MECHANICAL DRIVES:

Two groups,

They are

1. Drives that transmit power by means of friction: eg: belt drives and rope drives.
2. Drives that transmit power by means of engagement: eg: chain drives and gear drives.

   However, the selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, velocity ratio, shifting arrangement, Maintenance and cost.

GEAR DRIVES

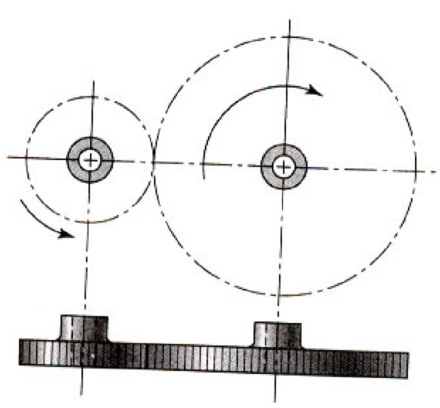
Gears are defined as toothed wheels, which transmit power and motion from one shaft to another by means of successive engagement of teeth

1. The centre distance between the shafts is relatively small.
2. It can transmit very large power
3. It is a positive, and the velocity ratio remains constant.
4. It can transmit motion at a very low velocity.

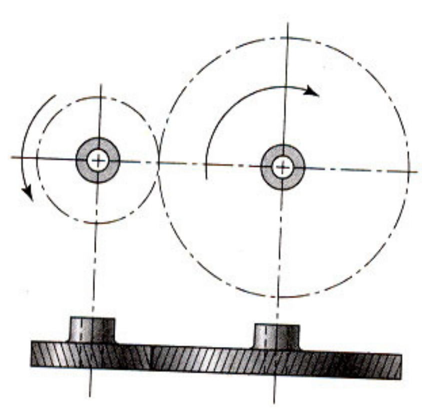
CLASSIFICATION OF GEARS:

Four groups:

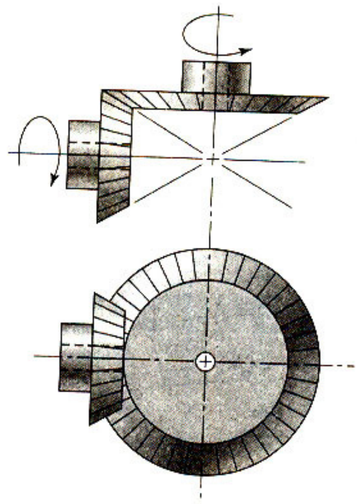
1) Spur Gears
2) Helical gears
3) Bevel gears and
4) Worm Gears



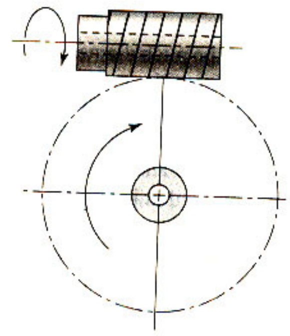
Spur Gear



Helical Gear



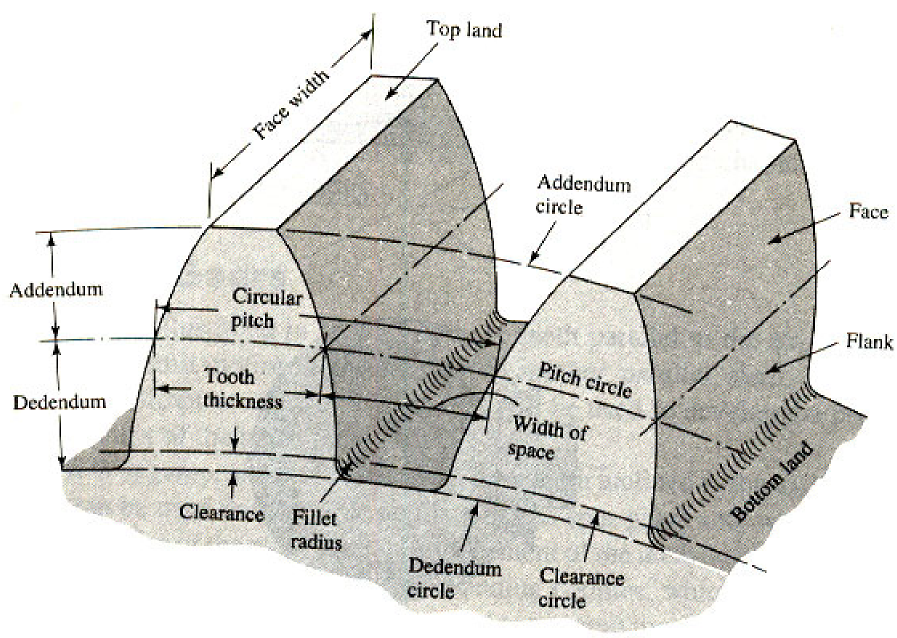
Bevel Gear



Worm Gear Set

NOMEN CLATURE
Spur gears are used to transmit rotary motion between parallel shafts. They are usually cylindrical is shape and the teeth are straight and parallel to the axis of rotation.

In a pair of gears, the larger is often called the GEAR and, the smaller one is called the PINION



Nomenclature of Spur Gear

1. **Pitch Surface:** The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together with out slipping.
2. **Pitch circle:** It is a theoretical circle upon which all calculations are usually based. It is an imaginary circle that rolls with out slipping with the pitch circle of a mating gear. Further, pitch circles of a mating gears are tangent to each other.

3. **Pitch circle diameter:** The pitch circle diameter is the diameter of pitch circle. Normally, the size of the gear is usually specified by pitch circle diameter. This is denoted by “d”
4. **Top land:** The top land is the surface of the top of the gear tooth
5. **Base circle:** The base circle is an imaginary circle from which the involute curve of the tooth profile is generated (the base circles of two mating gears are tangent to the pressure line)
6. **Addendum**: The Addendum is the radial distance between the pitch and addendum circles. Addendum indicates the height of tooth above the pitch circle.
7. **Dedendum**: The dedendum is the radial distance between pitch and the dedendum circles. Dedendum indicates the depth of the tooth below the pitch circle.
8. **Whole Depth**: The whole depth is the total depth of the tooth space that is the sum of addendum and Dedendum.
9. **Working depth:** The working depth is the depth of engagement of two gear teeth that is the sum of their addendums
10. **Clearance:** The clearance is the amount by which the Dedendum of a given gear exceeds the addendum of it’s mating tooth.
11. **Face:** The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called face of the tooth.
12. **Flank:** The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.
13. **Face Width:** is the width of the tooth measured parallel to the axis.
14. **Fillet radius:** The radius that connects the root circle to the profile of the tooth is called fillet radius.
15. **Circular pitch:** is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.
16. **Circular tooth thickness:** The length of the arc on pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically circular tooth thickness is half of circular pitch.
17. **Width of space:** (tooth space) The width of the space between two adjacent teeth measured along the pitch circle. Theoretically, tooth space is equal to circular tooth thickness or half of circular pitch
18. **Working depth:** The working depth is the depth of engagement of two gear teeth, that is the sum of their addendums
19. **Whole depth:** The whole depth is the total depth of the tooth space, that is the sum of addendum and dedendum and (this is also equal to whole depth + clearance)
20. **Centre distance:** it is the distance between centres of pitch circles of mating gears. (it is also equal to the distance between centres of base circles of mating gears)
21. **Line of action:** The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give smooth operation. The force is transmitted from the driving gear to the driven gear on this line.
22. **Pressure angle:** It is the angle that the line of action makes with the common tangent to the pitch circles.

23. **Arc of contact:** Is the arc of the pitch circle through which a tooth moves from the beginning to the end of contact with mating tooth.
24. **Arc of approach:** it is the arc of the pitch circle through which a tooth moves from its beginning of contact until the point of contact arrives at the pitch point.
25. **Arc of recess:** It is the arc of the pitch circle through witch a tooth moves from the contact at the pitch point until the contact ends.

26. **Contact Ratio?**

Velocity ratio: if the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

27. **Module:** It is the ratio of pitch circle diameter in mith meters to the number of teeth. it is usually denoted by ‘m’ Mathematically

$$m = \frac{D}{Z}$$

28. **Back lash**: It is the difference between the tooth space and the tooth thickness as measured on the pitch circle.

29. **Velocity Ratio**: Is the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

