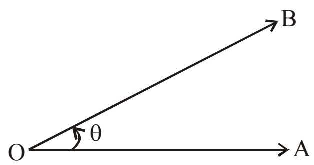
TRIGONOMETRY

In this chapter we intend to study an important branch of mathematics called '**trigonometry**'. It is the science of measuring angle of triangles, side of triangles.

Angle : -



Consider a ray OA if this ray rotate about its end point O and takes the position OB then we say that the angle ∠AOB has been generated.

Measure of an angle : The measure of an angle is the amount of rotation from initial side to the terminal side.

NOTE :

Relation between degree and radian measurement

π radians = 180 degree

$$\text{radian measure} = \frac{17}{180} \times \text{degree measure}$$

$$\text{degree measure} = \frac{180}{\pi} \times \text{radian measure}$$

1° = 60' (60 minutes)

1' = 60" (60 seconds)

Example 1 :

Find radian measure of 270°.

Solution :

$$\text{Radian meausre} = \frac{\pi}{180} \times 270 = \frac{3\pi}{2}$$

Example 2 :

Find degree measure of $\frac{5\pi}{9}$.

Solution :

$$\text{degree measure} = \frac{180}{\pi} \times \frac{5\pi}{9} = 100°$$

Example 3 :

If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

Solution :

$$60° = \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c \text{ and}$$

$$75° = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$$

$$\therefore \quad \frac{\pi}{3} = \frac{s}{r_1} \text{ and } \frac{5\pi}{12} = \frac{s}{r_2} \quad \left[\because \theta = \left(\frac{s}{r}\right)^c\right]$$

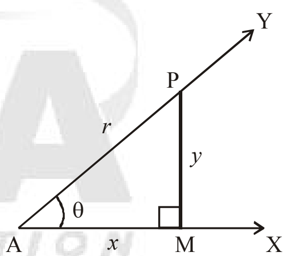
$$\Rightarrow \quad \frac{\pi}{3} r_1 = s \text{ and } \frac{5\pi}{12} r_2 = s$$

$$\Rightarrow \quad \frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$$

$$\Rightarrow \quad 4r_1 = 5r_2 \Rightarrow r_1 : r_2 = 5 : 4$$

Trigonometric ratios :

The most important task of trigonometry is to find the remaining side and angle of a triangle when some of its side and angles are given. This problem is solved by using some ratio of sides of a triangle with respect to its acute angle. These ratio of acute angle are called trigonometric ratio of angle. Let us now define various trigonometric ratio.



Consider an acute angle ∠YAX = θ with initial side AX and terminal side AY. Draw PM perpendicular from P on AX to get right angle triangle AMP. In right angle triangle AMP.

Base = AM = x

Perpendicular = PM = y and

Hypotenuse = AP = r.

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

We define the following six trigonometric Ratios:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{x}{r}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{y}{x}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{r}{y}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{r}{x}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{x}{y}$$

Important formula:-

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\text{cosec}^2\theta - \cot^2\theta = 1$
3. $\sec^2\theta + \tan^2\theta = 1$
4.

θ / T-ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

5. $\sin(90° - \theta) = \cos\theta$.
6. $\cos(90° - \theta) = \sin\theta$.
7. $\tan(90° - \theta) = \cot\theta \Rightarrow \cot(90° - \theta) = \tan\theta$.
8. $\text{cosec}(90° - \theta) = \sec\theta$.
9. $\sec(90° - \theta) = \text{cosec}\,\theta$.

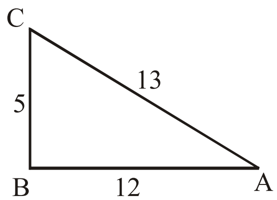
RELATION AMONG T-RATIONS

	sin θ	**cos θ**	**tan θ**	**cot θ**	**sec θ**	**cosec θ**
sin θ	sin θ	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\text{cosec}\,\theta}$
cos θ	$\sqrt{1-\sin^2\theta}$	cos θ	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{1}{\sqrt{\frac{\text{cosec}^2\theta-1}{\text{cosec}\,\theta}}}$
tan θ	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	tan θ	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\text{cosec}^2\theta-1}}$
cot θ	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	cot θ	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\text{cosec}^2\theta-1}$
sec θ	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1-\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	sec θ	$\frac{\text{cosec}\,\theta}{\sqrt{\text{cosec}^2\theta-1}}$
cosec θ	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	cosec θ

Example 4 :

In a Δ ABC right angled at B if AB = 12, and BC = 5 find sin A and tan A, cos C and cot C

Solution :



$$AC = \sqrt{(AB)^2 + (BC)^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{169} = 13$$

When we consider t-ratios of $\angle$ A we have

Base AB = 12

Perpendicular = BC = 5

Hypotenuse = AC = 13

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5}{12}$$

When we consider t-ratios of $\angle$C, we have

Base = BC = 5

Perpendicular = AB = 12

Hypotenuse = AC = 13

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{13}$$

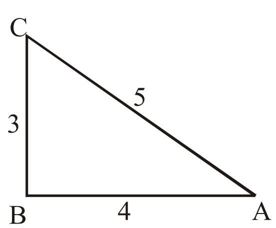
$$\cot C = \frac{\text{Base}}{\text{Perpendicular}} = \frac{5}{12}$$

Example 5 :

In a right triangle ABC right angle at B if $\sin A = \frac{3}{5}$ find all the six trigonometric ratios of $\angle$C

Solution :

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$$



$$\text{Base} = \sqrt{(\text{Hypotenuse})^2 - (\text{Perpendicualr})^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9} = \sqrt{16} = 4$$

Now

$$\sin C = \frac{BC}{AC} = \frac{4}{5}, \text{ cosec } C = \frac{5}{4}$$

$$\cos C = \frac{3}{5} = \frac{AB}{AC}, \ \sec C = \frac{5}{3}$$

$$\tan C = \frac{AB}{AC} = \frac{4}{3}, \ \cot C = \frac{3}{4}.$$

Example 6 :

Find the value of $2 \sin^2 30° \tan 60° - 3\cos^2 60° \sec^2 30°$

Solution :

$$2\left(\frac{1}{2}\right)^2 \times \sqrt{3} - 3\left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 2 \times \frac{1}{4} \times \sqrt{3} - 3 \times \frac{1}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3} - 2}{2}$$

Example 7 :

Find the value θ, $2 \sin 2\theta = \sqrt{3}$

Solution :

$$\sin 2\theta = \frac{\sqrt{3}}{2}$$
$$2\theta = 60$$
$$\theta = 30°$$

Example 8 :

Find the value of x.

