KISHORE VAIGYANIK PROTSAHAN YOJANA - 2013

Date : 27-10-2013 | **Duration : 3 Hours** | **Max. Marks : 160**

STREAM - SB/SX

GENERAL INSTRUCTIONS

- The Test Booklet consists of **120** questions.
- There are Two parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

MARKING SCHEME :

PART-I :

MATHEMATICS

Question No. **1 to 20** consist of **ONE (1)** mark for each correct response.

PHYSICS

Question No. **21 to 40** consist of **ONE (1)** mark for each correct response.

CHEMISTRY

Question No. **41 to 60** consist of **ONE (1)** mark for each correct response.

BIOLOGY

Question No. **61 to 80** consist of **ONE (1)** mark for each correct response.

PART-II :

MATHEMATICS

Question No. **81 to 90** consist of **TWO (2)** marks for each correct response.

PHYSICS

Question No. **91 to 100** consist of **TWO (2)** marks for each correct response.

CHEMISTRY

Question No. **101 to 110** consist of **TWO (2)** marks for each correct response.

BIOLOGY

Question No. **111 to 120** consist of **TWO (2)** marks for each correct response.

PART-I
One Mark Questions

MATHEMATICS

1. The sum of non-real roots of the polynomial equation $x^3 + 3x^2 + 3x + 3 = 0$.
(A) equals 0
(B) lies between 0 and 1
(C) lies between –1 and 0
(D) has absolute value bigger than 1

Sol. $f(x) = x^3 + 3x^2 + 3x + 3 = 0$

$f'(x) = 3x^2 + 6x + 3 = 3(x + 1)^2 \geq 0$

f(x) is Increasing function

Now $f(-3) = -6 < 0$

$f(-2) = 1 > 0$

$\therefore$ real root γ lies between – 3 and – 2

$\alpha + \beta + \gamma = -3; \quad -3 < \gamma < -2$

$\alpha + \beta - 3 < \alpha + \beta + \gamma < \alpha + \beta - 2$

$\alpha + \beta - 3 < -3 < \alpha + \beta - 2$

$-1 < \alpha + \beta < 0$

Ans.(C)

2. Let n be a positive integer such that

$\log_2\log_2\log_2\log_2\log_2(n) < 0 < \log_2\log_2\log_2\log_2(n)$.

Let ℓ be the number of digits in the binary expansion of n. Then the minimum and the maximum possible values of ℓ are
(A) 5 and 16 (B) 5 and 17 (C) 4 and 16 (D) 4 and 17

Sol. $\log_2\log_2\log_2\log_2\log_2 n < 0 < \log_2\log_2\log_2\log_2 n$

$16 < n < 2^{16}$

no. of digits in the binary expansion of 16 is 5

no. of digits in the binary expansion of 2^{16} is 17

so no. of digits in the binary expansion of n lies in 5 to 16

Ans. (A)

3. Let ω be a cube root of unity not equal to 1. Then the maximum possible value of $| a + bw + cw^2 |$ where a, b, c $\in \{+1, -1\}$ is
(A) 0 (B) 2 (C) $\sqrt{3}$ (D) $1+\sqrt{3}$

Sol. $|a + bw + cw^2|$

$|a - c + (b - c)w|$, for maximum value taking a = 1 , c = –1, b = 1

$|a + bw + cw^2| = |2 + 2w| = 2|w^2| = 2$

Ans. (B)

4. If a, b are positive real numbers such that the lines ax + 9y = 5 and 4x + by = 3 are parallel, then the least possible value of a + b is

(A) 13 (B) 12 (C) 8 (D) 6

Sol. $\frac{a}{4} = \frac{9}{b} \Rightarrow ab = 36$

using AM $\geq$ G.M

$\frac{a+b}{2} \geq \sqrt{ab} \Rightarrow a + b \geq 12$

Ans. (B)

5. Two line segments AB and CD are constrained to move along the x and y axes, respectively, in such a way that the points A, B, C, D are concyclic. If AB = a and CD = b, then the locus of the centre of the circle passing through A, B, C, D in polar coordinates is

(A) $r^2 = \frac{a^2+b^2}{4}$ (B) $r^2\cos 2\theta = \frac{a^2-b^2}{4}$

(C) $r^2 = 4(a^2 + b^2)$ (D) $r^2 \cos 2\theta = 4(a^2 - b^2)$

Sol. $2\sqrt{g^2 - c} = a$

$2\sqrt{f^2 - c} = b$

Polar coordinates of centre of circle be $(r\cos\theta, r\sin\theta)$

$g = -r\cos\theta$ and $g^2 - f^2 = \frac{a^2-b^2}{4}$

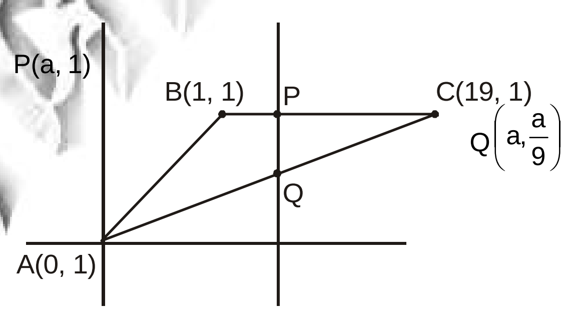
$f = -r\sin\theta$ $\therefore$ $r^2\cos 2\theta = \frac{a^2-b^2}{4}$

Ans. (B)

6. Consider a triangle ABC in the xy -plane with vertices A = (0,0), B = (1,1) and C = (9, 1). If the line x = a divides the triangle into two parts of equal area, then a equals

(A) 3 (B) 3.5 (C) 4 (D) 4.5

Sol.



area of PQC = $\frac{1}{2}$ area of ABC

$\frac{1}{2}(9-a)\left(1-\frac{a}{9}\right) = \frac{1}{2}\times\frac{1}{2}(8\times 1)$

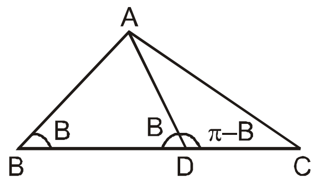
$(9 - a)^2 = 36 \Rightarrow a = 3$

Ans. (A)

7. Let ABC be an acute-angled triangle and let D be the midpoint of BC. If AB = AD, then tan(B)/tan(C) equals

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 3

Sol.



1 : 1

Using M - N Rule

(1 + 1) cot (π - B) = 1.cot B - cot C

3cotB = cot C

$\frac{\tan B}{\tan C} = 3$

Ans. (D)

8. The angles α, β, γ of a triangle satisfy the equations 2sinα + 3cosβ = $3\sqrt{2}$ and 3sinβ + 2cosα = 1. Then angle γ equals

(A) 150° (B) 120° (C) 60° (D) 30°

Sol. $2\sin\alpha + 3\cos\beta = 3\sqrt{2}$ ....(1)

$3\sin\beta + 2\cos\alpha = 1$ ....(2)

sum of squares of equation (1) and (2)

$4 + 9 + 12 \sin(\alpha + \beta) = 19$

$\sin(\alpha + \beta) = \frac{1}{2} \Rightarrow \alpha + \beta = 150°$ or $30°$

If $\alpha + \beta = 30° \Rightarrow \beta = 30 - \alpha$

put in equation (1) and (2)

we get $7 \sin\alpha + 3\sqrt{3}\cos\alpha = 6\sqrt{2}$

$7\cos\alpha - 3\sqrt{3}\sin\alpha = 2$

$\cos\alpha = \frac{7+9\sqrt{6}}{37} = .8 < \frac{\sqrt{3}}{2}$

$\cos\alpha < \cos 30°$ $\because \alpha > 30°$ $\therefore \alpha + \beta \neq 30°$

Ans. (D)

9. Let f : R → R be a function such that $\lim_{x\to\infty} f(x) = M > 0$. Then which of the following is **false?**

(A) $\lim_{x\to\infty} x\sin(1/x)\, f(x) = M$ (B) $\lim_{x\to\infty} \sin(f(x)) = \sin M$

(C) $\lim_{x\to\infty} x\sin(e^{-x})\, f(x) = M$ (D) $\lim_{x\to\infty} \frac{\sin x}{x}.f(x) = 0$

Sol. $\lim_{x\to\infty} x\sin(e^{-x})\, f(x) = \lim_{x\to\infty} \frac{\sin(e^{-x})}{e^{-x}} \frac{x}{e^{x}} f(x)$

$= 1 \times (0) \times M = 0$

Ans. (C)

10. For x, t $\in$ R let

$$p_t(x) = (\sin t)x^2 - (2\cos t)x + \sin t$$

be a family of quadratic polynomials in x with variable coefficients. Let $A(t) = \int_0^1 p_t(x)dx$. Which of the following statements are true?

(I) A(t) < 0 for all t.
(II) A(t) has infinitely many critical points.
(III) A(t) = 0 for infinitely many t.
(IV) A'(t) < 0 for all t.

(A) (I) and (II) only (B) (II) and (III) only (C) (III) and (IV) only (D) (IV) and (I) only

Sol. $A(t) = \int_0^1 \{(\sin t)x^2 - (2\cos t)x + \sin t\}\, dx$

$A(t) = \frac{\sin t}{3} - \cos t + \sin t = \frac{4}{3}\sin t - \cos t$

$A'(t) = \frac{4\cos t}{3} + \sin t$

St. I and IV are False

Ans. (B)

11. Let $f(x) = \sqrt{2 - x - x^2}$ and g(x) = cos x. Which of the following statements are true?

(I) Domain of $f((g(x))^2)$ = Domain of f(g(x))
(II) Domain of f(g(x)) + g(f(x)) = Domain of g(f(x))
(III) Domain of f(g(x)) = Domain of f(g(x))
(IV) Domain of $g((f(x))^3)$ = Domain of f(g(x))

(A) Only (I) (B) Only (I) and (II) (C) Only (III) and (IV) (D) Only (I) and (IV)

Sol. Domain of f(g(x)) is R

$\because$ $2 - \cos x - \cos^2 x \geq 0$

$(\cos x + 2)(\cos x - 1) \leq 0 \Rightarrow -2 \leq \cos x \leq 1$

$x \in R$

Domain of g(f(x)) is [-2, 1]

$\because \cos\left(\sqrt{2 - x - x^2}\right)$

$2 - x - x^2 \geq 0$

Domain of $f(g(x)^2)$ is R

$\because$ $2 - \cos^2 x - \cos^4 x \geq 0$

$(\cos^2 x + 2)(\cos^2 x - 1) \leq 0$

$-1 \leq \cos x \leq 1$

$x \in R$

Domain of $g(f^3(x)$ is Domain of g(f(x)

i.e., [-2, 1]

Ans. (B)

12. For real x with – 10 ≤ x ≤ 10 define f(x) = $\int_{-10}^{x} 2^{[t]}dt$, where for a real number r we denote by [r] the largest integer less than or equal to r. The number of points of discontinutiy of f in the interval (–10,10) is

(A) 0 (B) 10 (C) 18 (D) 19

Sol. Let r be an integer in (– 10, 10)

Now, LHL = $\lim_{x\to r^-}\int_{-10}^{x} 2^{[t]}dt$

$= \lim_{h\to 0^+}\left[\int_{-10}^{-9} 2^{[t]}dt + \int_{-9}^{-8} 2^{[t]}dt + ... + \int_{r-1}^{r-h} 2^{[t]}dt\right]$

$= \lim_{h\to 0}\left[2^{-10} + 2^{-9} + ... + 2^{r-1}(1-h)\right]$

$= 2^{-10} + 2^{-9} + ... + 2^{r-1}$ ... (1)

$\lim_{x\to r^+}\int_{-10}^{x} 2^{[t]}dt$

$= \lim_{h\to 0^+}\left[\int_{-10^2}^{-9} 2^{[t]}dt + \int_{-9}^{-8} 2^{[t]}dt + ... + \int_{r}^{r+h} 2^{[t]}dt\right]$

$= 2^{-10} + 2^{-9} + ... + 2^{r-1}$ ... (2)

$f(r) = \int_{-10}^{r} 2^{[t]}dt$

$= 2^{-10} + 2^{-9} + ... + 2^{r-1}$ ... (3)

From (1), (2) & (3)

f(x) is continous at all integers.