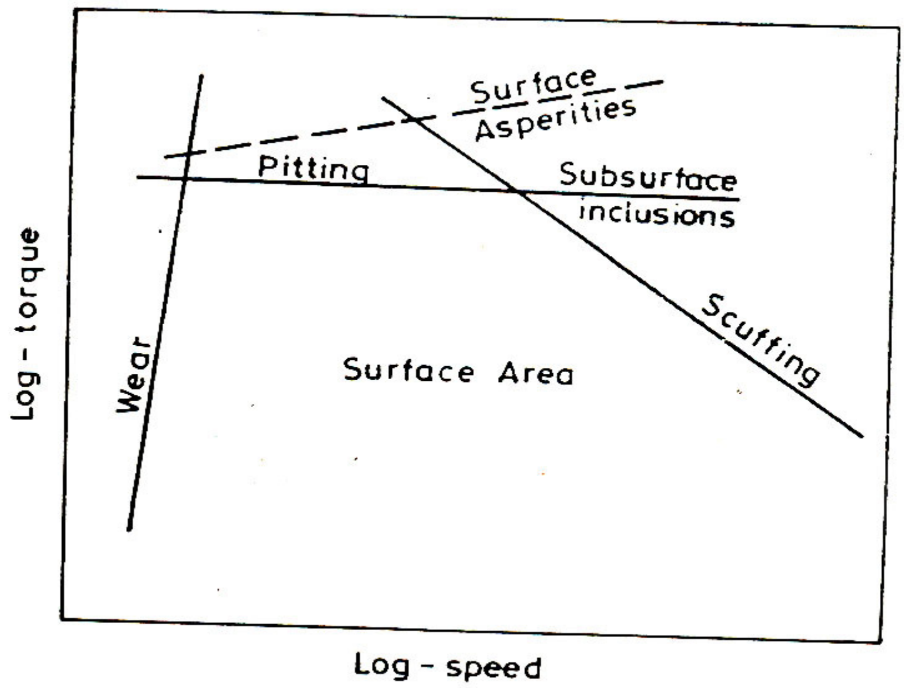
NOTATION

ENGLISH SYMBOLS

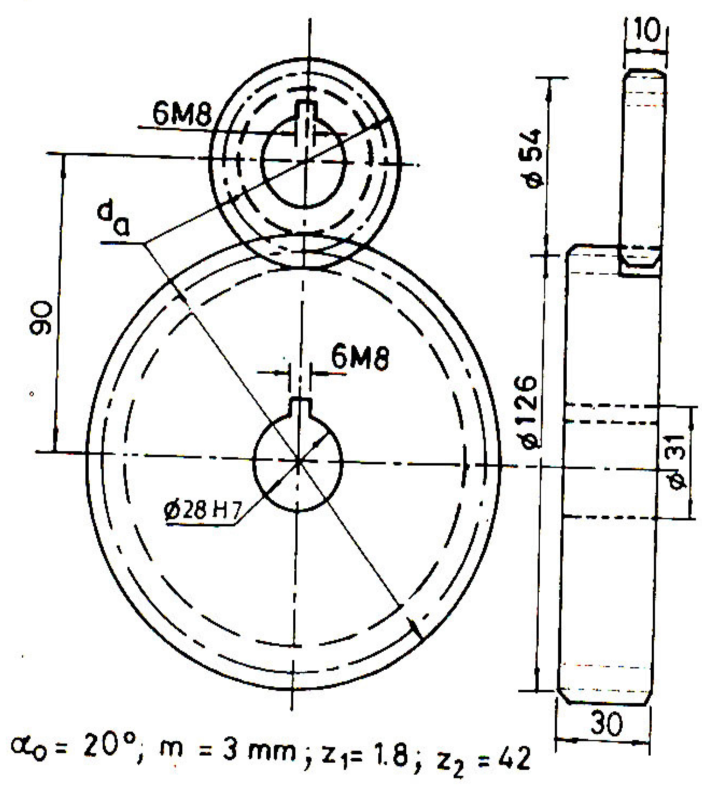
A_o	Centre distance
B	Face width
d_a	Addendem circle diameter
d_o	Pitch circke diameter
d_r	Root circle diameter
m	Module
r_a	Addendem circle radius
r_b	Base circle radius
r_o	Pitch circle radius
R	Radius of curvature of tooth profile
R_g	Gear ratio
Z	Number of teeth
α	Preasure angle
σ	Stress value
σ_b	Bending stress
σ_H	Hertz contact stress
σ_{HB}	Contact stress at the beginning of the engagement
σ_{HE}	Contact stress at the end of the engagement
σ_{HL}	Pitting limit stress
τ	Shear stress
ω	Angle velocity
Suffix 1	Pinion
Suffix 1	Gear

Nomenclature of Spur Gear

Failure Map of Involute Gears



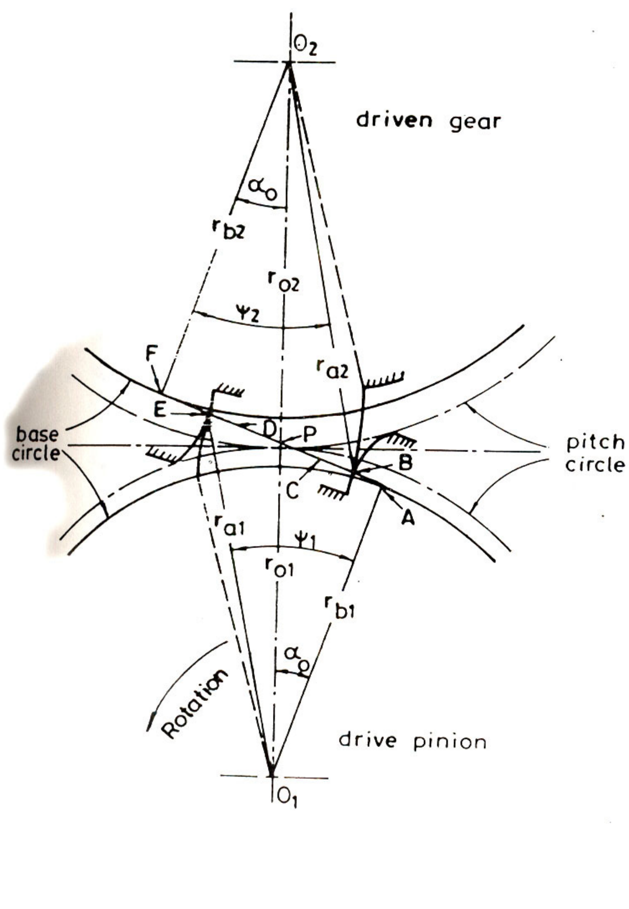
Failure Map of Involute Gears



Gear Set

Specification of Test Pinions and Gears

Variable	Symbol	Unit	Values of variables used in the experiments		
			Pinion		Gears
Module	m	(mm)		3.0	
Pressure angle	α_0	(deg)		20°	
Number of teeth	z	(--)	18		42
Pitch circle diameter	d	(mm)	54.0		126.0
Centre distance	a_o	(mm)		90.0	
Addendum circle diameter	d_a	(mm)	60.0		132.0
Root circle diameter	d_r	(mm)	46.5		118.5
Face width	B	(mm)	10.0		30.0



Different Phases of Gear Tooth Contact

Phase of contact	Position and number of pairs of teeth (J) in contact		Radius of curvature	
			R_1	R_2
Beginning of engagement	B	2	$C_3 - C_2$	C_2
Transition phase	C	2 to 1	$C_1 - C_6$	$C_3 - C_1 - C_6$
Pitch point	P	1	C_4	C_5
Transition phase	D	1 to 2	$C_3 - C_2 + C_6$	$C_2 - C_6$
End of engagement	E	2	C_1	$C_3 - C_1$

Expressions for the Calculation of Equivalent Radii of Curvature at Various Phases of Contact

Design consideration for a Gear drive

In the design of gear drive, the following data is usually given

i. The power to be transmitted
ii. The speed of the driving gear
iii. The speed of the driven gear or velocity ratio
iv. The centre distance

The following requirements must be met in the design of a gear drive

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions
(b) The gear teeth should have wear characteristics so that their life is satisfactory.
(c) The use of space and material should be recommended
(d) The alignment of the gears and deflections of the shaft must be considered because they effect on the performance of the gears
(e) The lubrication of the gears must be satisfactory

Selection of Gears:

The first step in the design of the gear drive is selection of a proper type of gear for a given application. The factors to be considered for deciding the type of the gear are

- ☞ General layout of shafts
- ☞ Speed ratio
- ☞ Power to be transmitted
- ☞ Input speed and
- ☞ Cost

1. Spur & Helical Gears – When the shaft are parallel

2. Bevel Gears – When the shafts intersect at right angles, and,

3. Worm & Worm Gears – When the axes of the shaft are perpendicular and not intersecting. As a special case, when the axes of the two shafts are neither intersecting nor perpendicular crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6: 1. On rare occasions this can be raised to 10: 1. When the velocity ratio increases, the size of the gear wheel increases. This results in an increase in the size of the gear box and the material cost increases. For high speed reduction two stage or three stage construction are used.

The normal velocity ratio for a pair of bend gears is 1: 1 which can be increased to 3: 1 under certain circumstances.

For high-speed reduction worm gears offers the best choice. The velocity ratio in their case is 60: 1, which can be increased to 100: 1. They are widely used in materials handling equipment due to this advantage.

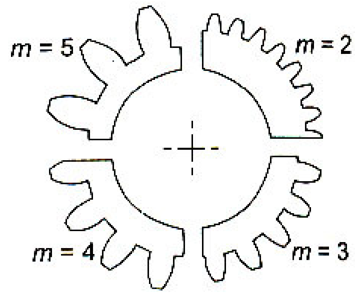
Further, spur gears generate noise in high-speed applications due to sudden contact over the entire face with between two meeting teeth. Where as, in helical gears the contact between the two meshing teeth begans with a point and gradually extends along the tooth, resulting in guite operations.

From considerations spurgears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

Low of Gearing:

The fundamental law of gearing states "The common normal to the both profile at the point of contact should always pass through a fixed point called the pitch point, in order to obtain a constant velocity ratio.

MODULE:
The module specifies the size of gear tooth. Figure shows the actual sizes of gear tooth with four different modules. It is observed that as the modules increases, the size of the gear tooth also increases. It can be said that module is the index of the size of gear tooth.



Standard values of module are as shown.

Recommended Series of Modules (mm)

Preferred (1)	Choice 2 (2)	Choice 3 (3)	Preferred (1)	Choice 2 (2)	Choice 3 (3)
1			8	7	(6.5)
1.25	1.125		10	9	
1.5	1.375		12	11	
2	1.75		16	14	
2.5	2.25		20	18	
3	2.75	(3.25)	25	22	
4	3.5		32	28	
5	4.5	(3.75)	40	36	
6	5.5		50	45	

Note: **The modules given in the above table apply to spur and helical gears. In case of helical gears and double helical gears, the modules represent normal modules**

The module given under choice 1, is always preferred. If that is not possible under certain circumstances module under choice 2, can be selected.

Standard proportions of gear tooth in terms of module m, for 20º full depth system.

Addendum = m
Dedendum = 1.25 m
Cl earance (c) = 0.25 m
Working depth = 2 m
Whole depth = 2.25 m
Tooth thickness = 1.5708 m $\left[\frac{\pi d}{2z} = \frac{\pi mz}{2z}\right] = 1.5708m$
Tooth space = 1.5708 m
Fillet radius = 0.4 m

Standard Tooth proportions of involute spur gear

Gear Terms	Proportions of Machine cut teeth		
	Circular pitch p	Diametral pitch P	Module m
Addendum	0.3183 p	1/P	m
Dedendum	0.3977 p	1.25/P	1.25 m
Tooth thickness	0.5 p	1.5708/P	1.5708 m
Tooth space	0.5 p	1.5708/P	1.5708 m
Working depth	0.6366 p	2/P	2 m
Whole depth	0.7160 p	2.25/P	2.25 m
Clearance	0.0794 p	0.25/P	0.25 m
Pitch diameter	zp/π	z/P	zm
Outside diameter	(z+2)p/π	(z+2)/P	(z+2) m
Root diameter	(z - 2.5)p/π	(z – 2.5)/P	(z – 2.5) m
Fillet radius	0.1273p	0.4/P	0.4 m

<u>Selection of Material :</u>

- The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or yield strength of the material.
- When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the deciding factor.
- The gear material should have sufficient strength to resist failure due to breakage of tte tooth.
- In many cases, it is wear rating rather than strength rating which decides the dimensions of gear tooth.
- The resistance to wear depends upon alloying elements, grawn size, percentage of carbon and surface hardness.
- The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
- For high-speed power transmission, the sliding velocities are very high and the material should have a low co-efficient of friction to avoid failure due to scoring.
- The amount of thermal distortion or wrapping during the heat treatment process is a major problem on gear application.
- Due to wraping the load gets concentrated at one corner of the gear tooth.
- Alloy steels are superior to plain carbon steel in this respect (Thermal distortion)

