UNIT-1: RANDOM PROCESS: Random variables: Several random variables. Statistical averages: Function of Random variables, moments, Mean, Correlation and Covariance function: Principles of autocorrelation function, cross – correlation functions. Central limit theorem, Properties of Gaussian process.

Introduction:

Transmission of information or a message from one point to another point is called communication. The two points that wants to interact is called transmitter and receiver points or can also be referred to as source and destination. Channel or a medium links any two points that wants to communicate. The channel can be wired (guided) or wireless (unguided). The information or a message is represented in the form of a signal on the channel depending on the nature of the channel. For example if the medium is a twisted pair or coaxial cable then the signal is electrical if the medium is optical then the signal is optical or light.

The signal can be analog or digital. If the signal is analog it is continuous and if it is digital it uses discrete representation. This version can be achieved with the application of base band approach like amplitude (AM), frequency (FM) and pulse (PM) modulation schemes for the analog signal representation. Digital signals use broad band approach like amplitude (ASK), frequency (FSK) and phase (PSK) shifting techniques. These signals are derived from sources that can also be classified as analog sources and digital sources. Examples of analog sources are microphone, TV cameras, and for digital source computer data is a best example. An AM radio system transmits electromagnetic waves with frequencies of around a few hundred kHz (MF band). The FM radio system must operate with frequencies in the range of 88-108 MHz (VHF band).

The information transfer can happen to a single point or to multiple points. If the signal transfer happens on a single link to only one receiving system the communication mode is called unicast communication eg:Telephone. If the signal transfer happens on multiple links to several or all receivers on the network, the communication mode is then called multicast or broadcast communication eg: Radio, Television. The Basic communication system model is shown in figure 1:

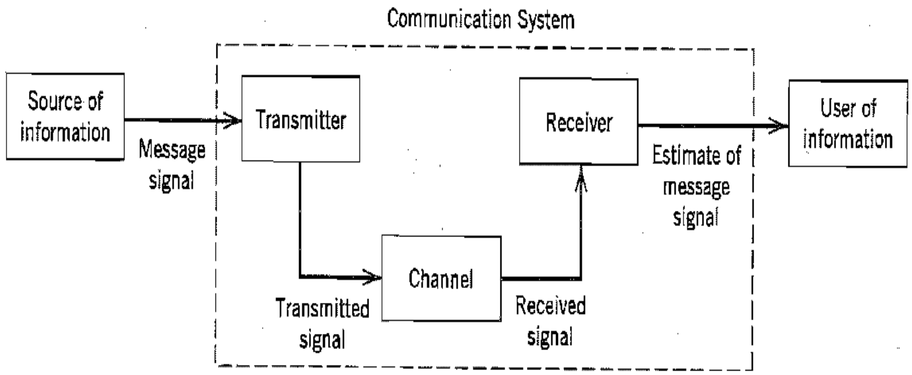


FIGURE 1 Elements of a communication system.

The majors communication resources of concerns are transmitted power, channel bandwidth, noise. The average power of the transmitted signal is referred to as the transmitted power. The Channel Bandwidth is the band of frequencies allotted for transmission and noise is any unwanted signals that disturb transmission of message signal. Hence designing a communication system that makes use of the resources efficiently is important. Hence the channel can be divided as power limited and band limited. Telephone channels are band limited where as satellite or radio channels are power limited.

Information Sources:

The communication environment is dominated by information sources like Speech, Music, Pictures, and Computer Data. Source of information can be characterized in terms of signals that carry information. Signal is a single valued function of time and an independent variable. The signal is single dimensional for speech, music and computer data. For pictures it's two dimensional and for video data it is three dimensional. For volume of data over time its four dimensional. The analog signal source produce continuous electrical signal with respect to time. The analog information source can be transformed into discrete source through the process of sampling and quantization. The discrete information source can be characterized as symbol rate, source alphabet, and source alphabet probabilities.

Communication Networks:

The communication networks consist of a number of nodes or stations or processors that perform the function of forwarding the data from one node/station to another node/station. The process of forwarding the data/message packets is called switching. There are three switching methods for data communication circuit switching, message switching, and packet switching. For circuit switching a dedicated path has to be provided. The link is fixed and reserves the bandwidth. Packet switching-uses store and forward, the path or bandwidth is allocated on demand.