Ans. (A)

13. For a real number x let [x] denote the largest integer less than or equal to x and {x} = x – [x]. The smallest possible integer value of n for which $\int_{1}^{n}[x]\{x\}dx$ exceeds 2013 is

(A) 63 (B) 64 (C) 90 (D) 91

Sol. $\int_{1}^{n}[r]\{r\}\,dx = \sum_{r=1}^{n-1}\int_{r}^{r+1} r(x-r)\,dx$

$\sum_{r=1}^{n-1} r\left[\frac{x^2}{2} - rx\right]_r^{r+1}$

$\sum_{r=1}^{n-1} r\left[\frac{(r+1)^2 - r^2}{2} - r.1\right]$

$\sum_{r=1}^{n-1} r\left[\frac{1}{2}\right] = \frac{1}{2}\frac{n(n-1)}{2}$

$\frac{n(n-1)}{4} \geq 2013$

$n(n - 1) \geq 4 \times 2013$

$\left(n - \frac{1}{2}\right)^2 \geq \frac{2013\times 16 + 1}{4}$

$n \geq \frac{\sqrt{32209}}{2} + \frac{1}{2}$

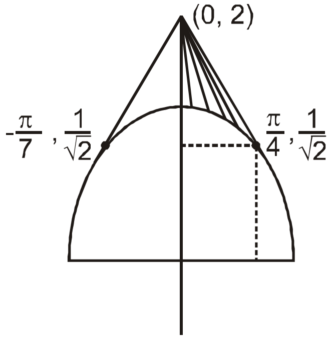
least n = 91

14. The area bounded by the curve y = cos x, the line joining (–π/4, cos(–π/4)) and (0, 2) and the line joining (π/4, cos(π/4)) and (0, 2) is

(A) $\left(\frac{4+\sqrt{2}}{8}\right)\pi-\sqrt{2}$ (B) $\left(\frac{4+\sqrt{2}}{8}\right)\pi+\sqrt{2}$

(C) $\left(\frac{4+\sqrt{2}}{4}\right)\pi-\sqrt{2}$ (D) $\left(\frac{4+\sqrt{2}}{4}\right)\pi+\sqrt{2}$

Sol.



required area = $2\left[\frac{1}{2}\left(\frac{1}{\sqrt{2}}+2\right).\frac{\pi}{4}-\int_0^{\pi/4}\cos x\,dx\right]$

$= \left(\frac{4+\sqrt{2}}{8}\right)\pi \; -\sqrt{2}$

Ans (A)

15. A box contains coupons labeled 1, 2, 3....n. A coupon is picked at random and the number x is noted. The coupon is put back into the box and a new coupon is picked at random. The new number is y. Then the probability that one of the numbers x, y divides the other is (in the options below [r] denotes the largest integer less than or equal to r)

(A) $\frac{1}{2}$ (B) $\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

(C) $-\frac{1}{n}+\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$ (D) $-\frac{1}{n}+\frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

Sol. Let x = 1

favourable out comes (1, 1), (1, 2) ....(1, n) no. of favourable out comes when x = 1

$= \left[\frac{n}{1}\right]$

∴ no. of favourable out comes when x = 1 or y = 1

$= 2\left[\frac{n}{1}\right]-1$

no. of favourable out comes when x = 2 or y = 2 but x ≠ 1, y ≠ 1

$2\left[\frac{n}{2}\right]-1$

Similarly

no. of favourable out comes when x = k or y = k but x, y ∉ {1, 2, ... k– 1}

$2\left[\frac{n}{k}\right]-1$

So probability = $\dfrac{\sum_{k=1}^{n}\left[\frac{n}{k}\right]-(1+1.......n \text{ times})}{n^2}$

$= \frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]-\frac{1}{n}$

16. Let $n \geq 3$. A list of numbers $0 < x_1 < x_2 < ... < x_n$ has mean μ and standard deviatiion σ. A new list of numbers is made as follows : $y_1 = 0$, $y_2 = x_2$, ...,$y_{n-1} = x_{n-1}$, $y_n = x_1 + x_n$. The mean and the standard deviation of the new list are $\hat{\mu}$ and $\hat{\sigma}$. Which of the following is necessarily true?

(A) $\mu = \hat{\mu}$, $\sigma \leq \hat{\sigma}$ (B) $\mu = \hat{\mu}$, $\sigma \geq \hat{\sigma}$ (C) $\sigma = \hat{\sigma}$ (D) μ may or may not be equal to $\hat{\mu}$

Sol. $\hat{\mu} = \dfrac{0 + x_2 + x_3 ....x_{n-1} + (x_1 + x_n)}{n} = \mu$

$$\hat{\sigma}^2 = \frac{1}{n}\Sigma y_i^2 - \hat{\mu}^2$$

$$= \frac{1}{n}\left[0^2 + x_2^2 + x_3^2 ....(x_{n-1})^2 + (x_1 + x_n)^2\right] - \mu^2$$

$$\hat{\sigma}^2 - \sigma^2 = \frac{2x_1x_n}{n} > 0$$

so $\hat{\sigma} > \sigma$

Ans. (A)

17. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ be unit vectors in the xy -plane, one each in the interior of the four quadrants. Which of the following statements is necessarily true?

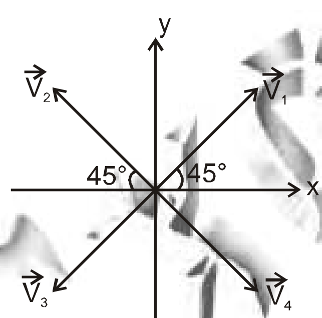
(A) $\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \vec{v}_4 = 0$

(B) There exist i, j with $1 \leq i < j \leq 4$ such $\vec{v}_i + \vec{v}_j$ is in the first quadrant

(C) There exist i, j with $1 \leq i < j \leq 4$ such that $\vec{v}_i . \vec{v}_j < 0$

(D) There exist i, j with $1 \leq i < j \leq 4$ such that $\vec{v}_i . \vec{v}_j > 0$

Sol.



In this case B, C, D are not possible.

Ans. (A)

18. The number of integers n with $100 \leq n \leq 999$ and containing at most two distinct digits is

(A) 252 (B) 280 (C) 324 (D) 360

Sol. Total Integers = 999 - 99 = 900

Total Integers in which all distinct digits

| 9 | 9 | 8 | = 648

so 900 - 648 = 252

Ans. (A)

19. For an integer n let $S_n = \{n + 1, n + 2, ..., n + 18\}$. Which of the following is true for all $n \geq 10$?

(A) S_n has a multiple of 19 (B) S_n has a prime

(C) S_n has at least four multiples of 5 (D) S_n has at most six primes

Sol. n + 1, n + 2, ... n + 18

(A) False, if n = 19

(C) False if n = 15

16 to 33

20, 25, 30 $\rightarrow$ only three multiples of 5

(D) no. of odd integers in $S_n = 9$

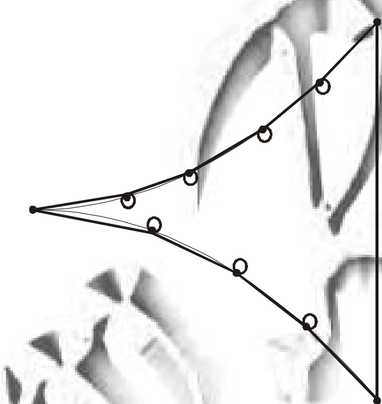
every third odd integer is multiple of 3

so maximum prime no. = 6.

Ans. (D)

20. Let P be a closed polygon with 10 sides and 10 vertices (assume that the sides do not intersect except at the vertices). Let k be the number of interior angles of P that are greater than 180°. The maximum possible value of k is

(A) 3 (B) 5 (C) 7 (D) 9

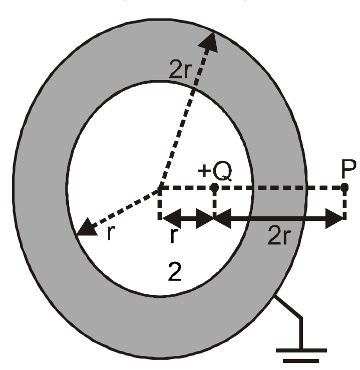


Sum of angles of closed polygon with 10 sides is 8π. So maximum number of possible obtuse angle is 7.

Ans. (C)

PHYSICS

21. Consider an initially neutral hollow conducting spherical shell with inner radius r and outer radius 2r. A point charge +Q is now placed inside the shell at a distance r/2 from the centre. The shell is then grounded by connecting the outer surface to the earth. P is an external point at a distance 2r from the point charge +Q on the line passing through the centre and the point charge +Q as shown in the figure.



The magnitude of the force on a test charge +q placed at P will be

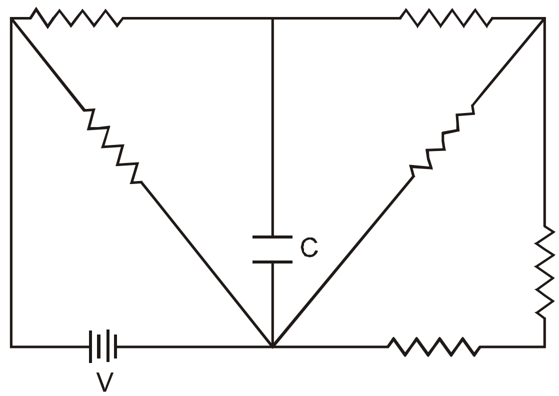
(A) $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{4r^2}$ (B) $\frac{1}{4\pi\varepsilon_0}\frac{9qQ}{100r^2}$ (C) $\frac{1}{4\pi\varepsilon_0}\frac{4qQ}{25r^2}$ (D) 0

Sol. Charge on outer most surface is zero

Hence force on q is also '0'

Ans. (D)

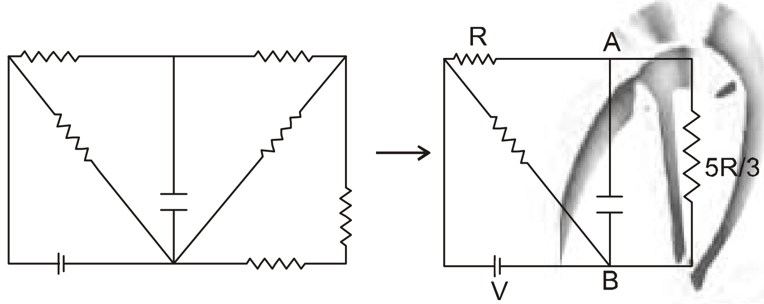
22. Consider the circuit shown in the figure below :



All the resistors are identical. The charge stored in the capacitor, once it is fully charged, is

(A) 0 (B) $\frac{5}{13}CV$ (C) $\frac{2}{3}CV$ (D) $\frac{5}{8}CV$

Sol.



$$V_{AB} = \frac{5\frac{R}{3}}{\frac{5R}{3}+R} \times V = \frac{5}{8}V$$

$$Q = \frac{5}{8}CV$$

Ans. (D)

23. A nuclear decay is possible if the mass of the parent nucleus exceeds the total mass of the decay particles. If M(A, Z) denotes the mass of a single neutral atom of an element with mass number A and atomic number Z, then the minimal condition that the β decay

$$X_Z^A \to Y_{Z+1}^A + \beta^- + \bar{v}_e$$

will occur is (m_e denotes the mass of the β particle and the neutrino mass m_v can be neglected) :

(A) $M(A, Z) > M(A, Z+1) + m_e$ (B) $M(A, Z) > M(A, Z+1)$

(C) $M(A, Z) > M(A, Z+1) + Zm_e$ (D) $M(A,Z) > M(A, Z+1) - m_e$

Ans. (A)

24. The equation of state of n moles of a non-ideal gas can be approximated by the equation

$$\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT$$

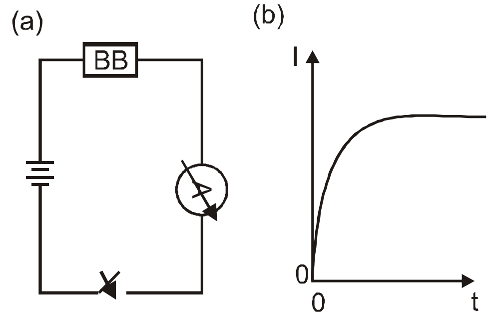
where a and b are constants characteristic of the gas. Which of the following can represent the equation of a quasistatic adiabat for this gas (Assume that C_V, the molar heat capacity at constant volume, is independent of temperature)?