Allowable Static Stresses σ_d to use in Lewis formulae

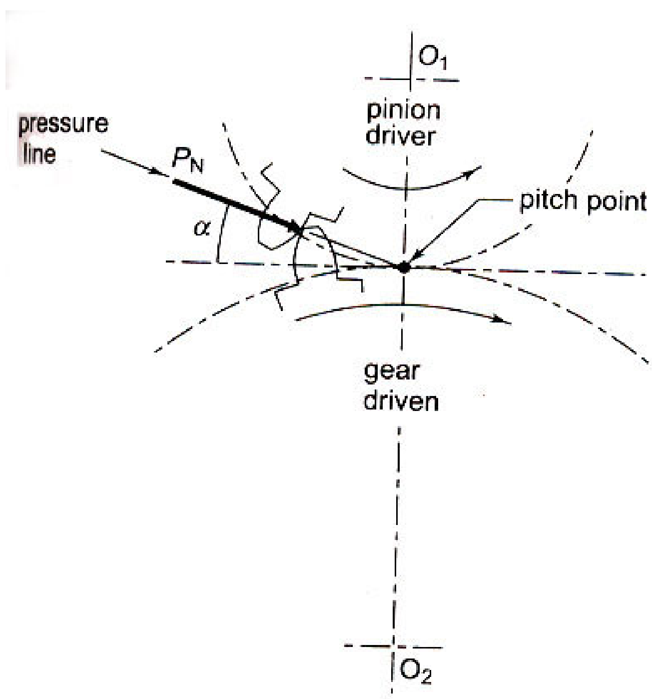
Material		Allowable static stress σ_d, MN/m^2 (kgf/mm^2)	BHN
Cast Iron Grade 20	..	47.1 (4.80)	200
Cast Iron Grade 25	..	56.4 (5.75)	220
Cast Iron Grade 35	..	56.4 (5.75)	225
Cast Iron Grade 35 (Heat treated)	..	78.5 (8.00)	300
Cast steel, 0.20%C, untreated	..	138.3 (14.10)	180
Cast steel, 0.20%C, heat treated	..	193.2 (19.70)	250
Bronze	..	68.7 (7.00)	80
Phosphor gear bronze	..	82.4 (8 .40)	100
Manganese bronze	..	138.3 (14.10)	100
Aluminium bronze	..	152.0 (15.50)	180
Forged steel, about 0.30%C (untreated)	..	172.6 (17.60)	150
Forged steel, about 0.30%C (heat treated)	..	220.0 (22.40)	200
Steel, C30 (heat treated)	..	220.6 (22.50)	300
Steel, C40, untreated	..	207.0 (21.10)	150
Steel, C45, untreated	..	233.4 (23.80)	200
Alloy steel, case hardened	..	345.2 (35.20)	650
Cr-Ni Steel, about 0.45%C heat treated	..	462.0 (47.10)	400
Cr-Va steel, about 0.45%C, heat treated	..	516.8 (52.70)	450
Rawhide, Fabroil, etc.	..	41.2 (4.20)	—
Plastic	..	58.8 (6.00)	—
Laminated phenolic materials (Bakelite, Micarta, Celoron)	..	41.2 (4.20)	—
Laminated steel (silent material)	..	82.4 (8.40)	—

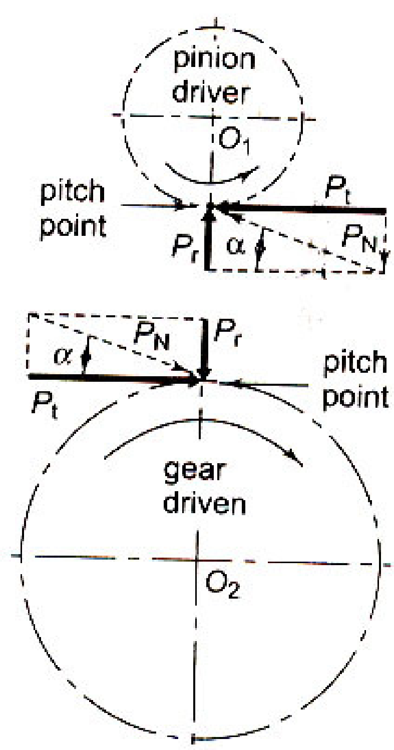
Force analysis – Spur gearing.

We know that, the reaction between the mating teeth occur along the pressure line, and the power is transmitted by means of a force exerted by the tooth of the driving gear on the meshing tooth of the driven gear. (i.e. driving pinion exerting force P_N on the tooth of driven gear).

According to fundamental law of gear this resultant force P_N always acts along the pressure line.

This resultant force $P_{N,}$ can be resolved into two components – tangential component P_t and radial components P_r at the pitch point.





The tangential component P_t is a useful component (load) because it determines the magnitude of the torque and consequently the power, which is transmitted.

The radial component P_r services no useful purpose (it is a separating force) and it is always directed towards the centre of the gear.

The torque transmitted by the gear is given by

$$M_t = \frac{P \times 60}{2\,\pi N_1}\ N-m$$

Where,

M_t = Torque transmitted gears (N- m)
PkW = Power transmitted by gears
N_1 = Speed of rotation (rev / mn)

The tangential component F_t acts at the pitch circle radius.

$$\therefore \quad M_t = F_t \frac{d}{2}$$

OR

$$F_t = \frac{2M_t}{d}$$

Where,

M_t = Torque transmitted gears N- mm
d = Pitch Circle diameter, mm

Further, we know,

Power transmitted by gears $= \dfrac{2\pi\ N\ M_t}{60}$ (kW)

Where

$$F_r = F_t\ \tan\alpha$$

and

resultant force,

$$FN = \frac{F_t}{Cos\alpha}$$

The above analysis of gear tooth force is based on the following assumptions.

i) As the point of contact moves the magnitude of resultant force P_N changes. This effect is neglected.

ii) It is assumed that only one pair of teeth takes the entire load. At times, there are two pairs that are simultaneously in contact and share the load. This aspects is also neglected.

iii) This analysis is valid under static conditions for example, when the gears are running at very low velocities. In practice there are dynamic forces in addition to force due to power transmission.

For gear tooth forces, It is always required to find out the magnitude and direction of two components. The magnitudes are determined by using equations

$$M_t = \frac{P \times 60}{2\pi N_1}$$

$$F_t = \frac{2M_t}{d_1}$$

Further, the direction of two components F_t and F_r are decided by constructing the free body diagram.

?

How

Minimum Number of Teeth:

The minimum number of teeth on pinion to avoid interference is given by

$$Z_{min} = \frac{2}{\sin^2 \alpha}$$

For 20˚full depth involute system, it is always safe to assume the number of teeth as 18 or 20

Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio $i = \frac{Z_2}{Z_1}$

Face Width:

In designing gears, it is required to express the face width in terms of module.

In practice, the optimum range of face width is $9.5m \leq b \leq 12.5m$

Generally, face width is assumed as ten times module

$$\therefore \boxed{b = 12.5m}$$

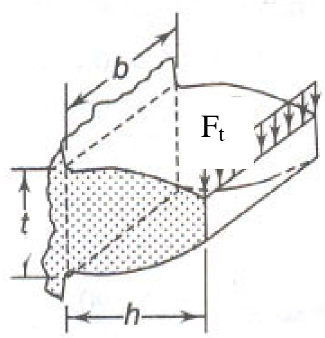
The LEWIS Bending Equation:

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth.

This equation announced in 1892 still remains the basis for most gear design today.

In the lewis analysis, the gear tooth is treated as a cantilever beam and the tangential component (F_t) causes the bending moment about the base of the tooth.

GEAR TOOTH AS CANTILEVER



The Lewis equation is based on the following assumption.

(i) The effect of radial component (F_r) which induces compressive stresses is neglected.

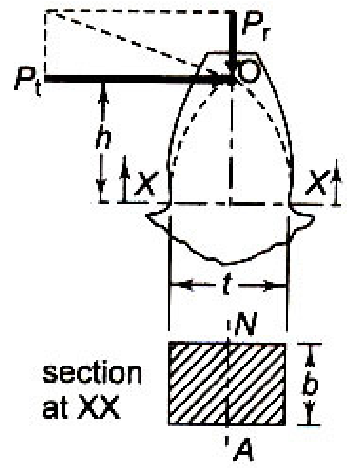
(ii) It is assumed that the tangential component (F_t) is uniformly distributed over the face width of the gear (this is possible when the gears are rigid and accurately machined.)

(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time only one pair of teeth is in contact and Takes total load

It is observed that the cross section of the tooth varies from free end to fixed end. Therefore, a parabola is constructed with in the tooth profile and shown in dotted lines.

Gear tooth as parabolic beam



The advantage of parabolic out lines is that it is a beam of uniform strength, and the stress at any cross section is uniform.

We know

$$M_b = F_t \times h$$

$$I = \frac{bt^3}{12} \; and \qquad y = \frac{t}{2}$$

$$\therefore \sigma_b = \frac{M_b y}{I} = \frac{M_b}{(I / y)} = \frac{M_b}{Z}$$ (Z= Section modulus)

$$= \frac{bt^3}{12} / \frac{t}{2} \qquad = \frac{bt^2}{6}$$

σ_b = Permissible bending stress (N/mm^2) $\quad \frac{6\,F_b \times h}{bt^2}$

$$\therefore F_t = \left[\frac{bt^2 \sigma_b}{6\,h} \right]$$

Multiplying the numerator and denominator of the right hand side by m, (m=Module)

$$F_t = mb\,\sigma_b \left(\frac{t^2}{6hm} \right)$$

The bracketed quantity depends on the from of the tooth and is termed as **lewis from stress factor Y**

Let $\qquad y = \frac{t^2}{6hm}$

Then the equation can be rewritten as $\quad F_t = mb\sigma_b y$

This y is called as lewis form factor

When the stress reaches the permissible magnitude of bending stresses, the corresponding force (F_t) is called the beam strength (S_b)

$$\therefore\ S_b = mb\,\sigma_b\,y$$

Where,

S_b = beam strength of gear tooth (N)

σ_b = Permissible bending stress (N/mm^2)

The above equation is known as LEWIS EQUATION

The values of the lewis from factor y is given in table below,

Values of Tooth-Form Factor (Lewis), Load at Tip of Tooth

z	y					
	14½-deg form	14½-deg variable centre distance	20-deg full depth form	20-deg stub-tooth form	Internal Gears: Spur pinion	Internal Gears: Internal gear
12	0.067	0.125	0.078	0.099	0.104	*
13	0.071	0.123	0.083	0.103	0.104	*
14	0.075	0.121	0.088	0.108	0.105	*
15	0.078	0.120	0.092	0.111	0.105	*
16	0.081	0.120	0.094	0.115	0.106	*
17	0.084	0.120	0.096	0.117	0.109	*
18	0.086	0.120	0.098	0.120	0.111	*
19	0.088	0.119	0.100	0.123	0.114	*
20	0.090	0.119	0.102	0.125	0.116	*
21	0.092	0.119	0.104	0.127	0.118	*
22	0.093	0.119	0.105	0.129	0.119	*
24	0.095	0.118	0.107	0.132	0.122	*
26	0.098	0.117	0.110	0.135	0.125	*
28	0.100	0.115	0.112	0.137	0.127	0.220
30	0.101	0.114	0.114	0.139	0.129	0.216
34	0.104	0.112	0.118	0.142	0.132	0.210
38	0.106	0.110	0.122	0.145	0.135	0.205
43	0.108	0.108	0.126	0.147	0.137	0.200
50	0.110	0.110	0.130	0.151	0.139	0.195
60	0.113	0.113	0.134	0.154	0.142	0.190
75	0.115	0.115	0.138	0.158	0.144	0.185
100	0.117	0.117	0.142	0.161	0.147	0.180
150	0.119	0.119	0.146	0.165	0.149	0.175
300	0.122	0.122	0.150	0.170	0.152	0.170
Rack	0.124	0.124	0.154	0.175		

*Internal gears with less than 28 teeth must be designed specially for the particular application, and their values of *y* must be determined for each one individually.

