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Crystal clear concepts by IITians

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Notes for School Exams

Physics XI

Fluid Mechanics

Mechanical Properties of Matter

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Prerequisite: Laws of motion (F. B.D.)

Content

NOTE: I have followed NCERT Physics book to prepare this notes. Even IIT-JEE (now JEE Advanced) follows NCERT. Please refer NCERT whenever two books disagree on a particular topic. All the best for Boards, JEE Main & JEE Advanced.

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Introduction

- A *fluid is any substance which flows because its particles are not rigidly attached to one another.*
  This includes liquids, gases and even some materials which are normally considered solids, such as glass.
- Fluid mechanics is the study of fluids either in motion (*fluid dynamics)* or at rest *(fluid statics)* and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids.

Fluids

- From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. **Both liquids and gases are fluids.**
- Thus all The technical distinction lies with the reaction of the two to an applied shear or tangential stress. A solid can resist a shear stress by a static deformation; a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied.
- Hence, we can define ***fluid as substance which cannot resist any shear stress applied to it.***

Density & Specific Volume

- **Density:** The mass per unit volume of material is called the density, which is generally expressed by the symbol ρ.

$$\rho = \frac{m}{V}$$

- The SI unit of density is **kg/m^3.**
- The density of water at 4°C and 1 atm (101325 Pa, standard atmospheric pressure) is1000 kg/m^3 or $1 g/cm^3$.
- The density of mercury is taken as 13600 kg/m^3.
- **Specific volume:** The reciprocal of density, i.e. the volume per unit mass, is called the specific volume.

$$\text{specific volume} = \frac{1}{\rho} = \frac{V}{m}$$

- SI Unit : m^3/kg

Specific Weight & Specific Gravity

- **Specific weight:** The specific weight of a fluid, denoted by γ, is its weight per unit volume.
- Therefore,

$$\gamma = \frac{mg}{V} = \rho g$$

- **Specific gravity or relative density (Important):** *Specific gravity, denoted by SG or RD, is the ratio of a fluid density to a standard reference fluid*, water (for liquids), and air (for gases):

$$\text{Relative density of a liquid} = \frac{\text{density of liquid}}{\text{density of water at } 4^\circ\text{C}}$$

$$SG_{liquid} = \frac{\rho_{liquid}}{\rho_{water}}$$

- Similarly, $SG_{gas} = \frac{\rho_{gas}}{\rho_{air}}$
- For example, the specific gravity of mercury (Hg) is SG_{Hg} = 13,600/1000 ≈ 13.6.

Exercise 1

1. Find the dimensions of a) density and b) specific gravity.
2. Find the specific density of a liquid whose density is a) 8300 kg/m^3 and b) 2 g/cm^3.
3. If the relative density of a fluid is 11.1 find its density in S.I. and CGS units.

Answers

1. a) $ML^{-3}T^0$ b) $M^0L^0T^0$
2. a) 8.3 b) 2
3. 11100 kg/m^3, 11.1 g/cm^3

Compressibility & viscosity

- **Compressibility:** Compressibility is the measure of the change in volume a substance undergoes when a pressure is exerted on the substance.
- Liquids are generally considered to be ***incompressible***, meaning *density of the liquid is independent of the variation in pressure and always remains constant.*

- **Viscosity:** *Viscosity is a fluid property that measures the resistance of the fluid to deforming due to a shear* force.
- Viscosity is the internal friction of a fluid which makes it resist flowing past a solid surface or other layers of the fluid. Viscosity can also be considered to be a measure of the resistance of a fluid to flowing. A thick oil has a high viscosity; water has a low viscosity.

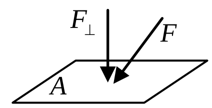
Ideal Fluid

- **An ideal fluid is one that is incompressible and has no viscosity.**
- Ideal fluids do not actually exist. We are going to *assume given fluid to be ideal, meaning incompressible and non-viscous fluid unless or otherwise stated.*

Pressure

1. ***Pressure (P)*** *is the force per unit area applied on a surface in a direction* ***perpendicular*** *to that surface.*
2. We can classify pressure in two categories: Average pressure and pressure at a point.
3. **Average Pressure**: Suppose a force F is applied to a surface of area A. Then, average pressure is defined as

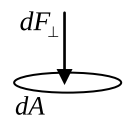
$$P = \frac{F_\perp}{A}$$



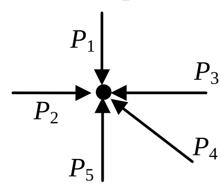
where $F_\perp$ is the component of force F perpendicular to the surface.

4. **Pressure at a point:** Suppose a infinitesimally small surface area dA centred at a point. Suppose a force $dF_\perp$ acts perpendicular to this force at that surface dA. Then, pressure at that point is given by

$$P = \frac{dF_\perp}{dA}$$



5. **Pressure in a fluid at a particular point acts equally in all direction.**



Here $P_1 = P_2 = P_3 = P_4 = P_5$

Important point about pressure

- **S.I unit** of pressure is **N/m^2** called **Pascal** (**Pa**).
  1 Pa = 1 N/m^2.
- Pressure is a **scalar** quantity (at school level).

6. Pressure in a fluid at a particular point acts equally in all direction.

- The atmosphere (atm) exerts certain pressure at a point depending on the column or height of atmosphere lying above that point. The average pressure of the atmosphere at sea level is known as atmospheric pressure (atm).
- The **atmospheric pressure** is about 1.013 ×10^5 Pa and is denoted by P_o. Thus, $\mathbf{P_o = 1.013 \times 10^5}$ **Pa**
- This gives us another unit for pressure, the atmosphere (atm), where **1 atm = 1.013** × **10^5 Pa**
- There is one more unit prevalent in laboratory, known as Bar. **1 Bar = 10^5 Pa**

Hydrostatics

- Hydrostatics is about the pressures exerted by a **fluid at rest.** Any fluid is meant, not just water.

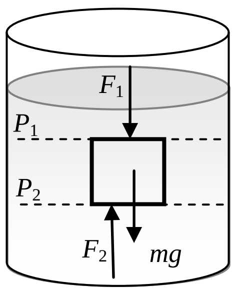
Pressure variation with depth in Hydrostatic

- Let us consider an **incompressible fluid** of **uniform density ρ** at **rest**.
- Consider an imaginary fluid volume (a cube, each face having area A) at rest. The sum of all the forces on this volume must be zero as it is in equilibrium.
- There are three vertical forces:

✓ The weight: $mg = \rho V g = \rho h A g$

✓ The upward force from the pressure P_2 on the bottom surface: $F_2 = P_2 A$

✓ The downward force from the pressure P_1 on the top surface: $F_1 = P_1 A$



i. Therefore, at equilibrium, we have, $F_2 - F_1 - mg = 0$

⇔ $P_2 A - P_1 A - mg = 0$

⇔ $(P_2 - P_1) A - \rho h A g = 0$

⇔ $(P_2 - P_1) - \rho h g = 0$

⇔ $P_2 - P_1 = \rho g h$

⇔ $\boxed{P_2 - P_1 = \rho g h}$

- Therefore, pressure P_2 at depth h is ρgh greater than pressure P_1.
- Hence, *pressure increases with depth.*

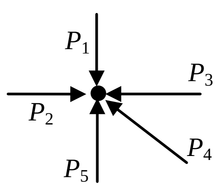
Important Points in Hydrostatics

1. Pressure in a continuously distributed uniform, incompressible static fluid varies only with vertical distance and is independent of the shape of the container. The pressure increases with depth in the fluid. Pressure P_2 at depth h is ρgh greater than pressure P_1. Thus,

   $P_2 - P_1 = \rho gh$

   Taking variation in acceleration due to gravity with depth or height into account we have, $\dfrac{dp}{dh} = -\rho g$

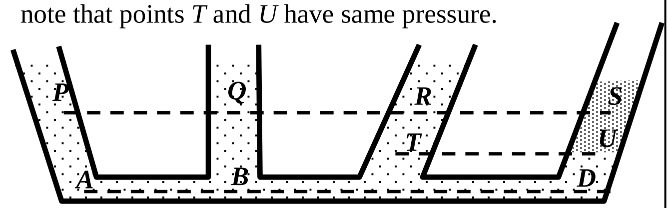
2. Pressure in a fluid at a particular point acts equally in all direction.

