf{R}\left\{ \left(E_g \angle\delta\right)\left(\frac{E_g \angle 90° - \delta}{x_d} - \frac{V_t \angle 90°}{x_d}\right)^* \right\}$$

$$= \frac{E_g^{\,2} \cos 90°}{x_d} - \frac{E_g V_t \cos\left(90° + \delta\right)}{x_d}$$

$$= \frac{E_g V_t \sin\delta}{x_d} \tag{5.39}$$

(Note- **R** stands for real part of). The graphical representation of (9.39) is called the power angle curve and it is as shown in Fig 5.3.

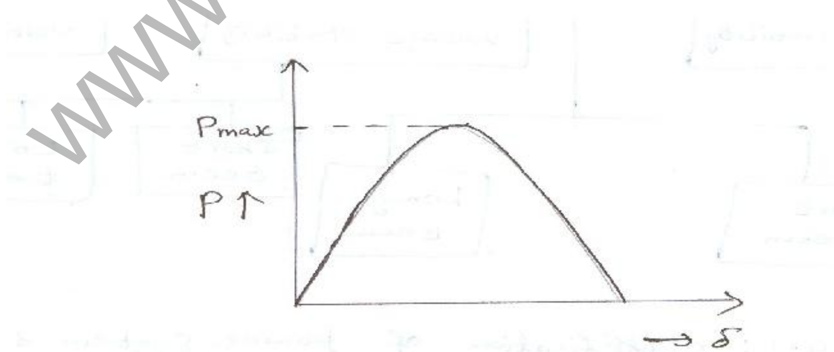


Fig 5.3 Power angle curve of a non – salient pole machine

The maximum power that can be transferred for a particular excitation is given by $\frac{E_g V_t}{x_d}$ at $\delta = 90^o$.

5.5.2 POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, I_d. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, I_q. The phasor diagram is shown in Fig 5, with same terminology as Fig 5.4 and R_a neglected.

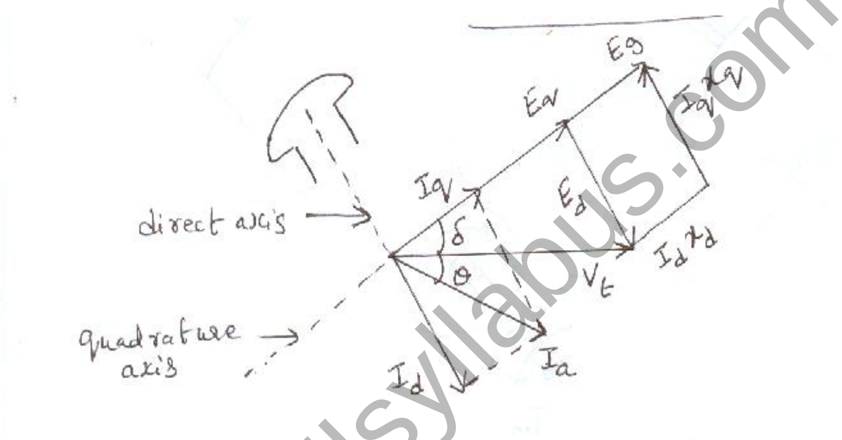


Fig 5.4 Phasor diagram of a salient pole machine

Power output $\quad P = V_t I_a \cos\theta$

$$= E_d I_d + E_q I_q \tag{5.40}$$

From Fig 5.4, $\quad E_d = V_t \sin\delta \;;\quad E_q = V_t \cos\delta$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \theta)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \theta) \tag{5.41}$$

Substituting (5.41) in (5.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^2 (x_d - x_q)\sin 2\delta}{2 x_d x_q} \tag{5.42}$$

the relation (5.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than 90^o.

5.6 POWER ANGLE RELATIONSHIP IN A SMIB SYSTEM:

Without loss of generality, many important conclusions on stability can be arrived at by considering the simple case of a Single Machine Infinite Bus (SMIB), where a generator supplies power to an infinite bus. The concept of an infinite bus arises from the fact that if we connect a generator to a much larger power system, it is reasonable to assume that the voltage and frequency of the larger system will not be affected by control of the generator parameters. Hence, the external system can be approximated by an infinite bus,