$\tan 3x = \sin 45° \cos 45° + \sin 30°$

Solution :

$$\tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$
$$= \frac{1}{2} + \frac{1}{2} = 1$$

$\Rightarrow$ $\tan 3x = 1$ $\Rightarrow$ $\tan 3x = \tan 45°$

$3x = 45°$

$x = 15°$

Example 9 :

If θ is an acute angle $\tan\theta + \cot\theta = 2$ find the value of $\tan^7 \theta + \cot^7 \theta$.

Solution :

$$\tan\theta + \cos\theta = 2$$

$$\tan\theta + \frac{1}{\tan\theta} = 2$$

$\Rightarrow$ $\tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow$ $\tan^2\theta - 2\tan\theta + 1 = 0$

$(\tan\theta - 1)^2 = 0$

$\tan\theta = 1$

$\theta = 45°$

Now, $\tan^7\theta + \cot^7\theta$.

$= \tan^7 45° + \cot^7 45°$

$= 1 + 1 = 2$

Example 10 :

Find the value of $\frac{\cos 37°}{\sin 53°}$

Solution :

We have

$$\frac{\cos 37°}{\sin 53°} = \frac{\cos(90° - 53°)}{\sin 53°} = \frac{\sin 53°}{\sin 53°} = 1$$

Example 11 :

Find the value of

$$\frac{\sin 36°}{\cos 54°} - \frac{\sin 54°}{\cos 36°}$$

Solution :

We have

$$= \frac{\sin 36°}{\cos 54°} - \frac{\sin 54°}{\cos 36°}$$

$$= \frac{\sin(90° - 54°)}{\cos 54°} - \frac{\sin(90° - 36°)}{\cos 36°}$$

$$= \frac{\cos 54°}{\cos 54°} - \frac{\cos 36°}{\cos 36°}$$

$= 1 - 1 = 0$

Example 12 :

Evaluate the cot 12° cot 38° cot 52° cot 60° cot 78°

Solution :

We have

cot 12° cot 38° cot 52° cot 60° cot 78°

= (cot 12° cot 78°) (cot 38° cot 52°) cot 60°

= [cot12° cot (90° – 12°)] [cot 38° cot (90° – 38°)] cot 60°

= [cot 12° tan 12°] [cot 38° tan 38°] cot 60°

$$= 1 \times 1 \times \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Example 13 :

If $\tan 2\theta = \cot(\theta + 6°)$, where 2θ and $\theta + 6°$ are acute angles find the value of θ.

Solution :

We have

$\tan 2\theta = \cot(\theta + 6°)$

$\cot(90° - 2\theta) = \cot(\theta + 6°)$

$90 - 2\theta = \theta + 6°$

$3\theta = 84°$

$\theta = 28°$

Example 14 :

Find the value of $(1 - \sin^2\theta)\sec^2\theta$.

Solution :

We have,

$(1 - \sin^2\theta)(\sec^2\theta)$

$= \cos^2\theta \sec^2\theta$

$$= \cos^2\theta \times \frac{1}{\cos^2\theta}$$

$= 1$

Example 15 :

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$$ find its value

Solution :

We have

$$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2\sec^2\theta.$$

Example 16 :

Find the value of $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

Solution :

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta.$$

Example 17 :

Find the value of $[(1 + \cot\theta) - \operatorname{cosec}\theta][1 + \tan\theta + \sec\theta]$

Solution :

$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

Example 18 :

If $\sin\theta = \frac{3}{5}$, find the value of $\sin\theta\cos\theta$.

Solution :

$$\sin\theta = \frac{3}{5}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\sin\theta \times \cos\theta = \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$$

Example 19 :

If $\cos\theta = \frac{1}{2}$, find the value if $\frac{2\sec\theta}{1+\tan^2\theta}$.

Solution :

$$\cos\theta = \frac{1}{2}$$

$$\sec\theta = 2$$

$$\frac{2\sec\theta}{1+\tan^2\theta} = \frac{2\sec\theta}{\sec^2\theta} = \frac{2}{\sec\theta} = \frac{2}{2} = 1$$

Example 20 :

If $\tan\theta = \frac{12}{5}$, find the value of $\frac{1+\sin\theta}{1-\sin\theta}$

Solution :

$$\tan\theta = \frac{12}{5}$$

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$= \sqrt{1+\left(\frac{12}{5}\right)^2} = \frac{13}{5}$$

$$\cos\theta = \frac{5}{13}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \frac{12}{13}$$

$$\text{thus } \frac{1+\sin\theta}{1-\sin\theta} = \frac{1+\frac{12}{13}}{1-\frac{12}{13}} = \frac{\frac{25}{13}}{\frac{1}{13}} = 25$$

Example 21 :

If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$ $0 < \theta < 90°$ find the value of $\tan\theta$.

