Total No. of Questions : 12] [Total No. of Printed Pages : 7

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F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

*(1) Answers to the **two sections** should be written in **separate answer books**.*

(2) Black figures to the right indicate full marks.

(3) Neat diagrams must be drawn wherever necessary.

(4) Assume suitable data, if necessary.

(5) Use of electronic pocket calculator is allowed.

SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where **[05]**

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations : **[06]**

$x + y - z + t = 2$

$2x + 3y + 4t = 9$

$y - 2z + 3t = 2$

(C) Verify Cayley Hamilton Theorem for the matrix **[07]**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR

Q.2) (A) Find Eigen Values and corresponding Eigen Vectors for the matrix **[07]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(B) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

$X_1 = (2, -2, 4)$, $X_2 = (-1, 3, -3)$, $X_3 = (1, 1, 1)$ **[05]**

(C) Find values of a, b, c so that the matrix

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$

becomes an orthogonal matrix. **[06]**

Q.3) (A) If $\frac{Z-1}{Z+i}$ is a purely imaginary number, then show that the locus of Z is a circle. **[06]**

(B) Show that the continued product of all values of $(1 + i\sqrt{3})^{\frac{1}{4}}$ is $2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$ **[05]**

(C) If α, β are roots of an equation,

$\sin^2\theta \; z^2 - (2\sin\theta\cos\theta)\; z + 1 = 0$, prove that

$\alpha^n + \beta^n = 2\cos n\theta \; \mathrm{cosec}^n\theta$, where n is an integer. **[05]**

OR

Q.4) (A) Find tanhx if $5 \sinh x - \cosh x = 5$ **[05]**

(B) If $u + iv = \sin(x + iy)$,

prove that :

(a) $u^2\mathrm{cosec}^2x - v^2\sec^2x = 1$

(b) $u^2\mathrm{sech}^2x + v^2\mathrm{cosech}^2x = 1$ **[05]**

(C) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point $3 + 2i$. Find the rest two vertices of the square. **[06]**

Q.5) (A) If $y = \dfrac{x^3}{x^2 - 1}$,

then find n^{th} order differential coefficient of y w.r. to x. **[05]**

(B) If $y = \sin^{-1}[3x - 4x^3]$,

prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0$. **[05]**

(C) Test convergence of the series : **(Any One)** **[06]**

(a) $1 + \dfrac{1}{2} + \dfrac{1 \cdot 3}{2 \cdot 4} + \dfrac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \ldots\ldots$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 1)\, x^n}{n^2}$

OR

Q.6) (A) If $y = (2x + 1) \log (4x + 3)$,

then find y_{20}. [05]

(B) If $y = \left[x + \sqrt{x^2 - 1}\right]^m$,

prove that $(x^2 - 1) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - m^2) y_n = 0$. [05]

(C) Test convergence of the series : **(Any One)** [06]

(a) $\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \ldots\ldots\ldots$

(b) $\sum_{n=1}^{\infty} \frac{4.7.10 \ldots\ldots (3n+1)}{1.2.3.4 \ldots\ldots\ldots n}$

SECTION - II

Q.7) (A) Expand $\frac{x}{e^x - 1}$ upto x^4. [05]

(B) Use Taylor's Theorem to obtain approximate value of $\sqrt{10}$ to four decimal places. [05]

(C) Solve : **(Any One)** [06]

(a) Find a and b, if

$$\lim_{x \to 0} \frac{a\sin^2 x + b\log\cos x}{x^4} = -\frac{1}{2}.$$

(b) Evaluate $\lim_{x \to 0} \left(\sin^2 \frac{\pi}{2 - ax}\right)^{\sec^2 \frac{\pi}{2 - bx}}$

OR

Q.8) (A) Expand $\sin^{-1}x$ in ascending powers of x. **[05]**

(B) Expland $3x^2 - 2x^2 + x - 4$ in powers of $(x + 2)$. **[05]**

(C) Solve : **(Any One)** **[06]**

(a) Evaluate $\lim_{x\to 0} \dfrac{\tanh x - 2\sin x + x}{x^5}$

(b) Evaluate $\lim_{x\to 0} \dfrac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$

Q.9) Solve : **(Any Two)** **[16]**

(A) Find value of n so that $u = r^n(3\cos^2\theta - 1)$

satisfies $\dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial u}{\partial r}\right) + \dfrac{1}{\sin\theta} \cdot \left(\sin\theta \dfrac{\partial u}{\partial \theta}\right)$.

(B) If $ux + vy = 0$; $\dfrac{u}{x} + \dfrac{v}{y} = 1$,

show that : $\left(\dfrac{\partial v}{\partial y}\right)_x - \left(\dfrac{\partial u}{\partial x}\right)_y = \dfrac{x^2 + y^2}{x^2 - y^2}$

(C) If $x = e^u \operatorname{cosec} v$; $y = e^u \cot v$, then

show that : $\left(\dfrac{\partial z}{\partial x}\right)^2 - \left(\dfrac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\dfrac{\partial z}{\partial u}\right)^2 - \sin^2 v\left(\dfrac{\partial z}{\partial v}\right)^2\right]$

OR

Q.10) Solve : **(Any Two)** **[16]**

(A) If $u = (2x + 3y)^n + \frac{1}{(x - y)^n}$,

show that : $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} + x u_x + y u_y = n^2 u$.

(B) If $x^3 + y^3 - 3axy = 0$,

show that : $\frac{d^2y}{dx^2} + \frac{2a^3 xy}{\left(y^2 - ax\right)^3} = 0$.

(C) If $f(xy^2, z - 2x) = 0$,

prove that : $x \frac{\partial z}{\partial x} - \frac{y}{2} \frac{\partial z}{\partial y} = 2x$.

Q.11) (A) The area of ΔABC is calculated using the formula $\Delta = \frac{1}{2} ab\sin C$. Errors of 2%, 3%, 4% are made in measuring a, b, C respectively. If the correct value of C is 30°, find % error in the calculated value of Δ. **[06]**

(B) If $x = u + v$; $y = v^2 + w^2$; $z = w^3 + u^3$,

show that : $\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}$. **[06]**

(C) Find Stationary values of $u = x + y + z$ if $xy + yz + zx = 3a^2$. **[06]**

OR

Q.12) (A) Verify $JJ' = 1$ for $x = e^{u}\tan v$ and $y = e^{u}\sec v$. **[06]**

(B) Examine for functional dependence/independence. If dependent, find relation between them :

$$u = \frac{x - y}{x + a} \quad ; \quad v = \frac{x + a}{y + a}$$ **[06]**

(C) The sum of three positive numbers is 'a'. Determine maximum value of their product. **[06]**
