



**FIREUP'S NUMBER SYSTEM**

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**FireUp's Free E-Book**

# NUMBER SYSTEM



## **FIREUP'S NUMBER SYSTEM**

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### **Introduction**

The study of number system is very important for every competitive examination including Common Admission Test (CAT). If we analyze last 10 years CAT pattern around 20 to 25 % of the question in Quantitative Ability are usually set on the properties of numbers.

FireUp's free e-book on number system starts from absolute basics and progresses to various advance concepts in Number Theory

This free e-book from [FireUp](http://fireup.co.in) is a gift to all serious CAT aspirants.

### **Definition of number system**

Number System is the combination of certain figures or symbols known as digits. We use common number system i.e. the decimal system. Only ten symbols (0-9) are used in this system. These symbols in a combined form gives a number known as numeral.

**They are divided mainly into two parts:**

1. Decimal Number System
2. Binary Number System (it includes 0&1 only).

There are other Number Systems too. They are discussed later.

### **Various important conceptual terms**

**Vulgar Fraction:** The fractions such as  $\frac{3}{11}$ ,  $\frac{5}{14}$ ,  $\frac{2}{10}$ ,  $\frac{67}{100}$  are called common or vulgar fractions.

**Imaginary Number:** Any number which is of the form  $\sqrt{-1} \times a$  is an imaginary number.



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**Real Number:** Any number which is not an imaginary number is a real number. Real Numbers may be divided into two categories.

**A: Rational Number:** A number which can be represented in the form of  $p/q$  where  $p$  and  $q$  are integers and  $q$  is not equal to 0.

**B: Irrational Number:** A number which cannot be expressed in the form of  $p/q$  is called an irrational number.

**Example:**  $\sqrt{5}$ ,  $\Pi$  (Exact value of  $\Pi$  is not  $22/7$ . It's just an approximated value)

**Even and Odd:** Any integer which is divisible by two is called an even number. Any integer which is not divisible by two is called an odd number.

**N.B.:** Zero has not been categorized in either of the two categories.

**Prime and Composite :** Any integer which is divisible by 1 and itself only is called a prime number. All integers which are not prime are called composite number as they are composed of two or more prime numbers.

**N.B.:** 1 is not a prime number.

### **Method to check if a number is prime number or not**

To determine if a number is prime or composite, follow these steps:

1. Find all factors of the number.
2. If the number has only two factors, 1 and itself, then it is prime.
3. If the number has more than two factors, then it is composite.

**For finding the factors you have to check the divisibility of each number smaller than the (square root of the given number + 1).** The divisibility rules are given later.



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**Positive and negative:** Any number which is greater than 0 is positive. Any number less than that is negative.

**Perfect numbers:** If the sum of the divisors of N, excluding N itself, is equal to N, then N is called a perfect number. e.g. 6, 28, 496, 8128 etc.

$$6 = 1 + 2 + 3;$$

$$28 = 1 + 2 + 4 + 7 + 14;$$

$$496 = 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248.$$

**N.B.:** The sum of the reciprocals of the divisors of a perfect number including that of its own is always =2.

E.g. For the perfect no. 28.

$$1/1+1/2+1/4+1/7+1/14+1/28=2$$

**Natural number:** Numbers used to count the objects is natural are called the natural numbers e.g. 1, 2, 3...

**Whole number:** All natural numbers and zero form the set of whole numbers. It is denoted by 'W'.

**Complex numbers:** Complex numbers can be represented as  $(a + I \times b)$  where a & b are real numbers and  $I = \sqrt{-1}$ , a is called the real part and b is called the imaginary part of the complex number. If a = 0, then the no. is purely imaginary. If b= 0 then it is purely real.

**Amicable Numbers:** Suppose, we start with any number and add its divisors to obtain a second no., then add the divisors of that second no. and continue the chain in the hope of eventually getting back to the original no. If the 1<sup>st</sup> step immediately restores the original no., then the chain has only one link and the no. is perfect. If the chain has two links, the two nos. are said to be amicable e.g. 220, 284.

220= 1,2,4,5,10,1,20,22,44,55,110 are the divisors of 220 the sum of the divisors = 284.