   Here $P_1 = P_2 = P_3 = P_4 = P_5$

3. The pressure is the same at all points on a given horizontal plane in the fluid.

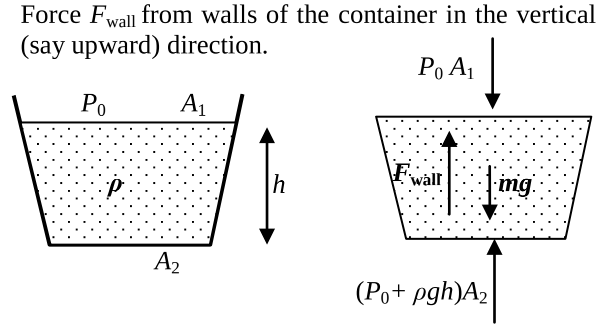
- An illustration of this is shown in the given Fig. The free surface of the container is atmospheric and forms a horizontal plane. Points *A, B, C, and D are at equal depth in a horizon*tal plane and are interconnected by the *same fluid*; therefore all points have the same pressure. The same is true of points *P, Q, and R which all have the* same lower pressure than at *A, B, C, and D. However, point S, although at the same depth* as *A, B, and C, has a different pressure because it lies beneath a different fluid.* Please note that points *T* and *U* have same pressure.



4. Forces acting on a fluid in equilibrium have to perpendicular to its surface, because it cannot sustain the shear stress.

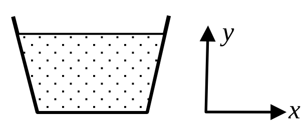
5. **Free body diagram of a liquid:** Forces on a fluids in equilibrium are (neglecting viscous forces) are:
   - Weight *mg* in downward direction
   - Force P_0A_1 from atmospheric pressure in downward direction
   - Normal force $(P_0+\rho gh)A_2$ from bottom surface in upward direction (how?)

   All of these forces are in vertical direction. By calculation it is found that net force is not zero. It means there must be some force in the vertical direction to maintain the equilibrium. Where does this force comes from? Yes! This force is due to walls of the container. Hence there is a fourth force:
   - Force F_{wall} from walls of the container in the vertical (say upward) direction.



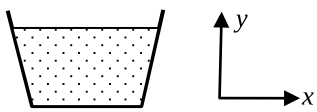
6. **Pressure difference in an accelerating fluids:** Consider a liquid kept at rest in a beaker as shown in figure. In this case we know that pressure do not change in horizontal direction (*x*-direction), it decreases upward along *y*-direction. So, we can write the equations,

   $\dfrac{dp}{dx} = 0$ and $\dfrac{dp}{dy} = -\rho g$



But, suppose the beaker is accelerated and it has components of acceleration a_x and a_y in x and y directions respectively, then the pressure decreases along both x and y direction. The above equation in that case reduces to,

$$\frac{dp}{dx} = -\rho a_x \quad \text{and} \quad \frac{dp}{dy} = -\rho\left(g + a_y\right)$$



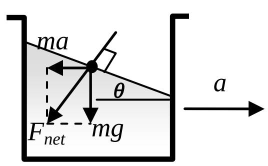
7. **Free surface of a liquid accelerated in horizontal direction:**

Consider a liquid placed in a beaker which is accelerating horizontally with an acceleration a . Then, $\tan\theta = \frac{a}{g}$

Proof: Consider a fluid particle of mass m at point P on the surface of liquid. From the accelerating frame of reference, two forces are acting on it,

(i) pseudo force (ma)

(ii) weight (mg)



- Net force in equilibrium should be perpendicular to the surface.

$$\therefore \tan\theta = \frac{ma}{mg}$$

or $$\tan\theta = \frac{a}{g}$$

Gauge Pressure and Vacuum Pressure

- **Absolute pressure (*P*)**: The pressure at a point is known as absolute pressure.
- **Gauge Pressure (when *P* > *P*$_o$)**: The excess pressure above atmospheric pressure is called as gauge pressure. Therefore,

  Gauge Pressure $= P - P_o$
- **Vacuum Pressure (when *P* < *P*$_o$)**

  Vacuum Pressure

  = Atmospheric Pressure – Absolute Pressure

⬄ ***Vacuum Pressure*** $= P_o - P$

The Mercury Barometer

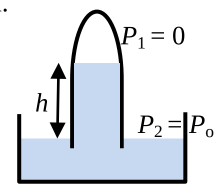
- Figure shows a very basic *mercury barometer, a device used to measure the pressure of the atmosphere.* The long glass tube is filled with mercury and inverted with its open end in a dish of mercury, as the figure shows. The space above the mercury column contains only mercury vapor, whose pressure is so small at ordinary temperature that it can be neglected.

  Thus

  $P_2 - P_1 = \rho g h$

⬄ $\boldsymbol{P_o = \rho g h}$

  where ρ = density of the mercury.



- The atmospheric pressure is often given as the length of mercury column in a barometer. Thus, *a pressure of 76cm of mercury means, 1 atmospheric pressure.*

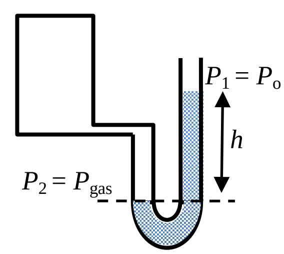
Manometer

- *Manometer is a simple device to measure the pressure in a closed vessel containing a gas.* It consists of a U-shape tube having some liquid. One end of the tube is open to the atmosphere and the other end is connected to the vessel as shown in figure. The pressure of the gas is equal to P_1. From hydrostatic,

$P_2 - P_1 = \rho g h$

⬄ $P_{gas} - P_o = \rho g h$

⬄ $P_{gas} = P_o + \rho g h$



where P_{gas} = pressure of the gas

P_o = the atmospheric pressure

h = difference in levels of the liquid in the two arms

ρ = the density of the liquid.

Pascal's Law

- Pascal's law states that "**if a pressure is applied to an *enclosed fluid*, it is transmitted *undiminished* to every portion of the fluid and the walls of the containing vessel.**"
- Applications of Pascal's law:

  Hydraulic lift

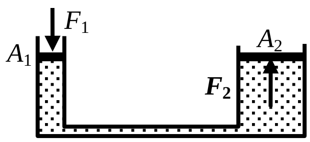
  Hydraulic brakes

  Cycle pump

Hydraulic Lift

- A hydraulic lift uses Pascal's principle. Hydraulic lift is used to raise heavy loads such as car. It contains of two vertical cylinders A and B of different cross sectional areas A_1 and A_2. Pistons are fitted in both the cylinders as shown in fig.
- A small force is applied F_1 to a small piston of area A_1 and cause a pressure increase on the fluid.
- According to Pascal's Law this increase in pressure P is transmitted to the larger piston of area A_2 and the fluid exerts a force F_2 on this piston.
- Thus, from Pascal's Law

$P = F_1/A_1 = F_2/A_2$



⇔ $F_2 = F_1(A_2/A_1)$

- Thus if $A_2 >> A_1$, even a small force F_1 is able to generate a large force F_2 which can raise the load.

Buoyant Force

- When an object is fully or partially submersed in a fluid, the surrounding fluid exerts a ***net upward force*** which is known as the **buoyant force** or **upthrust**.
- It is easier to lift a bucket immersed in water because of buoyant force.
- **NOTE:**

1. The buoyant force comes from the **pressure** exerted on the object by the surrounding fluid.
2. When showing F.B.D., we need to show either buoyant force in the upward direction or forces due to pressure. We never show both buoyant force and forces due to pressure in the same F.B.D.