Solution :

$$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\cos\theta = \sqrt{1-\frac{a^2}{a^2+b^2}} = \sqrt{\frac{b^2}{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{\sqrt{a^2+b^2}}}{\frac{b}{\sqrt{a^2+b^2}}} = \frac{a}{b}$$

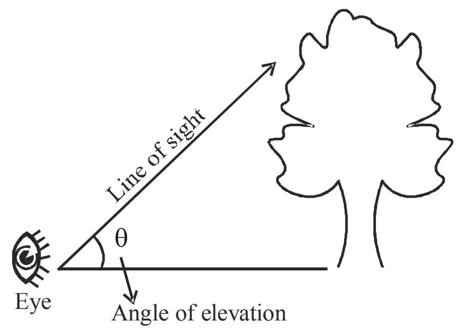
HEIGHT AND DISTANCE

Sometimes, we have to find the height of a tower, building, tree, distance of a ship, width of a river, etc. Though we cannot measure them easily, we can determine these by using trigonometric ratios.

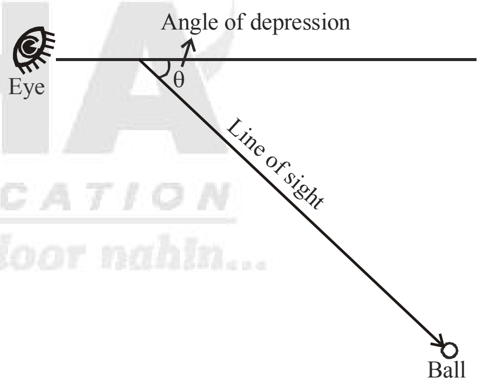
Line of Sight

The line of sight or the line of vision is a straight line from our eye to the object we are viewing.

If the object is above the horizontal from the eye, we have to lift up our head to view the object. In this process, our eye move through an angle. This angle is called **the angle of elevation** of the object.



If the object is below the horizontal from the eye, then we have to turn our head downwards to view the object. In this process, our eye move through an angle. This angle is called **the angle of depression** of the object.



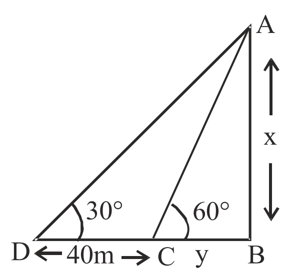
Example 22 :

A person observed the angle of elevation of the top of a tower is 30°. He walked 40 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60° Find the height of tower.

Solution :

Let height of tower AB = x m and BC = y m, DC = 40 m.

In ΔABC,



$$\frac{AB}{BC} = \tan 60° \Rightarrow \frac{x}{y} = \sqrt{3} \Rightarrow y = \frac{x}{\sqrt{3}} \qquad \text{....(i)}$$

Now In rt Δ ABD, $\frac{AB}{BD} = \tan 30°$

$$\Rightarrow \frac{x}{40+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}x = 40 + y \Rightarrow \sqrt{3}x = 40 + \frac{x}{\sqrt{3}} \qquad \text{[using (i)]}$$

$$\Rightarrow 3x = 40\sqrt{3} + x \Rightarrow 3x - x = 40\sqrt{3} \Rightarrow 2x = 40\sqrt{3}$$

$$x = 20\sqrt{3}\text{m}$$

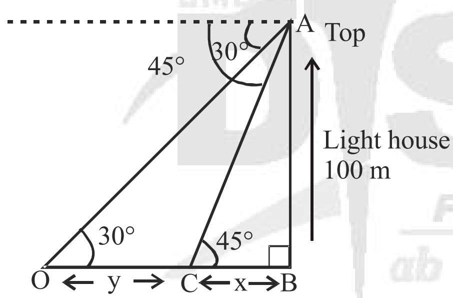
Example 23 :

As observed from top of a light house 100 m. high above sea level, the angle of depression of a ship sailing directly toward it changes from 30° to 45°. The distance travelled by the ship during the period of observation is

Solution :

Let 'y' be the required distance between two positions O and C of the ship

In rt. ΔABC,



$$\cot 45° = \frac{x}{100} \Rightarrow x = 100 \qquad \text{...(i)}$$

In Δ AOB, $\frac{y+x}{100} = \cot 30°$

$$\Rightarrow y + x = 100\sqrt{3} \Rightarrow y = 100\sqrt{3} - x$$

$$\Rightarrow y = 100\sqrt{3} - 100 \quad \text{[using (i)]}$$

$$\Rightarrow y = 100(\sqrt{3} - 1)$$

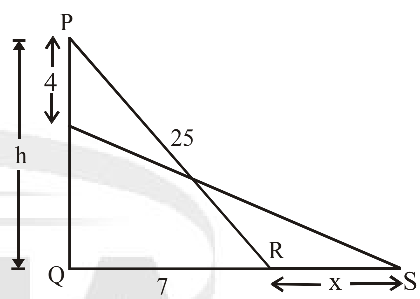
$$\Rightarrow y = 100(1.732 - 1) = 100 \times 0.732 = 73.20\text{ m.}$$

Example 24 :

A 25 m long ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4 m, then the foot of the ladder will slide by how much distance.

Solution :

Let the height of the wall be h.