(A) $T(V-nb)^{R/C_V}$ = constant (B) $T(V-nb)^{C_V/R}$ = constant

(C) $\left(T + \frac{ab}{V^2R}\right)(V-nb)^{R/C_V}$ = constant (D) $\left(T + \frac{n^2ab}{V^2R}\right)(V-nb)^{C_V/R}$ = constant

Ans. (A)

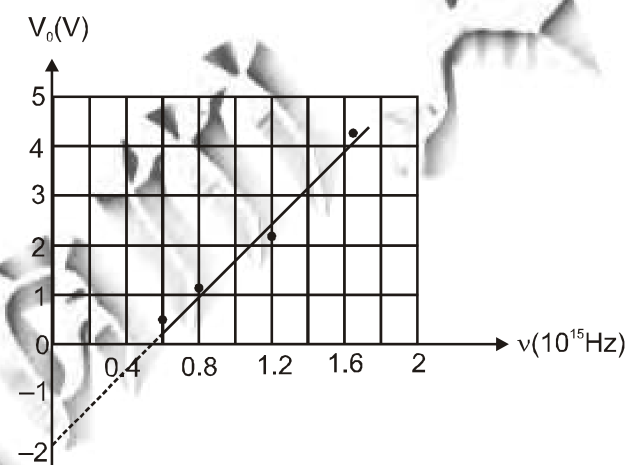
25. A blackbox (BB) which may contain a combination of electrical circuit elements (resistor, capacitor or inductor) is connected with other external circuit elements as shown below in the figure (a). After the switch (S) is closed at time t = 0, the current (I) as a function of time (t) is shown in the figure (b).



From this we can infer that the blackbox contains
(A) A resistor and a capacitor in series
(B) A resistor and a capacitor in parallel
(C) A resistor and an inductor in series
(D) A resistor and an inductor in parallel

Sol. I - t graph is for L-R series circuit.
Ans. (C)

26. In a photocell circuit the stopping potential, V_0, is a measure of the maximum kinetic energy of the photoelectrons. The following graph shows experimentally measured values of stopping potential versus frequency ν of incident light.



The values of Plank's constant and the work function as determined from the graph are (taking the magnitude of electronic charge to be $e = 1.6 \times 10^{-19}$ C)
(A) 6.4×10^{-34} Js, 2.0 eV
(B) 6.0×10^{-34} Js, 2.0 eV
(C) 6.4×10^{-34} Js, 3.2 eV
(D) 6.0×10^{-34} Js, 3.2 eV

Sol. $V = \frac{h\nu}{e} - \phi$

Hence from graph ϕ = 2eV

$$\frac{h}{e} = \text{slope} = \frac{6}{1.6 \times 10^{15}}$$

$$h = \frac{6 \times 1.6 \times 10^{-19}}{1.6 \times 10^{15}} = 6.0 \times 10^{-34}$$

Ans. (B)

27. An engine moving away from a vertical cliff blows a born at a frequency f. Its speed is 0.5% of the speed of sound in air. The frequency of the reflected sound received at the engine is

(A) 0.990 f (B) 0.995 f (C) 1.005 f (D) 1.010 f

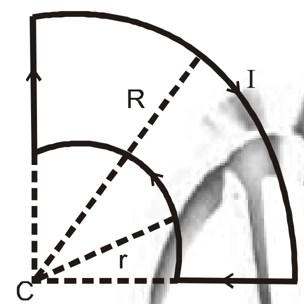
Sol. $f_1 = f_0\left[\frac{v}{v+v_s}\right]$

$f_R = f_1\left[\frac{v-v_s}{v}\right] = f_0\left[\frac{v-v_s}{v+v_s}\right] \simeq \left[1-\frac{2v_s}{v}\right]f$

= 0.990 f

Ans. (A)

28. An arangement with a pair of quarter circular coils of radii r and R with a common centre C and carrying a current I is shown.



The permeability of free space is μ_0. The magnetic field at C is

(A) μ_0 I(1/r – 1/R)/8 into the page (B) μ_0 I (1/r – 1/R)/8 out of the page

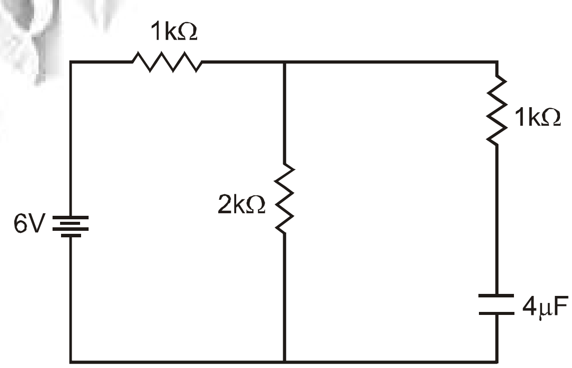
(C) μ_0 I(1/r + 1/R)/8 out of the page (D) μ_0 I(1/r + 1/R)/8 into the page

Sol. B due to Arc = $\frac{\mu_0 i\theta}{4\pi r}$

$\frac{\mu_0 i}{8}\left[\frac{1}{r}-\frac{1}{R}\right]$ out of the page

Ans. (B)

29. The circuit shown has been connected for a long time. The voltage across the capacitor is



(A) 1.2 V (B) 2.0 V (C) 2.4 V (D) 4.0 V

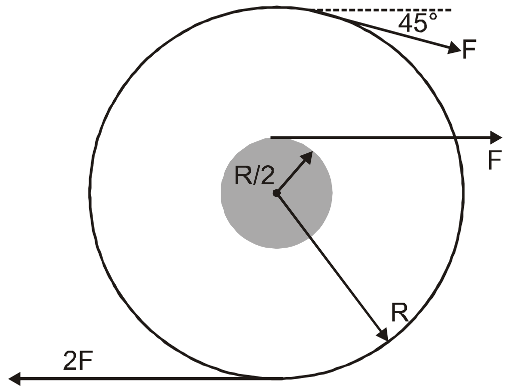
Sol. In stready state i through capacitor is zero.

Hence V across 2kΩ = V across capacitor

Vacross 2kΩ = $\frac{2}{2+1}\times 6 = 4V$

Ans. (D)

30. A wheel of radius R with an axle of radius R/2 is shown in the figure and is free to rotate about a frictionless axis through its centre and perpendicular to the page. Three forces (F, F, 2F) are exerted tangentially to the repective rims as shown in the figure.



The magnitude of the net torque acting on the system is nearly

(A) 3.5FR (B) 3.2 FR (C) 2.5 FR (D) 1.5 FR

Sol. $\tau_{Net} = F\left(\frac{R}{2}\right) + FR + 2FR = 3.5\ FR$

Ans. (A)

31. Two species of radioactive atoms are mixed in equal number. The disintegration of the first species is λ and of the second is $\lambda/3$. After a long time the mixture will behave as a species with mean life of approximately

(A) $0.70/\lambda$ (B) $2.10/\lambda$ (C) $1.00/\lambda$ (D) $0.52/\lambda$

Sol. avg life = $\frac{\int tdN_1 + tdN_2}{2N_0}$

where $dN_1 = \lambda N_o e^{-\lambda t} dt$

$dN_2 = \frac{\lambda}{3} N_o e^{-\frac{\lambda}{3}t} dt$

avg. life = $\frac{\int_o^\infty t(\lambda N_o e^{-\lambda t} dt + \frac{\lambda}{3} N_o e^{-\frac{\lambda}{3}t} dt}{2N_0}$

Intergrating we got

avg life = $\frac{2}{\lambda} \approx \frac{2.10}{\lambda}$

Ans. (B)

32. The bulk modulus of a gas is defined as B = –VdP/dV. For an adiabatic process the variation of B is proportional to P^n. For an idea gas, n is

(A) 0 (B) 1 (C) $\frac{5}{2}$ (D) 2

Sol. For adiabatic process

$PV^\gamma = C$

$P\gamma V^{\gamma-1} + \frac{dp}{dv} v^\gamma = 0$

$\gamma P = -V\frac{dp}{dv}$

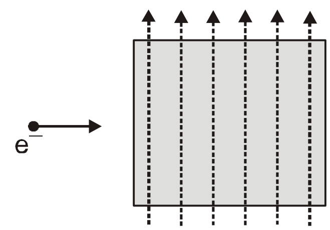
Hence n = 1

33. Photons of energy 7 eV are incident on two metals A and B with work functions 6 eV and 3 eV respectively. The minimum de Broglie wavelengths of the emitted photoelectrons with maximum energies are λ_A and λ_B, respectively where λ_A/λ_B is nearly

(A) 0.5 (B) 1.4 (C) 4.0 (D) 2.0

Ans. (D)

34. An electron enters a chamber in which a uniform magnetic field is present as shown. Ignore gravity.



During its motion inside the chamber

(A) the force on the electron remains constant

(B) the kinetic energy of the electron remains constant

(C) the momentum of the electron remains constant

(D) the speed of the electron increases at a uniform rate

Sol. $\because$ work done = 0

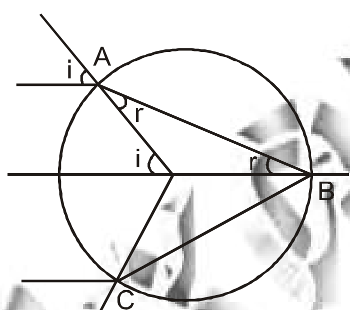
Hence kinetic energy = constant

Ans. (B)

35. A ray of light incident on a glass sphere (refractive index $\sqrt{3}$) suffers total internal reflection before emerging out exactly parallel to the incident ray. The angle of incidence was

(A) 75° (B) 30° (C) 45° (D) 60°

Sol.



$\angle i = 2\angle r$

$\frac{\sin i}{\sin r} = \sqrt{3}$

$2\cos r = \sqrt{3}$

r = 30° & i = 60°

Note : But for r = 30° TIR cannot take place at B.

Ans. (D)

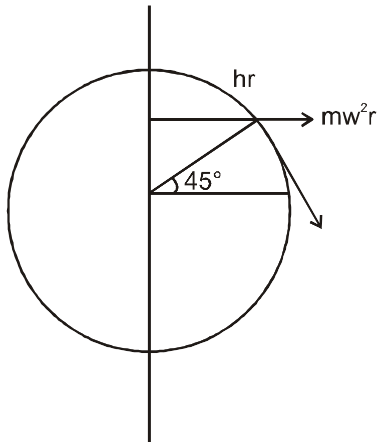
36. Young-Laplace law states that the excess pressure inside a soap bubble of radius R is given by $\Delta P = 4\sigma/R$ where σ is the coefficient of surface tension of the soap. The Eötvös number E_0 is a dimensionless number that is used to describe the shape of bubbles rising through a surrounding fluid. It is a combination of g, the acceleration due to gravity, ρ, the density of the surrounding fluid, σ and a characterstic length scale L which could be the radius of the bubble. A possible expression for E_0 is

(A) $\frac{\rho g}{\sigma L^3}$ (B) $\frac{\rho L^2}{\sigma g}$ (C) $\frac{\rho g L^2}{\sigma}$ (D) $\frac{g L^2}{\sigma \rho}$

37. A plank is resting on a horizontal ground in the northern hemisphere of the Earth at a 45° latitude. Let the angular speed of the Earth be ω and its radius r_e. The magnitude of the frictional force on the plank will be

(A) $mr_e\omega^2$ (B) $\frac{mr_e\omega^2}{\sqrt{2}}$ (C) $\frac{mr_e\omega^2}{2}$ (D) Zero

Sol.