Archimedes Principle
(Buoyant Forces)

- Archimedes' Principle states that a body which is completely or partially submerged in a fluid experiences a net upward force called the buoyant force, B , which is equal in magnitude to the weight of the fluid displaced by the object. Thus,

Buoyant force = weight of the displaced liquid

$B = V_{im}\rho_{liquid}\, g$ (Buoyant force)

where,

B = magnitude of Buoyant force

V_{im} = volume of displaced liquid = immersed volume of solid

ρ_{liquid} = density of liquid

Proof:

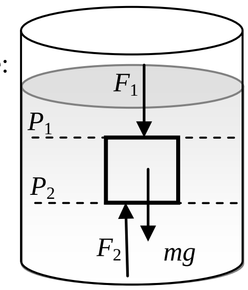
- As shown in Fig., consider a body of height h lying inside a liquid of density ρ,. Area of cross-section of the body is A. The forces on the sides of the body cancel out.
- There are two vertical forces due to pressures:

✓ The upward force from the pressure P_2 on the bottom surface:

$F_2 = P_2A$

✓ The downward force from the pressure P_1 on the top surface:

$F_1 = P_1A$



- The resultant force ($F_2 - F_1$) is acting on the body in the upward direction and is called upthrust or buoyant force (B).

∴ $B = F_2 - F_1 = P_2A - P_1A = (P_2 - P_1)A = h\rho gA$

($\because P_2 - P_1 = \rho gh$)

- But $Ah = V$, the volume of the body = volume of liquid displaced

∴ $B = V\rho g = Mg$

($\because M = V\rho$ = mass of liquid displaced)

i.e., upthrust or buoyant force= Weight of liquid displaced

- This proves the Archimedes' principle.

Law of floatation

- Consider an object of volume V and density ρ_{solid} floating in a liquid of density ρ_{liquid}. Let V_{im} be the volume of object immersed in the liquid. For equilibrium of the object,

Weight = upthrust

$V\rho_{solid}\, g = V_{im}\rho_{liquid}\, g$

⇔ $\dfrac{V_{im}}{V} = \dfrac{\rho_{solid}}{\rho_{liquid}}$ (fraction of volume immersed in liquid)

- This is the fraction of volume immersed in liquid.
- Three possibilities may arise:

i. $\boldsymbol{\rho_{solid} < \rho_{liquid}}$: Body is ***partially submerged*** in liquid. The fraction submerged is given by the relation

$$\frac{V_{im}}{V} = \frac{\rho_{solid}}{\rho_{liquid}}$$

ii. $\boldsymbol{\rho_{solid} = \rho_{liquid}}$: Body is ***completely submerged*** in liquid. Body remains floating in liquid.

iii. $\boldsymbol{\rho_{solid} > \rho_{liquid}}$: Body will ***sink*** in liquid.

Apparent Weight

- If an object is placed inside a fluid then,
  Apparent weight = (Actual weight) – (Buoyant force)

Buoyant Force in Accelerating Fluids

- Suppose a body is dipped inside a liquid of density ρ_{liquid} placed in an elevator moving with an acceleration $\vec{a}$. The buoyant force F in this case becomes,

$$F = V_{im}\rho_{liquid}\, g_{eff}$$

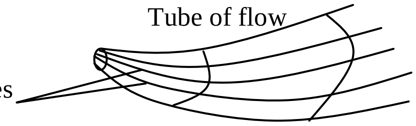
Here, $g_{eff} = |\vec{g} - \vec{a}|$

- Concept of g_{eff} is explained in chapter Simple Harmonic Motion.
- For example, if the lift is moving upwards with an acceleration a, the value of g_{eff} is $g + a$ and if it is moving downwards with acceleration a, the g_{eff} is $g - a$. In a freely falling lift g_{eff} is zero (as $a = g$) and hence, net buoyant force is zero. This is why, in a freely falling vessel filled with some liquid, the air bubbles do not rise up (which otherwise move up due to buoyant force).

Space for notes:

Fluids in Motion

- All fluid flow is classified into one of two broad categories or regimes. These two flow regimes are laminar flow and turbulent flow.
- **Laminar Flow:** Laminar flow is also referred to as **streamline** or **viscous flow** or **steady flow**. *When a liquid flows such that each particle of the passing a given point moves along the same path and has the same velocity as its predecessor, the flow is called streamline flow or steady flow.*
- **Streamline:** A streamline may be defined as the path, the tangent to which at any point gives the direction of the flow of liquid at that point.
- **Tube of flow.** *A bundle of streamline forming a tubular region is called a tube of flow.*



- **Turbulent Flow:** Turbulent flow is characterized by the irregular movement of particles of the fluid. The particles travel in irregular paths with no observable pattern and no definite layers.
- **Critical velocity.** The critical velocity of a liquid is that limiting (maximum) value of its velocity of flow upto which the flow is streamlined and above which the flow becomes turbulent.

Reynolds Number

- The Reynolds number is a dimensionless number comprised of the physical characteristics of the flow. The flow regime (either laminar or turbulent) is determined by evaluating the Reynolds number of the flow .
- The Reynolds number for fluid flow is given by

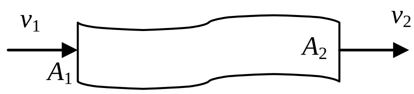
$$R_e = \frac{\rho v D}{\eta} \qquad \text{(Reynolds number)}$$

where
R_e = Reynolds number (dimensionless; have not any unit)
v = average velocity
D = diameter of pipe
η = viscosity of fluid (to be studied later)
ρ = fluid density

- **Important point to note about Reynolds number:**
  i. For practical purposes (as per NCERT), if the Reynolds number is less than 1000, the flow is laminar. If it is greater than 2000, the flow is turbulent.
  ii. Flows with Reynolds numbers between 1000 and 2000 are sometimes referred to as unsteady flows.
  iii. R_e represents the ratio of inertial force (force due to inertia i.e. mass of moving fluid or due to inertia of obstacle in its path) to viscous force.

Equation of Continuity

- Equation of continuity states that total mass of fluids going into the tube through any cross-section should be equal to the total mass coming out of the same tube from any other cross section in the same time. The continuity equation results from conservation of mass.
- Let us consider mass is entering with speed v_1 at left end and flowing out with speed v_2.



- Clearly, in a time interval Δt, mass entering
  = (mass per unit time) × time = $\rho A_1 v_1 \Delta t$.
  (Hint: mass = $\rho V = \rho A l$
  Hence, mass per unit time = m/t = $\rho A l/t = \rho Av$)
- And, in same time interval Δt, mass leaving
  = (mass per unit time) × time
  = $\rho A_2 v_2 \Delta t$
- Hence, from conservation of mass we have,
  $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$

⇔ $A_1 v_1 = A_2 v_2$ (Equation of Continuity)

- *The product of the area of cross section and the speed remains the same at all points of a tube of flow***.** This is called the "equation of continuity" and expresses the law of conservation of mass in fluid dynamics.

Bernoulli's Principle

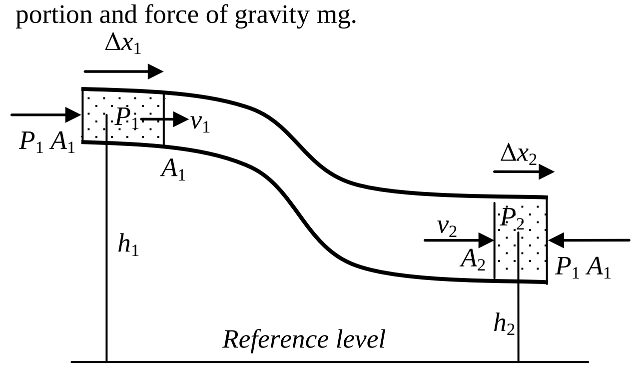
- Bernoulli's Principle relates the speed of a fluid at a point the pressure at that point and the height of that point above a reference level. It is just the application of work-energy theorem in the case of fluid flow.
- We here consider the case of *irrotational* and *steady flow* of an *incompressible* and *non viscous liquid*.
- **Bernoulli's Principle states that the sum of pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume of an incompressible, non-viscous fluid in a streamlined irrotational flow remains constant along a streamline.** Mathematically,

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

$$\Leftrightarrow p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Proof of Bernoulli's Principle

- One end of the pipe is horizontal at a height h_1 above some reference level and has uniform cross-sectional area A_1 up to some length. The other end is at a height h_2 from the reference level and has uniform cross-sectional area A_2.
- Now consider the portion of fluid shown by shaded volume as the system. Suppose the system of fluid gets displaced from the position1 shown in figure to that in position 2 in a small time interval.
- Now, we shall find out the work done by different forces to use Work-Kinetic Energy theorem.
- Here four forces are acting on the system. Normal force from wall, force P_1A_1 on left portion, force P_2A_2 on right portion and force of gravity mg.