284 = 1, 2, 4, 71, 142, are the divisors. Sum of divisors = 220

**Rational Numbers:** Every integers and fractions are rational numbers .It can always be denoted as p/q, where p and q are integers and q is not equal to 0.

**Irrational Numbers:** Infinite non-recurring decimal number can be expressed as an



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irrational number.

**Cyclic Numbers:** It implies to integers of  $n$  digits which if multiplied by any number from 1 through  $n$  will give same  $n$  digits as the original number was, they will be in the same cyclic order in the product

**N.B.:-** 1) More than 1000 amicable pairs are known.

2) Every odd amicable pair so far discovered is a multiple of 3.



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### **Tests of divisibility**

**Divisibility by 2:** If its unit's digit is any of 0,2,4,6,8.

Ex: 100 is divisible by 2 while 101 is not.

**Divisibility by 3:** If the sum of its digits is divisible by 3.

Ex: 309 is divisible by 3, since sum of its digits =  $(3+0+9) = 12$ , which is divisible by 3.

**Divisibility by 4:** If the number formed by the last two digits is divisible by 4.

Ex: 2648 is divisible by 4, since the number formed by the last two digits is 48 which is divisible by 4.

**Divisibility by 5:** If its units digit is either 0 or 5.

Ex: 20825 and 50545 are divisible by 5.

**Divisibility by 6:** If it is divisible by both 2 & 3.

Ex: 53256 is divisible by 6 because it is divisible by 2 as well as 3.

**Divisibility by 7:** If after subtraction of a number consisting of the last three digits from a number consisting of the rest of its digits the result is a number that can be divided by 7 evenly

Ex.: 414141 is divisible 7 as  $414-141= 273$  is divisible by 7

Many different ways to test divisibility by seven have been devised. Some are long and complex, a few involve rewriting the digits, and one even consists of a grid-like box. We have chosen one of the more simplistic versions even though in almost every case it is quicker to merely perform long division.

**Divisibility by 8:** If the last three digits of the number are divisible by 8.

Ex: 3652736 is divisible by 8 because last three digits (736) is divisible by 8.

**Divisibility by 9:** If the sum of its digit is divisible by 9.

Ex: 672381 is divisible by 9, since sum of digits =  $(6+7+2+3+8+1) = 27$  is divisible by 9.

**Divisibility by 10:** If the digit at unit's place is 0 it is divisible by 10.

Ex: 69410, 10840 is divisible by 10.





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**Divisibility by 11:** If the difference of the sum of its digits at odd places and sum of its digits at even places, is either 0 or a number divisible by 11.

Ex: 4832718 is divisible by 11, since:  
(Sum of digits at odd places) – (sum of digits at even places)  
=  $(8+7+3+4)-(1+2+8) = 11$

**Divisibility by 12:** A number is divisible by 12 if it is divisible by both 4 and 3.

Ex: 34632

(i) The number formed by last two digits is 32, which is divisible by 4

(ii) Sum of digits =  $(3+4+6+2) = 18$ , which is divisible by 3.

**Divisibility by 13:** Divide and check. There are methods but they are time consuming and hence not feasible.

**Divisibility by 14:** If a number is divisible by 2 as well as 7.

**Divisibility by 15:** If a number is divisible by both 3 & 5.

**Divisibility by 16:** If the number formed by the last 4 digits is divisible by 16.

Ex: 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

**Divisibility by 24:** If a number is divisible by both 3 & 8.

**Divisibility by 40:** If it is divisible by both 5 & 8.

**Divisibility by 80:** If a number is divisible by both 5 & 16.

Before getting into the problem solving mode you should know these:

You should memorize these tables. They will come very handy and useful.



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### **Reference Tables**

#### *Table of Squares*

<b>Number</b>	<b>Square</b>	<b>Number</b>	<b>Square</b>	<b>Number</b>	<b>Square</b>
1	1	16	256	31	961
2	4	17	289	32	1024
3	9	18	324	33	1089
4	16	19	361	34	1156
5	25	20	400	35	1225
6	36	21	441	36	1296
7	49	22	484	37	1369
8	64	23	529	38	1444
9	81	24	576	39	1521
10	100	25	625	40	1600
11	121	26	676		
12	144	27	729		
13	169	28	784		
14	196	29	841		
15	225	30	900		