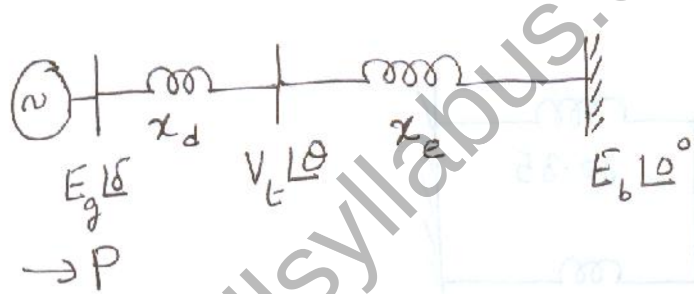


Fig. 5.5 SMIB System

In Fig. 5.5, the infinite bus voltage is taken as reference and δ is the angle between E_g and E_b. The generator is assumed to be connected to the infinite bus through a lossless line of reactance x_e. The power transferred (using classical model) is given by

$$P = \frac{E_g\, E_b}{x_d + x_e} \sin\delta \tag{5.43}$$

and using salient pole model,

$$P = \frac{E_g\, E_b}{x_d + x_e} \sin\delta + \frac{E_b^{\,2}\left(x_d - x_q\right)}{2\left(x_d + x_e\right)\left(x_q + x_e\right)} \sin 2\delta \tag{5.44}$$

An important measure of performance is the *steady state stability limit*, which is defined as *the maximum power that can be transmitted in steady state without loss of synchronism*, to the receiving end. If transient analysis is required, respective transient quantities namely E_g', x_d' and x_q' are used in (5.43) and (5.44) to calculate the power output.

5.7 TRANSIENT STABILITY

Transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network.

Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different (See example 9.8). Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely $P_S > 0$). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased.

Critical clearing time is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. One of the important methods for studying the transient stability of an SMIB system is the application of equal-area criterion.

5. 8 EQUAL- AREA CRITERION

Transient stability assessment of an SMIB system is possible without resorting to actual solution of the swing equation, by a method known as equal–area criterion. In a SMIB system, if the system is unstable after a fault is cleared, $\delta(t)$ increases indefinitely with time, till the machine loses synchronism. In contrast, in a stable system, $\delta(t)$ reaches a maximum and then starts reducing as shown in Fig.5.6.

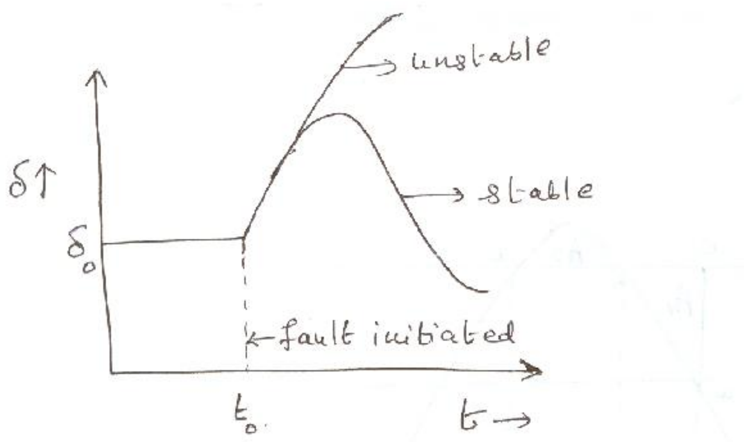


Fig.5.6 Swing Curve (δ V_S t) for stable and unstable system

Mathematically stated,

$$\frac{d\delta(t)}{dt} = 0$$

some time after the fault is cleared in a stable system and $\frac{d\delta}{dt} > 0$, for a long time after the fault is cleared in an unstable system.

Consider the swing equation (21)

$$M\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Multiplying both sides by $2\frac{d\delta}{dt}$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

This may be written as

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

Integrating both sides we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_o}^{\delta} P_a \, d\delta$$

$$\text{or } \frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_o}^{\delta} P_a \, d\delta} \tag{5.45}$$

For stability $\frac{d\delta}{dt} = 0$, some time after fault is cleared. This means

$$\int_{\delta_o}^{\delta} P_a \, d\delta = 0 \tag{5.46}$$

The integral gives the area under the $P_a - \delta$ curve. The condition for stability can be, thus stated as follows: A SMIB system is stable if the area under the $P_a - \delta$ curve, becomes zero at some value of δ. This means that the accelerating (positive) area under $P_a - \delta$ curve, must equal the decelerating (negative) area under $P_a - \delta$ curve. Application of equal area criterion for several disturbances is discussed next.

5.9 SUDDEN CHANGE IN MECHANICAL INPUT

Consider the SMIB system shown in Fig. 5.7.