- **Work done W_N by normal force from the walls:**
  $W_N = 0$ …(i)
  (because normal force is perpendicular to motion of fluid)
- **Work done W_1 by force P_1A_1 at the left end:**
  W_1 = force × displacement = $P_1 A_1 \Delta x_1$ …(ii)
- **Work done W_2 by force P_2A_2 at the right end:**
  W_2 = force × displacement = $- P_2 A_2 \Delta x_2$ …(iii)
  (negative sign because force P_2A_2 & displacement Δx_2 are in opposite directions)
- **The work done on the system W_3, by the gravitational force mg**:
  W_g = force × displacement = $mg(h_1 - h_2)$ …(iv)
- ∴ the total work done on the system, by using Work- KE theorem, we have:
  $W_T = \Delta KE$

⇔ $W_N + W_1 + W_2 + W_g = ½ mv_2^2 - ½ mv_1^2$

⇔ $P_1A_1\Delta x_1 - P_2A_2\Delta x_2 + mg(h_1 - h_2) = ½ mv_2^2 - ½ mv_1^2$ … (v)

- As the liquid is **incompressible (means density ρ is uniform)**, the mass flow rate at both the ends must be the same.
  $\rho A_1 \Delta x_1 = \rho A_2 \Delta x_2 = m$
  (because, mass = density x volume
  = density x area x length)
- ⇔ $A_1 \Delta x_1 = A_2 \Delta x_2 = m/\rho$ … (vi)
- From equations (v) and (vi), we have
  $P_1 m/\rho - P_2 m/\rho + mg(h_1 - h_2) = ½ mv_2^2 - ½ mv_1^2$

⇔ $P_1 - P_2 + \rho g(h_1 - h_2) = ½ \rho v_2^2 - ½ \rho v_1^2$
(multiplying both sides by ρ/m)

⇔ ⇔ $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$
(rearranging)

⇔ $p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Memory Map for proof of Bernoulli's theorem

- **Step 1:** Find out the work done by normal force, forces due to pressures at left and right ends and force due to gravity.
- **Step 2: Use Work Kinetic Energy theorem:**
  $W_T = \Delta KE$

⇔ $W_N + W_1 + W_2 + W_g = ½ mv_2^2 - ½ mv_1^2$

⇔ $P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 + mg(h_1 - h_2) = ½ mv_2^2 - ½ mv_1^2$

- **Step 3: Mass flow rate at both the ends must be the same.**
  $\rho A_1 \Delta x_1 = \rho A_2 \Delta x_2 = m$

⇔ $A_1 \Delta x_1 = A_2 \Delta x_2 = m/\rho$

- Use these two equations and rearrange to get Bernoulli's theorem.

Another form of Bernoulli's Equation

- Bernoulli's Equation,

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

- If we divide both sides by **ρg** we get
- $\frac{p}{\rho g} + \frac{1}{2}\frac{V^2}{g} + h = \text{constant}$
- Each term in the above equation has the dimension of length and hence every term is known as 'head'.

✓ The first term $\frac{p}{\rho g}$ is called 'pressure head'.

✓ The second term $\frac{1}{2}\frac{V^2}{g}$ is called 'velocity head'.

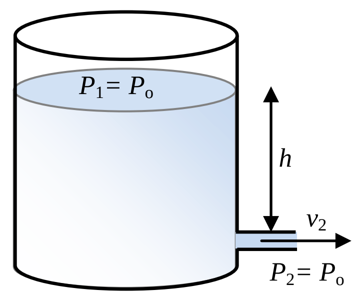
✓ The third term h is known as the 'elevation head'.

Torricelli's Theorem

(Speed of Efflux)

- Consider liquid of density ρ in a tank of large cross sectional area A_1. There is a **very small hole** of cross-sectional area A_2 at the bottom with liquid flowing out as shown in figure. Such a hole is called an **orifice**.
- Let v_1 and v_2 be the speed and P_1 and P_2 be the speed of the liquid at position 1 and 2 respectively.
- The idea here is that both the tank and the narrow opening (orifice) are open to the atmosphere. The pressure will be the same at 1 and 2 because they are open to the atmosphere. Therefore, $P_1 = P_2 = P_o$ = atmospheric pressure.
- From the equation of continuity, we get

  $A_1 v_1 = A_2 v_2$ or $v_1 = \frac{A_2}{A_1} v_2$
- As $A_1 >> A_2$, so the liquid may be taken at rest at the top, *i.e.*, $v_1 = 0$. Applying Bernoulli's equation at points 1 and 2, we get



- Now, applying Bernoulli's equation at positions 1 and 2, we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

⇔ $P_0 + \frac{1}{2}\rho(0)^2 + \rho g(h) = P_0 + \frac{1}{2}\rho v_2^2 + \rho g(0)$

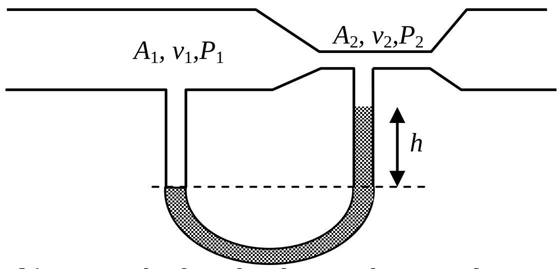
⇔ $\rho g h = \frac{1}{2}\rho v_2^2$

⇔ **$v_2 = \sqrt{2gh}$** (speed of efflux)

- *The speed of liquid coming out though a small hole (orifice) at a depth 'h' below the free surface is same as that of a particle fallen freely through the height 'h' under gravity. This is known as Torricelli's theorem.*
- *The speed of the liquid coming out is called the speed of efflux.*

Venturimeter

- **Venturimeter:** *It is a device used to measure the rate of flow of a liquid through a pipe. It is an application of Bernoulli's principle*. It is also called flow meter or venture tube.
- **Construction.** It consists of a horizontal tube having wider opening of cross-section A_1 and a narrow neck of cross-section A_2. These two regions of the horizontal tube are connected to a manometer, containing a liquid of density ρ_m.



- **Working.** Let the liquid velocities be v_1 and v_2 at the wider and the narrow portions. Let P_1 and P_2 be the liquid pressures at these regions. By the equation of continuity,

$$A_1v_1 = A_2v_2 \Rightarrow \frac{A_1}{A_2} = \frac{v_2}{v_1}$$

- If the liquid has density ρ and is flowing horizontally, then from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad (\because h_1 = h_2)$

or $P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

$$= \frac{1}{2}\rho v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right)$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \qquad \left[\because \frac{A_1}{A_2} = \frac{v_2}{v_1}\right]$$

$$= \frac{1}{2}\rho v_1^2\left(\frac{A_1^2 - A_2^2}{A_2^2}\right)$$

- If h is the height difference in the two arms of the manometer tube, then

$$P_1 - P_2 = \rho_m hg$$

- $$\therefore \rho_m hg = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2 - A_2^2}{A_2^2}\right)$$

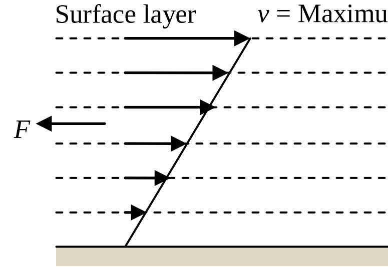
$$\therefore \quad v_1 = \sqrt{\frac{2h\rho_m g}{\rho} \times \frac{A_2^2}{A_1^2 - A_2^2}}$$

- Volume flow rate of the liquid,

$$Q = A_1v_1 = A_1 A_2\sqrt{\frac{2h\rho_m g}{\rho\left(A_1^2 - A_2^2\right)}}.$$

Viscosity

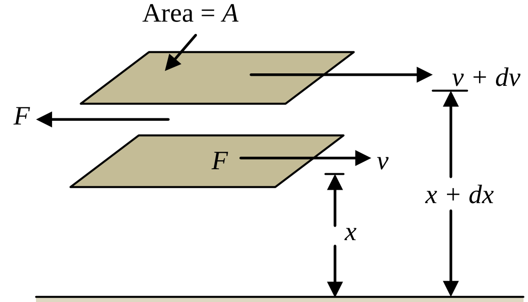
- Informally, *viscosity is the quantity that describes a fluid's resistance to flow.*
- Fluids resist the relative motion of immersed objects through them as well as to the motion of layers with differing velocities within them.
- **In a laminar flow, the relative velocity between the consecutive layers of fluid results in tangential force at the surfaces of the layers known as viscous force and the property of the fluid causing it is known as viscosity.**



- *The layer of the liquid in contact with the surface remain stuck to it due to adhesive force and has zero velocity. The velocity of layer gradually increases on moving upwards from the surface and is the largest at the top.*
- If a liquid flow easily, it means it has less viscosity; e.g., kerosene is less viscous than diesel. Similarly, honey is more viscous than water.
- Hard materials such as rock can be considered as liquids, because they can flow - although extremely slowly. Glass windows in very old buildings are often thicker at the bottom than the top because over hundreds of years the glass has flowed downwards.