#### *Cube of 1 to 10*

<b>Number</b>	<b>Cube</b>
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000



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### **Fractions**

#### ***Decimal Fractions***

fractions, in which denominators are powers of 10, are known as decimal fractions. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

#### ***Addition & subtraction of Decimal fractions***

**Rule:** The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in a usual way.

#### ***Multiplication of a decimal Fraction by a Power of 10***

**Rule:** Shift the decimal point to the right by as many places of decimal as is the power of 10.

#### ***Multiplication of Decimal fractions***

**Rule:** Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal in the given numbers.

#### ***Dividing a Decimal fraction By a Counting Number***

**Rule:** Divide the given number without considering the decimal point by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as are there in the dividend.

#### ***Dividing a Decimal fraction By a Decimal Fraction***

**Rule:** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.



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### ***H. C. F. & L. C. M. of Decimal fractions***

**Rule:** In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers as without decimal point, find H. C. F. or, L.C.M., as the case may be. Now, in the result, make off as many decimal places as are there in each of the given numbers.

### ***Comparison of fractions***

Suppose, some fractions are to be arranged in ascending or descending order of magnitude.

**Rule:** Convert each one of the given fractions in the decimal form. Now, arrange them in ascending order as per requirements.

### ***Recurring Decimal***

If, in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a recurring decimal. In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set.

### ***Pure Recurring Decimal***

A decimal fraction, in which all the figures after the decimal point are repeated, is called a pure recurring decimal e.g.  $\frac{2}{3} = 0.666\dots = 0.6$ .

### ***Converting a Pure Recurring Decimal into Vulgar Fraction***

**Rule:** Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

### ***Decimal***

A decimal fraction in which some figures do not repeat and some of them are repeated is called a mixed recurring decimal, e.g.  $0.173333\dots = 0.173$ .



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### ***Converting a Mixed recurring Decimal into Vulgar Fraction***

**Rule:** In the numerator, take the difference between the number formed by all the digits after decimal point ( taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

### ***Equivalent Fractions***

When two fractions give out the same ratio or number it is said to be equivalent, e.g.  $2/7 = 6/21$

## **BASE SYSTEM**

### ***Understanding different type of Number Systems***

In our decimal number system, the rightmost position represents the "ones" column, the next position represents the "tens" column, the next position represents "hundreds", etc. Therefore, the number 123 represents 1 hundred and 2 tens and 3 ones, whereas the number 321 represents 3 hundreds and 2 tens and 1 one.

The values of each position correspond to powers of the base of the number system. So for our decimal number system, which uses base 10, the place values correspond to powers of 10:

... 1000 100 10 1  
...  $10^3$   $10^2$   $10^1$   $10^0$

### ***Converting from other number bases to decimal***

Other number systems use different bases. The binary number system uses base 2, so the place values of the digits of a binary number correspond to powers of 2. For example, the value of the binary number 10011 is determined by computing the place value of each of



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the digits of the number:

1 0 0 1 1 the binary number  $2^4$   $2^3$   $2^2$   $2^1$   $2^0$  place values.

So the binary number 10011 represents the value

$$(1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ = 16 + 0 + 0 + 2 + 1 = 19$$

The same principle applies to any number base. For example, the number 2132 base 5 corresponds to

2 1 3 2 number in base 5

$5^3$   $5^2$   $5^1$   $5^0$  place values

So the value of the number is

$$(2 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0) \\ = (2 \times 125) + (1 \times 25) + (3 \times 5) + (2 \times 1) \\ = 250 + 25 + 15 + 2 \\ = 292.$$

### ***Converting from decimal to other number bases***

In order to convert a decimal number into its representation in a different number base, we have to be able to express the number in terms of powers of the other base. For example, if we wish to convert the decimal number 100 to base 4, we must figure out how to express 100 as the sum of powers of 4.

$$100 = (1 \times 64) + (2 \times 16) + (1 \times 4) + (0 \times 1) \\ = (1 \times 4^3) + (2 \times 4^2) + (1 \times 4^1) + (0 \times 4^0)$$

Then we use the coefficients of the powers of 4 to form the number as represented in base 4:

$$100 = 1\ 2\ 1\ 0 \text{ base } 4$$

One way to do this is to repeatedly divide the decimal number by the base in which it is to be converted, until the quotient becomes zero. As the number is divided, the remainders - in reverse order - form the digits of the number in the other base.