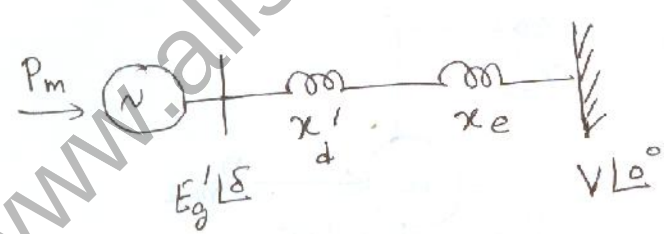


Fig.5.7 SMIB System

The electrical power transferred is given by

$$P_e = P_{\max} \sin \delta$$

$$P_{\max} = \frac{E_g' V}{x_d' + x_e}$$

Under steady state $P_m = P_e$. Let the machine be initially operating at a steady state angle δ_o, at synchronous speed ω_s, with a mechanical input P_{mo}, as shown in Fig.5.8 (point *a*).

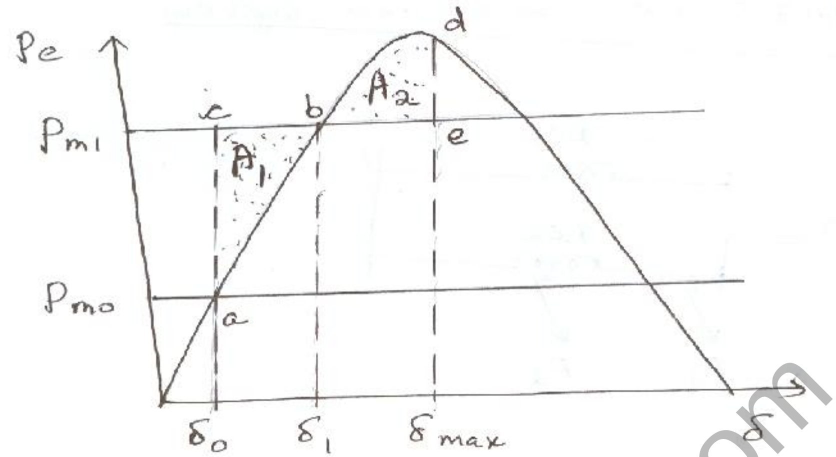


Fig.5.8 Equal area criterion–sudden change in mechanical input

If there is a sudden step increase in input power to P_{m1} the accelerating power is positive (since $P_{m1} > P_{mo}$) and power angle δ increases. With increase in δ, the electrical power P_e increases, the accelerating power decreases, till at $\delta = \delta_1$, the electrical power matches the new input P_{m1}. The area A_1, during acceleration is given by

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta$$

$$= P_{m1}(\delta_1 - \delta_0) - P_{max}(\cos\delta_0 - \cos\delta_1) \qquad (5.47)$$

At *b*, even though the accelerating power is zero, the rotor is running above synchronous speed. Hence, δ and P_e increase beyond *b*, wherein $P_e < P_{m1}$ and the rotor is subjected to deceleration. The rotor decelerates and speed starts dropping, till at point *d*, the machine reaches synchronous speed and $\delta = \delta_{max}$. The area A_2, during deceleration is given by

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m1})\, d\delta = P_{max}(\cos\delta_1 - \cos\delta_{max}) - P_{m1}(\delta_{max} - \delta_1) \qquad (5.48)$$

By equal area criterion $A_1 = A_2$. The rotor would then oscillate between δ_0 and δ_{max} at its natural frequency. However, damping forces will reduce subsequent swings and the machine finally settles down to the new steady state value δ_1 (at point *b*). Stability can be maintained only if area A_2 at least equal to A_1, can be located above P_{m1}. The limiting case is shown in Fig.5.9, where A_2 is just equal to A_1.

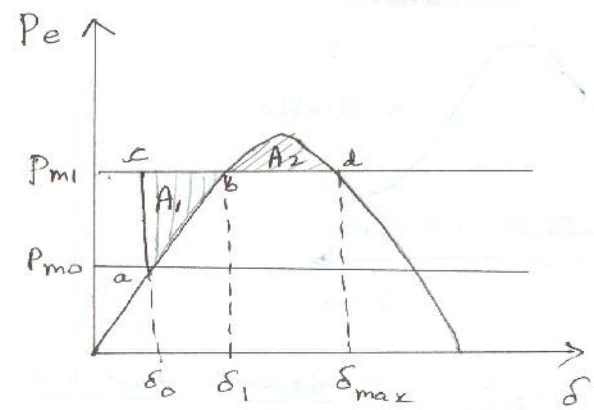


Fig.5.9 Maximum increase in mechanical power

Here δ_{max} is at the intersection of P_e and P_{m1}. If the machine does not reach synchronous speed at *d*, then beyond *d*, P_e decreases with increase in δ, causing δ to increase indefinitely. Applying equal area criterion to Fig.5.9 we get

$A_1 = A_2$.