Coefficient of viscosity

- Consider the steady flow of liquid on some horizontal stationary surface as shown in the figure.



- **According to Newton, the viscous force F between two adjacent layers of a laminar flow at a given temperature is**

i. **directly proportional to the area (A) of the layers in contact**

$F \propto A$ …(i)

ii. **directly proportional to the velocity gradient** $\frac{dv}{dx}$

$F \propto \frac{dv}{dx}$ …(ii)

Combining (i) & (ii)

$F \propto A\frac{dv}{dx}$

⇔ $F = -\eta A\frac{dv}{dx}$ (viscous force)

where η is the constant of proportionality known as the coefficient of viscosity of the fluid. Its magnitude depends on the type of the fluid and its temperature.

- Negative sign states that the direction of viscous force is opposite to that of relative velocity of the layer wrt another layer.
- **S.I. Unit of η** : Ns/m^2 which is same as Pa-s
- **CGS Unit of η** : dyne-s/cm^2 which is also known as **poise.**
  1 Pa-s = 10 poise
- Its dimensional formula is $M L^{-1} T^{-1}$.

Definition of η

- Taking A = 1 square unit and $\frac{dv}{dx}$ = 1 unit in the equation

  $F = \eta A\frac{dv}{dy}$, we get $\eta = F$.

  Thus, *the coefficient of viscosity can be defined as the viscous force acting per unit surface area of contact and per unit velocity gradient between two adjacent layers in a laminar flow of a fluid.*
- Note that the co-efficient of viscosity of liquids decrease with increase in temperature, while that of gases increase with the increase in temperature.

Stokes Law

- When an object moves through a fluid, it experiences a viscous force which acts in opposite direction of its velocity.
- The resistive force (viscous force) on a small, smooth, solid spherical body of radius R, moving with velocity v through a laminar viscous medium of large dimensions, having co-efficient of viscosity η is given by

$F = 6\pi\eta R v$ (Stokes law)

- This equation is called Stokes' Law which can be verified using dimensional analysis. This relationship is valid only for 'laminar flow'.
- This viscous force F acts opposite to velocity v of the object.

Proof:

- Viscous force depends on R, v and η.
- Let $F = kR^a v^b \eta^c$

  $\Rightarrow [F] = k[R]^a [v]^b [\eta]^c$

  $\Rightarrow [MLT^{-2}] = k[L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$
- Comparing coefficients of M, L and T

  $c = 1$

  $a + b - c = 1$

  $-b - c = -2$
- Solving we get,

  $a = b = c = 1$.
- Thus,

  $F = kR^1 v^1 \eta^1 = kRv\eta$
- Value of k is 6π. Therefore,

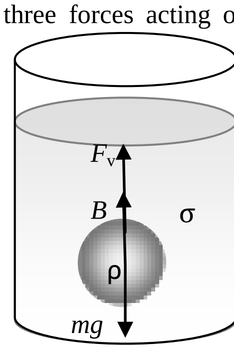
  $F = 6\pi\eta Rv$

Terminal velocity

- In fluid dynamics, **terminal velocity or settling velocity is the velocity at which the net force acting on an object moving through fluid becomes zero.**
- Terminal velocity varies directly with the ratio of viscosity to weight. More viscosity means a lower terminal velocity, while increased weight means a higher terminal velocity.

Terminal velocity of a small sphere

- Suppose a small, smooth, solid sphere of radius r of material having density ρ falls freely in a laminar fluid of density σ (< ρ) and co-efficient of viscosity η as shown in the figure.
- Let its terminal velocity be v in the downward direction.
- The FBD in this figure lists three forces acting on the sphere:



- **Weight mg downward:**

  Therefore, weight

  $mg = \left(\frac{4}{3}\right)\rho\pi r^3 g$ …(i)
- **Buoyant force B upward:**

  $B = V_{im}\,\rho_{liquid}\,g$

⇔ $B = \left(\frac{4}{3}\right)\pi r^3 \sigma g$ …(ii)

- **Viscous force F_v upward:**

  From Stokes' Law :

  $F_v = 6\,\pi\,\eta\,r\,v$ …(iii)

- At terminal velocity there is no acceleration, therefore from Newton's 2nd law, we have:

  $mg - B - F_v = 0$

  $$\left(\frac{4}{3}\right)\rho\pi r^3 g - \left(\frac{4}{3}\right)\pi r^3\sigma g - 6\pi\eta r v = 0$$

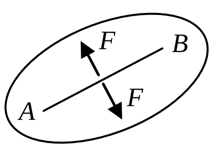
  ⇔ $$v = \left(\frac{2}{9}\right) r^2 g\left(\frac{\rho - \sigma}{\eta}\right)$$ (terminal velocity)

Surface Tension

- Liquids sometimes form drops, and sometimes spread over a surface and wet it. Why does this happen, and why are raindrops never a metre wide?
- The reason is that **a fluids try to occupy minimum surface area**. This is because of a fluid property known as surface tension.

Definition of surface tension

- Let us consider an imaginary line *AB* drawn in any direction in a liquid surface. The surface on either side of the liquid exerts a pulling force *F* on the other side.
- This *force F is perpendicular to the line AB and tangential to surface of the fluid.*



- We give the definition of surface tension as:
  "The force exerted by the molecules lying on one side of an imaginary line of unit length, on the molecules lying on the other side of the line, which is perpendicular to the line and parallel to the surface is defined as the surface tension (*S*) of the liquid."
- In simple words, surface tension is perpendicular force (from either side of line *AB*) per unit length.
- Thus, if *F* be the force acting on either side of the line *AB* of length *L*, then the surface tension *S* is given by:

  $$S = \frac{F}{L}$$ (Surface tension)

- Clearly, the **SI unit** of surface tension is **N/m**.

Surface Potential Energy

- Surface energy. The free surface of a liquid possesses minimum area due to surface tension. To increase the surface area, molecules have to be brought from interior to the surface. Work has to be done against the forces of attraction. This work is stored as the potential energy of the molecules on the surface. So the molecules at the surface have extra energy compared to the molecules in the interior.
- *The extra energy possessed by the molecules of surface film of unit area compared to the molecules in the interior is called surface energy.* It is equal to the work done in increasing the area of the surface film by unit amount.

  $$\text{Surface energy} = \frac{\text{work done}}{\text{increase in surface area}}$$

- Surface tension can also be defined as **"the potential energy (U) stored in the surface of the liquid per unit area . "**

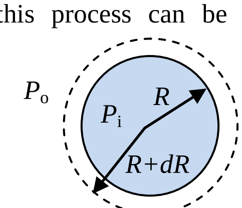
  $$S = \frac{U}{A}$$

  ⇔ $\boldsymbol{U = AS}$ (surface potential energy)

- By this definition, its SI unit is $\mathbf{Jm^{-2}}$ which is the same as $\mathbf{Nm^{-1}}$.

Drops and bubbles

- Let us assume that the pressures inside and outside are P_i *and* P_0 ($P_i > P_0$) respectively.
- **The pressure on the concave surface is always more than that on the convex surface.**
- Let the surface tension of the liquid forming the wall of the bubble be *T*.
- Suppose, on blowing the bubble, its radius increases from *R* to *R+dR*. The work done in this process can be calculated in two ways.