In order to avoid the breakage of gear tooth due to bending, the beam strength should be more than the effective force between the meshing teeth

In design of gears, It is required to decide the weaker between pinion and gear.

When the same material is used for pinion and gear, the pinion is always weaker than the gear-------------- **Why?**

- We know that $S_b = mb\sigma_b y$
- It can be observed that 'm' and 'b' are same for pinion and as well as for gear in a gear pair,
- When different materials are used, the product $\sigma_b . y$ decides the weaker between the pinion and gear
- The lewis form factor y is always less for pinion compared to gear
- Therefore, when the same material is used for pinion and gear, the pinion is always weaker than the gear.

Effective load-Calculation

Earlier we have seen how to determine the tangential component of the resultant force between two meshing teeth.

This component can be calculated by using

I. $$M_t = \frac{P \times 60}{2\pi N_1}$$

And

II. $$F_t = \frac{2M_t}{d_1}$$

The value of the tangential component, depends upon rated power and rated speed.

In gear design, the maximum force (due to maximum torque) is the criterion. This is accounted by means of a factor called service factor – (C_s)

This service factor (C_s) is defined as

$$C_s = \frac{Maximum\,Torque}{Rated\ Torque}$$

$$\therefore C_s = \frac{(M_t)_{\max}}{M_t} = \frac{(F_t)_{\max}}{F_t}$$

Where, (F_t) is the tangential force due to rated torque (M_t)

$$(F_t)_{\max} = C_s F_t$$

The values of service factors are given in table…

Service factor C_s for gears

Type of load	Type of service		
	Intermittent or 3 h per day	8 to 10 h per day	Continuous 24 h/day
Steady	0.80	1.0	1.25
Ligh shocks	1.00	1.25	1.50
Medium shocks	1.25	1.50	1.80
Heavy shocks	1.50	1.80	2.00

We know, that

σ_b is permissible static bending stress which is modified to $C_v \sigma_b$ where, C_v is the velocity factor used for taking into account the fatigue loading

This velocity factor C_v developed by. **Carl. G. Barth,** expressed in terms of pitch line velocity.

The values of velocity factor are as below

(i) $C_v = \frac{3}{3+V}$, for ordinary and commercially cut gears (made with form cutters) and V<10 m / Sec

(ii) $C_v = \frac{6}{6+V}$, For accurately hobbed and generated gears and V < 20 m/Sec.

(iii) $C_v = \frac{5.6}{5.6+\sqrt{v}}$, For precision gears with shaving grinding and lapping and V > 20 m/Sec

Where, v = pitch line Velocity (m/Sec)

$$= \frac{\pi dn}{60 \times 10^3}$$ d, mm

n, rev/min

(The velocity factor is an empirical relationship developed by past experience).

Dynamic effects (Dynamic Tooth Load)

When gears rotate at very low speed, the transmitted load P_t can be considered to be the actual force present between two meshing teeth

However in most of the cases the gears rotate at appreciable speed and it becomes necessary to consider the dynamic force resulting from impact between mating teeth.

The **Dynamic force** is induced due to the following factors

1. Inaccuracies of the tooth profile
2. Errors in tooth spacings
3. Misalignment between bearings
4. Elasticity of parts, and
5. Inertia of rotating masses.

There are two methods to account for Dynamics load.

I. Approxinate estimation by the velocity factor in the preliminary stages of gear design

II. Precise estimation by **Bucking Hams** equation in the final stages of gear design.

Note: Approximate estimation, Using velocity factor (C_v) developed by Barth discussed earlier.

In the final stages of gear desing when gear dimensions are known errors specified and quality of gears determined, the Dynamic load is calculated by equation derived by

Earle Buckingham

Where, F_d = Dynamic load

$= F_t + F_i$

Where, F_t = Tangential tooth load

F_i = Inevement load due to dynamic action

$$F_d = F_t + \frac{k_3 V(Cb + F_t)}{k_3 V + \sqrt{Cb + F_t}}$$

Where,

V = Pitch line Velocity (m/Sec)
C = Dynamic factor (N/mm^2) depending upon machining errors
e = measured error in action between gears in mm
b = face width of tooth (mm)
F_t = tangential force due to rated torque (N)
K_3 = 20.67 in SI units

The Dynamic factor C, depends upon modulus of elasticity of materials for pinion and gear and the form tooth or pressure angle and it is given by

$$C = \frac{e}{K_1\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

Where

K = Constant depending upon the form of tooth – (take from DDH)
E_1 = Modulus of elasticity of pinion material (N/mm^2)
E_2 = Modulus of elasticity of gear material (N/mm^2)

The Values of K, for various tooth forms are given as.

The error, e, in the dynamic load equation is measured error in action between gears in mm

This error depends upon the quality of gear and manufacturing methods.

WEAR TOOTH LOAD

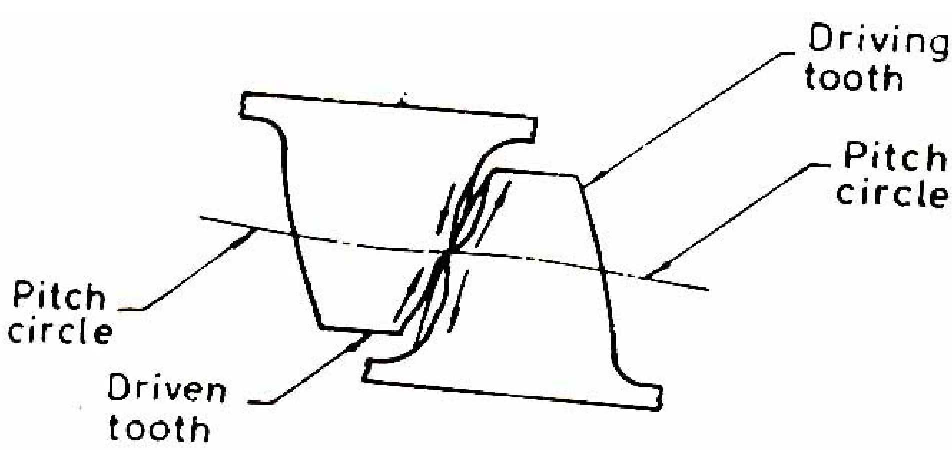
WEAR:
For gears wear is defined as loss of material from contacting surfaces of teeth.

It is further classified as

- Normal wear
- Moderate wear
- Destructive wear
- Abrasive wear
- Scratching and etc.

Generally, normal wear (Polishinging in) does not constitute failure because it involves loss of metal at a rate too slow to affect performance

- Moderate wear refers to loss of metal more rapid than normal wear.
- This need not necessarily be destructive and may develop on heavily loaded gear teeth.
- Destructive wear usually results from loading that is excessive for the lubricant employed.
- The effect of destructive wear on the tooth profile of an involute gear is depicted in the figure.



PITTING
Pitting is the principal mode of failure of rolling surfaces. The details of the process vary with the material and operating conditions, but in all cases it manifests itself by the initiation and propagation of cracks in the near surface layer until microscopic pieces detach themselves to form a pit or a spall.

In spur gears surface pitting has long been recognised as one of the failure modes. This is often refereed to as “Pitch line Pitting”

The main factors affecting pitting type of failure,

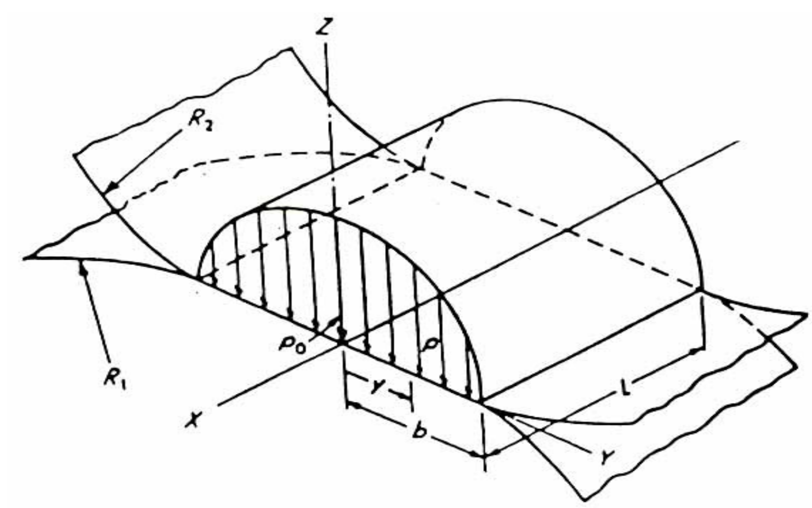
- Contact stress.
- Material pouring and hardness.
- Surface finish and lubrication

Contact stress was originally conceived By “HERTZ” (1896) in whose name it is often referred to as Hertz Contact Stress.

The failure of the gear tooth due to pitting occurs when the contact stress between two meshing teeth exceeds the surface endurance strength the material

In order to avoid this type of failure, the proportions of the gear tooth and surface properties such as surface hardness should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.

The Hertz stress is based on the assumptions of electric and isometric material behaviours, load is compressive and normal to the contacting surfaces which are stationary and the size of contacting area whose dimensions are relatively smaller compared with the curvature radius of the contacting bodies



The above figure,
Illustrates the contact are and corresponding stress distribution between two cylinders.

Here the area of contact stress which is theoretically rectangular with one dimension being the cylinder length L. (i.e. corresponding to face width of the gear)

The distribution of pressure is represented by a semi elliptical prism and the maximum contact pressure Po exists on the load axis,

The current gear design practice is to estimate the contact stress at the pitch point of the teeth by assuming line contact between two cylinders whose radil of contact depends on the gear geometry at the pitch point.

The analysis of wear strength was done by Earle Buckingham and was accepted by AGMA (American Gear Manufacturing Association) in 1926. This Buckingham equation gives the wear strength of the gear tooth based on Hertz theory of contact stress.

Hence, the maximum tooth load from wear consideration as evaluated from Hertz contact stress equation applied for pitch point contact is given by

$$F_t = d_1 bQK$$

Where,

d_1 = Pitch circle diameter of pinion in mm.

b = Face width of the pinion in mm.

$$Q = \text{Ratio factor} = \frac{2VR}{VR+1}$$

$$= \frac{2\,Z_2}{Z_2 + Z_1}$$

and

K = Load stress factor (also known as material combination factor in N / mm^2)

This load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle, and the modulus of elasticity of the materials of the gear.