Probability Theory:

Statistics is branch of mathematics that deals with the collection of data. It also concerns with what can be learned from data. Extension of statistical theory is Probability Theory. Probability deals with the result of an experiment whose actual outcome is not known. It also deals with averages of mass phenomenon. The experiment in which the Outcome cannot be predicted with certainty is called Random Experiment. These experiments can also be referred to as a probabilistic experiment in which more than one thing can happen. Eg: Tossing of a coin, throwing of a dice.

Deterministic Model and Stochastic Model or Random Mathematical Mode can be used to describe a physical phenomenon. In Deterministic Model there is no uncertainty about its time dependent behavior. A sample point corresponds to the aggregate of all possible outcomes. Sample space or ensemble composed of functions of time-Random Process or stochastic Process.

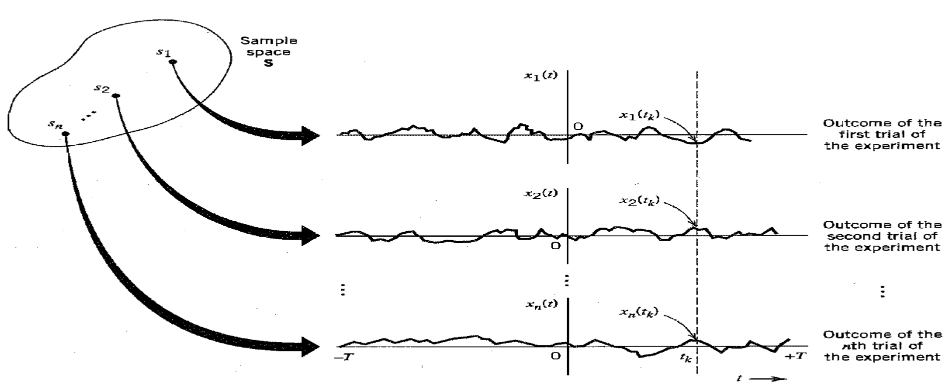


FIGURE 1.1 An ensemble of sample functions.

x1(t) is an outcome of experiment 1
x2(t) is the outcome of experiment 2
...
xn(t) is the outcome of experiment n

Each sample point in S is associated with a sample function x(t). X(t; s) is a random process is an ensemble of all time functions together with a probability rule. X(t; sj) is a realization or sample function of the random process.Probability rules assign probability to any meaningful event associated with an observation An observation is a sample function of the random process.

{x1(tk); x2(tk); :::; xn(tk)g = f X(tk; s1);X(tk; s2); :::;X(tk; sn)}

$X(t_k; s_j)$ constitutes a random variable. Outcome of an experiment mapped to a real number. An oscillator with a frequency ω0 with a tolerance of 1%.The oscillator can take values between ω0 (1±0.01). Each realization of the oscillator can take any value between (ω0) (0.99) to (ω0) (1.01). The frequency of the oscillator can thus be characterized by a random variable.

Statistical averages are important in the measurement of quantities that are obscured by random variations. As an example consider the problem of measuring a voltage level with a noisy instrument. Suppose that the unknown voltage has value a and that the instrument has an uncertainty x. The observed value may be y = a + x. Suppose that n independent measurements are made under identical conditions, meaning that neither the unknown value of the voltage nor the statistics of the instrument noise change during the process. Let us call the n measurements y_i, $1 \leq i \leq n$. Under our model of the process, it must be the case that $y_i = a + x_i$.

Now form the quantity

$$\tilde{y}(n) = 1/n \, \Sigma \, y_i \qquad \text{where } i=1\ldots n$$

This is the empirical average of the observed values. It is important to note that ỹ (n) is a random variable because it is a numerical value that is the outcome of a random experiment. That means that it will not have a single certain value. We expect to obtain a different value if we repeat the experiment and obtain n new measurements. We also expect that the result depends upon the value of n, and have the sense that larger values of n should give better results.