**Example:** Convert the decimal number 82 to base 6:

**Solution:**



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$$82/6 = 13 \text{ remainder } 4$$

$$13/6 = 2 \text{ remainder } 1$$

$$2/6 = 0 \text{ remainder } 2$$

The answer is formed by taking the remainders in reverse order: 2 1 4 base 6

## **Theorems**

### ***Rule of Simplification***

The rule is known as the rule of 'VBODMAS' where V, B, O, D, M, A and S stand for Vinculum, Bracket, Of, Division, Multiplication, Addition, and Subtraction respectively.

### ***Ascending or Descending Orders in Rational Numbers***

**Thumb Rule:** When the numerator and the denominator of the fractions increase by a constant value, the last fraction is the biggest.

**Example:** Which one of the following fractions is the greatest?

- a.  $1/8$
- b.  $4/9$
- c.  $7/10$

**Solution:** From above, Answer =  $7/10$  as the numerator increases by 3 (a constant a value) and the denominator also increases by a constant value (1), so the last fraction i.e.  $7/10$  is the greatest.

### ***Some Rules on Counting Numbers***

**I.** Sum of all the first n natural numbers =  $n(n+1)/2$ .

**Example:**  $1+2+3 \dots\dots\dots+ 105 = 105(105+1)/2 = 5565$

**II.** Sum of first n odd numbers =  $n^2$



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**Example:**  $1+3+5+7 = 4^2 = 16$  (as there are four odd numbers).

For Example:  $1+3+5+\dots+20^{\text{th}}$  odd number (i.e.  $20 \times 2-1$ ) = 39

**III.** Sum of first n even numbers =  $n(n+1)$

**Example:**  $2+4+6+8+\dots+100$  (or  $50^{\text{th}}$  even number)

$$= 50 \times (50 + 1) = 2550$$

**IV.** Sum of squares of first n natural numbers =  $\{n(n+1)(2n+1)\}/6$

Example:  $1^2+ 2^2+ 3^2+\dots+10^2 = 10(10+1)(2 \times 10+1)$

$$= 10 \times 11 \times 21 / 6 = 385$$

**V.** Sum of cubes of first n natural numbers =  $[n \times (n+1) / 2]^2$

**Example:**  $1^3+ 2^3+ \dots+ 6^3 = [6 \times (6+1) / 2]^2 = 21^2 = 441$

**Note:** In the first n counting numbers, there are  $n/2$  odd and  $n/2$  even numbers provided n, the number of numbers, is even. If n, the number of numbers, is odd, there are  $\frac{1}{2}(n+1)$  odd numbers and  $\frac{1}{2}(n-1)$  even numbers.

**Note:** In the first n counting numbers, there is  $n/2$  odd and  $n/2$  even numbers provided n, the number of numbers, is even. If n, the number of numbers, is odd then there are  $\frac{1}{2}(n+1)$  odd numbers and  $\frac{1}{2}(n-1)$  even numbers.

**Example:** from 1 to 400 there are  $400/2 = 200$  even numbers or from 1 to 50, there are  $50/2 = 25$  odd numbers.

## **Power and Index**

If a number 'p' is multiplied by itself n times, the product is called nth power of 'p' and is written as  $P^n$ . In  $P^n$ , p is called the base and n is called the index of the power.

*It is discussed under a separate chapter – INDICES*





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### Cyclicity

To understand cyclicity let us take a simple example.

Take any two numbers say 43 & 97.

If they are multiplied, the answer is 4171. The last digit of the product is same as the last digit of  $3 \times 7$ .

Hence, it is 1.

This concept could be extended to a host of situations. An interesting pattern emerges when we look at the exponents of the numbers. We would find conclusions as given below.

The last digits of the exponents of all numbers have cyclicity i.e. every Nth power of the base shall have the same last digit, if N is the cyclicity of the number. All numbers ending with 2, 3, 7, 8 have a cyclicity of 4.

For instance,

$2^1$  ends with 2

$2^2$  ends with 4

$2^3$  ends with 8

$2^4$  ends with 6

$2^5$  end with 2 again.