According to Buckingham, this load stress factor is given by

$$K = \frac{(\sigma_{es})^2 \; Sin\;\alpha}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

Where, σ_{es} = Surface endurance limit of a gear pair. (N / mm^2)

= [2.75 (BHN) -70]

Where BHN = the average Brinall hardness number of gear and pinion for the steels

Design procedure for spur gears:

(i) Find design tangential tooth load, from power transmitted and pitch line velocity

$$= F_T = \frac{1000p\;C_s}{V}.$$

(ii) Apply lewis relationship i.e. $F_t = \sigma_d . P_C . b . y = \sigma_d\, C_v\, b\pi\; m\; y$

a) This lewis equation is applied only to the weaker of the two wheels

b) When both the pinion and gear are made of the same material, then pinion is weaker.

c) When the pinion and gear are made of different materials then the product of (σ_o x y) is the deciding factor. The lewis equation is used to that wheel for which (σ_o x y) is less.

d) The product of [(σ_o . C_v) .y] is called as strength factor of the gear

e) The face width may be taken as 9.5m to 12.5m for cut teeth and 6.5m to 9.5 m for cast teeth.

(iii) Calculate the dynamic load (F_d) on the tooth by using Backingham equation, i.e.,

$$F_D = F_t + F_i$$

by

(iv) Find the static tooth load (i.e.,. Beam strength or the endurance strength of the tooth) by using the relation,

$$F_s = \sigma_e \; b \; p_c \; y$$

$$= \sigma_e \; b \; \pi \; m \; y$$

for safety against breakage $F_S > F_d$

(v) Finally find the wear tooth load by using the relation

$$F_W = d_1 b \; Q \; K$$

The wear load (F_W) should not be less than the dynamic load (F_D)

Design a pair of spur gears to transmit 20kW of power while operating for 8 – 10 hrs/day sustaining medium shock, from shaft rotating at 1000 rev/min to a parallel shaft which is to rotate at 310 rev/min. Assume the number of teeth on pinion to be 31 and 20° full depth involute tooth profile. if load factor C = 522.464 N/mm and also for wear load taking load stress factor, K = 0.279 N/mm^2. Suggest suitable hardness. Boththe pinio gears are made of cast steel 0.2% corbon (untreated) whose σ_d = 137.34 N/mm
check the design for dynamic load if

Given: P = 20kW, =20x10^{3}W, Z_1 = 31, Z_2 = 100, V. R = 1:3.225, ∝ = 20°Full depth.
N_1 = 1000 rev/min, N_2 = 310 rev/min

Material: Cast steel 0.2% C, (untreated) σ_d = 137.34 N/mm^2

Type of load: Medium skock, with 8-10hrs/day.

C = dynamic factors depending up on machining errors 522.464 N/mm

K = load stress factor (wear) = 0.279 N/mm^2

<u>Solution:</u>

σ_{d1} **=** Allowable static stress = 207.0 N/mm2(Pinion)
σ_{d2} **=** 138.3N/mm2(Gear)

Let $C_v = \dfrac{3.05}{3.05+V}$ **(assume)**

V = pitch lin velocity $= V = \dfrac{\pi \, d_1 \, N_1}{60}$

$$= \frac{\pi\ m\ Z_1 N_1}{60}$$

$$= \frac{\pi \times m \times 31 \times 1000}{60} = 1623.15m\ \ mm/Sec$$

$$\boxed{V = 1.623m \quad mm/Sec}$$

For, Medium shock, with 08- 10 hrs/day the service factor C_s, for gear, C_s =1.5

The tangential tooth load $= F_t = \frac{1000\ p}{V}.C_s$ P, in kW,

V, m/ Sec

$$= \frac{20 \times 10^3}{1.623m}. \times 1.5$$

$$= \frac{18484}{m}.\ N$$

Now $C_v = \frac{3.05}{3.05 + 1.623\ m.}$

W.K.T, Tooth form factor for the pinion,

$$= \ 0.154 - \frac{0.912}{Z_1} \quad \text{(for 20° full depth)}$$

$$= 0.154 - \frac{0.912}{31}$$

= 0.154 - 0.0294

= 0.1246

and Tooth from factor, for the gear

$$= \ 0.154 - \frac{0.912}{Z_2}$$

$$= 0.154 - \frac{0.912}{100} \qquad (\because Z_2 = 100)$$

= 0.154 - 0.00912

= 0.1448

σ_{d1} x y_1 = 137.34 x 0.01246 = 17.11

σ_{d1} x y_2 = 137.34 x 0.1448 = 19.88

Since (σ_d .y_2)is less than (σ_{d2} .y_2), there fore PINION is WEAKER

Now, by using the lewis equation to the pinion, We have,

$$F_t = \sigma_d C_v b\pi m y_1$$

$$= \frac{18484}{m} = 207 \times \left(\frac{3.05}{3.05 + 1.623 m} \right) \times 10m \times \pi m \times 0.1246$$

$$= \frac{18484}{m} = \frac{2471.37 m^2}{3.05 + 1.623 m}$$

By hit and trial methd,

m =	LHS	RHS
01	18484	528.86
02	9242	1570.12
03	6161	2808.75
04	4621	4143.98
05	3696	5533.74
Let m =	4107	4833.65

M 04.5 Hence, m = module = 4.5 is OK.

But the standard module is 5.0 mm

∴ Let us take **m = 5.0 mm**

Face width = b = 10m (assumed)
= 10 x 5 =50mm

Pitch circle diameter of

i) Pinion , $d_1 = mz_1$
= 5 x 31 = 155mm

ii)Gear, $d_2 = mz_2$
= 5 x 100 = 500mm

$$= F_t = \frac{18484}{m} . = \frac{18484}{5} = 3696.8\ N_s$$

$V = 1.623m = 1.623 \times 5$
$= 8.115$ m/sec

Checking the gear for dynamic and wear loads

We know, that, the Dynamic load on gear tooth

$$F_d = F_t + F_i$$

$$= F_i = \frac{K_3 v\ (Cb + F_t)}{K_3 v + \sqrt{Cb + F_t}}.$$

$$\therefore F_d = 3696.8 + \frac{20.67 \times 8.115\ (522.464 \times 50 + 3696.9)}{20.67 \times 8.115 + \sqrt{522.464 \times 50 + 3696.8}}.$$

$$= 3696.8 + \frac{167.73\ (29820)}{167.73 + 129820}. = 3696.8 + \frac{5001708.6}{167.73 + 172.68}{}_s$$

$$= 3696.8 + \frac{5001708.6}{340.41}.$$

$$= 3696.8 + 14693.18$$

$$\boxed{\therefore\ F_d = 18389.98\text{ N}}$$

Assuming:

$\sigma_{en} = 259.0$ N /mm^2

State tooth load or endurance strength of the tooth $\Big\}$ $= F_{en} = \sigma_{en} .b\pi$

$F_{en} = 259 \times 50 \times \pi \times 5 \times 0.1246$

$= 25345.89$ N

For medium shock taking $F_{en} = 1.35\ F_d$

$= 1.35 \times 18389.98$

$= 24826.473$

i.e., $\frac{F_{en}}{F_d} = \frac{25345.89}{18389.98} = 1.378$

Design is safe

Wear load

W.K.T,

$$Q = \text{Ratio factor} = = \frac{2VR}{VR + 1}.$$

$$= \frac{2 \times 1.3225}{1.3225 + 1}. = 1.138$$

F_w = d, b Q K

= 155 x 50 x 1.138 x 0.279

= 2460.6 N

Is the design is safe from the point of wear?

$\therefore$ find new k

$$= \frac{F_d}{155 \times 50 \times 0.138}$$

$$= \frac{18389}{8819.5}$$

$\therefore$ k = 2.08

Heat treaded for 350 BHN

$\therefore$ $F_w > F_d$ design is safe

A pair of carefully cut spur gears with 20° stub involute profile is used to transmit a maximum power 22.5 kW at 200 rev/min. The velocity ratio is 1:2. The material used for both pinion and gear is medium cast iron, whose allowable, static stress may be taken as 60 Mpa. The approximate center distance may be taken as 600 mm, determine module and face width of the spur pinion and gear. Check the gear pair for dynamic and wear loads

The dynamic factor or deformations factor in Bucking hams dynamic load equation may be taken as 80, and material combination/load stress factor for the wear may be taken as 1.4

Given: VR = 2, N_1 = 200 rev/min, N_2 = 100 rev/ min, P = Power transmitted, 22.5 kW

Center distance = L = 600mm $\sigma_{d_1} = \sigma_{d_2}$ = 60Mpa, C= 80, K= 1.4

Assumption:

i) b = face width = 10m

ii) Steady load condition and 8 – 10 hrs/day

$\therefore$ C_s = 1.0

Both the gear and pinion are mode of the same material. Therefore pinion is weaker and the design will be based on pinion.

W.K.T,
Centre distance between the shafts (L) = 600mm

$$= \frac{d_1 + d_2}{2}.$$

and $$= \frac{d_1 + 2d}{2}. = 600\ mm$$

$\therefore$ d_1 = 400mm = 0.4 m
d_2 = 800mm = 0.8m

V_1 = Pitch line velocity of pinion $= \frac{\pi\ d_1\ N_1}{60}.$

$$V = \frac{\pi \times 0.4 \times 200}{60}. = 4.2 m/\sec$$

Since V_i = pitch line velocity is less than 12 *m/sec* the velocity factor = C_v, may be taken as

$$= \frac{3.05}{3.05 + v}.$$

$$= \frac{3.05}{3.05 + v}. = \frac{3.05}{3.05 + 4.2}. = \frac{3.05}{7.25}.$$

$$= 0.421$$

Now, $$Z_1 = \frac{d_1}{m} = \frac{400}{m}.$$

$\therefore$ y_1 = tooth form factor = $0.175 - \frac{0.910}{Z_1}$ (for 20° stub systems)

$$0.175 - \frac{0.910}{400} m$$

$$= 0.175 - 0.002275\ m$$

W.K.T,

Design tangential tooth load $= F_t = \frac{P \times 10^3}{v\ .} \times C_s$

$$= \frac{22.5 \times 10^3}{4.2} \times 1.0$$

$$= 5357\ N$$

W.K.T,

$F_t = \sigma_d . C_v \, b\pi \, m \times y_1$

$= 60 \times 0.421 \times 10m \times \pi m \times (0.175 - 0.002275m)$

Solving for m, we get m = 6.5

$\therefore$ m = 8.0 (standard)

Face width = b = 10m = 10 x 8 = 80mm

$$Z_1 = \frac{d_1}{m} = \frac{400}{m}. = 50$$

$$Z_2 = \frac{d_2}{m} = \frac{800}{8}. = 100$$

Checking two gears for dynamic and wear load

W.K.T

(i) Dynamic load = $F_d = F_T + F_i$

$$g = F_T + \frac{20.67 \times 4.2\ (80 \times 80 + 53.57)}{20.67 \times 4.2 + \sqrt{(80 \times 80 + 53.57)}}$$

= 5357 + 5273

= 10630 N

W.K.T,

y_1 = Tooth form factor for pinion = 0.175 – 0.002275m

= [0.175 – 0.002275 x 8]