$F_r = m\omega^2 r \cos 45°$
where r = Rcos 45°

$F_r = \frac{m\omega^2 R}{2}$

Ans. (C)

38. The average distance between molecules of an ideal gas at STP is approximately of the order of
(A) 1 nm (B) 100 nm (C) 100 cm (D) 1 μm

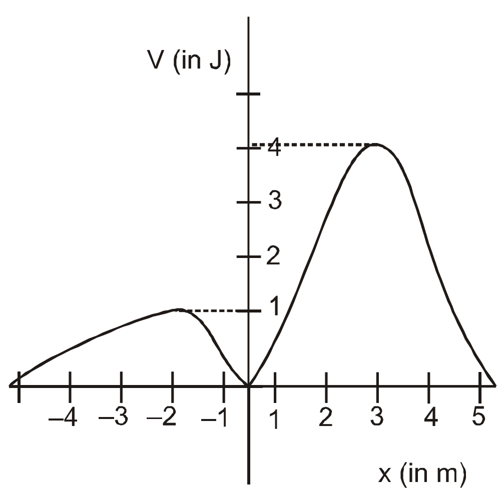
Sol. $n = \frac{PV}{RT} = \frac{10^5 \times 1}{\frac{25}{3} \times 300} = 40$

$N = 40 \times 6.023 \times 10^{23} \simeq 24 \times 10^{24}$

Average sep. $= \left(\frac{1}{N}\right)^{1/3} \simeq 1\text{nm}$

Ans. (A)

39. A point particle of mass 0.5 kg is moving along the x-axis under a force described by the potential energy V shown below. It is projected towards the right from the origin with a speed v.
What is the minimum value of v for which the particle will escape infinitely fasr away from the origin ?

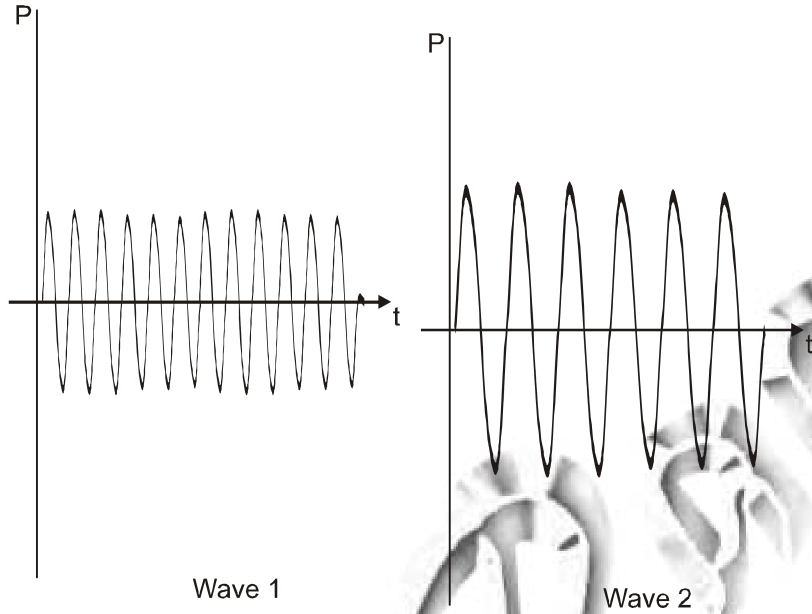


(A) $2\sqrt{2}$ ms^{-1} (B) 2 ms^{-1} (C) 4 ms^{-1} (D) The particle will never escape

Sol. $\frac{1}{2}mv^2 + 0 = 0 + 1$

$v^2 = 4$ or v = 2 m/s

40. The figure below shows pressure variation in two different sound waves in air with time at a given position. Both the figures are drawn to the same scale.



Which of the following statements is true?
(A) Wave 1 has lower frequency and smaller amplitude compared to wave 2
(B) Wave 1 has higher frequency and greater amplitude compared to wave 2
(C) Wave 1 has shroter wavelength and greater amplitude compared to wave 2
(D) Wave 1 has shorter wavelength and smaller amplitude compared to wave 2

Ans. (D)

CHEMISTRY

41. Among the following, the set of isoelectronic ions is
(A) Na^+, Mg^{2+}, F^-, Cl^- (B) Na^+, Ca^{2+}, F^-, O^{2-}
(C) Na^+, Mg^{2+}, F^-, O^{2-} (D) Na^+, K^+, S^{2-}, Cl^-

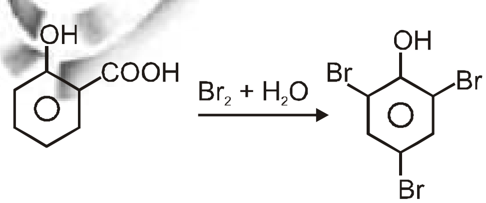
Sol. Na^+, Mg^{2+}, F^- & O^{2-} are isoelectronic(10 electron speceis)
Ans. (C)

42. For a zero-order reaction with rate constant k, the slope of the plot of reactant concentration against time is
(A) k/2.303 (B) k (C) –k/2.303 (D) – k

Sol. $C_t = C_0 - kt$; slope = – k
Ans. (D)

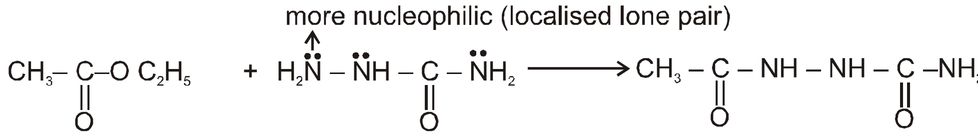
43. The compound which reacts with excess bromine to produce 2, 4, 6-tribromophenol, is
(A) 1, 3-cyclohexadiene (B) 1, 3-cyclohexanedione
(C) salicylic acid (D) cyclohexanone

Sol.



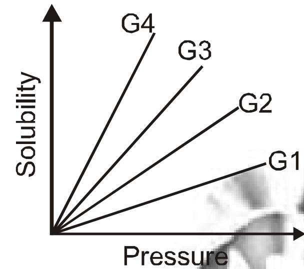
Ans. (C)

44. Ethyl acetate reacts with $NH_2NHCONH_2$ to form
(A) $CH_3CONHCONHNH_2$ (B) $CH_3CON(NH_2)CONH_2$
(C) $CH_3CONHNHCONH_2$ (D) $CH_3CH_2NHNHCONH_2$

Sol.


Ans. (C)

45. The variation of solubility of four different gases (G1, G2, etc.) in a given solvent with pressure at a constant temperature is shown in the plot.



The gas with the highest value of Henry's law constant is
(A) G4 (B) G2 (C) G3 (D) G1

Sol. $\because\ P = K_H.X_{(g)}$

$$X_{(g)} = P \times \frac{1}{K_H}$$

$$\text{slope} = \frac{1}{K_H}$$

Ans. (D)

46. For the reaction, $A \rightleftharpoons nB$ the concentration of A decreases from 0.06 to 0.03 mol L^{-1} and that of B rises from 0 to 0.06 mol L^{-1} at equilibrium. The values of n and the equilibrium constant for the reaction, respectively, are
(A) 2 and 0.12 (B) 2 and 1.2 (C) 3 and 0.12 (D) 3 and 1.2

Sol.

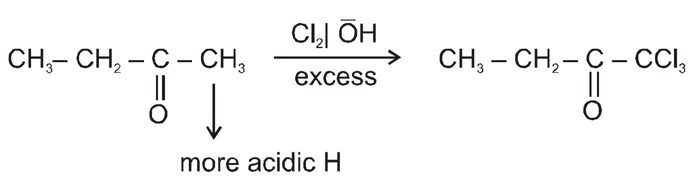
	A $\rightleftharpoons$	n B
t = 0	0.06	0
t = t	0.03	0.06

$\alpha c = 0.03$
$n\alpha c = 0.06$
$n = 2$

$$K_C = \frac{(0.06)^2}{0.03} = \frac{(0.06)^2}{0.03} = 0.12$$

Ans. (A)

47. The reaction of ethyl methyl ketone with Cl_2/excess OH^- gives the following major product
(A) $ClCH_2CH_2COCH_3$ (B) $CH_3CH_2COCCl_3$
(C) $ClCH_2CH_2COCH_2Cl$ (D) $CH_3CCl_2COCH_2Cl$

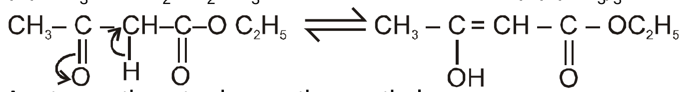
Sol.


Ans. (B)

48. The compound that readily tautomerizes is

(A) $CH_3COCH_2CO_2C_2H_5$ (B) $CH_3COCH_2CH_2CH_3$
(C) $CH_3COCH_2CH_2CH_3$ (D) $(CH_3)_3CCOC(CH_3)_3$

Sol.



Aceto acetic ester has active methylene group.

Ans. (A)

49. Hydrolysis of BCl_3 gives X which on treatment with sodium carbonate produces Y, X and Y, respectively, are

(A) H_3BO_3 and $NaBO_2$ (B) H_3BO_3 and $Na_2B_4O_7$
(C) B_2O_3 and $NaBO_2$ (D) B_2O_3 and $Na_2B_4O_7$

Sol.

$BCl_3 + 3H_2O \longrightarrow H_3BO_3 + 3HCl$

$4H_3BO_3 + Na_2CO_3 \longrightarrow Na_2B_4O_7 + CO_2 + 6H_2O$

Ans. (B)

50. The numbers of lone pair(s) on Xe in XeF_2 and XeF_4 are, respectively,

(A) 2 and 3 (B) 4 and 1 (C) 3 and 2 (D) 4 and 2

Sol. XeF_2 ; no. of lp on Xe = 3
XeF_4 ; no. of lp on Xe = 2

Ans. (C)

51. The entropy change in the isothermal reversible expansion of 2 moles of an ideal gas from 10 to 100 L at 300 K is

(A) 42.3 JK^{-1} (B) 35.8 J K^{-1} (C) 38.3 J K^{-1} (D) 32.3 J K^{-1}

Sol. $\Delta S = \frac{nRT}{T} \ln \frac{v_2}{v_1}$

$= nR \ln \frac{v_2}{v_1}$

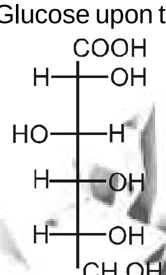
$= 2.303\ nR \log \frac{v_2}{v_1}$

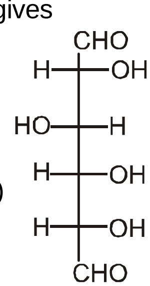
$= 2.303 \times 2 \times 8.314 \times 1$

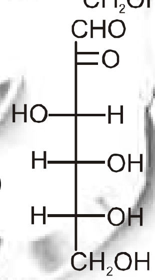
= 38.3 J/k

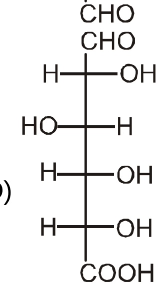
Ans. (C)

52. D-Glucose upon treatment with bromine-water gives

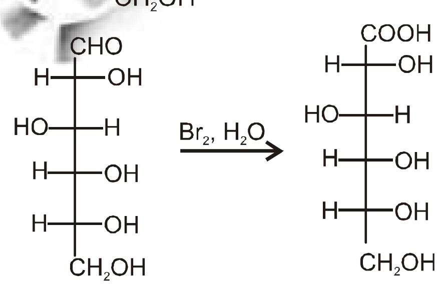
(A) 

(B) 

(C) 

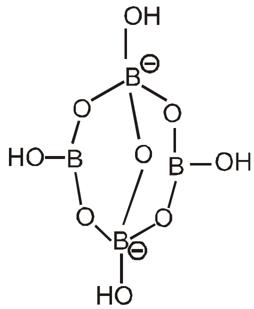
(D) 

Sol.



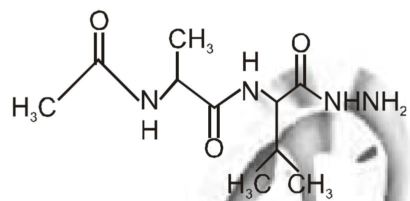
53. In the structure of borax, the numbers of boron atoms and B-O-B units, respectively, are
(A) 4 and 5 (B) 4 and 3 (C) 5 and 4 (D) 5 and 3

Sol.



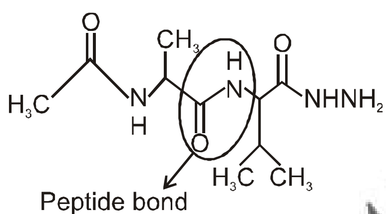
Ans. (A)

54. The number of peptide bonds in the compound



is
(A) 1 (B) 2 (C) 3 (D) 4

Sol.