From (5.47) and (5.48) we get

$$P_{m1}(\delta_{max} - \delta_0) = P_{max}(\cos\delta_0 - \cos\delta_{max})$$

Substituting $P_{m1} = P_{max}\sin\delta_{max}$, we get

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \qquad (5.49)$$

Equation (5.49) is a non-linear equation in δ_{max} and can be solved by trial and error or by using any numerical method for solution of non-linear algebraic equation (like Newton-Raphson, bisection etc). From solution of δ_{max}, P_{m1} can be calculated. $P_{m1} - P_{mo}$ will give the maximum possible increase in mechanical input before the machine looses stability.

5.10 NUMERICAL EXAMPLES

Example 1: A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e = 100 - 75 = 25$ MW

$$M = \frac{GH}{180 f} = \frac{1350}{180 \times 50} = 0.15 \ \text{MJ} - \text{s}/^{o}\text{e}$$

$$0.15 \frac{d^2 \text{¢}}{dt^2} = 25$$

$$\text{Acceleration} \ \sim \ = \frac{d^2 \text{¢}}{dt^2} = \frac{25}{0.15} = 166.6 \ ^{o}\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \ \ ^{o}\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \ rps/\text{s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note ^{o}e = electrical degree; ^{o}m = mechanical degree; P=number of poles.

(c) 10 cycles = $\frac{10}{50} = 0.2$ s

$$N_S = \text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2 = 1500 + 13.88 \times 0.2 = 1502.776$ rpm.

Example 2: Two 50 Hz generating units operate in parallel within the same plant, with the following ratings: Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA; Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA. Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}} \qquad = 4 \times \frac{500}{100} = 20 \ \text{MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}} \qquad = 5 \times \frac{1000}{100} = 50 \ \text{MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

Example 3: Obtain the power angle relationship and the generator internal emf for (i) classical model (ii) salient pole model with following data: $x_d = 1.0$ pu : $x_q = 0.6$ pu : $V_t = 1.0$ pu : $I_a = 1.0$ pu at upf

Solution:

(i) **Classical model**: The phasor diagram is shown in Fig P3.

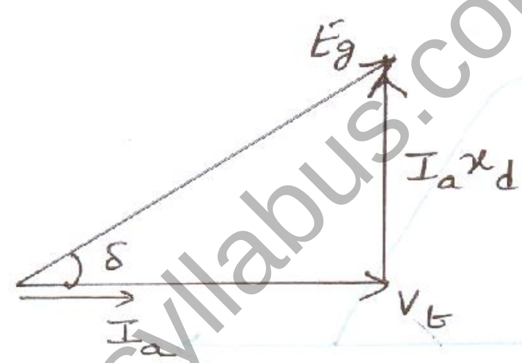


Fig.P3 Example 3, case(i)

$$\left|E_g\right| = \sqrt{V_t^2 + (I_a\, x_d)^2} = \sqrt{(1.0)^2 + (1.0 \times 1.0)^2} = 1.414$$

$$\delta = \tan^{-1}\frac{I_a\, x_d}{V_t} = \tan^{-1}\frac{1.0}{1.0} = 45°$$

$$\therefore\ E_g = 1.414\ \angle 45°.$$

If the excitation is held constant so that $\left|E_g\right|$ = 1.414, then power output

$$P = \frac{1.414 \times 1.0 \sin\delta}{1.0} = 1.414 \sin\delta$$

(ii) **Salient pole:** From Fig (5), we get using (41a) to (41d)

$$E_g = E_q + I_d\, x_d = V_t \cos\delta + I_d\, x_d$$

$$= V_t \cos\delta + I_a \sin\delta\, x_d$$

(* $\theta = 0^0$, since we are considering upf)