= 0.175 – 0.018200

= 0.1568

Let flexural endurance limit (σe) for cast iron may be taken as 85 Mpa = (85 N/mm)

$$\therefore Fen = \sigma_{en\,.} . b\ \pi\ my$$

$$= 85 \times 80 \times \pi \times 8 \times 0.1568$$

= 26720 N

For steady loads $Fen = 1.25 fd.$

1.25×10610

13287.5 N

W.K.T,

$$Q = \text{Ratio factor} = \frac{2VR}{VR + 1} = \frac{2 \times 2}{2 + 1}$$

$= 1.33$

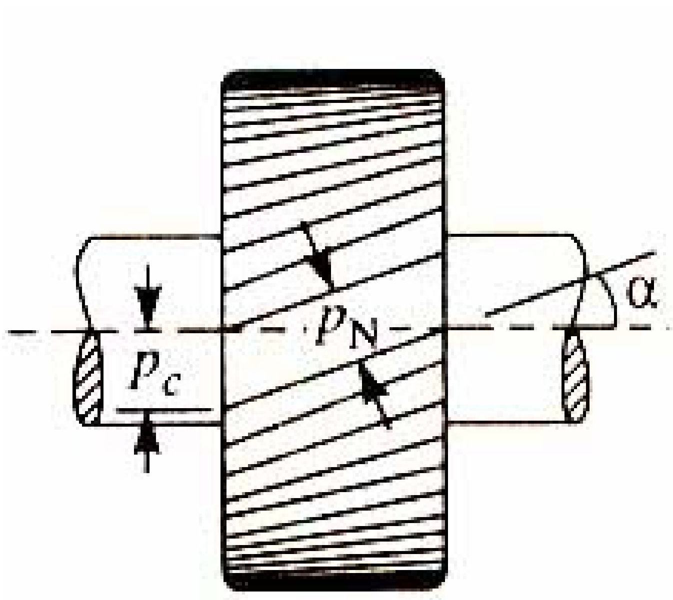
$F_w = d_1\ b\ Q\ K$

$= 400 \times 80 \times 1.33 \times 1.4 = 59584$ N

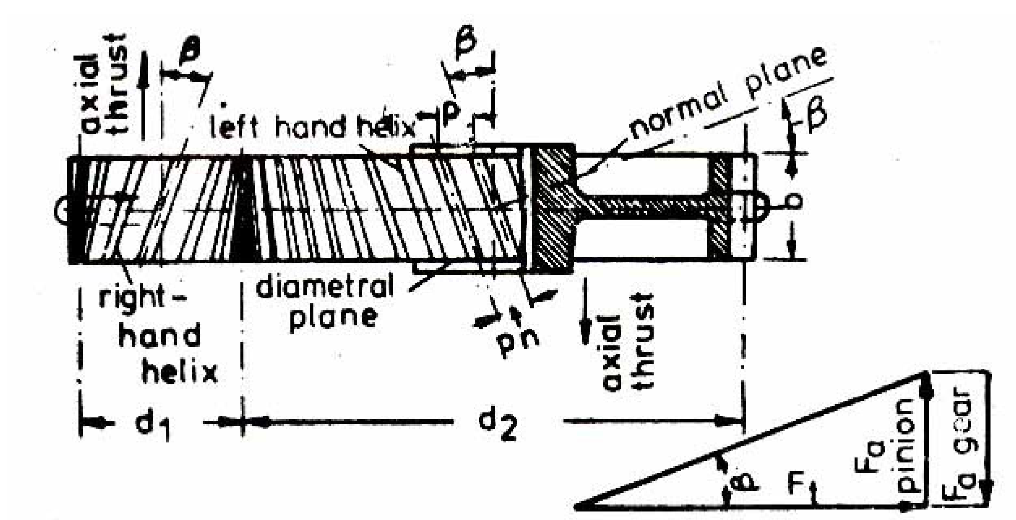
Since both F_{en} and F_w are greater than F_d, the design is safe

Helical Gears:

A Helical gear has teeth in the form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spurgearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contacts as in spurgearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with high efficiency of transmission.



The helical gears may be single helical type or double helical type. In case of single helical type there is some axial thrust between the teeth which is a disadvantage. In order to eliminate this axial trust double helical gears (i.e., herning bone gears) are used. It is equivalent to two single helical gears, In which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.



Terms used:

Helix angle: It is constant angle made by the helices with the axis of rotation

Axial pitch: It is the distance parallel to the axis between similar faces of adjacent teeth. It is same as circular pitch and is therefore denoted by P_C.

Normal pitch: It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by P_N.

$$P_N = P_C \cos \beta$$

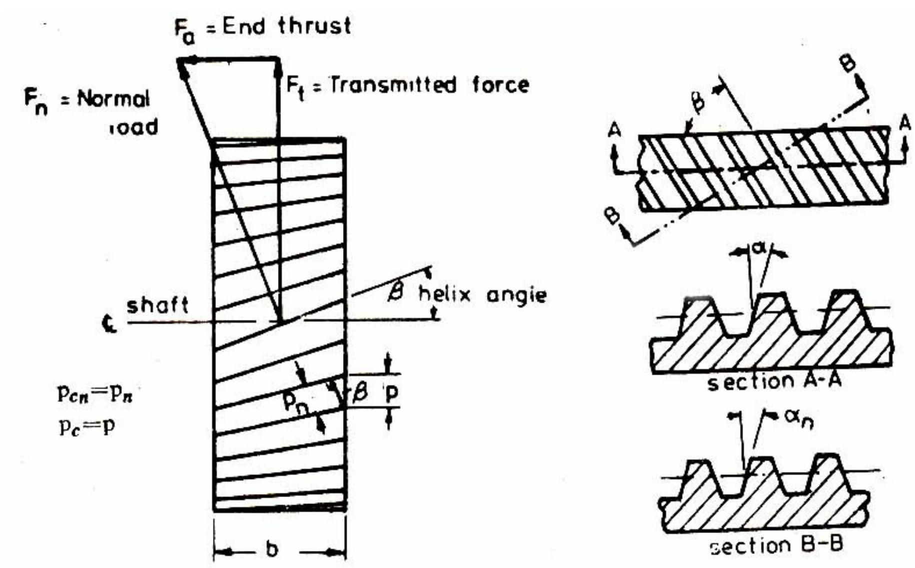
again

$$\tan \propto_N = \tan \propto \cos\beta$$

$\propto_N$ = Normal pressure angle
$\propto$ = Pr. angle

Face width: In order to have more than one pair of teeth in contact, the tooth displacement (i.e., the advancement of one end of tooth over the other end) or over lap should be atleast equal to the axial pitch such that, over lap $P_C = b \tan \beta$--------- (i)

The normal tooth load (F_N) has two components, one is tangential component (F_t) and the other axial component (F_A) as shown in fig



The axial or end thrust is given by

$$F_A = F_N \sin \beta = F_t \tan \beta \text{ --------(ii)}$$

From the above equation (i), we see that as the helix angle increases then the tooth over lop increases. But at the same time the end thrust as given by the equation (ii) also increases which is not desirable. It is usually recommended that the over lop should be 15% of the circular pitch.

Over lop = b tan β = 1.11 P_C

$$\therefore \quad b = \frac{1.11\, P_c}{\tan \beta} \qquad (\because = p_c = \pi_m) \qquad b = \text{minimum face width}$$

m = Module,

Note:

1. The maximum face width may be taken as 12.5 to 20.0m
2. In case of double helical or herring bone gears the minimum face width is given by

$$b = \frac{2.3 P_c}{\tan \beta} \qquad = b = \frac{2.3 \times \pi\ m}{\tan \beta} \qquad \geq = \frac{2.3 \times \pi\ m}{\sin \beta}$$

3. In a single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears it may be made up to 45°

$b = 12.5\ m_n$ To $20.m_n$.

Formative or equivalent number of teeth for helical gear:

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth an a helical gear

$Z_E = Z / \cos^3. \beta$

Z = Actual number of teeth on a helical gear and
β = helix angle.

Proportion of Helical Gears:

AGMA Recommendations.

Pressure angle in the plane of rotation	$\propto$ = 15° to 25°
Helix angle,	β = 20 - 45°
Addendum	= 0.8 m (maximum)
Dedendum	= 1.0 m
Minimum total depth	= 1.8 m (maximum)
Minimum clearance	= 0.2 m
Thickness of tooth	= 1.5708 m

STRENGTH OF HELICAL GEARS: (P962 K/G)

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore, in order to find the strength of helical gear, a modified lewis equation is used.

It is given by, $F_T = \sigma_o. C_V\, b\, \pi\, m\, y'$.

Where

(i) F_T, σ_o, C_V, b, π, m, as usual , with same meanings,

And
y' = Tooth from factor or lewis factor corresponding to the **FORMATIVE OR VIRTUAL OR EQUIVALENT NUMBER OF TEETH.**

The values of C_V, velocity factor, from equation, (D.D.H)

Item	Equation
(a) For low-angle helical gears when v is less than 5 m/s	$C_v \frac{4.58}{4.58 + v}$
(b) For all helical and herringbone gears when v is 5 to 10 m/t	$C_v \frac{6.1}{6.1 + v}$
(c) For gears when v is 10 to 20 m/s (Barth's formula)	$C_v \frac{15.25}{15.25 + v}$
(d) For precision gear with v greater than 20 m/s	$C_v \frac{5.55}{5.55 + \sqrt{v}}$
(e) For non metallic gears	$C_v \frac{0.7625}{1.0167 + v} + 0.25$

(ii) The dynamic tooth load, $F_d = F_t + F_i$

Where $$F_i = \frac{K_3 v \left(cb \cos^2 \beta + F_t\right) \cos \beta}{K_3 v + \left(cb \cos^2 \beta + F_t\right)^{1/2}}$$

K_S = 20.67 in SI units
= 6.60 in metric units,

(iii) The static tooth load or endurance strength of the tooth is given by

$$F_S = \sigma_e \, b \, \pi \, m y' \geq F_d$$

The maximum or limiting wear tooth load for helical gears is given by,

$$F_w = \frac{d_1 \, b \, Q \, K}{\cos^2 \beta} \geq F_d$$

Where d_1. b, Q and K have usual meanings as discussed in spur gears

In this case,
Where K = The load stress factor

$$K = \frac{(\sigma_{es})^2 \sin \alpha_N}{1.4} \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

Pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10,000 rev/min and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 Mpa. Determine the suitable module and face width from static strength considerations and check the gears for dynamic and wear loads. given σ_{es} = 618 MPa

Given: P = 15kW= 15x10^3 W, $\propto$ = 20°, β = 45°, N_1 = 10,000 rev/min,
d_1 = 80mm = 0.08 m, d_2 = 320 mm = 0.32 m, $\sigma_{d1} = \sigma_{d2}$ = 100 MPa = 100 N/mm^2,
σ_{es} = 618 MPa = 618 N/mm^2

Since, both the pinion and gear are made of the same material (i.e., cast steel) the pinion is weaker. Thus the design is based on the pinion.