Now, $h = \sqrt{25^2 - 7^2}$

$= \sqrt{576} = 24\text{m}$

QS $= \sqrt{625 - 400}$

$= \sqrt{225} = 15\text{m}$

Required distance, $x = (15 - 7) = 8$ m

EXERCISE

1. If $\tan\theta = 1$, then find the value of $\frac{8\sin\theta + 5\sin\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$
   (a) 2 (b) $2\frac{1}{2}$
   (c) 3 (d) $\frac{4}{5}$
2. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^2\theta + \tan^4\theta$ is
   (a) $\frac{3}{2}$ (b) 1
   (c) $\frac{1}{2}$ (d) 0
3. The value of tan4°. tan43°. tan47°. tan86° is
   (a) 0 (b) 1
   (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
4. If $\tan 15° = 2-\sqrt{3}$, the value of tan 15°. cot 75° + tan 75° . cot 15° is.
   (a) 14 (b) 12
   (c) 10 (d) 8
5. The value of $(\sin^2 1° + \sin^2 3° + \sin^2 5° + ...+ \sin^2 85° + \sin^2 87° + \sin^2 89°)$
   (a) $21\frac{1}{2}$ (b) 22
   (c) $22\frac{1}{2}$ (d) $23\frac{1}{2}$
6. If $\sin\theta - \cos\theta = \frac{7}{13}$ and $0 < \theta < 90°$, then the value of $\sin\theta + \cos\theta$ is.
   (a) $\frac{17}{13}$ (b) $\frac{13}{17}$
   (c) $\frac{1}{13}$ (d) $\frac{1}{17}$
7. The minimum value of $\cos 2\theta + \cos\theta$ for real values of θ is–
   (a) –9/8 (b) 0
   (c) –2 (d) None of these
8. If $5\tan\theta - 4 = 0$, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta} = ?$
   (a) $\frac{5}{3}$ (b) $\frac{5}{6}$
   (c) 0 (d) $\frac{1}{6}$
9. The value of $\tan^6 20° - 33\tan^4 20° + 27\tan^2 20°$ is :
   (a) 2 (b) 3
   (c) 4 (d) 5
10. If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = ?$
    (a) $\frac{5}{7}$ (b) $\frac{3}{7}$
    (c) $\frac{1}{12}$ (d) $\frac{3}{4}$
11. If $y = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$ then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is equal to
    (a) 1/y (b) y
    (c) 1 – y (d) 1 + y
12. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°; when he retreats 20m from the bank, he finds the angle to be 30°. The height of the tree and the breadth of the river are –
    (a) $10\sqrt{3}$ m, 10 m (b) 10 ; $10\sqrt{3}$ m
    (c) 20 m, 30 m (d) None of these
13. If θ is an acute angle such that $\tan^2\theta = \frac{8}{7}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is
    (a) $\frac{7}{8}$ (b) $\frac{8}{7}$
    (c) $\frac{7}{4}$ (d) $\frac{64}{49}$
14. If 3 cos θ = 5 sin θ, then the value of $\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is equal to
    (a) $\frac{271}{979}$ (b) $\frac{376}{2937}$
    (c) $\frac{542}{2937}$ (d) None of these
15. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then $x^2 + y^2 =$
    (a) 1 (b) 2 (c) 0 (d) None
16. If $1 + \sin^2 A = 3\sin A\cos A$, then what are the possible values of tan A?
    (a) 1/4, 2 (b) 1/6, 3
    (c) 1/2, 1 (d) 1/8, 4
17. The value of $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$ is
    (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) 2

18. If $\dfrac{x\,\text{cosec}^2 30°.\sec^2 45°}{8\cos^2 45°.\sin^2 60°} = \tan^2 60° - \tan^2 30°$, then x = ?
(a) 1 (b) – 1
(c) 2 (d) 0

19. If $\theta + \phi = \dfrac{\pi}{6}$, what is the value of $(\sqrt{3}+\tan\theta)$ $(\sqrt{3}+\tan\phi)$?
(a) 1 (b) – 1
(c) 4 (d) – 4

20. If θ is an acute angle such that $\sec^2\theta = 3$, then $\dfrac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta}$ is
(a) $\dfrac{4}{7}$ (b) $\dfrac{3}{7}$ (c) $\dfrac{2}{7}$ (d) $\dfrac{1}{7}$

21. What should be the height of a flag where a 20 feet long ladder reaches 20 feet below the flag (The angle of elevation of the top of the flag at the foot of the ladder is 60°)?
(a) 20 feet (b) 30 feet
(c) 40 feet (d) $20\sqrt{2}$ feet

22. If θ & 2θ – 45° are acute angles such that sin θ = cos (2θ – 45°) then tan θ is equal to
(a) 1 (b) – 1
(c) $\sqrt{3}$ (d) $\dfrac{1}{\sqrt{3}}$

23. If 5θ & 4θ are acute angles satisfying sin 5θ = cos 4θ then 2 sin 3θ – $\sqrt{3}$ tan 3θ = ?
(a) 1 (b) 0
(c) – 1 (d) $\dfrac{1}{\sqrt{3}}$

24. A vertical pole with height more than 100 m consists of two parts, the lower being one-third of the whole. At a point on a horizontal plane through the foot and 40 m from it, the upper part subtends an angle whose tangent is $\dfrac{1}{2}$. What is the height of the pole ?
(a) 110m (b) 200m
(c) 120m (d) 150m

25. If sec θ + tan θ = x, then sec θ = ?
(a) $\dfrac{x^2+1}{x}$ (b) $\dfrac{x^2+1}{2x}$
(c) $\dfrac{x^2-1}{2x}$ (d) $\dfrac{x^2-1}{x}$

26. The correct value of the parameter 't' of the identity $2(\sin^6 x + \cos^6 x) + t(\sin^4 x + \cos^4 x) = -1$ is:
(a) 0 (b) –1
(c) –2 (d) –3

27. If a cos θ – b sin θ = c, then a cos θ + b sin θ = ?
(a) $\pm\sqrt{a^2+b^2+c^2}$ (b) $\pm\sqrt{a^2+b^2-c^2}$
(c) $\pm\sqrt{c^2-a^2-b^2}$ (d) None of these

28. $\dfrac{\tan\theta}{\sec\theta-1} + \dfrac{\tan\theta}{\sec\theta+1}$ is equal to
(a) 2 tan θ (b) 2 sec θ
(c) 2 cosec θ (d) 2 tan θ . sec θ.