Substituting given values we get

$$E_g = \cos\delta + \sin\delta.$$

Again from Fig (9.5) we have

$$E_d = V_t \sin\delta = I_q x_q$$

$$\therefore\ V_t \sin\delta - I_q x_q = 0$$

$$V_t \sin\delta - I_a \cos\delta\, x_q = 0$$

Substituting the given values we get

$$0 = \sin\delta - 0.6\cos\delta$$

We thus have two simultaneous equations.

$$E_g = \cos\delta + \sin\delta$$

$$0 = \sin\delta - 0.6\cos\delta$$

Solving we get $\delta = 30.96^o$

$$E_g = 1.372 \text{ pu}$$

If the excitation is held constant, then from (42)

$$P = 1.372 \sin\delta + 0.333 \sin 2\delta$$

Example 4: Determine the steady state stability limit of the system shown in Fig 8, if V_t = 1.0 pu and the reactances are in pu.

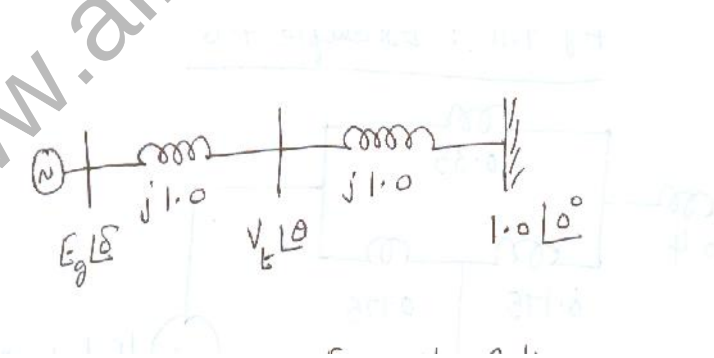


Fig. P4 Example 4

Solution:

$$\text{Current I} = \frac{V_t\angle\theta - 1.0\angle 0^\circ}{j1.0} = \frac{1.0\angle\theta - 1.0\angle 0^\circ}{j1.0}$$

$$E_g \angle\delta = V_t\angle\theta + j1.0(I)$$

$$= 1\angle^{-} + \frac{j1.0\left(1.0\angle^{-} - 1.0\angle 0^\circ\right)}{j1.0}$$

$$= \cos\theta + j\sin\theta + \cos\theta + j\sin\theta - 1.0$$

$$= 2\cos\theta - 1 + j\,2\sin\theta$$

When maximum power is transferred $\delta = 90^o$; which means real part of E = 0

$$\therefore\ 2\cos\theta - 1 = 0$$

$$\theta = \cos^{-1} 0.5 = 60^o$$

$$\left|E_g\right| = 2\times\sin 60^o = 1.732$$

$$E_g = 1.732\angle 90^\circ \text{ (for maximum power)}$$

$$\text{Steady state stability limit} = \frac{1.732\times 1.0}{1.0 + 1.0} = 0.866 \text{ pu}$$

Example 5: A 50 Hz synchronous generator having an internal voltage 1.2 pu, H = 5.2 MJ/MVA and a reactance of 0.4 pu is connected to an infinite bus through a double circuit line, each line of reactance 0.35 pu. The generator is delivering 0.8pu power and the infinite bus voltage is 1.0 pu. Determine: maximum power transfer, Steady state operating angle, and Natural frequency of oscillation if damping is neglected.

Solution: The one line diagram is shown in Fig P5.

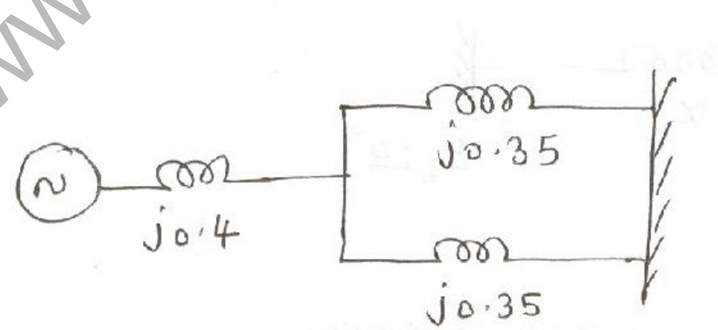


Fig . P5 Example 6

$$\text{(a) } X = 0.4 + \frac{0.35}{2} = 0.575 \text{ pu}$$

$$P_{max} = \frac{E_g\, E_b}{X} = \frac{1.2 \times 1.0}{0.575} = 2.087 \text{ pu}$$

(b) $P_e = P_{max\ sin}\, \delta_o$

$$\therefore \delta_o = \sin^{-1}\frac{P_e}{P_{max}} = \sin^{-1}\left(\frac{0.8}{2.087}\right) = 22.54°.$$