The same set of the last digits shall be repeated for the subsequent powers. So, if we want to find the last digit of (say)  $2^{45}$ , divide 45 by 4.

The remainder is 1

So the last digit would be the same as last digit of  $2^1$ , which is 2

Working out similarly for all other digits we get

CYCLICITY TABLE

1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1



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Find out the last digit of

1)  $3^{57}$

2)  $7^{23} \times 8^{13}$

3)  $235^{1000}$



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### SOLVED EXAMPLES

(1) What is the number in the unit place in  $(729)^{59}$ ?

**Solution:** When 729 is multiplied twice, the number in the unit place is 1. Thus the number in the unit place in  $(729)^{58}$  is 1.

$\therefore (729)^{59} = (729)^{58} \times (729) = (\dots 1) \times (729) = 9$  in the unit place.

**Note:** When you are solve this type of questions (for odd numbers) try to get the last Digit 1 , as has been done in the above two examples.

(2) Find the number in the unit place in  $(98)^{40}$ ,  $(98)^{42}$ ,  $(98)^{43}$

Solution:  $(98)^4 = (\dots 6)$

$$\therefore (98)^{4n} = (\dots 6)$$

Thus ,  $(98)^{40} = (\dots 6) = 6$  in the unit place

$$(98)^{42} = (98)^{4 \times 10} \times (98)^2 = (\dots 6) \times (\dots 4) = 4 \text{ in the unit place}$$

$$(98)^{43} = (98)^{4 \times 10} \times (98)^3 = (\dots 6) \times (\dots 4) = 2 \text{ in the unit place}$$

**Note:** When there is an even number in the unit place of base, try to get 6 in the place.

#### Rule I : For Odd numbers

When there is an odd digit in the unit place (except 5), multiply the number by itself until you get 1 in the unit place.

$$(\dots 1)^n = (\dots 1)$$

$$(\dots 3)^{4n} = (\dots 1)$$

$$(\dots 7)^{4n} = (\dots 1)$$

where  $n = 1, 2, 3, \dots$

#### Rule 2: For even numbers

When there is an even digit in the unit place, multiply the number by itself until you get 6 in the unit place.

$$(\dots 2)^n = (\dots 6)$$

$$(\dots 4)^{4n} = (\dots 6)$$

$$(\dots 6)^{4n} = (\dots 6)$$

$$(\dots 8)^{4n} = (\dots 6)$$

where  $n = 1, 2, 3, \dots$



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**Note:** If there is 1,5 or 6 in the unit place of the given number, then after any times of its multiplication, it will have the same digit in the unit place, i.e. ,

$$(\dots 1)^n = (\dots 1)$$

$$(\dots 5)^n = (\dots 5)$$

$$(\dots 6)^n = (\dots 6)$$

**(3) The digit in the unit place of the number represented by  $(7^{95} - 3^{58})$  is**

A. 7

B. 0

C. 6

D. 4

**Answer:** D (4)

Cycle of 7 is

$$7^1 = 7$$

$$7^2 = 49$$

$$7^3 = 343$$

$$7^4 = 2401$$

If we divide 95 by 4, the remainder will be 3.

So the last digit of  $(7)^{95}$  is equals to the last digit of  $(7)^3$  i.e. 3.

Cycle of 3 is

$$3^1 = 3$$

$$3^2 = 9$$

$$3^3 = 27$$

$$3^4 = 81$$

$$3^5 = 243$$

If we divide 58 by 4, the remainder will be 2.



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So the last digit of  $(7)^{95}$  is equals to the last digit of  $(3)^2$  i.e. 9.

So the unit digit of  $(795 - 3^{58})$  is

$$\dots 3 - \dots 9 = \dots 4$$

So the unit digit is 4.

**(4) A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.**

- A. 1                      B. 4                      C. 3                      D. 5

**Answer:** D (5)

899 are completely divisible by 29. 63 engender a remainder of 5 when divided by 29 .So the number leaves a remainder 5 when divided by 29.

**(5) If  $n^2 = 123456787654321$ , what is n?**

- A. 12344321                      B. 1235789  
C. 1111111                      D. 1111112

**Answer:**

$$11^2 = 121$$

$$111^2 = 12321$$

$$1111^2 = 1234321$$

.

.