- **1st way:**
  W = (force) × (displacement)
  ⇔ W = pressure difference × area × displacement
  ⇔ $W = (P_i - P_o)\,4\,\pi\,R^2\,dR$ … (i)
- **2nd way:**
- The surface area of the bubble of radius R is, $S = 4\,\pi\,R^2$
- ∴ the increase in the surface area is,
  $dS = 4\,\pi\,(R + dR)^2 - 4\,\pi\,R^2$
  ⇔ $dS = 4\,\pi\,(R^2 + dR^2 + 2R.\,dR - R^2)$
  ⇔ $dS = 4\,\pi\,(dR^2 + 2R.\,dR)$
  ⇔ $dS = 8\,\pi\,R.\,dR$
  (because dR^2 is very small, we can ignore it)

- W = surface tension × total increase in area

⇔ $W = 8 \pi S R\, dR$ (ii)

- Equating equations (i) and (ii), we get:

$(P_i - P_o)\, 4 \pi R^2 \cdot dR = 8 \pi S R\, dR$

⇔ $p_i - p_o = \frac{2S}{R}$ (Excess pressure inside a liquid drop)

- For a soap bubble which has tow surface areas,

$p_i - p_o = \frac{4S}{R}$ (Excess pressure inside a soap bubble)

- Try to prove it yourself. Hint: Equation (ii) will be

W = 16 π S R dR as soap bubble has two surfaces.

Drops and bubbles (Quick recap)

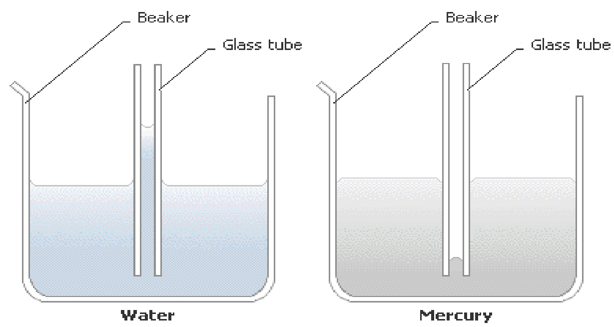
- The pressure on the concave surface is always more than that on the convex surface. $P_i > P_o$
- For the case of **air bubble inside water,** only one surface is formed. Therefore, for **air bubble** (or for any bubble or drop where single surface is formed) $p_i - p_o = \frac{2S}{R}$
- For the case of **soap bubble** where two surfaces are formed: $p_i - p_0 = \frac{4S}{R}$

Shape of Liquid Surface

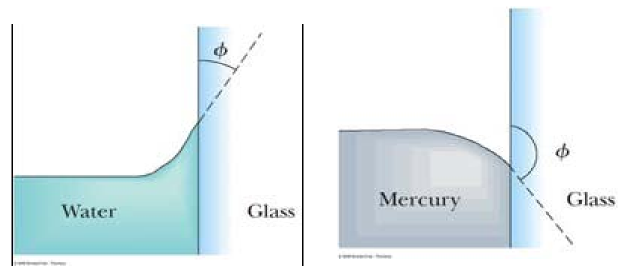
- You must have seen water wets a glass container whereas mercury does not. Why?
- Before that let us familiarize with two new kind of intermolecular forces.:
- **Cohesive force :** Inter-molecular attractive force between molecules of the same matter.
- **Adhesive force :** Attractive force between molecules of different matters.
- Water molecules, for instance, are more attracted to glass than they are to one another. *It means in case of water cohesive forces are stronger than adhesive forces.* Water will therefore climb up a narrow glass tube that is dipped into a beaker of water, because the water would rather be in contact with the glass than with itself.
- Mercury molecules, on the other hand, are more attracted to each other than they are to glass. *It means in case of mercury cohesive forces are lesser than adhesive forces.* Mercury will avoid contact with a narrow glass tube that is dipped into a beaker of mercury.

Capillary Action and Contact angle

- **Capillarity:** Liquids display a behavior called **capillarity** or **capillary action** (capillary is a kind of narrow glass tube) because their molecules are more or less attracted to the surface they contact than they are to themselves.
- **"The phenomenon of rise or fall of a liquid in a capillary, held vertical in a liquid, due to its property of surface tension is called capillarity."**
- Capillary action is the result of surface tension and adhesive forces.



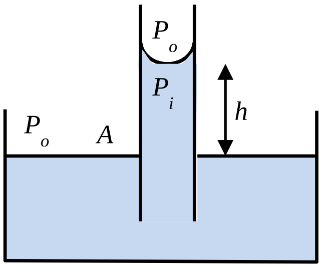
- **Contact Angle:** The tangent drawn at a point , where the surface of meniscus is in contact with wall of the capillary, makes an angle θ with the wall. θ is known as the contact angle of the liquid with the matter of the capillary.

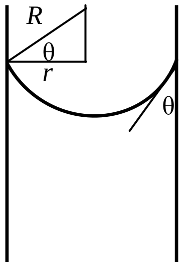


- **Case 1: The adhesive forces (liquid-glass) are greater than the cohesive forces (liquid-liquid)**
- The liquid clings to the walls of the container the liquid "wets" the surface, e.g., water. The meniscus of water in the capillary is **concave.**
- In this case contact angle, **$\theta < 90°$.**
- **Case 2: The adhesive forces (liquid-glass) are lesser than the cohesive forces (liquid-liquid)**
- The liquid curves downward the liquid does not "wet" the surface, e.g., mercury. The meniscus of mercury in the capillary is **convex.**
- In this case contact angle, **$\theta > 90°$.**

CAPILLARY RISE

- We are going to derive capillary rise h (the height or depth) above which liquid rises or falls in a capillary tube when it is inserted in fluid).
- Suppose liquid rises to height h in a capillary of radius r held vertical in the liquid as shown in the figure.
- The radius of concave meniscus of liquid in the capillary is R.





- From second figure, it is clear that

$$\cos\theta = \frac{r}{R}$$

⇔ $R = \dfrac{r}{\cos\theta}$ … (i)

- The pressure on the concave surface of the meniscus (P_o) is greater than the pressure on the convex surface (P_i).

$$\therefore\ P_o - p_i = \frac{2S}{R} \quad \text{… (ii)}$$

(because, the liquid has one free surface.)

- Also, for equilibrium, the pressure at point B is the same as at point A which is P_o as both are at the same horizontal level.

$$\therefore\ P_o - P_i = h\rho g \quad \text{… (iii)}$$

where ρ = density of the liquid and g = acceleration due to gravity.

- Comparing equations (2) and (3),

$$\frac{2S}{R} = h\rho g$$

⇔ $h = \dfrac{2RS}{\rho g} = \dfrac{2S\cos\theta}{r\rho g}$

(putting the value of R from equation (i))

- Hence, capillary rise is given by

$$h = \frac{2S\cos\theta}{r\rho g} \quad \text{(Capillary rise)}$$

- For mercury and glass, θ > 90°. Hence, cos θ is negative. Therefore, mercury falls in a glass capillary and its meniscus is convex.

Mechanical Properties of Matter

Elasticity

- **Elasticity:** If a body regains its original size and shape after the removal of deforming force, it is said to be elastic body and this property is called elasticity.
- **Perfectly elastic body:** If a body regains its original size and shape completely and immediately after the removal of deforming force, it is said to be a perfectly elastic body. The nearest approach to a perfectly elastic body is quartz fibre.
- **Plasticity:** If a body does not regain its original size and shape even after the removal of deforming force, it is said to be a plastic body and this property is called plasticity.
- **Perfectly plastic body:** If a body does not show any tendency to regain its original size and shape even after the removal of deforming force, it is said to be a perfectly plastic body. Putty and paraffin wax are nearly perfectly plastic bodies.
- **Note:** No body is perfectly elastic or perfectly plastic. All the bodies found in nature lie between these two limits. When the elastic behavior of a body decreases, its plastic behavior increases.

Stress

- **Stress:** *The internal restoring force set up per unit area of cross-section of the deformed body is called stress.* As the restoring force is equal and opposite to the external deforming force F under equilibrium, therefore

$$\text{Stress} = \frac{F}{A} \quad \text{(stress)}$$

The SI unit of stress is Nm^{-2} and the CGS unit is dyne cm^{-2}. The dimensional formula of stress is $[ML^{-1}T^{-2}]$.