(c) $P_s = P_{max} \cos \delta_o = 2.087 \cos (22.54^o)$

$= 1.927$ MW (pu)/ elec rad.

$$M \text{ (pu)} = \frac{H}{\Pi f} = \frac{5.2}{\Pi \times 50} = 0.0331\ s^2 / elec\ \ rad$$

Without damping $s = \pm j\sqrt{\frac{P_S}{M}} = \pm j\sqrt{\frac{1.927}{0.0331}}$

$= \pm$ j 7.63 rad/sec = 1.21 Hz

Natural frequency of oscillation $\omega_n = 1.21$ Hz.

Example 6: In example .6, if the damping is 0.14 and there is a minor disturbance of $\Delta\delta = 0.15$ rad from the initial operating point, determine: (a) ω_n (b) ξ (c) ω_d (d) setting time and (e) expression for δ.

Solution:

(a) $\omega_n = \sqrt{\frac{P_S}{M}} = \sqrt{\frac{1.927}{0.0331}} = 7.63$ rad/sec = 1.21 Hz

(b) $\xi = \frac{D}{2}\sqrt{\frac{1}{M P_S}} = \frac{0.14}{2}\sqrt{\frac{1}{0.0331 \times 1.927}} = 0.277$

(c) $\omega_d = \omega_n\sqrt{1-\xi^2} = 7.63\sqrt{1-(0.277)^2} = 7.33$ rad/sec = 1.16 Hz

(d) Setting time $= 4\tau = 4\frac{1}{\xi\omega_n} = 4 \times \frac{1}{0.277 \times 7.63} = 1.892$ s

(e) $\Delta\delta_o = 0.15$ rad $= 8.59^o$

$\theta = \cos^{-1}\xi = \cos^{-1} 0.277 = 73.9^o$

$$\delta = \delta_o + \frac{\Delta\delta_o}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_d t + \theta)$$

$$= 22.54^o + \frac{8.59}{\sqrt{1-0.277^2}} e^{-0.277 \times 7.63t} \sin(7.33t + 73.9^o)$$

$= 22.54^o + 8.94\, e^{-2.11t} \sin (7.33t + 73.9^o)$

The variation of delta with respect to time is shown below. It can be observed that the angle reaches the steady state value of 22.54^o after the initial transient. It should be noted that the magnitudes of the swings decrease in a stable system with damping.

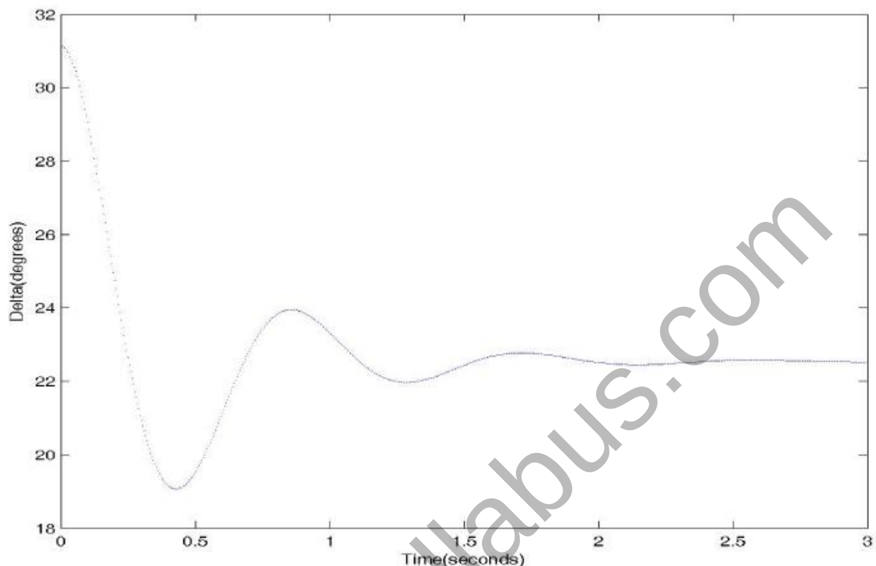


Fig.P6 Swing Curve for example 7

Example 7: In example 6, find the power angle relationship

(i) For the given network

(ii) If a short circuit occurs in the middle of a line

(iii) If fault is cleared by line outage

Assume the generator to be supplying 1.0 pu power initially.