29. If a cos θ + b sin θ = m and a sin θ – b cos θ = n, then $a^2 + b^2$ =
(a) $m^2 - n^2$ (b) $m^2 n^2$
(c) $n^2 - m^2$ (d) $m^2 + n^2$

30. The angular elevation of a tower CD at a place A due south of it is 60° ; and at a place B due west of A, the elevation is 30°. If AB = 3 km, the height of the tower is
(a) $2\sqrt{3}$ km (b) $3\sqrt{6}$ km
(c) $\dfrac{3\sqrt{3}}{2}$ km (d) $\dfrac{3\sqrt{6}}{4}$ km

31. If sin α + cos β = 2 (0° ≤ β < α ≤ 90°), then $\sin\left(\dfrac{2\alpha+\beta}{3}\right)$ =
(a) $\sin\dfrac{\alpha}{2}$ (b) $\cos\dfrac{\alpha}{3}$
(c) $\sin\dfrac{\alpha}{3}$ (d) $\cos\dfrac{2\alpha}{3}$

32. If $\cos^4\theta - \sin^4\theta = \dfrac{2}{3}$, then the value of $2\cos^2\theta - 1$ is
(a) 0 (b) 1
(c) $\dfrac{2}{3}$ (d) $\dfrac{3}{2}$

33. If sin α sec (30° + α) = 1 (0 < α < 60°), then the value of sin α + cos 2α is
(a) 1 (b) $\dfrac{2+\sqrt{3}}{2\sqrt{3}}$
(c) 0 (d) $\sqrt{2}$

34. The minimum value of $2\sin^2\theta + 3\cos^2\theta$ is
(a) 0 (b) 3
(c) 2 (d) 1

35. If cosec 39° = x, the value of $\dfrac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51° - \dfrac{1}{\sin^2 51° \sec^2 39°}$ is
(a) $\sqrt{x^2-1}$ (b) $\sqrt{1-x^2}$
(c) $x^2 - 1$ (d) $1 - x^2$

36. If $A = \sin^2\theta + \cos^4\theta$, for any value of θ, then the value of A is
(a) $1 \le A \le 2$ (b) $\dfrac{3}{4} \le A \le 2$
(c) $\dfrac{13}{16} \le A \le 1$ (d) $\dfrac{3}{4} \le A \le \dfrac{13}{16}$

37. If tan 2θ, tan 4θ = 1, then the value of tan 3θ is
(a) $\sqrt{3}$ (b) 0
(c) 1 (d) $\dfrac{1}{\sqrt{3}}$

38. If $\tan(\theta_1+\theta_2)=\sqrt{3}$ and $\sec(\theta_1-\theta_2)=\frac{2}{\sqrt{3}}$, then the value of $\sin 2\theta_1+\tan 3\theta_2$ is equal to (assume that $0<\theta_1-\theta_2<\theta_1+\theta_2<90°$)
(a) 0 (b) 3
(c) 1 (d) 2

39. If $\sec\theta = x+\frac{1}{4x}$ $(0°<\theta<90°)$, then $\sec\theta+\tan\theta$ is equal to
(a) $\frac{x}{2}$ (b) 2x
(c) x (d) $\frac{1}{2x}$

40. If x = a secθ cos ϕ, y = b secθ sin ϕ, z = c tan θ, then, the value of $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}$ is :
(a) 1 (b) 4
(c) 9 (d) 0

41. If $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=\frac{5}{3}$, then sin θ is equal to :
(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

42. If $0\le\theta\le\frac{\pi}{2}$ and $\sec^2\theta+\tan^2\theta=7$, then θ is
(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian
(c) $\frac{\pi}{5}$ radian (d) $\frac{\pi}{6}$ radian

43. The simplest value of
$\sin^2x+2\tan^2x-2\sec^2x+\cos^2x$ is
(a) 1 (b) 0
(c) –1 (d) 2

44. A kite is flying at a height of 50 metre. If the length of string is 100 metre then the inclination of string to the horizontal ground in degree measure is
(a) 90 (b) 60
(c) 45 (d) 30

45. From the top of a light-house at a height 20 metres above sea-level, the angle of depression of a ship is 30°. The distance of the ship from the foot of the light-house is
(a) 20m (b) $20\sqrt{3}$ m
(c) 30m (d) $30\sqrt{3}$ m

46. If x = a sin θ and y = b tanθ then prove that $\frac{a^2}{x^2}-\frac{b^2}{y^2}$ is
(a) 1 (b) 2
(c) 3 (d) 4

47. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\,\text{cosec}\theta = 3$, then the relation between x and y is
(a) $2x^2+y^2=2$ (b) $x^2+4y^2=4$
(c) $x^2+4y^2=1$ (d) $4x^2+y^2=4$

48. If $\sec\theta+\tan\theta=\sqrt{3}$, then the positive value of sinθ is
(a) 0 (b) $\frac{1}{2}$
(c) $\frac{\sqrt{3}}{2}$ (d) 1

49. The radian measure of $63°14'51''$ is
(a) $\left(\frac{2811\pi}{8000}\right)^c$ (b) $\left(\frac{3811\pi}{8000}\right)^c$
(c) $\left(\frac{4811\pi}{8000}\right)^c$ (d) $\left(\frac{5811\pi}{8000}\right)^c$

50. If $\frac{\cos^4\alpha}{\cos^2\beta}+\frac{\sin^4\alpha}{\sin^2\beta}=1$, then the value of $\frac{\cos^4\beta}{\cos^2\alpha}+\frac{\sin^4\beta}{\sin^2\alpha}$ is
(a) 4 (b) 0 (c) $\frac{1}{8}$ (d) 1

51. $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}\left(\text{where } \theta\ne\frac{\pi}{2}\right)$ is equal to
(a) $\frac{1+\sin\theta}{\cos\theta}$ (b) $\frac{1-\sin\theta}{\cos\theta}$
(c) $\frac{1-\cos\theta}{\sin\theta}$ (d) $\frac{1+\cos\theta}{\sin\theta}$

52. The angles of elevation of the top of a tower standing on a horizontal plane from two points on a line passing through the foot of the tower at a distance 9 ft and 16 ft respectively are complementary angles. Then the height of the tower is
(a) 9 ft (b) 12 ft
(c) 16 ft (d) 144 ft