Ans. (A)

55. For the isothermal reversible expansion of an ideal gas
(A) $\Delta H > 0$ and $\Delta U = 0$ (B) $\Delta H > 0$ and $\Delta U < 0$
(C) $\Delta H = 0$ and $\Delta U = 0$ (D) $\Delta H = 0$ and $\Delta U > 0$

Sol. $\Delta U = 0$, $\Delta H = 0$

Ans. (C)

56. If the angle of incidence of X-ray of wavelength 3Å which produces a second order diffracted beam from the (100) planes in a simple cubic lattice with interlayer spacing a = 6 Å is 30°, the angle of incidence that produ ces a first-order diffracted beam from the (200) planes is
(A) 15° (B) 45° (C) 30° (D) 60°

Ans. (C)

57. The number of ions produced in water by dissolution of the complex having the empirical formular, $COCl_3 4NH_3$, is
(A) 1 (B) 2 (C) 4 (D) 3

Sol. Coordination number = 6

$[Co(NH_3)_4Cl_2]Cl \rightarrow [Co(NH_3)_4Cl_2]^+ + Cl^-$

Ans. (B)

58. The spin-only magnetic moments of $[Fe(NH_3)_6]^{3+}$ and $[FeF_6]^{3-}$ in BM are, respectively,
(A) 1.73 and 1.73 (B) 5.92 and 1.73 (C) 1.73 and 5.92 (D) 5.92 and 5.92

Sol. $Fe^{3+} = 3d^5$

1 1 1 1 1   _   _ _ _
3d   4s   4p

$\overset{\ominus}{F}$ is WFL so $\mu = 5.92$ While NH_3 is SFL & pairing of electrons takes place so $\mu = 1.73$

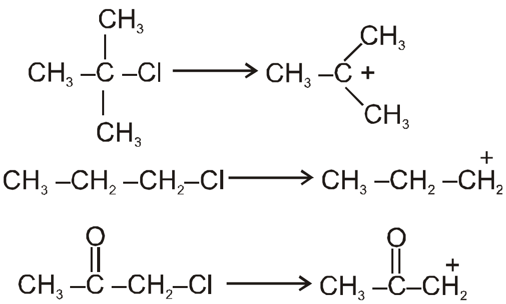
Ans. (C)

59. The order of S_N1 reactivity in aqueous acetic acid solution for the compounds

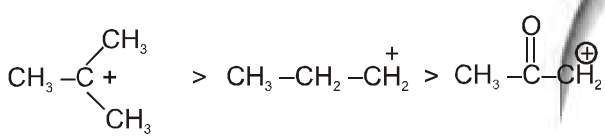
is

(A) 1 > 2 > 3 (B) 1 > 3 > 2 (C) 3 > 2 > 1 (D) 3 > 1 > 2

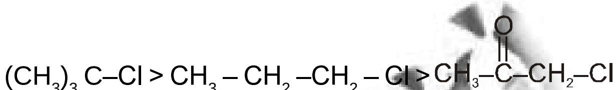
Sol.



Carbocation stability order



So reactivity order is



Ans. (C)

60. An ionic compound is formed between a metal M and a non-metal Y. If M occupies half the octahedral voids in the cubic close-packed arrangement formed by Y, the chemical formula of the ionic compound is

(A) MY (B) MY_2 (C) M_2Y (D) MY_3

Sol.

	Z (effective no. of atoms/ unit cell)
Y	4
M	2

then formula of ionic compound is MY_2

Ans. (B)

BIOLOGY

61. Human fetal haemoglobin differs from the adult haemoglobin in that it has

(A) higher affinity for oxygen (B) lower affinity for oxygen

(C) two subunits only (D) is glycosylated

Sol. HbF have more affinity for oxygen and this property help in extracting O_2 from mother.

Ans. (A)

62. Nucleolus is an organelle responsible for the production of

(A) carbohydrates (B) messenger RNA (C) lipids (D) ribosomal RNA

Sol. In nucleolus RNA-PI is present.

Ans. (D)

63. The sequences of four DNA molecules are given below:

i. TATATATATATATA ATATATATATATAT
ii. TTTCCCGGGAAA AAAGGGCCCTTT
iii. TTGCGTTGCCC AACGCAACGGG
iv. GCCGGATCCGGC CGGCCTAGGCCG

Which one of these DNA moelcules will have the highest melting temperature (T_m)?
(A) i (B) ii (C) iii (D) iv

Sol. In GC Pairing 3 H bonds are present.

Ans. (D)

64. If DNA codons are ATG GAA, insertion of thymine after the first codon results in,
(A) non-sense mutation (B) mis-sense mutation
(C) frameshift mutation (D) silent mutation

Sol. Entry of T Cause formation of stop codon.

Ans.(A)

65. Genetic content of a cell reduces to half during
(A) meiotic prophase I (B) mitotic prophase
(C) meiotic prophase II (D) meiotic telophase

Ans.(D)

66. Which one of the following techniques is used for the detection of proteins ?
(A) Northern blotting (B) Western blotting (C) Southern blotting (D) In-situ hybridization

Ans.(B)

67. Fission yeasts are
(A) Archaebacteria (B) Eubacteria (C) Prokaryotes (D) Eukaryotes

Sol. Schizosaccharomyces pombe is called fission yeast.

Ans.(D)

68. In green leaves, the light and dark reactions occur in
(A) stroma and grana respectively
(B) grana and stroma respectively
(C) cristae and matrix respectively
(D) both occur in cytoplasm

Sol. Grana bear photosystem and stroma have enzymes of dark reaction.

Ans. (B)

69. According to Mendel, ... segregate and .... assort independently.
(A) alleles of a gene; alleles of different genes
(B) alleles of different genes; alleles of a gene
(C) dominanat traits; recessive traits
(D) recessive traits; recessive traits

Sol. 2 alleles of a gene separate in neiosis and Non linked different genes segregate independently.
Ans. (A)

70. The two enzymatic activities associated with RUBISCO are
(A) oxidase and oxygenase (B) oxygenase and carboxylase
(C) oxidase and carboxylase (D) oxygenase and carbamylation
Ans. (B)

71. Chlorofluorocarbons (CFCs) are belived to be associated with cancers because,
(A) CFCs react with DNA and cause mutations
(B) CFCs react with proteins involved in DNA repair
(C) CFCs destroy the ozone layer and permit harmful UV rays to reach the earth
(D) CFCs react with DNA polymerase and reduce fidelity of DNA replication

Sol. UV rays damage DNA causing thymine dimerisation.

72. Morphogenetic movements take place predominantly during the following embryonic stage

(A) blastula (B) Morula (C) Gastrula (D) Fertilized eggs

Sol. In gastrulation cell migration occur forming 3 germ layers.

Ans. (C)

73. The only organ which is capable of producing Fructose in humans is

(A) liver (B) pancreas

(C) seminal vesicles (D) muscle

Ans. (C)

74. Stroke could be prevented/treated with

(A) balanced diet (B) clotting factors (C) insulin (D) blood thinners

Sol. Thinners prevent cardiovascular accidents which can cause stroke.

Ans. (D)

75. In orange and lemon, the edible part of the fruit is

(A) placenta (B) thalamus

(C) hairs of the ovary wall (D) succulent Mesocap

Ans. (C)

76. Which one of the following statements about nitrogenase is correct?

(A) It is sensitive to CO_2 and therefore present in isolated nodules.

(B) It requires O_2 and therefore functional during the day

(C) It is sensitive to O_2 and therefore is functional in anaerobic environments

(D) It is sensitive to light and therefore functions only in dark.

Ans. (C)

77. Part of epidermis that keeps out unwanted particles is called

(A) columnar epithelium (B) squamous epithelium

(C) ciliated epithelium (D) cuboidal epithelium

Ans. (C)

78. Species that are most effective at colonising new habitats show

(A) low reproductive ability (B) high dispersal ability

(C) slow growth and maturation (D) high competitive ability

Ans. (D)

79. In a large isolated population, alleles p and q at a locus are at Hardy Weinberg equilibrium. The frequencies are p = 0.6 and q = 0.4. The proportion of the heterozygous genotype in the population is

(A) 0.24 (B) 1 (C) 0.48 (D) 0.12

Sol. Heterozygous population = 2pq = .48

Ans. (C)

80. In vertebrates 'glycogen' is stored chiefly in

(A) heart and blood (B) spleen and stomach

(C) bones and lymph (D) liver and muscles

Ans. (D)

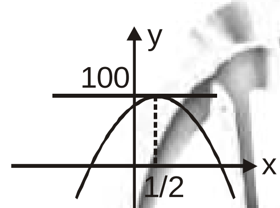
PART-II
Two Mark Questions

MATHEMATICS

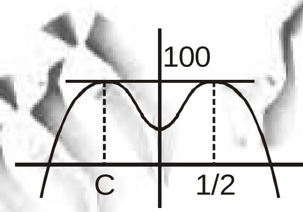
81. Let f(x) be a non-constant polynomial with real coefficients such that $f\left(\frac{1}{2}\right) = 100$ and $f(x) \le 100$ for all real x. Which of the following statements is NOT necessarily true ?
(A) The coefficient of the highest degree term in f(x) is negative
(B) f(x) has at least two real roots
(C) If $x \neq 1/2$ then $f(x) < 100$
(D) At least one of the coefficients of f(x) is bigger than 50.

Sol. Coefficient of highest degree term must be negative becuase if it is positive, then $x \to \infty$, $y \to \infty$ and it is not possible, since $f(x) \le 100$.

Now, graph will be like



at least two real roots will be there, & if $x \neq \frac{1}{2}$, then f(x) < 100, it is not always true, as the graph can be like this also



Now, let the highest coefficients, it can have is 49

then, $f\left(\frac{1}{2}\right) = 49 + \frac{49}{2} + \frac{49}{2^2} + \ldots$

But the sum cannot be equal to 100.

Ans. (C)

82. Let a, b, c, d be real numbers such that

$$\sum_{k=1}^{n}(ak^3 + bk^2 + ck + d) = n^4$$

for every natural number n. Then | a | + | b | + | C | + | d | is equal to
(A) 15 (B) 16 (C) 31 (D) 32

Sol. $\sum_{k=1}^{n}(ak^3 + bk^2 + ck + d) = n^4$

$a\sum_{k=1}^{n}k^3 + b\sum_{k=1}^{n}k^2 + c\sum_{k=1}^{n}k + \sum_{k=1}^{n}d = n^2$

$n^4(12 - 3a) - n^3(4b + 6a) - n^2(6c + 6b + 3a) - n(6c + 2b + 12d) = 0$

$12 - 3a = 0, \quad 4b + 6a = 0, \quad 6c + 6b + 3a = 0, \quad 6c + 2b + 12d = 0$

$\Rightarrow a = 4, b = -6, c = 4, d = -1$

$|a|+|b|+|c|+|d| = 15$

83. The vertices of the base of an isosceles triangle lie on a parabola $y^2 = 4x$ and the base is a part of the line $y = 2x - 4$. If the third vertex of the triangle lies on the x-axis, its coordinates are

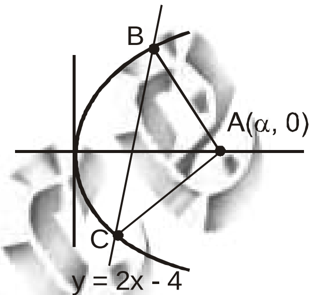
(A) $\left(\frac{5}{2}, 0\right)$ (B) $\left(\frac{7}{2}, 0\right)$ (C) $\left(\frac{9}{2}, 0\right)$ (D) $\left(\frac{11}{2}, 0\right)$

Sol.

$(2x - 4)^2 = 4x$

$(x - 2)^2 = x$
$x^2 - 5x + 4 = 0$

$x = 1, 4$



C(1, -2)
B(4, 4) $\because$ AB = AC

$$\sqrt{(\alpha - 4)^2 + 16} = \sqrt{(\alpha - 1)^2 + 4}$$

On solving, we get $\alpha = \frac{9}{2}$

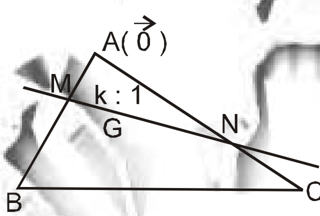
Ans. (C)

84. In a triangle ABC, let G denote its centroid and let M, N be points in the interiors of the segments AB, AC, respectively, such that M, G, N are collinear. If r denotes the ratio of the area of triangle AMN to the area of ABC then

(A) r = 1/2 (B) r > 1/2 (C) $4/9 \le r < 1/2$ (D) $4/9 < r$

Sol.