So, we can conclude  $11111111^2 = 123456787654321$



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**(6)  $(UV)^2 = ILU$ . If, the letters I, L, U and V stand for distinct digits, then I equal:**

- A. 1                      B. 6                      C. 3                      D. 9

**Answer: C (3)**

$$19^2 = 361$$

Only satisfies the given condition.

So I = 3.

**(7) Let a, b, c be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb' both defined under the usual decimal number system. If  $(ab)^2 = ccb$  and  $ccb > 300$  then the value of b is**

- A. 1                      B. 0                      C. 5                      D. 6

**Answer. A (1)**

$$ccb > 300 .$$

Therefore, C can be any of 3 , 4 ,.....,8,9 . And ccb must be a perfect square.

$$18^2 = 324$$

$$19^2 = 381$$

$$20^2 = 400$$

$$21^2 = 441$$

.

.

$$31^2 = 961$$



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You would see 441 is the only perfect square whose decimal and hundredths digits are the same.

Therefore,  $c^2 b = 441$  and  $b = 1$ .

**(8) If  $|r - 6| = 11$  and  $|2q - 12| = 8$ , what is the minimum possible value of  $q/r$ ?**

- A.  $-2/5$                       B.  $2/17$                       C.  $10/17$                       D. None of these

**Answer.** D (None of these)

$$q = 10, 2.$$

$$r = 17, -5.$$

So minimum value would be  $10 / -5 = -2$ . Which is not in the option.

**(9) The unit's digit in the product (274 x 318 x 577 x 313) is**

- (1) 2                      (2) 3                      (3) 4                      (4) 5

Start multiplying only the unit's digit of the first two numbers of the above sum. i.e.  $4*8=32$ .

Multiply unit's digit of next number with the unit's digit of the result i.e. 7 (of 577) with 2 (of 32) =  $7*2=14$ . Proceeding in the same manner 3 (of 313) with 4 (of 14) =  $3*4=12$ . So the unit's digit will be the unit's digit of the result. i.e. 2.

*Shortcut:* unit's place of product of unit's digit of all the number.  $4*8*7*3 = -2$

Answer: Option (1)

**(10) The unit's digit in the product  $(3127)^{173}$  is**

- (1) 1                      (2) 3                      (3) 7                      (4) 9



## **FIREUP'S NUMBER SYSTEM**

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Consider only the digit at unit's place. Check its cyclicity again.

Cyclicity explained: The power raised to which the unit's place gets repeated is called the cyclicity for that number.

Cycle for 1 is 1

Cycle for 2 is 4

*Explained*

$$2^1=2$$

$$2^2=4$$

$$2^3=8$$

$$2^4=16$$

$$2^8=51$$





## FIREUP'S NUMBER SYSTEM

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### **BASIC LEVEL EXERCISE**

Given below are few simple questions which we have categorized as basic level exercise. Before progressing to advance level exercise with difficult questions, answers the questions below and evaluate yourself with the answer key at the end of exercise

1. How many times does the digit 6 appear when you count down from 11 to 100?
  - A. 9
  - B. 10
  - C. 19
  - D. 20
  
2. The third number from the beginning, out of X consecutive integral where N is the largest one, would be
  - A.  $N - X + 1$
  - B.  $N - X - 1$
  - C.  $N - X + 3$
  - D.  $N - X - 3$
  
3.  $(n^2 - 81n + 199)$  is a prime for n is equal to
  - A. 78
  - B. 79
  - C. 80
  - D. 81



### **FIREUP'S NUMBER SYSTEM**

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4. The unit's place of  $2^{44} \times 3^{53} \times 6^{11} \times 7^{23}$ :
- A. 1  
B. 6  
C. 8  
D. 4
5. The average of 5 consecutive numbers a, b, c, d and e is 15. What will be value of e?
- A. 11  
B. 19  
C. 17  
D. Data inadequate
6. The population of a settlement trebled in two years. What is the percent increase of the population?
- A. 50%  
B.  $66 \frac{2}{3}\%$   
C. 73%  
D. 75%
7. Which of the following is the highest fraction?
- A.  $\frac{7}{15}$