Types of Random Variable(RV):

1. Discrete Random Variable: An RV that can take on only a finite or countably infinite set of outcomes.

2. Continuous Random Variable: An RV that can take on any value along a continuum (but may be reported "discretely").Random Variables are denoted by upper case letters (*Y*).Individual outcomes for RV are denoted by lower case letters (*y*)

Discrete Random Variable:

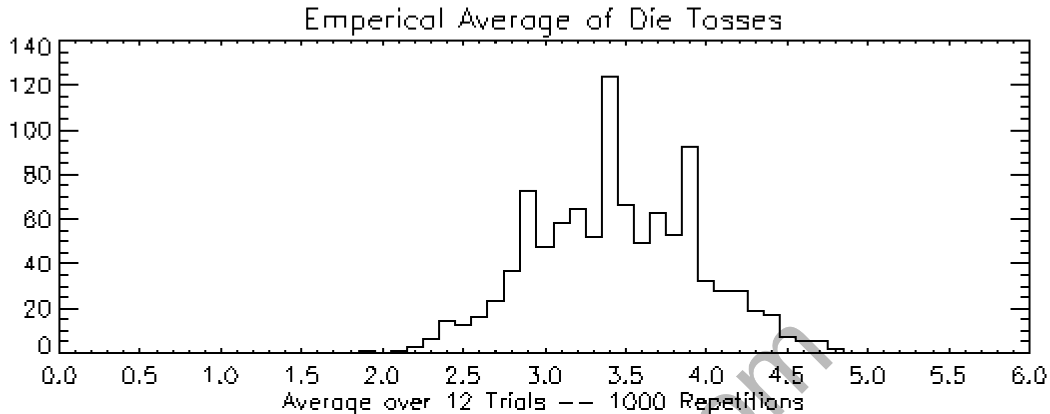
Discrete Random Variable are the ones that takes on a countable number of values this means you can sit down and list all possible outcomes without missing any, although it might take you an infinite amount of time.

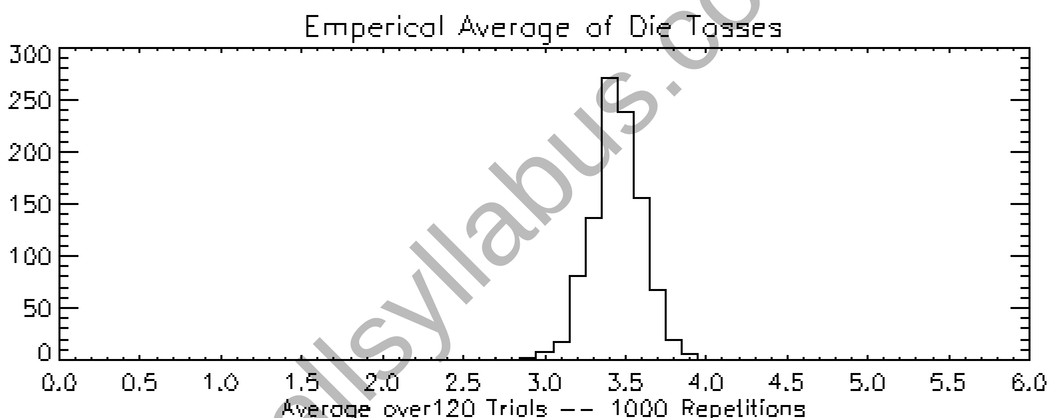
X = values on the roll of two dice: X has to be either 2, 3, 4, …, or 12.
Y = number of accidents on the UTA campus during a week: Y has to be 0, 1, 2, 3, 4, 5, 6, 7, 8, ……………"real big number"

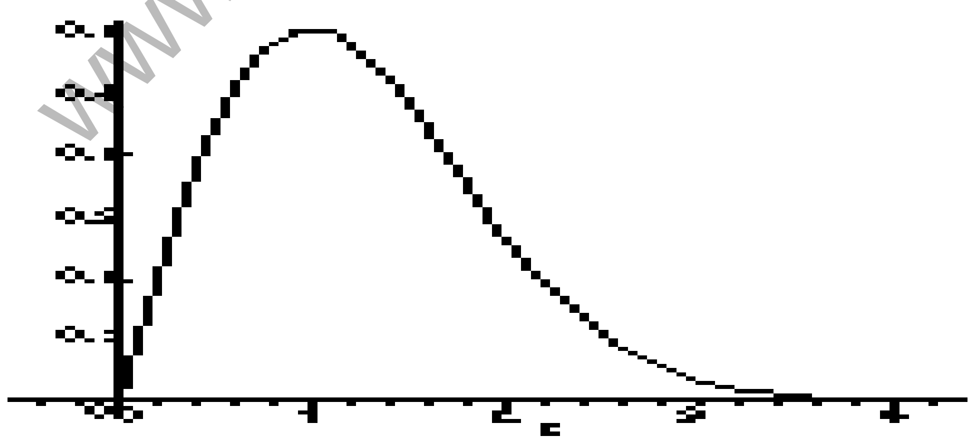
Probability Distribution: Table, Graph, or Formula that describes values a random variable can take on, and its corresponding probability (discrete RV) or density (continuous RV).