53. If $\sin^2\alpha=\cos^3\alpha$, then the value of $(\cot^6\alpha-\cot^2\alpha)$ is
(a) 1 (b) 0
(c) – 1 (d) 2

54. The simplified value of
$(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\theta)$ is
(a) – 2 (b) 2
(c) 1 (d) – 1

55. The value of $\frac{\sin 53°}{\cos 37°}\div\frac{\cot 65°}{\tan 25°}$ is
(a) 2 (b) 1
(c) 3 (d) 0

56. The value of $\frac{\cos 60°+\sin 60°}{\cos 60°-\sin 60°}$ is
(a) – 1 (b) $\sqrt{3}+2$
(c) $-(2+\sqrt{3})$ (d) $\sqrt{3}-2$

57. The value of
$$\frac{\cot 5°.\cot 10°.\cot 15°.\cot 60°.\cot 75°.\cot 80°.\cot 85°}{(\cos^2 20°+\cos^2 70°)+2}$$ is
(a) $\frac{9}{\sqrt{3}}$ (b) $\frac{1}{9}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{9}$

58. In a triangle, the angles are in the ratio 2 : 5 : 3. What is the value of the least angle in the radian ?

(a) $\frac{\pi}{20}$ (b) $\frac{\pi}{10}$

(c) $\frac{2\pi}{5}$ (d) $\frac{\pi}{5}$

59. If x = a cos θ – b sin θ, y = b cos θ + a sin θ, then find the value of $x^2 + y^2$.

(a) a^2 (b) b^2

(c) $\frac{a^2}{b^2}$ (d) $a^2 + b^2$

60. If tan α + cot α = 2, then the value of $\tan^7\alpha + \cot^7\alpha$ is

(a) 2 (b) 16

(c) 64 (d) 128

61. From 125 metre high towers, the angle of depression of a car is 45°. Then how far the car is from the tower ?

(a) 125 metre (b) 60 metre

(c) 75 metre (d) 95 metre

62. The value of $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$ is equal to

(a) –1 (b) 1

(c) 2 (d) 0

63. The shadow of a tower standing on a level plane is found to be 30 m longer when the Sun's altitude changes from 60° to 45°. The height of the tower is

(a) $15(3+\sqrt{3})$m (b) $15(\sqrt{3}+1)$m

(c) $15(\sqrt{3}-1)$m (d) $15(3-\sqrt{3})$m

64. If $\sin 17° = \frac{x}{y}$, then sec 17° – sin 73° is equal to

(a) $\frac{y}{\sqrt{y^2-x^2}}$ (b) $\frac{y^2}{\left(x\sqrt{y^2-x^2}\right)}$

(c) $\frac{x}{\left(y\sqrt{y^2-x^2}\right)}$ (d) $\frac{x^2}{\left(y\sqrt{y^2-x^2}\right)}$

65. If θ is a positive acute angle and cosec θ + cot θ = $\sqrt{3}$, then the value of cosec θ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) 1

66. If cos α + sec α = $\sqrt{3}$, then the value of $\cos^3\alpha + \sec^3\alpha$ is

(a) 2 (b) 1

(c) 0 (d) 4

67. If sin θ + cos θ = $\sqrt{2}$ cos θ, then the value of cot θ is

(a) $\sqrt{2}+1$ (b) $\sqrt{2}-1$

(c) $\sqrt{3}-1$ (d) $\sqrt{3}+1$

68. The value of $\sin^2 1° + \sin^2 2° + \sin^2 3° + ... + \sin^2 89°$ is

(a) 22 (b) 44

(c) $22\frac{1}{2}$ (d) $44\frac{1}{2}$

ANSWER KEY

1	(a)	**11**	(b)	**21**	(b)	**31**	(b)	**41**	(a)	**51**	(a)	**61**	(a)
2	(b)	**12**	(a)	**22**	(a)	**32**	(c)	**42**	(b)	**52**	(b)	**62**	(c)
3	(b)	**13**	(a)	**23**	(b)	**33**	(a)	**43**	(c)	**53**	(a)	**63**	(a)
4	(a)	**14**	(a)	**24**	(c)	**34**	(b)	**44**	(d)	**54**	(b)	**64**	(d)
5	(c)	**15**	(a)	**25**	(b)	**35**	(c)	**45**	(b)	**55**	(b)	**65**	(c)
6	(a)	**16**	(c)	**26**	(d)	**36**	(b)	**46**	(a)	**56**	(c)	**66**	(c)
7	(a)	**17**	(c)	**27**	(b)	**37**	(c)	**47**	(b)	**57**	(d)	**67**	(a)
8	(c)	**18**	(a)	**28**	(c)	**38**	(d)	**48**	(b)	**58**	(d)	**68**	(d)
9	(b)	**19**	(c)	**29**	(d)	**39**	(b)	**49**	(a)	**59**	(d)		
10	(d)	**20**	(d)	**30**	(d)	**40**	(a)	**50**	(d)	**60**	(a)		

HINTS & EXPLANATIONS

1. (a) $\tan\theta = 1$

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+1} = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}$$

Now, $\dfrac{8\sin\theta+5\sin\theta}{\sin^3\theta-2\cos^3\theta+7\cos\theta}$

$$= \frac{8\times\frac{1}{\sqrt{2}}+5\times\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^3-2\times\left(\frac{1}{\sqrt{2}}\right)^3+7\times\frac{1}{\sqrt{2}}}$$

$$= \frac{\frac{8+5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}}-\frac{2}{2\sqrt{2}}+\frac{7}{\sqrt{2}}} = \frac{\frac{(8+5)}{\sqrt{2}}}{\frac{1-2+14}{2\sqrt{2}}} = \frac{13\times 2}{13} = 2$$