Let $\vec{AB} = \vec{b}, \vec{AC} = \vec{c}$



$\vec{AM} = \lambda \vec{b}$

$\vec{AN} = m \vec{c}$

Let G divides MN in the ratio K : 1

So $\frac{k\mu \vec{c} + \lambda \vec{b}}{k+1} = \frac{\vec{b} + \vec{c}}{3}$

$\Rightarrow \frac{k\mu}{k+1} = \frac{1}{3}$ $\frac{\lambda}{k+1} = \frac{1}{3}$

$\Rightarrow k = \frac{\lambda}{\mu}$

$\Rightarrow \frac{1}{\lambda} + \frac{1}{\mu} = 3$

AM $\ge$ GM

$$\frac{\frac{1}{\lambda} + \frac{1}{\mu}}{2} \ge \frac{1}{\sqrt{\lambda\mu}} \Rightarrow \left(\frac{2}{3}\right)^2 \le \lambda\mu \qquad \text{... (1)}$$

Now, $\frac{\text{area of } \Delta AMN}{\text{area of } ABC} = \frac{\frac{1}{2}\lambda\mu\left|\vec{b} \times \vec{c}\right|}{\frac{1}{2}\left|\vec{b} \times \vec{c}\right|}$

$= \lambda\mu$

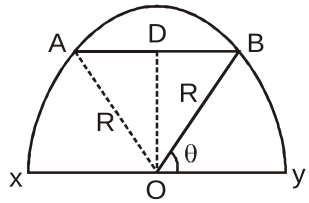
using $\frac{1}{\lambda} + \frac{1}{\mu} = 3 \Rightarrow$ Ratio $= \frac{\lambda}{3\lambda - 1}\lambda \in [0,1]$ maximum value of ratio $= \frac{\lambda^2}{3\lambda - 1}$ attain when $\lambda = 1$ using derivative but λ is not 1 becuase M is an interior point.

so $\frac{4}{9} \le$ ratio $< \frac{1}{2}$

85. Let XY be the diameter of a semicircle with centre O. Let A be a variable point on the semicircle and B another point on the semicircle such that AB is parallel to XY. The value of ∠BOY for which the inradius of triangle AOB is maximum, is

(A) $\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{5}$

Sol.



$OD = R\sin\theta$

$AB = 2R\cos\theta$

$$r_{OAB} = \frac{ar(\Delta OAB)}{semi-perimeter}$$

$$= \frac{\frac{1}{2}\times OD\times AB}{\frac{2R+AB}{2}} = \frac{\frac{1}{2}\times R\sin\theta 2R\cos\theta}{\frac{2R+2R\cos\theta}{2}}$$

$$r_{OAB} = \frac{R\sin\theta\cos\theta}{(1+\cos\theta)}$$

$$\frac{dr_{OAB}}{d\theta} = \frac{(1+\cos\theta)\cos 2\theta - \sin\theta\cos\theta(-\sin\theta)}{(1+\cos\theta)^2} = 0$$

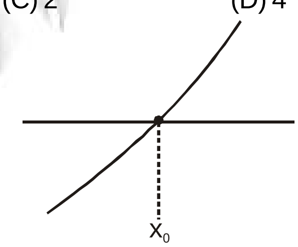
at $\cos\theta = \frac{\sqrt{5}-1}{2}$

Ans. (A)

86. Let $f(x) = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$. The number of real roots of f(x) = 0 is

(A) 0 (B) 1 (C) 2 (D) 4

Sol. $f(x) = 1 + x + \frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}$



$f'(x) = 1 + x + \frac{x^2}{2}+\frac{x^3}{6}$

$f''(x) = 1 + x + \frac{x^2}{2} > 0$

$\Rightarrow f'(x)$ is an increasing f^n

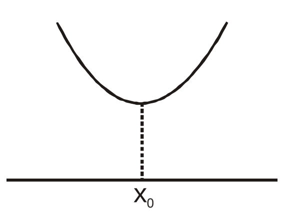
$\Rightarrow f'(x) = 0$ at $x = x_0$

$\Rightarrow f'(x_0) = 0 \Rightarrow 1 + x_0 + \frac{x_0^2}{2}+\frac{x_0^3}{6} = 0$ .... (1)

$f'(-2)\, f'(-1) < 0$

$\Rightarrow x_0 \in (-2, -1)$

$f(x_0) = 1 + x_0 + \frac{x_0^2}{2}+\frac{x_0^3}{6}+\frac{x_0^4}{24} = \frac{x_0^4}{24} > 0$

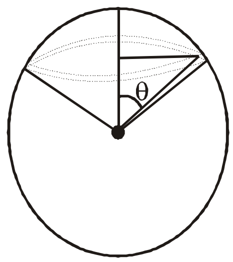


no solution

87. Suppose that the earth is a sphere of radius 6400 kilometers. The height from the earth's surface from where exactly a fourth of the earth's surface is visible, is

(A) 3200 km (B) $3200\sqrt{2}$ km (C) $3200\sqrt{3}$ km (D) 6400 km

Sol.



AP = $R\sin\theta$

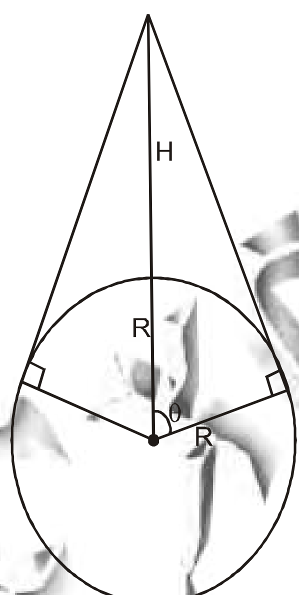
area of ring = $(2\pi R\sin\theta).Rd\theta$

Total area required, $\pi R^2 = \left|\int_0^{\theta} 2\pi R^2 \sin\theta d\theta\right|$

$\frac{1}{2} = 1 - \cos\theta$

$\cos\theta = \frac{1}{2}$

Now,



$\cos\theta = \frac{R}{R+H}$

$\frac{1}{2} = \frac{R}{R+H} = R = H = 6400$

Ans. (D)

88. Let n be a positive integer. For a real number x, let [x] denote the largest integer not exceeding x and {x} = x – [x]. Then $\int_1^{n+1} \frac{(\{x\})^{[x]}}{[x]} dx$ is equal to

(A) $\log_e(n)$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $1+\frac{1}{2}+....+\frac{1}{n}$

Sol. $\int_1^{n+1} \frac{\{x\}^{[x]}}{[x]} dx = \int_1^{2} \frac{\{x\}^{[x]}}{[x]} dx + \int_2^{3} \frac{\{x\}^{[x]}}{[x]} dx + ... + \int_1^{n+1} \frac{\{x\}^{[x]}}{[x]} dx$

$= \sum_{r=1}^{n} \int_r^{r+1} \frac{\{x\}^{[x]}}{[x]} dx$

$= \sum_{r=1}^{n} \int_r^{r+1} \frac{(x-r)^r}{r} dx$

$= \sum_{r=1}^{n} \left[\frac{(x-r)^{r+1}}{r(r+1)} \right]_r^{r+1}$

$= \sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$

Ans. (C)

89. A box contains coupons labelled 1,2,...,100. Five coupons are picked at random one after another without replacement. Let the numbers on the coupons be $x_1, x_2, ..., x_5$. What is the probability that $x_1 > x_2 > x_3$ and $x_3 < x_4 < x_5$?

(A) 1/120 (B) 1/60 (C) 1/20 (D) 1/10

Sol. $\frac{{}^{100}C_5 [1 \times 2(3)]}{{}^{100}C_5 \times 5!}$ Suppose 1, 2, 3, 4, 5 are selected coupons.

		1		

$= \frac{1}{20}$

Place of 1 is fixed

Total arrangements of 5 is 2

and

		1		5

arrangements of 2, 3, 4, are

2	3	1	4	5	3 way
4	2	1	3	5	
4	3	1	2	5	

Ans. (A)

90. In a tournament with five teams, each team plays against every other team exctly once. Each game is won by one of the playing teams and the winning team scores one point, while the losing team scores zero. Which of the following is NOT necessarily true?

(A) There are at least two teams which have at most two points each.

(B) There are at least two teams which have at least two points each.

(C) There are at most three teams which have at least threee points each

(D) There are at most four teams which have at most two points each

Sol. Let teams be T_1, T_2, T_3, T_4 & T_5

Now, we can have 5 teams with the scores of 2 points each matches are

(I) $T_1 T_2$ (II) $T_1 T_3$ (III) $T_1 T_4$ (IV) $T_1 T_5$ (V) $T_2 T_3$ (VI) $T_2 T_4$

(VII) $T_2 T_5$ (VIII) $T_3 T_4$ (IX) $T_3 T_5$ (X) $T_4 T_5$

Match No.	T_1	T_2	T_3	T_4	T_5
I	1	0			
II	1		0		
III	0			1	
IV	0				1
V		1	0		
VI		1		0	
VII		0			1
VIII			1	0	
IX			1		0
X				1	0
Total Score	2	2	2	2	2

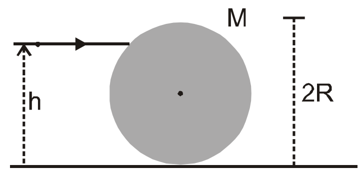
This score board contradicts, option D

$\therefore$ D is not always necssarily true.

Ans. (D)

PHYSICS

91. A bullet of mass m is fired horizontally into a large sphere of mass M and radius R resting on a smooth horizontal table.



The bullet hits the sphere at a height h from the table and sticks to its surface. If the sphere starts rolling without slippng immediately on impact, then

(A) $\frac{h}{R}=\frac{4m+3M}{2(m+M)}$ (B) $\frac{h}{R}=\frac{m+3M}{m+2M}$ (C) $\frac{h}{R}=\frac{10m+7M}{5(m+M)}$ (D) $\frac{h}{R}=\frac{4m+3M}{m+M}$

Sol. Apply conservation of linear momentum

$mv = (m + M)\, v_0$

$$mv \sin\theta R = \left(\frac{2}{5}MR^2 + mR^2\right)\omega_0$$

$$mv\left(\frac{h-R}{R}\right)R = \frac{(2M+5M)}{5}\omega_0 R^2$$

$$(m + M)\,(h - R)\omega_0 R = \frac{(2M+5M)}{5}\omega_0 R^2$$

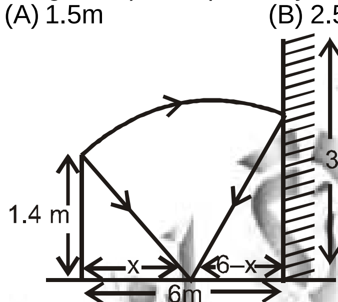
$$\frac{h}{R} = \frac{(10m+7M)}{5(m+M)}$$

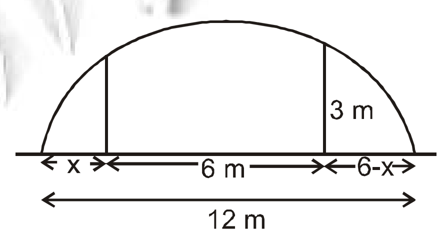
Ans. (C)

92. A small boy is throwing a ball towards a wall 6 in front of him. He releases the ball at a height of 1.4 m from the ground. The ball bounces from the wall at a height of 3 m, rebounds from the ground and reaches the boy's hand exactly at the point of release. Assuming the two bounces (one from the wall and the other from the ground) to be perfectly elastic, how far ahead of the boy did the ball bounce from the ground?