W K T,
Torque transmitted by the pinion

$$T = \frac{P \times 60}{2 \pi N_1} = \frac{15 \times 10^3 \times 60}{2 \pi \times 10,000} = 14.32 \text{ N-m}$$

$\therefore$ Tangential tooth load on the pinion $F_t = \frac{T}{d_1/2} = \frac{14.32}{0.08/2} = 358$ N

W.K.T

Number of teeth on the pinion $= Z_1 = \frac{d_1}{m} = \frac{80}{m}$

And formative or equivalent number of teeth for pinion $= Z_{E1} = \frac{Z_1}{\cos^3 \beta}$

$$= \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}.$$

$\therefore$ Tooth from factor for pinion for 20°stub teeth

$$y'_1 = 0.175 - \frac{0.841}{Z_{E1}}$$

$$= 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\, m$$

W.K.T

$$V = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.08 \times 10,000}{60} = 42\ m/Sec$$

$\therefore$ $$C_v = \frac{5.55}{5.55 + \sqrt{V}} = \frac{5.55}{5.55 + \sqrt{42}}$$ $\because$ V is greater than 20 m/sec

$$C_v = \frac{5.55}{5.55 + 6.48} = \frac{5.55}{12.03} = 0.461$$

Since maximum face width,(b) for helical gear may be taken as 12.5 m to 20.0 m.
Let us take b = 12.5 m

W.K.T
tangential tooth load (F_t)

$= 358 = (\sigma_{d1}.\ C_V)\ b\ \pi\ m\ y_1^1$

$= (100 \text{ x } 0.461) \text{ x } 12.5m \text{ x } \pi \text{ x } m \text{ x } (0.175-0.003)$

$= 72m^2 - 1.5m^3$

By Trial and hit method,
Solution for m, =
m = 2.1 say 2.5 mm (standard)

and face width = b = 12.5 m = 12.5 x 2.5 = 31.25 mm say 32.0 mm

cheeking the gear for wear:

WKT. $V.R = \frac{d_2}{d_1} = \frac{320}{80} = 4$

$$Q = \frac{2 \times VR}{VR+1} = \frac{2\times 4}{4+1} = \frac{8}{5} = 1.6$$

WKT. $\tan \alpha_N = \tan \propto \cos \beta$
$= \tan 20° \cos 45°$
$= 0.2573$

$\therefore \alpha_N = 14.4°$

Since, both the gears are made of same material (i.e., cast steel).

Therefore, let

$E_1 = E_2 = 200 \text{ x } 10^3 \text{ N/ mm}^2$

Load stress factor = $K = \frac{\sigma_{es}^2\ .\ Sin\ \alpha_N}{1.4}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$

$$= \frac{618^2 \times Sin\ 14.4}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right)$$

$$= 0.678\ N/mm^2$$

W.K.T,

$$= F_W = \frac{d_1 \; b \; Q \; K}{Cos^2 \; \beta} = \frac{80 \times \; 32 \times 1.6 \times 0.678}{Cos^2 \; 45} \; 5554 \text{ N}$$

Since maximum load for wear is much more than the tangential load on the tooth. Design is satisfactory for wear consideration.

Seminally try for F_d = dynamic load

$$F_d = F_t + F_i$$

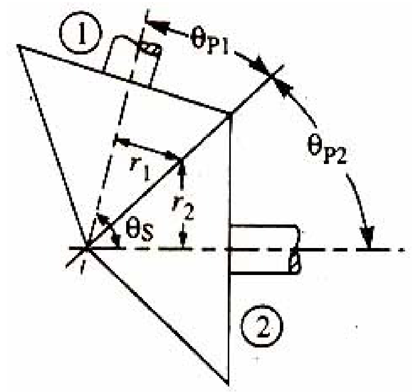
$$= F_t + \frac{k_3 \; v \; (C_b \; \cos^2 \beta) \cos \beta}{k_3 \; v + \sqrt{C_b \; Cos^2 \; \beta + F_t}}$$

C= dynamic factor depending upon machine error (for an error of 0.04) } = 712.0

$$= 358 + \frac{20.67 \times 42 \; (712 \times 32 \; Cos^2 \; 45 + 358) \cos 45}{(20.67 \times 42) + \sqrt{(712 \times 32 \; Cos^2 \; 45 + 358) \cos 45}}$$

$$= F_D = ?$$

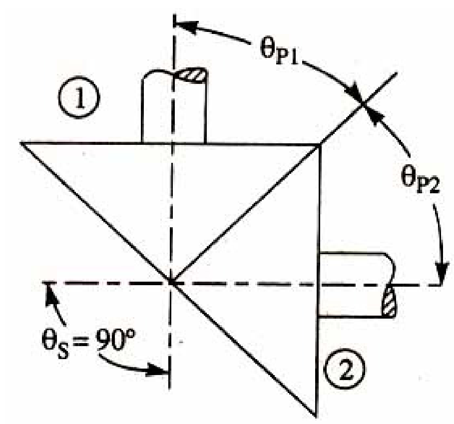
Bevel gears:
The bevel gears are used to transmit power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones.



CLASSIFICATION OF BEVEL GEARS:

Classified depending upon the angles between the shafts and the pitch surfaces.

(i) **Miter gears:** when equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angles as shown, then they are known as miter gear.



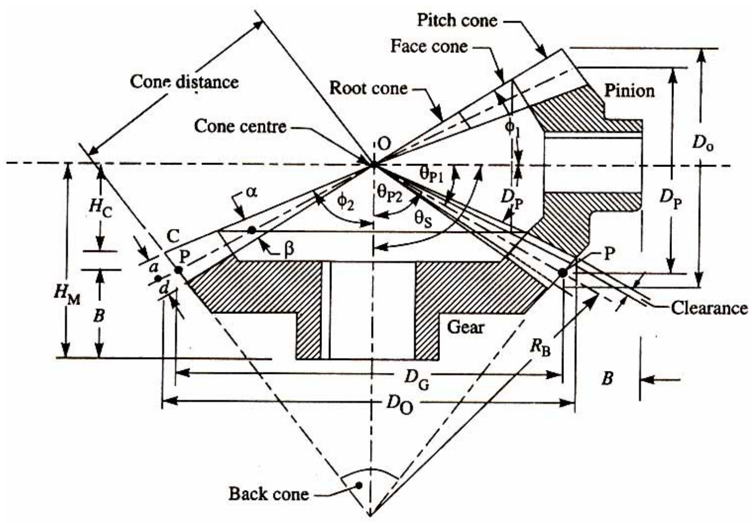
(ii) **Angular bevel gears:** when the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as angular bevel gears.

(iii)**Crown bevel gears:** when bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90° then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing as shown.

(iv)**Internal bevel gears:** when the teeth on the bevel gear are cut on the inside of the pitch cone then they are known ass inter bevel gears.

Note: The bevel gears may have straight or spiral teeth. It may be assumed, unless other wise stated that the bevel gear has straight teeth and and the axes of the shafts intersect at right angle.

TERMS USED IN BEVEL GEARS:



A sectional view of two bevel gears in mesh is as shown. The following terms are important from the subject point of view.

(i) **Pitch cone:** It is a cone containing the pitch elements of the teeth.

(ii) **Cone centre:** It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

(iii) **Pitch angle:** It is the angle made by the pitch line with the axis of the shaft. It is denoted by (i.e, δ_1 & δ_2)

(iv) **Cone distance:** It is the length of the pitch cone element. It is also called as a pitch cone radius. It is denoted by 'OP' Mathematically cone distance or pitch cone radius

$$= OP = \frac{\text{pitch radius}}{\sin O_p} = \frac{D_p/2}{\sin O_{p1}} = \frac{D_G/2}{\sin O_{p2}}$$

(v) **Addendum angle:** It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by θ_a. Mathematically addendum angle.

$$\tan\ \theta_a = \frac{2h_{a1}\ Sin\ \delta_1}{d_1}$$

$$= \frac{2h_{a2}\ Sin\ \delta_2}{d_2}$$

(vi) **Dedendum angle:** It is the angle subtended by the Dedendum of the tooth at the cone centre. It is denoted by θ_d. Mathematically,

$$\tan \theta_d = \frac{2h_{f1} \sin \delta_1}{d_1}$$

$$= \frac{2h_{f2} \sin \delta_2}{d_2}$$

Where, h_{a1}, h_{a2} = addendum of the pinion and gear respectively, mm

h_{f1}, h_{f2} = dedendum of pinion and gear respectively, mm

(vii) **Face angle:** It is the angle subtended by the face of the tooth at the cone centre. The face angle is equal to the pitch angle plus addendum angle.

(viii) **Root angle:** It is the angle subtended by the root of the tooth at the cone centre. It is equal to the pitch angle minus dedendum angle

(ix) **Back cone:** (Normal cone): It is the imaginary cone perpendicular to the pitch cone at the end of the tooth.

(x) **Crown height:** It is the distance of the crown point C, from the cone centre O, parallel to the axis of the gear. It is the denoted by C

(xi) **Mounting height:** It is the distance of the back of the boss from the cone centre. It is denoted by ' m'

(xii) **Pitch diameter:** It is the diameter of the largest pitch circle.

(xiii) **Outside or addendum cone diameter**: It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically outside dia ,

$$d_{O1} = d_1 + 2ha_1,\ \text{Cos}\ \delta_1$$

$$d_{O2} = d_2 + 2ha_2,\ \text{Cos}\ \delta_2$$

Proportions of Bevel gears:

The proportion for the bevel gear may be taken as

(i) Addendum:	a = 1.0 m
(ii) Dedendum:	d=1.2 m
(iii) Clearance	= 0.2 m
(iv) Working depth	= 2.0m
(v) Tooth thiknes	= 1.5708

Formative or Equivalent number of teeth for Bevel Gears: (Tredgold's approximation)

$$Z_e = Z/\text{Cos}\delta$$

STRENGTH OF BEVEL GEARS:

The strength of a bevel gear tooth is obtained in a similar way as discussed in the previous articles. The modified form of the lewis equation for the tangential tooth load is given as follows

$$= F_t = (\sigma_d \times C_v)\ b\ \pi\ m\ y^1 \left(\frac{L - b}{L} \right)$$

y^1= lewis form factor based on formative or equivalent number of teeth

L = Slant height of pitch cone (or cone distance)

$$= \frac{1}{2} \sqrt{d_1^2 + d_2^2}$$

Where d_1 and d_2 are the pitch circle diameters on the larger diameter of pinion and gears respectively

(i) The factor i.e, $\frac{L - b}{L}$ may be called as bevel factor

(ii) For satisfactory operation of bevel gears the face width should be from 6m to 10 m. Also ratio L/b should not exceed 3, (i.e., $b \leq L / 3$)for this the number of teeth in the pinion must not be less than $\frac{48}{\sqrt{1 + (vR)^2}}$

(iii) The dynamic loads for bevel gears may be obtained in the same similar manner as discussed for spur gears.

(iv) The static tooth load or endurance strength of the tooth for bevel gears is given by

$$= F_e = \sigma_e\ b\ \pi\ m\ y^1 \left(\frac{L - b}{L} \right)$$

The value of flexural endurance limit (σ_e) may be taken from table

(v) The maximum or limiting load for wear for bevel gears is given by

$$= F_w = \frac{D_1\, b\, Q_e\, k}{Cos\ \ \delta_1}$$

Where,
D_1, b, Q, k, have usual meanings as discussed in spur gears except that Q_e is based on formative or equivalent number of teeth, such that,

$$Q = \frac{2\, Ze_2}{Ze_2 + Ze_1}$$

A pair of bevel gears to connect two shafts at right angles and transmit 9 kW. The allowable static stress for pinion and gear materials may be taken respectively as 85 MPa and 55 MPa and brinill hardness of 200 and 160. The speed may be assumed as 1200/420 and number of teeth may be assumed as 21 for pinion and 60 for gear. Tooth profile may be taken as 20° full depth involute. Check the design for dynamic and wear loads.