2. (b) Given, $\cos^2\theta+\cos^4\theta = 1$
or, $\cos^4\theta = 1-\cos^2\theta$ $[\because \sin^2\theta+\cos^2\theta = 1]$
$\cos^4\theta = \sin^2\theta.$

or, $1 = \dfrac{\sin^2\theta}{\cos^2\theta}.\dfrac{1}{\cos^2\theta}$

$\Rightarrow$ $\tan^2\theta.\sec^2\theta = 1$
or, $\tan^2\theta.(1+\tan^2\theta) = 1$ $[\because \sec^2\theta-\tan^2\theta = 1]$

or, $\boxed{\tan^2\theta+\tan^4\theta = 1}$

3. (b) $\tan 4°.\tan 43°.\tan 47°.\tan 86°$
$= \tan 4°.\tan 43°.\tan(90°-43°)\tan(90°-4°)$
$= \tan 4°.\tan 43°.\cot 43°\cot 4°$ $[\because \tan(90-\theta) = \cot\theta]$

$= \tan 4°\times\tan 43°\times\dfrac{1}{\tan 43°}\times\dfrac{1}{\tan 4°}$ $\left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$

$= 1$

4. (a) $\tan 15°.\cot 75°+\tan 75°.\cot 15°$
$= \tan 15.\cot(90°-15°)+\tan(90°-15°)\cot 15°$
$= \tan 15°.\tan 15°+\cot 15°.\cot 15°$
$= (\tan 15)^2+(\cot 15)^2$

$= (\tan 15°)^2+\dfrac{1}{(\tan 15)^2}$

Putting the value of $\tan 15° = 2-\sqrt{3}$

$$= (2-\sqrt{3})^2+\left(\frac{1}{2-\sqrt{3}}\right)^2$$

$$= (2-\sqrt{3})^2+\left[\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}\right]^2$$

$$= (2-\sqrt{3})^2+\left(\frac{2+\sqrt{3}}{4-3}\right)^2$$

$$= (2-\sqrt{3})^2+(2+\sqrt{3})^2$$

$= 2\left[2^2+(\sqrt{3})^2\right]$ $[\because (a+b)^2+(a-b)^2 = 2(a^2+b^2)]$

$= 2(4+3) = 2\times 7 = 14$

5. (c) To find total number of terms
First term = 1, last term = 89, common diff = 2.
$a_n = a_1+(n-1)d$
$89 = 1+(n-1)2$
$\Rightarrow$ $88 = (n-1)2$
$\Rightarrow$ $n-1 = 44$
$\Rightarrow$ 45 terms.
Now, $\sin^2 1°+\sin^2 3°+\sin^2 5°+.........+\sin^2 85°+\sin^2 87°+\sin^2 89°$
$= (\sin^2 1°+\sin^2 89°)+(\sin^2 3°+\sin^2 87°)+.....$ 22 terms $+\sin^2 45°$
$= (\sin^2 1°+\cos^2 1°)+(\sin^2 3°+\cos^2 3°)+.....$ 22 terms $+\left(\dfrac{1}{\sqrt{2}}\right)^2$

$= (1+1+..........$ 22 terms$)+\dfrac{1}{2}$

$= 22+\dfrac{1}{2} = 22\dfrac{1}{2}$

6. (a) $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2$
$= (\sin^2\theta+\cos^2\theta)+2\sin\theta.\cos\theta+(\sin^2\theta+\cos^2\theta)-2\sin\theta.\cos\theta.$
$= 1+1 = 2$
So, $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2 = 2$

or, $(\sin\theta+\cos\theta)^2+\left(\dfrac{7}{13}\right)^2 = 2$

or, $(\sin\theta-\cos\theta)^2 = 2-\dfrac{49}{169} = \dfrac{289}{169}$

$\sin\theta+\cos\theta = \sqrt{\left(\dfrac{17}{13}\right)^2} = \dfrac{17}{13}.$

7. (a) Let $S = \cos 2\theta+\cos\theta = 2\cos^2\theta-1+\cos\theta$

$$= -1+2\left(\cos^2\theta+\frac{1}{2}\cos\theta+\frac{1}{16}\right)-\frac{1}{8}$$

$$= -\frac{9}{8}+2\left(\cos\theta+\frac{1}{4}\right)^2 \geq -\frac{9}{8}$$

So, the minimum value $S = -9/8$

8. (c) Given, $5\tan\theta - 4 = 0$

$\Rightarrow \tan\theta = \frac{4}{5}$

Expression, $\dfrac{\dfrac{(5\sin\theta - 4\cos\theta)}{\cos\theta}}{\dfrac{(5\sin\theta + 4\cos\theta)}{\cos\theta}} = \dfrac{5\tan\theta - 4}{5\tan\theta + 4}$

$= \dfrac{5\times\frac{4}{5} - 4}{5\times\frac{4}{5} + 4}$

$= \dfrac{4-4}{4+4} = \dfrac{0}{8} = 0$

9. (b) $\sqrt{3} = \tan 60° = \tan(3\times 20°) = \dfrac{3\tan 20° - \tan^3 20°}{1 - 3\tan^2 20°}$

Squaring, $3 = \dfrac{9t^2 + t^6 - 6t^4}{1 + 9t^4 - 6t^2}$, $\tan 20° = t$

$\Rightarrow t^6 - 33t^4 + 27t^2 = 3$

$\Rightarrow \tan^6 20° - 33\tan^4 20° + 27\tan^2 20° = 3$

10. (d) Given, $\tan\theta = \dfrac{1}{\sqrt{7}}$

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\dfrac{1}{\sqrt{7}}\right)^2} = \sqrt{\dfrac{8}{7}}$

$\csc\theta = \dfrac{\sec\theta}{\tan\theta} = \dfrac{\sqrt{\frac{8}{7}}}{\sqrt{\frac{1}{7}}} = \sqrt{8}$

$\therefore \dfrac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \dfrac{(\sqrt{8})^2 - \left(\sqrt{\frac{8}{7}}\right)^2}{(\sqrt{8})^2 + \left(\sqrt{\frac{8}{7}}\right)^2}$