Solution:

(i) From example 6, $P_{max} = 2.087$, $P_e = 2.087 \sin \delta$.

(ii) If a short circuit occurs in the middle of the line, the network equivalent can be draw as shown in Fig. 12a.

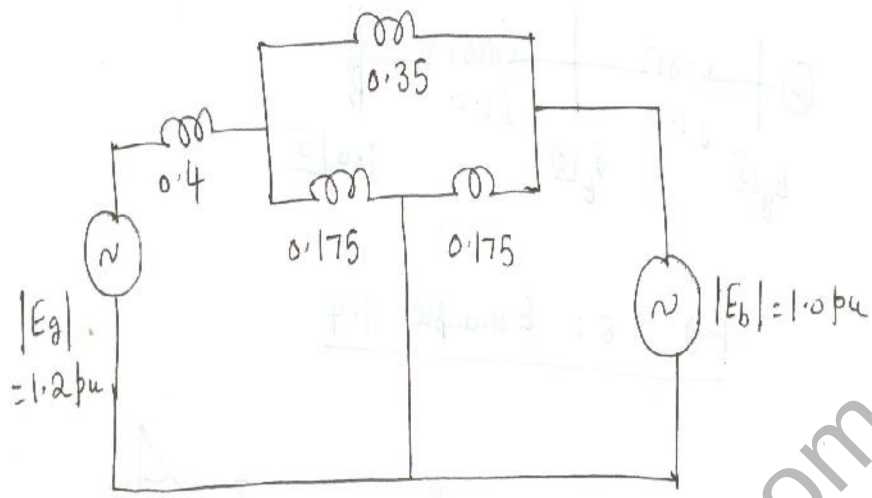
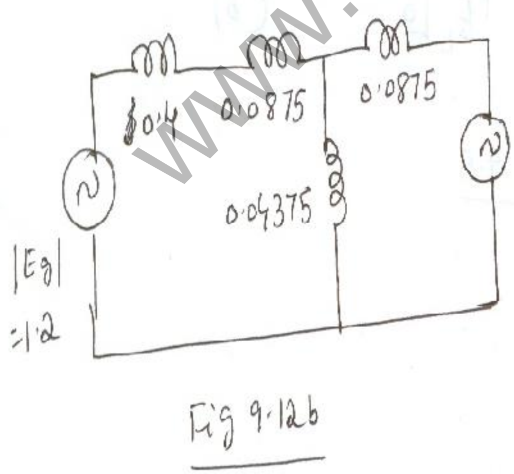


Fig.P7a   Short circuit in middle of line

The network is reduced by converting the delta to star and again the resulting star to delta as shown in Fig P7a, P7b and P7c.



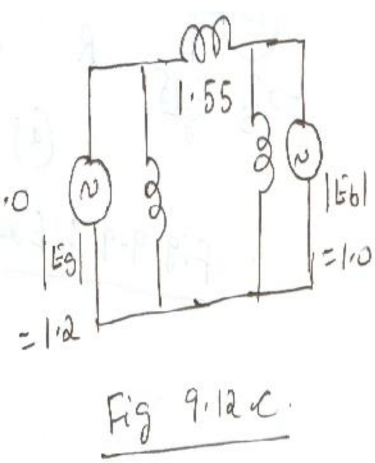


Fig.P7b **Fig.P7c**

The transfer reactance is 1.55 pu. Hence,

$$P_{max} = \frac{1.2 \times 1.0}{1.55} = 0.744 \quad ; \quad P_e = 0.744 \sin \delta$$

(iii) When there is a line outage

$$X = 0.4 + 0.35 = 0.75$$

$$P_{max} = \frac{1.2 \times 1.0}{0.75} = 1.6$$

$$P_e = 1.6 \sin \delta$$

Example 8: A generator supplies active power of 1.0 pu to an infinite bus, through a lossless line of reactance x_e = 0.6 pu. The reactance of the generator and the connecting transformer is 0.3 pu. The transient internal voltage of the generator is 1.12 pu and infinite bus voltage is 1.0 pu. Find the maximum increase in mechanical power that will not cause instability.