- **Types of stress:**

(a) **Tensile stress:** It is the restoring force set up per unit cross-sectional area of a body when the length of the body increases in the direction of the deforming, force. It is also known as *longitudinal stress*.

(b) **Compressive stress:** It is the restoring force set up per unit cross-sectional area of a body when its length decreases under a deforming force.

(c) **Hydrostatic stress**: If a body is subjected to a uniform force from all sides, then the corresponding stress is called *hydrostatic stress* or *volume stress*.

(d) **Tangential or Shearing stress:** When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of the body. The tangential force applied per unit area is equal to the tangential stress.

Strain

- **Strain:** The ratio of the change in any dimension produced in the body to the original dimension is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Strain has no units and dimensions.

Types of strain

(a) **Longitudinal strain:** It is defined as the increase in length per unit original length, when the body is deformed by external forces.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$$

(b) **Volumetric strain**: It is defined as the change in volume per unit original volume, when the body is deformed by external forces.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$

(c) **Shear strain:** It is defined as the angle θ (in radian), through which a face originally perpendicular to the fixed face gets turned on applying tangential deforming force.

$$\text{Shear strain} = \theta = \tan\theta = \frac{\text{Relative displacement between 2 parallel planes}}{\text{Distance between parallel planes}}$$

- **Elastic limit:** The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit. If the deforming force exceeds the elastic limit, the body acquires a permanent set or deformation and is said to be *overstrained*.

HOOKE'S LAW & MODULUS OF ELASTICITY

- **Hooke's law:** It states that within the elastic limit, the stress is directly proportional to strain. Thus within the elastic limit,

Stress $\propto$ Strain

$$\Rightarrow \frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

Modulus of elasticity: *The modulus of elasticity or coefficient of elasticity of a body is defined as the ratio of stress to the corresponding strain, within the elastic limit.*

$$E = \frac{\text{Stress}}{\text{Strain}}$$

The SI unit of modulus of elasticity is Nm^{-2} and its dimensions are $[ML^{-1}T^{-2}]$.

Units and dimensions of moduli of elasticity: The SI unit of moduli of elasticity is Nm^{-2} and its CGS unit is dyne cm^{-2}. Its dimensional formula is $[ML^{-1}T^{-2}]$. Its value depends on the nature of the material of the body and the manner in which it is deformed.

- **Different types of moduli of elasticity**.: Corresponding to the three types of strain, we have three important moduli of elasticity:

(a) **Young's modulus (*Y*):** Within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus. Thus,

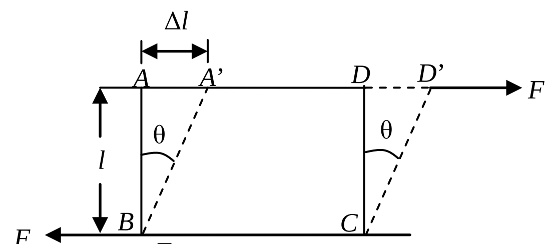
$$Y = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}} \qquad \text{(Young's modulus)}$$

- $\therefore Y = \frac{F/A}{\Delta l / l}$

$\Rightarrow Y = \frac{F}{A}.\frac{l}{\Delta l}$

(b) **Modulus of rigidity or shear modulus or torsional modulus (η):** Within the elastic limit, the ratio of shear stress to shear strain is called modulus of rigidity. Thus

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}}$$



Shear stress $= \frac{F}{A}$

- Shear strain $= \theta \approx \tan\theta = \frac{AA'}{AB} = \frac{\Delta l}{l}$

- The modulus of rigidity is given by

$$\eta = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A}.\frac{l}{\Delta l}$$

(c) **Bulk modulus (*B*)**: Within the elastic limit, the ratio of volume stress to the volumetric strain is called bulk modulus of elasticity.

- Consider a body of volume *V* and surface area *A*. Suppose a force *F* acts uniformly over the whole surface of the body and it decreases the volume by Δ*V*, then bulk modulus of elasticity is given by

$$B = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} \qquad \text{(Bulk modulus)}$$

$$\therefore B = -\frac{F/A}{\Delta V/V} = \frac{F}{A}.\frac{V}{\Delta V}$$

- $$\Rightarrow \boxed{B = -\frac{pV}{\Delta V}}$$

  where p (= F/A) is the normal pressure. Negative sign shows that the volume decreases with the increase in stress.

- **Compressibility (K):** The reciprocal of the bulk modulus of a material is called its compressibility. Thus,

$$K = \frac{1}{B} = -\frac{\Delta V}{pV} \qquad \text{(Compressibility)}$$

Poisson's ratio

- When a wire is loaded, its length increases but its diameter decreases. The strain produced in the direction of applied force is called longitudinal strain and that produced in the perpendicular direction is called lateral strain.
- **Definition:** Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

  Suppose the length of the loaded wire increases from l to $l+\Delta l$ and its diameter decreases from D to $D–\Delta D$.

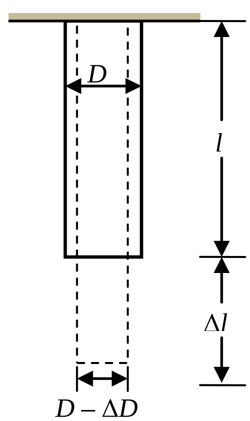
  Longitudinal strain $= \frac{\Delta l}{l}$

  Lateral strain $= -\frac{\Delta D}{D}$

  Poisson's ratio is

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \sigma = \frac{-\Delta D/D}{\Delta l/l}$$



- The negative sign indicates that longitudinal and lateral strains are in opposite sense.
- As the Poisson's ratio is the ratio of two strains, it has no units and dimensions.

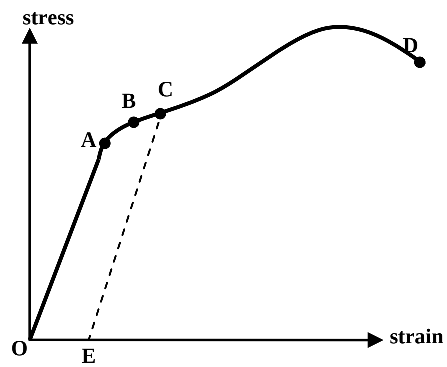
SPACE FOR NOTES

Stress-strain curve

- Figure shows a stress-strain curve for a metal wire which is gradually being loaded.

(a) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A Hooke's law is obeyed. The point A is called the **proportional limit.** In this region, the wire is perfectly elastic.

(b) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called **elastic region** and the point B is called **elastic limit or yield point**. The stress corresponding to the yield point is called yield strength (S_y).



(c) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CE. Even on reducing the stress to zero, a residual strain equal to OE is left in the wire. The material is said to have acquired a **permanent set**. The fact that the stress-strain curve is not retraced on reversing the strain is called **elastic hysteresis.**

(d) If the load is increases beyond the point C, there is large increase in the strain or the length of the wire. In this region, the constrictions (called necks and waists) develop at few points along the length of the wire and the wire ultimately breaks at the point D, called the **fracture point.** In the region between B and D, the length of wire goes on increasing even without any addition of load. This region is called ***plastic region*** and the material is said to undergo *plastic flow* or *plastic deformation*. The stress corresponding to the breaking point is called *ultimate strength* or *tensile strength* of the material.

