BTC – 11

I Semester B.Tech. Examination, Feb./March 2010
ENGINEERING MATHEMATICS – I

Time : 3 Hours Max. Marks : 80

Instructions *: 1) Answer* ***all*** *questions in Part* ***A****,* ***6*** *out of 8 questions in Part* ***B*** *and* ***3*** *out of 5 questions in Part* ***C****.*
2) Part ***A*** *: Questions from* ***1*** *to* ***8*** *carry* ***1*** *mark* ***each*** *and* ***9*** *to* ***14*** *carry* ***2*** *marks* ***each****.*
3) Part ***B*** *:* ***Each*** *question carries* ***5*** *marks.*
4) Part ***C*** *:* ***Each*** *question carries* ***10*** *marks.*

PART – A

1. Find the real part of Cosh(x + iy).
2. If z is a complex number then find the imaginary part of log z.
3. State Cayley-Hamilton theorem.
4. Define permutation groups.
5. If x = a Sint, y = a Sinpt then find $\frac{dy}{dx}$.
6. With usual notations, write the pedal equation of the curve r = $f(\theta)$.
7. Evaluate $_{-1}\int^{1} x\, e^{-x}\, dx$.
8. State the condition for exactness of a differential equation Mdx + Ndy = 0.
9. Find the real and imaginary part of $(1 - \cos\alpha + i\sin\alpha)$.
10. Obtain the characteristic polynomial of $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$.

P.T.O.

11. Find the vector perpendicular to the $6\hat{i} - 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j} - 2\hat{k}$.

12. If $Z = \log\left[\sqrt{(x^2 + y^2)}\right]$ then find $\frac{dy}{dx}$.

13. Evaluate: $\int_0^1 \frac{(\text{Sin}^{-1}x)^2}{\sqrt{1-x^2}}\,dx$.

14. Solve: $y\sqrt{(1 - x^2)}\,dy + x\sqrt{(1 - y^2)}\,dx = 0$.

PART – B

1. Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

2. If $\text{Cos}\alpha + \text{Cos}\beta + \text{Cos}\gamma = 0 = \text{Sin}\alpha + \text{Sin}\beta + \text{Sin}\gamma$ then prove that

   i) $\text{Cos}3\alpha + \text{Cos}3\beta + \text{Cos}3\gamma = 3\text{Cos}(\alpha + \beta + \gamma)$

   ii) $\text{Sin}3\alpha + \text{Sin}3\beta + \text{Sin}3\gamma = 3\text{Sin}(\alpha + \beta + \gamma)$.

3. Solve by Crammer's rule the following equations.

   $5x - 2y - 3z = 17$, $3x - y + z = 15$, $x + y - 6z = -13$.

4. If $y = a\,\text{Cos}(\log x) + b\,\text{Sin}(\log x)$ then prove that

   $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

5. If $y = \tan^{-1}x$, then prove that $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$.

6. In a group G having more than one element, if $x^2 = x$ for every $x \varepsilon G$, prove that G is abelian.

7. Prove that every group of order 3 is abelian.

8. Evaluate : (i) $\text{Lim}_{x\to 0} \frac{x^2\text{Sin}(Y_x)}{\text{Sin}x}$

   (ii) $\text{Lim}_{x\to\pi/2} (\text{Cos}x)^{(\pi/2 - x)}$.

PART – C

1. Verify Cayley-Hamilton theorem for A = $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

   Also find A^4, Verify whether A^{-1} exists, if so find A^{-1}.

2. Solve: $\frac{dy}{dx} = \frac{4x - 6y - 1}{2x - 3y + 2}$.

3. i) Find the values of 'a' and 'b' such that

   $$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x = 1}{x^3}$$

   ii) $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$.

4. Obtain the reduction formula for $I_{m,n} = \int \sin^m x \cos^n x \, dx$.

5. With usual notation prove that $\tan \phi = r \frac{d\theta}{dr}$.