Given: $\theta_s = 90°$, $P = 9kW = 9000W$, $Z_1 = 21$, $Z_2 = 60$, $\sigma_{d1} = 85$ MPa, $\sigma_{d2} = 55$ MPa, $N_1 = 1200$ rev/min, $N_2 = 420$ rev/min, $\alpha = 20°$(full depth involute)

Find, module, and check the design for dynamic and wear loads,

Since, the shafts are at right angles,
Therefore, pitch angle for pinion,

$$= \tan \delta_1 = \frac{d_1}{d_2} = \frac{Z_1}{Z_2} = \frac{1}{i}$$

$$\therefore \quad \delta_1 = \tan^{-1}\left(\frac{21}{60}\right) = 19.3°$$

$$\boxed{\therefore \quad \delta_1 = 19.3°}$$

and the pitch angle for gear $= \delta_2 = 90 - \delta_1$

$$\boxed{\therefore \quad \delta_2 = 70.7°}$$

W.K.T,

formative number of teeth for pinion

$$Z_{e_1} = \frac{Z_1}{\cos \delta_1} = \frac{21}{\cos 19.3}.$$

$$= \frac{Z_1}{0.9438} = 22.25°$$

$$\boxed{Z_{e_1} = 22.25°}$$

and $$Z_{e_2} = \frac{Z_2}{\cos \delta_2} = \frac{60}{\cos 70.7} = \frac{60}{0.3305}$$

$$\boxed{Z_{e_2} = 181.54°}$$

W.K.T.

for 20° full depth involute system tooth from factor

For pinion $= y_1^1 = 0.154 - \frac{0.912}{Z_{e_1}}$

$$= 0\ 154 - \frac{0.912}{22\ .\ 25} = (0\ 154 - 0.0405) = 0.11350$$

and for gear $= y_2^1 = 0\ 154 - \frac{0.912}{181.54}$

$$0.154 - 0.00502 = 0.14898$$

$$\sigma_{d_1} \times y_1^1 = 85 \times 0.11350 = 9.6475$$
$$\sigma_{d_2} \times y_2^1 = 55 \times 0.14898 = 8.1939$$

Since the product, $\sigma_{d_2} \times y_2^1$ is less than $\sigma_{d_1} \times y_1^1$ therefore Gear is weaker, and thus the design should be based on gear only.

W.K.T.

$$F_t = \frac{P \times 10^3}{v}$$

Here $v = \frac{\pi . d_2\ N_2}{60} \qquad = \frac{\pi . mZ_2\ N_2}{60}$

$$= \frac{\pi \times m \times 60 \times 420}{60} = 1320m \quad mm/Sec$$

$$\boxed{v = 1.320m \quad m/Sec}$$

Now

$$F_t = \frac{P \times 10^3}{v} = \frac{9 \times 10^3}{1.320\, m} = \frac{6818.18}{m}$$

$$\boxed{F_t = \frac{6818.18}{m} \text{ N}}$$

Taking velocity factor

$$= C_v = \frac{6.1}{6.1 + v}$$

(taking into consideration that gears are very accurately cut and ground gears having a pitch line velocity from 6 m/ sec to 20 m/sec)

$$\therefore = C_v = \frac{6.1}{6.1 + 1.32\, m}$$

W.K.T,

Length of pitch cone element

$$= L = \frac{d_2}{2 \ Sin \ \delta_2} = \frac{m \times 60}{2 \ Sin \ 70.7}$$

$$= \frac{m \times 60}{2 \times 0.9438} = 31.78 \ m$$

$$\therefore \boxed{L = 31.78 \text{ m}}$$

Assuming the face width b = 1/3rd of the length of the pitch cone element L,

$$\therefore \ b = \frac{L}{3} = \frac{31.78 \ m}{3} = 10.60\, m$$

$$\therefore \quad \boxed{b = 10.60 \text{ m}}$$

W.K.T,

The tangential tooth load on gear

$$F_t = (\sigma_{d2} \times C_v)\ b\ \pi\ m\ y_2^1 \left(\frac{L-b}{L}\right)$$

$$= \frac{6818.18}{m} = 55 \times \frac{6.1}{6.1 + 1.32m} \times 10.60\ m$$

$$\times \pi \times m \times 1.4898 \times \left(\frac{31.78\ m - 10.60m}{31.78\ m}\right)$$

$$= \frac{6818.18}{m} = \frac{1109m^2}{6.1 + 1.32\ m}$$

$$41590 + 8999\ m = 1109\ m^3$$

Solving this by hit and trial method we get, m = 4.58

$\therefore$ m = 5.0 (Standard)

and b = 10.60 x m

= 10.60 x 5 = 53.0 mm

$\therefore$ b = face width = 53.0 mm

Thus, d_2 = m x 60 = 5 x 60 = 300 mm

d_1 = m x 21 = 5 x 21 = 105 mm

& L = 31.78 m = 31.78 x 5 = 158.9

Check for dynamic load

W.K.T,

Pitch line velocity

V = 1.320 m m/sec

= 1.32 x 5

v = 6.600 m / Sec

and tangential tooth load on the gear

$$= F_t = \frac{6818.18}{m} \quad N$$

$$= \frac{6818.18}{5}$$

F_t = 1363.63 N

From table the tooth error in action for first class commercial gears having module 5 mm is

e = 0.0555

Take K_1 = 9.0 for 20° full depth teeth

and E1 = 210 x 10^3 N/mm^2 and

E_2 = 84 x 10^3 N/mm^2

C = dynamic factor depending upon machining errors

$$= \frac{e}{k_1(1/E_1 + 1/E_2)}$$

$$= \frac{0.0555}{9.0 \times \left[\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}\right]}$$

$$= \frac{6.166 \times 10^{-3}}{(4.76 \times 10^{-6} + 1.190 \times 10^{-5})}$$

$$= \frac{6.166 \times 10^{-3}}{(0.476 + 1.190)10^{-5}}$$

$$= \frac{6.166 \times 10^{-3} \times 10^{5}}{1.666}$$

$$= \frac{6.166 \times 10^{-2}}{1.666} = 370.1\ N/m$$

$\therefore$ C = dynamic factor = 370.1.N/m

W.K.T

Dynamic load on the gear

$$= F_d = F_t + F_i$$

$$= F_t + \frac{k_3 v\ (cb + F_t)}{k_3\ v + \sqrt{c_b + F_t}}$$

$$= 1363.63 + \frac{20.67 \times 6.6\ (370.1 \times 53 + 1863.63)}{20.67 \times 6.6 + \sqrt{370.1 \times 53 + 1363.63}}$$

$$= 1363.63 + \frac{136.422\ (19615.3 + 1363.63)}{136.422 + \sqrt{19615.3 + 1363.63}}$$

$$= 1363.63 + \frac{136.422 \times 20978.93}{136.422 + \sqrt{20978.93}}$$

$$= 1363.63 + \frac{2861987.588}{136.422 + 144.841}$$

$$= 1363.63 + \frac{2861987.588}{281.263}$$

$$= 1363.63 + 10175.485$$

$$F_d = 11539.115\ N$$

σ_{en} = for gear material of BHN = 160, is taken as 83.5 N/mm^2

Further, we know static tooth load or endurance strength of the tooth

$$= F_S = \sigma_{en}\ b\,\pi m y_2^1 \left(\frac{L-b}{L}\right)$$

$$= 83.5 \times 53 \times \pi \times 5 \times 0.14898$$

$$\times \left(\frac{158.9 - 53}{158.9}\right)$$

$$= 83.5 \times 53 \times \pi \times 5 \times 0.14898 \times 0.666$$

$$\boxed{F_S = 6902.116\ \text{N}}$$

Since $F_S < F_d$, the design is not satisfactory from the standpoint of dynamic load.

It is known that $F_S \geq 1.25\ F_d$ for steady loads

i.e., F_d – dynamic load on gear must be reduced

i.e, by assuming for a satisfactory design against dynamic load, let us take the precision gears (class III) having tooth error in action

$$e = 0.0150\ \text{mm}$$

$$\boxed{\therefore\ C = 100.02\ \text{N/mm}}$$

$$\therefore\ F_D = 1363.63 + \frac{20.67 \times 6.6\ (100.02 \times 53 + 1363.63)}{20.67 \times 6.6 + \sqrt{100.02 \times 53 + 1363.63}}$$

$$= 1363.63 + \frac{136.422\ (5300 + 1363.63)}{136.422 + \sqrt{5300 + 1363.63}}$$

$$= 1363.63 + \frac{136.422 \times 6663.63}{136.422 + \sqrt{6663.63}}$$

$$= 1363.63 + \frac{136.422 \times 6663.63}{136.422 + 81.63}$$

$$= 1363.63 + \frac{909065.7319}{281.052}$$

$$= 1363.63 + 4169.0318$$

$$F_D = 5532.66 \text{ N}$$

From the above we see that by taking precision gear, F_S is greater than F_D, therefore, the design is satisfactory from the standpoint of dynamic load

$$\therefore \text{ here } = \frac{F_s}{F_D} = \frac{6902.166}{5532.66} = 1.2475$$

(Hence, design is safe)

<u>Check for wear load</u>

For 180 BHN, σ_{es} may be 617.8 N /mm^2 normally steel for pinion and cost iron for gear of 200 & 160 , (Hence in the table take 180 & 180)

$$\therefore \text{ k = load stress factor } = \frac{(\sigma_{es})^2 \; Sin\phi}{1.4}\left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

$$= \frac{(617.8)^2 \; Sin 20}{1.4}\left[\frac{1}{210\times10^3} + \frac{1}{84\times10^3}\right]$$

$$= \frac{381676 \times 0.342}{1.4}\left[\frac{1 + 2.5}{210\times10^3}\right]$$

$$= \frac{130533.192}{1.4}\left[\frac{3.5}{210\times10^3}\right]$$

$$= 93237.994 \times \frac{0.01666}{10^3}$$

$$= 93.2379 \text{ x } 0.01666$$

$$1.553 \text{ N / mm}^2$$

$$\therefore \text{ k} = 1.553 \text{ N / mm}^2$$

and

Q_e = ratio factor

$$= \frac{2\,Ze_2}{Ze_2 + Ze} = \frac{2 \times 181.54}{22.25 + 181.54}$$

$$= \frac{363.08}{203.79} = 1.78$$

W.K.T

Maximum or limiting load for wear

$= F_W = d_1^{*}\, b\, Q_e\, K$ (*? For pinion, please explain)

$= 105 \times 53 \times 1.78 \times 1.553$

$$F_w = 15397.70 \text{ N}$$

Since, F_w is greater than F_D the design is satisfactory from the standpoint of wear also