(A) 1.5m (B) 2.5 m (C) 3.5 m (D) 4.5 m

Sol.





$$3 = (6 + x)\tan\theta\left[1-\frac{6+x}{12}\right]$$

$$3=\frac{(6+x)(6-x)\tan\theta}{12}$$

$$1.4 = x\tan\theta\left[1-\frac{x}{12}\right]$$

$$1.4 = \frac{x(12-x)}{12}\tan\theta$$

$$\frac{30}{14} = \frac{36-x^2}{12x-x^2}$$

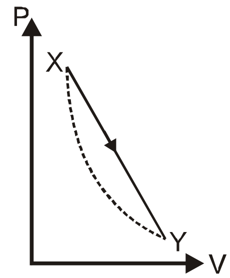
$360x - 30x^2 = 36 \times 14 - 14x^2$

$16x^2 - 360x + 36 \times 14 = 0$

$$x = \frac{360 \pm \sqrt{(360)^2 - 4\times 36\times 14\times 16}}{32}$$

$$x = \frac{360 \pm 312}{32} = \frac{48}{32} = 1.5$$

93. In the P-V diagram below the dashed curved line is an adiabat.



For a process that is described by a straight line joining two points X and Y on the adiabat (solid line in the diagram) heat is : (hint : Consider the variations in temperature from X to Y along the straitght line)

(A) absorbed throughtout from X to Y (B) released throughout from X to Y

(C) absorbed from X up to an intermediate point Z (not shown in the figure) and then released from Z to Y

(D) released from X up to an intermediate point Z (not shown in the figure) and then absorbed from Z to Y

Sol. from graph

Ans. (C)

94. A singly ionized helium atom in an excited state (n = 4) emits a photon of energy 2.6 eV. Given that the ground state energy of hydrogen atom is –13.6 eV, the energy (E_t) and quantum number (n) of the resulting state are respectively,

(A) E_t = – 13.6eV, n = 1 (B) E_t = – 6.0 eV, n = 3

(C) E_t = – 6.0 eV, n = 2 (D) E_t = – 13.6 eV, n = 2

Sol. $2.6 = 13.6\ z^2\left[\frac{1}{n^2}-\frac{1}{4^2}\right]$

$$\frac{2.6}{13.6\times 4} = 2^2\left[\frac{1}{n^2}-\frac{1}{4^2}\right]$$

$$\frac{2.6}{13.6\times 4} = \frac{1}{n^2}-\frac{1}{16}$$

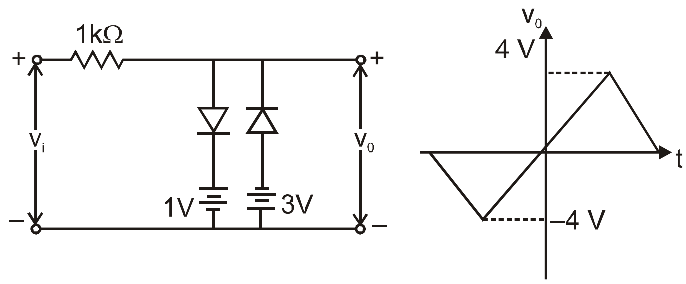
n = 3 now Energy

$$E = \frac{13.6Z^2}{n^2}eV$$

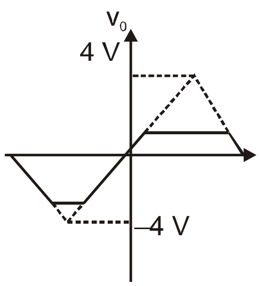
$$-\frac{13.6\times 4}{9}eV = -6eV$$

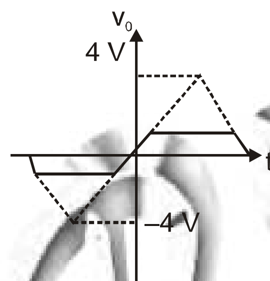
Ans. (B)

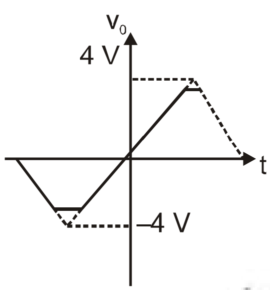
95. The figure below shows a circuit and its input voltage v_i as function of time t.

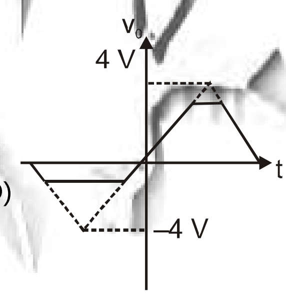


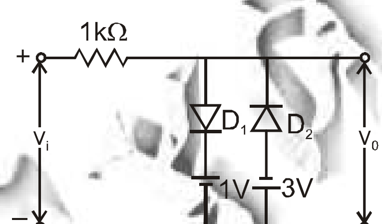
Assuming the diodes to be ideal, which of the following graphs depicts the output voltage v_0 as function of time t ?

(A)


(B)


(C)


(D)


Sol.


$V_0 = V_i$ when no current flow through 1kΩ \

for negative values of V_i

i from D_1 = 0 (always)

i from D_2 = 0 upto 3V

So from 0 to – 3V $V_i = V_0$

from – 3 to – 4 V V_0 = – 3V

For positive values of V_i

i from D_1 = 0 upto 1V

i from D_2 = 0 (always)

Hence 0 to 1V $V_i = V_0$

1 to 4 V V_0 = 1 volt

Correct graph is (A)

Ans. (A)

96. A ball is rolling without slipping in a spherical shallow bowl (radius R) as shown in the figure and is executing simple harmonic motion. If the radius of the ball is doubled, the period of oscillation

(A) increases slightly
(B) is reduced by a factor of 1/2
(C) is increased by a factor of 2
(D) decreases slightly

Sol. $mg\sin\theta - F_r = ma$

$$F_r = \frac{2}{5}mr^2\frac{a}{r^2}$$

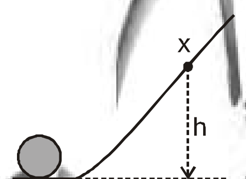
$$\Rightarrow a = \frac{5}{7}\frac{g\sin\theta}{R-r}$$

$$\omega = \sqrt{\frac{5g}{7(R-r)}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{7(R-r)}{5r}}$$

Ans. (D)

97. A solid sphere rolls without slipping, first horizontal and then up to a point X at height h on an inclined plane before rolling down, as shown.



The initial horizontal speed of the sphere is

(A) $\sqrt{10gh/7}$
(B) $\sqrt{7gh/5}$
(C) $\sqrt{5gh/7}$
(D) $\sqrt{2gh}$

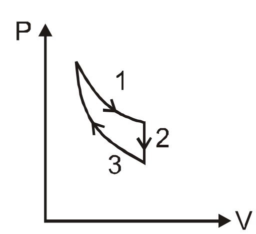
Sol. $$mgh = \frac{1}{2}m\nu^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{\nu^2}{R^2}$$

$$mgh = \frac{7}{10}m\nu^2$$

$$\nu = \sqrt{\frac{10gh}{7}}$$

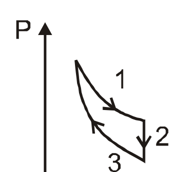
Ans. (A)

98. The three processes in a thermodynamic cycle shown in the figure are : Process $1 \rightarrow 2$ is isothermal; Process $2 \rightarrow 3$ is isochoric (volume remains constant); Process $3 \rightarrow 1$ is adiabatic. The total work done by the ideal gas in this cycle is 10 J. The internal energy decreases by 20 J in the isochoric process. The work done by the gas in the adiabatic process is –20 J. The heat added to the system in the isothermal process is



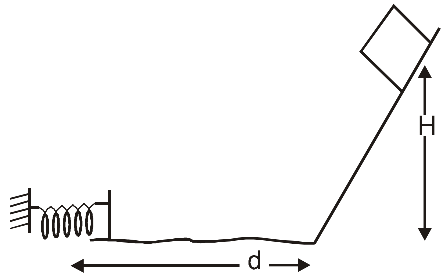
(A) 0 J
(B) 10 J
(C) 20 J
(D) 30 J

Sol.



$\Delta Q_{1\rightarrow 2} = \Delta w_{12}$
$w_{Total} = \Delta w_{12} + \Delta w_{31}$
$10 = \Delta w_{12} - 20$
$\Delta Q_{1\rightarrow 2} = \Delta w_{12} = 30$ J

99. A block of mass m slides from rest at a height H on a frictionless inclined plane as shown in the figure. It travels a distance d across a rough horizontal surface with coefficient of kinetic friction μ, and compresses a spring of spring k by a distance x before coming to rest momentarily. Then the spring extends and the block travels back attaining a final height of h. Then



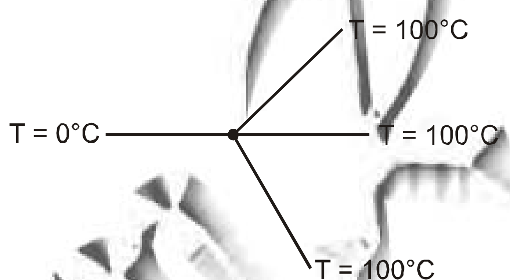
(A) $h = H - 2\mu(d + x)$
(B) $h = H + 2\mu(d + x)$
(C) $h = H - 2\mu d + kx^2/mg$
(D) $= H - 2\mu(d + x) + kx^2/2mg$

Sol. $mgH - 2\mu\, mg(d + x) - mgh = 0$

$h = H - 2\mu(d + x)$

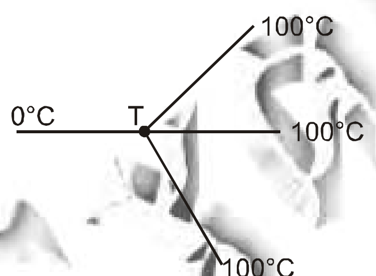
Ans. (A)

100. A metallic prong consists of 4 rods made of the same material, cross-section and same lengths as shown. The three forked ends are kept at 100° C and the handle end is at 0°C. The temperature of the junction is



(A) 25°C
(B) 50°C
(C) 60°
(D) 75°C

Sol.



$$3\frac{kA}{\ell}[100 - T] = \frac{kA}{\ell}[T - 0]$$

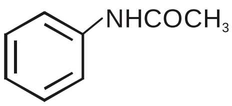
3w – 3T = T

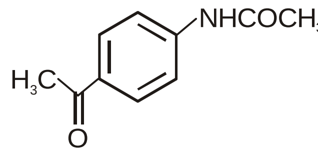
T = 75° C

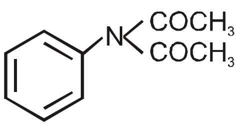
Ans. (D)

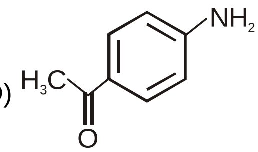
CHEMISTRY

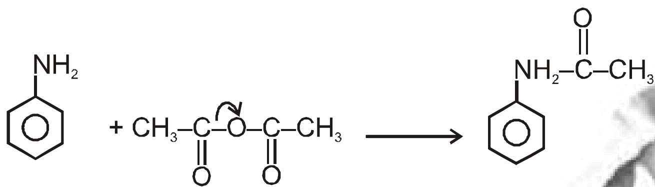
101. The major product obtained in the reaction of aniline with acetic anhydride is

(A) 

(B) 

(C) 

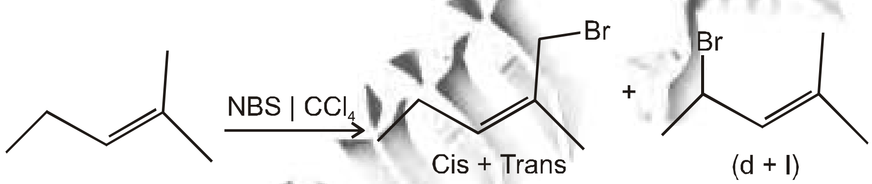
(D) 

Sol. 

Ans. (A)

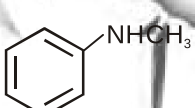
102. The maximum number of isomers that can result from monobromination of 2-methyl-2-pentene with N-bromosuccinimide in boiling CCl_4 is

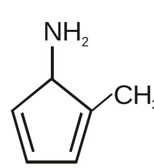
(A) 1 (B) 2 (C) 3 (D) 4

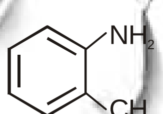
Sol. 