## **FIREUP'S NUMBER SYSTEM**

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B.  $11/16$

C.  $7/9$

D.  $2/13$

**8.** If  $x$  is an odd integer all of the following are odd except

A.  $x - 2$

B.  $6x + x$

C.  $x^2 + 2x$

D.  $x^2 + x$

**9.** The number nearest to 7030 that is exactly divisible by 53 is:

A. 6996

B. 7026

C. 7044

D. 7049

**10.** Find the greatest number that will divide 578, 503, and 528 leaving the same remainder in each case.

A. 5

B. 15

C. 25



**FIREUP'S NUMBER SYSTEM**

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D. 50

**11.** The total number of prime factors of the product  $(8)^{20} (15)^{24} (17)^{15}$  will be

A. 59

B. 118

C. 121

D. 123

**12.** A heap of stones can be made up exactly into groups of 25. But when made into groups of 18, 27 and 32, there are always 11 left. Find the least number of stones that may be contained in such a heap.

A. 535

B. 825

C. 875

D. 1035

**13.** A number  $1568a35b$  is divisible by 88. Find a and b.

A. 3, 2

B. 4, 6

C. 6, 2

D. 2, 8



**FIREUP'S NUMBER SYSTEM**

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**14.** A gardener wants to plant 17956 trees and arranges them in such a way that there are as many rows as there are trees in a row. Find the number of trees in each row.

- A. 117
- B. 131
- C. 134
- D. 137

**15.** If  $x$  is a positive integer such that  $2x + 12$  is perfectly divisible by  $x$ , then the number of possible values of  $x$  is

- A. 2
- B. 5
- C. 6
- D. 12

**16.** What is the greatest power of 5 which can divide  $80!$  Exactly  $[x! = x(x-1)(x-2) \dots (2)(1)]$

- A. 16
- B. 20
- C. 19
- D. None of these



### **FIREUP'S NUMBER SYSTEM**

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**17.** When  $P^2 + 17$  are divided by 12 ( $P$  is a prime number greater than 3) the remainder is

- A. 6
- B. 1
- C. 0
- D. 8

**18.** It is desired to extract the maximum power of 3 from  $24!$  Where  $n! = n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1$ . What will be the exponent of 3?

- A. 8
- B. 9
- C. 11
- D. 10

**19.** Find the least number which when divided by 6, 15, 17 leaves a remainder 1 but when divided by 7 leaves no remainder?

- A. 211
- B. 511
- C. 1022
- D. 86

**20.** What is the smallest number which when increased by 5 is completely divisible by 8, 11 and 24?



**FIREUP'S NUMBER SYSTEM**

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- A. 264
- B. 259
- C. 269
- D. None of these

***ANSWERS KEY: BASIC LEVEL EXERCISE***

1	C	2	C	3	D	4	D	5	C	6	C	7	C	8	D	9	D	10	C
11	D	12	C	13	C	14	C	15	C	16	C	17	A	18	D	19	B	20	B



## FIREUP'S NUMBER SYSTEM

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### CAT LEVEL EXERCISE

1. If  $x$  and  $y$  are unequal positive integers and  $\sqrt{xy} > 5$ , which of the following cannot be the value of  $(x + y)$  ?

- (a) 16
- (b) 10
- (c) 20
- (d) 25

2. If  $a = bc$  and  $(a + b)^2 = c^2$  where  $a, b, c$  are all positive, which of the following is true?

- A.  $b > 1$
- B.  $b = 1$
- C.  $b < 1$
- D. can't determine

3.  $2|x| + 4y = 8$ ;  $6|y| - 2x = 18$ ; which of the following values of  $(x, y)$  satisfy both these equations?

- A.  $(1, -6)$
- B.  $(-6, -1)$
- C.  $(-2, 1)$
- D.  $(-2, -1)$





### **FIREUP'S NUMBER SYSTEM**

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4. If  $|x - 3| > 1$ , then the range of  $x$  is:

- A.  $3 < X < \infty$
- B.  $4 < X < \infty$
- C.  $-\infty < X < 2$
- D. Both (b) & (c)

5. If  $|3x + 5| < 9$ , then

- A.  $-14/3 < x < 4/3$
- B.  $-4/3 < x < 14/3$
- C.  $x < 14/3$
- D.  $x > -4/3$