Elastic potential energy of a stretched wire

- **To prove:**

Elastic potential energy $=\frac{1}{2}\times$Stress × strain × volume

Proof:

- Suppose a force F applied on a wire of length l increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl, the internal force increases from zero to F (= applied force).
  ∴ Average internal force for an increase in length Δl of wire $=\frac{0+F}{2}=\frac{F}{2}$
- Work done on the wire is
  W = Average force × increase in length $=\frac{F}{2}\times\Delta l$
- This work done is stored as elastic potential energy U in the wire.
  $\therefore U=\frac{1}{2}F\times\Delta l=\frac{1}{2}$ Stretching force × increase in length
- Let A be the area of cross-section of the wire. Then
  $\therefore U=\frac{1}{2}\frac{F}{A}\times\frac{\Delta l}{l}\times Al$
  ∴ $U=\frac{1}{2}\times$ Stress × strain × volume
- Elastic potential energy per unit volume of the wire or elastic energy density is
  $u=\frac{U}{\text{Volume}}$
  or $u=\frac{1}{2}\text{stress}\times\text{strain}$
- But stress = Young's modulus × strain

$\therefore u=\frac{1}{2}\times$Young's modulus × strain^2

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C = O n̂ CEPT
Crystal clear concepts by IITians

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Physics Classes by Pranjal Sir

(Admission Notice for XI & XII - 2014-15)

Batches for Std XIIth

Batch 1 (Board + JEE Main + Advanced): (Rs. 16000)
Batch 2 (Board + JEE Main): (Rs. 13000)
Batch 3 (Board): (Rs. 10000)
Batch 4 (Doubt Clearing batch): Rs. 8000

About P. K. Bharti Sir (Pranjal Sir)

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- Visiting faculty at D. P. S. Bokaro
- Produced AIR 113, AIR 475, AIR 1013 in JEE - Advanced
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Physics Class Schedule for Std XIIth (Session 2014-15) by Pranjal Sir

Sl. No.	Main Chapter	Topics	Board level	JEE Main Level	JEE Adv Level
	Basics from XIth	Vectors, FBD, Work, Energy, Rotation, SHM	3rd Mar to 4th Apr 14		
1.	**Electric Charges and Fields**	**Coulomb's Law**	**5th & 6th Apr**	**5th & 6th Apr**	**5th & 6th Apr**
		Electric Field	**10th & 12th Apr**	**10th & 12th Apr**	**10th & 12th Apr**
		Gauss's Law	**13th & 15th Apr**	**13th & 15th Apr**	**13th & 15th Apr**
		Competition Level	**NA**	**17th & 19th Apr**	**17th & 19th Apr**
2.	**Electrostatic Potential and Capacitance**	**Electric Potential**	**20th & 22nd Apr**	**20th & 22nd Apr**	**20th & 22nd Apr**
		Capacitors	**24th & 26th Apr**	**24th & 26th Apr**	**24th & 26th Apr**
		Competition Level	**NA**	**27th & 29th Apr**	**27th & 29th Apr, 1st, 3rd & 4th May**
PART TEST 1		Unit 1 & 2	4th May	NA	NA
			NA	11th May	11th May
3.	**Current Electricity**	**Basic Concepts, Drift speed, Ohm's Law, Cells, Kirchhoff's Laws, Wheatstone bridge, Ammeter, Voltmeter, Meter Bridge, Potentiometer etc.**	**6th, 8th, 10th, 13th May**	**6th, 8th, 10th, 13th May**	**6th, 8th, 10th, 13th May**
		Competition Level	**NA**	**15th & 16th May**	**15th, 16th, 17th, 18th & 19th May**
PART TEST 2		Unit 3	**18th May**	**NA**	**NA**
			NA	**20th May**	**20th May**
SUMMER BREAK		**21st May 2013 to 30th May 2013**			
4.	**Moving charges and Magnetism**	**Force on a charged particle (Lorentz force), Force on a current carrying wire, Cyclotron, Torque on a current carrying loop in magnetic field, magnetic moment**	**31st May, 1st & 3rd Jun**	**31st May, 1st & 3rd Jun**	**31st May, 1st & 3rd Jun**
		Biot Savart Law, Magnetic field due to a circular wire, Ampere circuital law, Solenoid, Toroid	**5th, 7th & 8th Jun**	**5th, 7th & 8th Jun**	**5th, 7th & 8th Jun**
		Competition Level	**NA**	**10th & 12th Jun**	**10th, 12th, 14th & 15th Jun**
PART TEST 3		Unit 4	**15th Jun**	**NA**	**NA**

			NA	**22nd Jun**	**22nd Jun**
5.	**Magnetism and Matter**		**17th, 19th & 21st Jun**	**17th, 19th & 21st Jun**	**Not in JEE Advanced Syllabus**
6.	**Electromagnetic Induction**	**Faraday's Laws, Lenz's Laws, A.C. Generator, Motional Emf, Induced Emf, Eddy Currents, Self Induction, Mutual Induction**	**24th, 26th & 28th Jun**	**24th, 26th & 28th Jun**	**24th, 26th & 28th Jun**
		Competition Level	**NA**	**29th Jun & 1st Jul**	**29th Jun, 1st, 3rd & 5th Jul**
PART TEST 4		Unit 5 & 6	**6th Jul**	**NA**	**NA**
			NA	**13th Jul**	**13th Jul**
7.	**Alternating current**	**AC, AC circuit, Phasor, transformer, resonance,**	**8th, 10th & 12th Jul**	**8th, 10th & 12th Jul**	**8th, 10th & 12th Jul**
		Competition Level	**NA**	**15th July**	**15th & 17th July**
8.	**Electromagnetic Waves**		**19th & 20th July**	**19th & 20th July**	**Not in JEE Advanced Syllabus**
PART TEST 5		Unit 7 & 8	27th Jul	27th Jul	27th Jul
Revision Week		Upto unit 8	**31st Jul & 2nd Aug**	**31st Jul & 2nd Aug**	**31st Jul & 2nd Aug**
Grand Test 1		Upto Unit 8	3rd Aug	3rd Aug	3rd Aug
9.	**Ray Optics**	**Reflection**	**5th & 7th Aug**	**5th & 7th Aug**	**5th & 7th Aug**
		Refraction	**9th & 12th Aug**	**9th & 12th Aug**	**9th & 12th Aug**
		Prism	**14th Aug**	**14th Aug**	**14th Aug**
		Optical Instruments	**16th Aug**	**16th Aug**	**Not in JEE Adv Syllabus**
		Competition Level	**NA**	**19th & 21st Aug**	**19th, 21st, 23rd, 24th Aug**
10.	**Wave Optics**	**Huygens Principle**	**26th Aug**	**26th Aug**	**26th Aug**
		Interference	**28th & 30th Aug**	**28th & 30th Aug**	**28th & 30th Aug**
		Diffraction	**31st Aug**	**31st Aug**	**31st Aug**
		Polarization	**2nd Sep**	**2nd Sep**	**2nd Sep**
		Competition Level	**NA**	**4th & 6th Sep**	**4th, 6th, 7th, 9th, 11th Sep**
	PART TEST 6	Unit 9 & 10	**14th Sep**	**14th Sep**	**14th Sep**
REVISION ROUND 1 (For JEE Main & JEE Advanced Level): 13th Sep to 27th Sep					
Grand Test 2		Upto Unit 10	28th Sep	28th Sep	28th Sep
DUSSEHRA & d-ul-Zuha Holidays: 29th Sep to 8th Oct					
11.	**Dual Nature of Radiation and Matter**	**Photoelectric effect etc**	**9th & 11th Oct**	**9th & 11th Oct**	**9th & 11th Oct**
Grand Test 3		Upto Unit 10	12th Oct	12th Oct	12th Oct
12.	**Atoms**		**14th & 16th Oct**	**14th & 16th Oct**	**14th & 16th Oct**
13.	**Nuclei**		**18th & 19th Oct**	**18th & 19th Oct**	**18th & 19th Oct**
	X-Rays		**NA**	**21st Oct**	**21st & 25th Oct**
	PART TEST 7	Unit 11, 12 & 13	**26th Oct**	**NA**	**NA**
14.	**Semiconductors**	**Basic Concepts and Diodes, transistors, logic gates**	**26th, 28th, 30th Oct & 1st Nov**	**26th, 28th, 30th Oct & 1st Nov**	**Not in JEE Adv Syllabus**
15.	**Communication System**		**2nd & 4th Nov**	**2nd & 4th Nov**	**Not in JEE Adv Syllabus**
	PART TEST 8	Unit 14 & 15	**9th Nov**	**9th Nov**	**NA**
	Unit 11, 12 & 13	Competition Level	**NA**	**8th, 9th & 11th Nov**	**8th, 9th, 11th, 13th & 15th Nov**
	PART TEST 9	Unit 11, 12, 13, X-Rays	NA	16th Nov	16th Nov
	Revision Round 2 (Board Level)	Mind Maps & Back up classes for late registered students	18th Nov to Board Exams	18th Nov to Board Exams	18th Nov to Board Exams
	Revision Round 3 (XIth portion for JEE)		18th Nov to JEE	18th Nov to JEE	18th Nov to JEE
	30 Full Test Series	Complete Syllabus	Date will be published after Oct 2014		