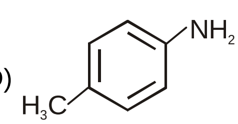
Ans. (D)

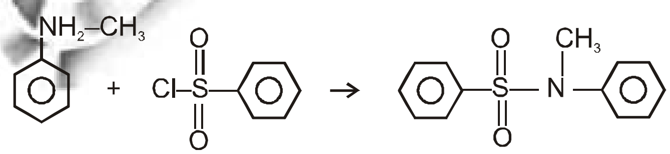
103. The compound X (C_7H_9N) reacts with benzensulfonyl chloride to give Y ($C_{13}H_{13}NO_2S$) which is insoluble in alkali. The compound X is

(A) 

(B) 

(C) 

(D) 

Sol. 

No acidic hydrogen (insoluble in alkali)

Ans. (A)

104. In 108 g of water, 18 g of a non-volatile compound is dissolved. At 100°C the vapor pressure of the solution is 750 mm Hg. Assuming that the compound does not undergo association or dissociation, the molar mass of the compound in g mol^{-1} is

(A) 128 (B) 182 (C) 152 (D) 228

Sol. $\frac{n}{N} = \frac{P_O - P_S}{P_O}$

$\frac{18}{m} \times \frac{18}{108} = \frac{760 - 750}{760}$

$\frac{18}{6m} = \frac{10}{760}$

6m = 18 × 76

m = 228

Ans. (D)

105. The standard electrode potential of Zn^{2+}/Zn is –0.76 V and that of Ca^{2+}/Cu is 0.34 V. The emf (V) and the free energy change (kJ mol^{-1}), respectively, for a Daniel cell will be

(A) – 0.42 and 81 (B) 1.1 and –213 (C) –1.1 and 213 (D) 0.42 and –81

Sol. $E^0_{Cell} = E^0_{Cu^{2+}|Cu} + E^0_{Zn|Zn^{2+}}$

$E^0_{Cell} = 0.34 + 0.76 = +1.1V$ $\because \Delta G = -nFE^0_{Cell}$

$\Delta G = -2 \times 96500 \times 1.1 = -213$ V

Ans. (B)

106. Consider the equilibria (1) and (2) with equilibrium constants K_1 and K_2, respectively

$SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$ .... (1)

$2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$ .... (2)

K_1 and K_2 are related as

(A) $2K_1 = K_2^2$ (B) $K_1^2 = \frac{1}{K_2}$ (C) $K_2^2 = \frac{1}{K_1}$ (D) $K_2 = \frac{2}{K_1^2}$

Sol. $K_1 = \frac{[SO_3]}{[O_2]^{\frac{1}{2}}[SO_2]}$

$K_2 = \frac{[SO_2]^2[O_2]}{[SO_3]^2}$

Hence: $K_2 = \frac{1}{K_1^2}$

$K_1^2 = \frac{1}{K_2}$

Ans. (B)

107. Aqueous solution of metallic nitrate X reacts with NH_4OH to form Y which dissolves in excess NH_4OH. The resulting complex is reduced by acetaldehyde to deposit the metal. X and Y, respectively, are

(A) $Cs(NO_3)$ and CsOH (B) $Zn(NO_3)_2$ and ZnO

(C) $AgNO_3$ and Ag_2O (D) $Mg(NO_3)_2$ and $Mg(OH)_2$

Sol. $AgNO_3 + NH_4OH \longrightarrow AgOH \xrightarrow[\text{(excess)}]{NH_4OH} [Ag(NH_3)_2]^+$

Tollen's reagent + R – CHO ⟶ R – COOH

108. The density of eq. wt of a metal are 10.5 g cm^{-3} and 100, respectively. The time required for a current of 3 amp to deposit a 0.005 mm thick layer of the same metal on an area of 80 cm^2 is closest to

(A) 120 s (B) 135 s (C) 67.5 s (D) 270 s

Sol. $w = \dfrac{E \times i \times t}{96500}$

$w = \dfrac{100 \times 3 \times t}{96500}$

$d = \dfrac{m}{V}$ $m = dV$

$w = 10.5 \times 80 \times 5 \times 10^{-4} = 42 \times 10^{-2}$

$42 \times 10^{-2} = \dfrac{100 \times 3 \times t}{96500}$

t = 135 sec.

Ans. (B)

109. The amount of $Na_2S_2O_3.5H_2O$ required to completely reduce 100 mL of 0.25 N iodine solution, is

(A) 6.20 g (B) 9.30 g (C) 3.10 g (D) 7.75 g

Sol. $N_1V_1 = N_2V_2$

mili eq. of hypo = 0.25 × 100

mili eq. of hypo= 25

eq. of hypo = 0.025

mole of hypo = 0.025 × 1 $\because V_f = 1$

weight of hypo = 0.025 × 248 = 6.2 g

Ans. (A)

110. In aqueous solution, $[Co(H_2O)_6]^{2+}$ (X) reacts with molecular oxygen in the presence of excess liquor NH_3 to give a new complex Y. The number of unpaired electrons in X and Y are, respectively

(A) 3, 1 (B) 3, 0 (C) 3, 3 (D) 7, 0

Sol. $[CO(H_2O)_6]^{2+} + 6NH_3 \longrightarrow [CO(NH_3)_6]^{2+}$

Paramagnetic
n = 3 ; μ = 3.9

O_2 ↓ $-e^-$ (oxidation)

$[CO(NH_3)_6]^{3+}$
Diamagnetic
μ = 0 ; n = 0

Ans. (B)

BIOLOGY

111. 10^9 bacteria were spread on an agar plate containing penicillin. After incubation overnight at 37°C, 10 bacterial colonies were observed on the plate. That the colonies are likely to be resistant to penicillin can be tested by

(A) measuring their growth rate

(B) observing the colour of the colonies

(C) checking their ability to grow on another plate containing penicillin.

(D) checking their ability to cause disease

112. Watson and Crick model of DNA is

(A) B-form DNA with a spiral length of 34 Å and a diameter of 20 Å

(B) A-form DNA with a spiral length of 15 Å and a diameter of 20 Å

(C) Z-form DNA with a spiral length of 34 Å and a diameter of 20 Å

(D) B-form DNA with a spiral length of 28 Å and a diameter of 14 Å

Ans. (A)

113. Eco RI and Rsa I restriction endonucleases require 6 bp and 4 bp sequences respectively for cleavage. In a 10 kb DNA fragment how many probable cleavage sites are present for these enzymes

(A) 0 Eco RI and 10 Rsa I

(B) 1 Eco RI and 29 Rsa I

(C) 4 Eco RI and 69 Rsa I

(D) 2 Eco RI and 39 Rsa I

Sol. For EcoR I $\left(\frac{1}{4}\right)^6 \times 10000 = 2.44$ sites present

For RsaI $\left(\frac{1}{4}\right)^4 \times 10000 = 39.06$

Using probability rules

Ans. (D)

114. From an early amphibian embryo the cells that would give rise to skin in adults were transplanted into the developing brain region of another embryo. The transplanted cells developed into brain tissue in the recipient embryo. What do you infer from this experiment?

(A) Cell fate is permanently determined during early embryonic development.

(B) Developmental fate of donor cells is influenced by the surrounding cells.

(C) Developmental fate of donor cells is not influenced by recipient cells.

(D) Any cell which is transplanted into another embryo always develops into a brain.

Sol. Early embryonic cells have yet not committed and differentiated.

Ans. (B)

115. Presence of plastids in Plasmodium suggests

(A) it is a plant species

(B) it is a parasite with a cynobacterium as an endosymbiont

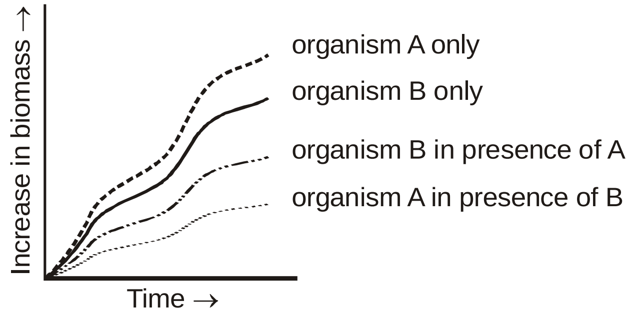
(C) it is a parasite with a archebacterium as an endosymbiont

(D) it is a plant species with a archebacterium as an endosymbiont

Sol. (Secondary Endosymbiont) Apicoplast i.e: non photosynthetic plastid is present in plasmodium (protista)

Ans. (B)

116. The figure below demonstrates the growth curves of two organisms A and B growing in the same area. What kind of relation exists between A and B?

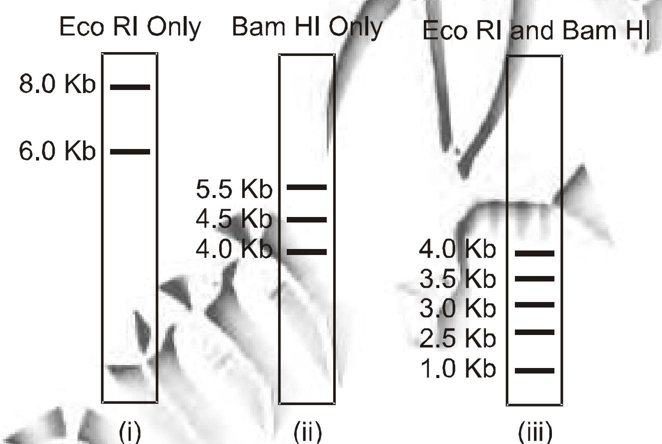


(A) Competition
(B) Symbiosis
(C) Commensalisms
(D) Mutualism

Sol. Competition is harmful for both species (A is affected more)

Ans. (A)

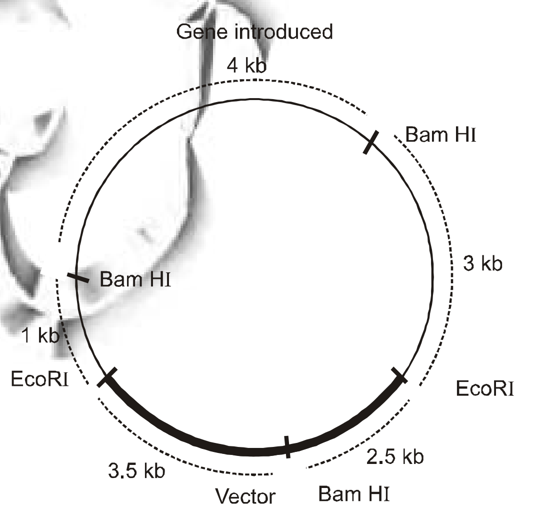
117. A scientist has cloned an 8 Kb fragment of a mouse gene into the Eco RI site of a vector of 6 Kb size. The cloned DNA has no other Eco RI site within. Digestions of the cloned DNA is shown below.



Which one of the following sets of DNA fragments generated by digestion with both Eco RI and Bam HI as shown in (iii) is from the gene?

(A) 1 Kb and 4 Kb
(B) 1 Kb and 2.5 Kb
(C) 1 Kb and 3 Kb
(D) 1 Kb and 3.5 Kb

Sol.



Restriction map as per data is shown above. Thus 1 kb, 3 kb, 4 kb are fragments of gene

Ans. (A,C)

118. Brown fat is a specialised adipose tissue with abundant mitochondria and rich blood supply. Brown fat

(A) insulates animals that are acclimatised to cold.

(B) is the major source of heat production of birds.

(C) provides energy to muscles.

(D) produces heat without producing ATP.

Sol. Brown fat is abundant in bibernating mammals and new born babies (mammals only)

Ans. (D)

119. In some species, individuals forego reproduction and help bring up another individual's offspring. Such altruistic behaviour CANNOT be explained by which of the following?

(A) An individual helps relatives only and gets indirect genetic benefits.

(B) The individual benefits because it can later inherit the breeding position.

(C) The individual benefits because it gets access to resources, such as food and security from predators, in return.

(D) The species benefits from a reduction in competition among offspring.

Ans. (C)

120. Lions in India are currently restricted to Gir, Gujarat. Efforts are being made to move them to other parts of the country. This is because they are MOST susceptible to extinction due to infectious diseases under the following conditions when present as

(A) several small, isolated populations

(B) one large population

(C) several large, connected populations

(D) several large, isolated populations

Ans. (B)