6. The number of positive integer valued pairs  $(x, y)$ , satisfying  $4x - 17y = 1$  and  $x < 1000$  is;

- A. 59
- B. 57
- C. 55
- D. 58

7. Let  $a, b, c$  be distinct digits. Consider a two digit number 'ab' and a three digit number 'ccb' both defined under the usual decimal number system. If  $(ab)^2 = ccb$  and  $ccb > 300$  then the value of  $b$  is



### **FIREUP'S NUMBER SYSTEM**

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A. 1

B. 0

C. 5

D. 6

**8.** If a number 774958A96B is to be divisible by 8 and 9, the value of A and B, respectively, will be:

A. 7, 8

B. 8, 0

C. 5, 8

D. None of these

**9.** A is the set of positive integers such that when divided by 2, 3, 4, 5, 6 leaves the remainder 1, 2, 3, 4, 5 respectively. How many numbers between 0 and 200 belongs to set A?

A. 0

B. 1

C. 2

D. 3

**10.** If x and y are unequal positive integers and  $\sqrt{xy} > 5$ , which of the following cannot be the value of (x + y)?

A. 16

B. 10



## FIREUP'S NUMBER SYSTEM

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C. 20

D. 25

**11.** If  $a = bc$  and  $(a + b)^2 = c^2$  where  $a, b, c$  are all positive, which of the following is true?

A.  $b > 1$

B.  $b = 1$

C.  $b < 1$

D. can't determine

**12.**  $2 |x| + 4y = 8$ ;  $6 |y| - 2x = 18$ ; which of the following values of  $(x, y)$  satisfy both these equations?

A.  $(1, -6)$

B.  $(-6, -1)$

C.  $(-2, 1)$

D.  $(-2, -1)$

**13.** Which is the least number that must be subtracted from 1856, so that remainder when divided by 7, 12 and 16 will leave the same remainder 4?

A. 137

B. 1361

C. 140

D. 172



### **FIREUP'S NUMBER SYSTEM**

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**14.** A ninety-nine digit number is formed by writing first 54 natural number, in front of each as (123456789101112.....5354). Find the remainder when the number is divided by 8.

- A. 4
- B. 7
- C. 2
- D. 0

**15.**  $n^3$  is odd, which of the following statement (s) is (are) true

(A)  $n$  is odd            (B)  $n^2$  is odd            (C)  $n^2$  is even

- A. A only
- B. B only
- C. A & B only
- D. A & C only

**16.** A certain number when divided by 899 leaves the remainder 63. Find the remainder when the same number is divided by 29.

- A. 5
- B. 4
- C. 1
- D. Can't be determined



## FIREUP'S NUMBER SYSTEM

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**17.** Five digit numbers are formed using only 0, 1, 2, 3 and 4 exactly once. What is the difference between the maximum and minimum number that can be formed?

- A. 19800
- B. 41976
- C. 32976
- D. None of these

**18.** How many numbers can be formed from 1, 2, 3, 4, 5 (without repetition) when the digit at the units place must be greater than that in the tenth places?

- A. 54
- B. 60
- C.  $5! / 3$
- D.  $2 \times 4!$

**19.**  $(BE)^2 = MPB$  where B, E, M & P are distinct integers, then  $M = ?$

- A. 2
- B. 3
- C. 9
- D. None of these

**20.** Find the product  $n(n + 1)(2n + 1)$ ;  $n \in \mathbb{N}$ , which one is necessarily false?



**FIREUP'S NUMBER SYSTEM**

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- A. Always even
- B. Divisible by 3
- C. Always divisible by sum of squares of first natural numbers
- D. Never divisible by 237

21.  $n^2 - 81n + 199$  is a prime for  $n$  is equal to

- A. 78
- B. 79
- C. 80
- D. 81

22. If  $a = bc$  and  $(a + b)^2 = c^2$  where  $a, b, c$  are all positive, which of the following is true?

- A.  $b > 1$
- B.  $b = 1$
- C.  $b < 1$
- D. can't determine

**ANSWER KEY : CAT LEVEL EXERCISE**

1	B	2	C	3	B	4	B	5	A	6	A	7	A	8	B	9	D	10	B
11	C	12	B	13	D	14	C	15	C	16	A	17	C	18	B	19	B	20	D
21	D	22	C																



## **FIREUP'S NUMBER SYSTEM**

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### **What's Next?**

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