Solution:

$$P_{max} = \frac{1.12 \times 1.0}{0.9} = 1.244 \text{ pu}$$

$$P_{mo} = P_{eo} = 1.0 = P_{max} \sin \delta_o = 1.244 \sin \delta_o$$

$$\therefore \ \delta_o = \sin^{-1} \frac{1.0}{1.244} = 53.47^o = 0.933 \text{ rad.}$$

The above can be solved by N–R method since it is of the form f(δ_{max}) = K. Applying N–R method, at any iteration 'r', we get

$$\Delta \delta_{max}^{(r)} = \frac{K - f\left(\delta_{max}^{(r)}\right)}{\dfrac{df}{d\delta_{max}^{(r)}}}$$

$$\frac{df}{d\delta_{max}^{(r)}} = \left(\delta_{max}^{(r)} - \delta_o\right) \cos \delta_{max}^{(r)}$$

(This is the derivative evaluated at a value of $\delta = \delta_{max}^{(r)}$) $\delta_{max}^{(r+1)} = \delta_{max}^{(r)} + \Delta \delta_{max}^{(r)}$

Starting from an initial guess of δ_{max} between $\frac{\pi}{2}$ to π, the above equations are solved iteratively till $\Delta \delta_{max}^{(r)} \leq \in$. Here K = cos δ_o = 0.595. The computations are shown in table P8, starting from an initial guess $\delta_{max}^{(1)}$ = 1.745 rad.

Table P8

Interaction r	$¢_{\max}^{(r)}$	$\dfrac{df}{d¢_{\max}^{(r)}}$	$f\left(¢_{\max}^{(r)}\right)$	$\Delta ¢_{\max}^{(r)}$	$¢_{\max}^{(r+1)}$
1	1.745	– 0.1407	0.626	0.22	1.965
2	1.965	– 0.396	0.568	– 0.068	1.897
3	1.897	– 0.309	0.592	– 0.0097	1.887
4	1.887	– 0.2963	0.596	– 0.0033	1.883

Since $\Delta ¢_{\max}^{(r)}$ is sufficient by small, we can take

δ_{max} = 1.883 rad = 107.88°

δ_1 = 180 – $¢_{\max}$ = 72.1°

$P_{m1} = P_{max} \sin \delta_{max}$ = 1.183

Maximum step increase permissible is $P_{m1} - P_{mo}$, = 1.183 – 1.0 = 0.183 pu

Example 9: Transform a two machine system to an equivalent SMIB system and show how equal area criterion is applicable to it.

Solution: Consider the two machine system show in Fig.P9.

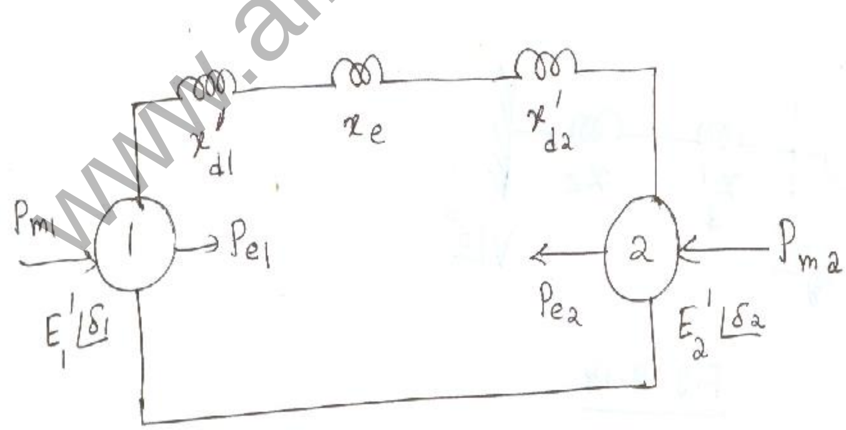


Fig.P9 Two machine system under steady state (neglecting losses)

$P_{m1} = -P_{m2} = P_m$; $P_{e1} = -P_{e2} = P_e$

The swing equations are

$$\frac{d^2\delta_1}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} = \frac{P_m - P_e}{M_1}$$

$$\frac{d^2\delta_2}{dt^2} = \frac{P_{m2} - P_{e2}}{M_2} = \frac{P_e - P_m}{M_2}$$

Simplifying, we get

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{M_1 + M_2}{M_1 M_2}(P_m - P_e)$$

or $$M_{eq}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where $M_{eq} = \dfrac{M_1 M_2}{M_1 + M_2}$

$\delta = \delta_1 - \delta_2$

$$P_e = \frac{E_1' E_2'}{x_{d1}' + x_e + x_{d2}'}\sin\delta$$

This relation is identical to that of an SMIB system in form and can be used to determine the relative swing ($\delta_1 - \delta_2$) between the two machines to assess the stability.