1. Discrete Probability Distribution: Assigns probabilities (masses) to the individual outcomes
2. Continuous Probability Distribution: Assigns density at individual points, probability of ranges can be obtained by integrating density function
3. Discrete Probabilities denoted by: $p(y) = P(Y=y)$
4. Continuous Densities denoted by: $f(y)$
5. Cumulative Distribution Function: $F(y) = P(Y \leq y)$

For a discrete random variable, we have a probability mass function (pmf).The pmf looks like a bunch of spikes, and probabilities are represented by the heights of the spikes. For a continuous random variable, we have a probability density function (pdf). The pdf looks like a curve, and probabilities are represented by areas under the curve.







Continuous Random Variable:

Continuous Random Variable is usually measurement data [time, weight, distance, etc] the one that takes on an uncountable number of values this means you can never list all possible outcomes even if you had an infinite amount of time.

X = time it takes you to drive home from class: X > 0, might be 30.1 minutes measured to the nearest tenth but in reality the actual time is 30.10000001…………………. minutes?)
A continuous random variable has an infinite number of possible values & the probability of any one particular value is zero.

Random Process:

A (one-dimensional) random process is a (scalar) function y(t), where t is usually time, for which the future evolution is not determined uniquely by any set of initial datalor at least by any set that is knowable to you and me. In other words, \random process" is just a fancy phrase that means \unpredictable function". Random processes y take on a continuum of values ranging over some interval, often but not always -∞ to +∞. The generalization to y's with discrete (e.g., integral) values is straightforward.

Examples of random processes are:

(i) the total energy E(t) in a cell of gas that is in contact with a heat bath;
(ii) the temperature T(t) at the corner of Main Street and Center Street in Logan, Utah;
(iii) the earth-longitude _(t) of a specific oxygen molecule in the earth's atmosphere.

One can also deal with random processes that are vector or tensor functions of time. Ensembles of random processes. Since the precise time evolution of a random process is not predictable, if one wishes to make predictions one can do so only probabilistically. The foundation for probabilistic predictions is an ensemble of random processes |i.e., a collection of a huge number of random processes each of which behaves in its own, unpredictable way. The probability density function describes the general distribution of the magnitude of the random process, but it gives no information on the time or frequency content of the process. Ensemble averaging and Time averaging can be used to obtain the process properties

- Ensemble averaging : Properties of the process are obtained by averaging over a collection or 'ensemble' of sample records using values at corresponding times
- Time averaging :Properties are obtained by averaging over a single record in time

Stationary random processes:

A random process is said to be stationary if its statistical characterization is independent of the observation interval over which the process was initiated. Ensemble averages do not vary with time. An ensemble of random processes is said to be stationary if and only if its probability distributions p_n depend only on time differences, not on absolute time:

$p_n(y_n; t_n + \tau ; : : : ; y_2; t_2 + \tau ; y_1; t_1 + \tau) = p_n(y_n; t_n; : : : ; y_2; t_2; y_1; t_1)$:

If this property holds for the absolute probabilities p_n. Most stationary random processes can be treated as ergodic. A random process is ergodic if every member of the process carries with it the complete statistics of the whole process. Then its ensemble averages will equal appropriate time averages. Of necessity, an ergodic process must be stationary, but not all stationary processes are ergodic.

Nonstationary random processes:

Nonstationary random processes arise when one is studying a system whose evolution is influenced by some sort of clock that cares about absolute time. For example, the speeds v(t) of the oxygen molecules in downtown Logan, Utah make up an ensemble of random processes regulated in part by the rotation of the earth and the orbital motion of the earth around the sun; and the influence of these clocks makes v(t) be a nonstationary random process. By contrast, stationary random processes arise in the absence of any regulating clocks. An example is the speeds v (t) of oxygen molecules in a room kept at constant temperature.