$= \dfrac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \dfrac{8\left(1-\frac{1}{7}\right)}{8\left(1+\frac{1}{7}\right)} = \dfrac{\frac{6}{7}}{\frac{8}{7}} = \dfrac{6}{8} = \dfrac{3}{4}$

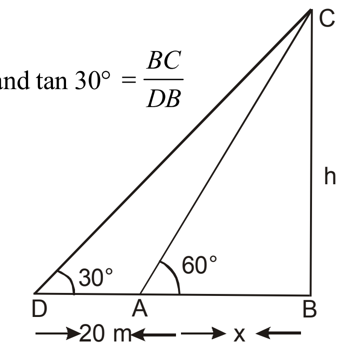
11. (b) $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha} = \dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\dfrac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$= \dfrac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \dfrac{(1+\sin^2\alpha + 2\sin\alpha) - (1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$

$= \dfrac{2\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)} = \dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha} = y$

12. (a) From right angled Δs ABC and DBC, we have



$\tan 60° = \dfrac{BC}{AB}$ and $\tan 30° = \dfrac{BC}{DB}$

$\Rightarrow \sqrt{3} = \dfrac{h}{x}$

and $\dfrac{1}{\sqrt{3}} = \dfrac{h}{x+20}$

$\Rightarrow h = x\sqrt{3}$

and $h = \dfrac{x+20}{\sqrt{3}}$

$\Rightarrow x\sqrt{3} = \dfrac{x+20}{\sqrt{3}} \Rightarrow 3x = x + 20 \Rightarrow x = 10\,m$

Putting $x = 10$ in $h = \sqrt{3}\,x$, we get $h = 10\sqrt{3}$

Hence, height of the tree = $10\sqrt{3}$ m and the breadth of the river = 10 m.

13. (a) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\tan^2\theta} = \dfrac{1}{\frac{8}{7}} = \dfrac{7}{8}$.

14. (a) Given, $3\cos\theta = 5\sin\theta \Rightarrow \tan\theta = \dfrac{3}{5}$.

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\left(\dfrac{3}{5}\right)^2} = \sqrt{\dfrac{25+9}{25}} = \dfrac{\sqrt{34}}{5}$.

In expression, dividing the numerator & denominator by $\cos\theta$,

$= \dfrac{5\tan\theta - 2\sec^4\theta + 2}{5\tan\theta + 2\sec^4\theta - 2}$

$= \dfrac{5\times\frac{3}{5} - 2\times\left(\frac{\sqrt{34}}{5}\right)^4 + 2}{5\times\frac{3}{5} + 2\times\left(\frac{\sqrt{34}}{5}\right)^4 - 2}$

$= \dfrac{3 - 2\times\frac{1156}{625} + 2}{3 + 2\times\frac{1156}{625} - 2} = \dfrac{5 - \frac{2312}{625}}{1 + \frac{2312}{625}}$

$= \dfrac{813}{2937} = \dfrac{271}{979}$

15. (a) We have, $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ....(i)

and $x\sin\theta = y\cos\theta$ ...(ii)

Equation (i) may be written as

$$x\sin\theta.\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$$

$$\Rightarrow y\cos\theta = \sin\theta\cos\theta$$

$\therefore\ y = \sin\theta$ ...(iii)

Putting the value of y from (iii) in (ii), we get

$x\sin\theta = \sin\theta.\cos\theta \Rightarrow x = \cos\theta$ ...(iv)

Squaring (iii) and (iv), and adding , we get

$$x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$$

16. (c) In the given equation,

$1 + \sin^2 A = 3\sin A\cos A$

Dividing both sides by $\cos^2 A$,

We get $\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} = 3.\frac{\sin A}{\cos A}$

$$\Rightarrow \sec^2 A + \tan^2 A = 3\tan A$$

$$\Rightarrow 1 + \tan^2 A + \tan^2 A = 3\tan A$$

$$\Rightarrow 2\tan^2 A - 3\tan A + 1 = 0$$

$$\Rightarrow 2\tan^2 A - 2\tan A - \tan A + 1 = 0$$

$$\Rightarrow 2\tan A(\tan A - 1) - 1(\tan A - 1) = 0$$

$$\Rightarrow (2\tan A - 1)(\tan A - 1) = 0$$

$$\Rightarrow \tan A = \frac{1}{2}, 1$$

17. (c) $\cos 20° = \cos(90° - 70°) = \sin 70°$

$\cos 70° = \sin 20°$

$$\therefore \frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°} = \frac{\sin^3 70° - \sin^3 20°}{\sin^3 70° - \sin^3 20°} = 1$$

18. (a) $$\frac{x \times 2^2.(\sqrt{2})^2}{8 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

or, $$\frac{x \times 4 \times 2}{8 \times \frac{1}{2} \times \frac{3}{4}} = 3 - \frac{1}{3} \Rightarrow \frac{8x}{3} = \frac{9-1}{3}$$

or, $$\frac{8}{3}x = \frac{8}{3}$$

$$\boxed{x = 1}$$

19. (c) Given that $\theta + \phi = \frac{\pi}{6}$

$$\Rightarrow \tan(\theta + \phi) = \tan\frac{\pi}{6}$$

$$\Rightarrow \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \sqrt{3}\tan\theta + \sqrt{3}\tan\phi = 1 - \tan\theta\tan\phi$ ...(1)

$$(\sqrt{3} + \tan\theta)(\sqrt{3} + \tan\phi)$$

$$= 3 + \sqrt{3}\tan\theta + \sqrt{3}\tan\phi + \tan\theta\tan\phi$$

$$= 3 + 1 - \tan\theta\tan\phi + \tan\theta\tan\phi = 4$$

20. (d) $\sec^2\theta = 3 \Rightarrow \sec\theta = \sqrt{3}$

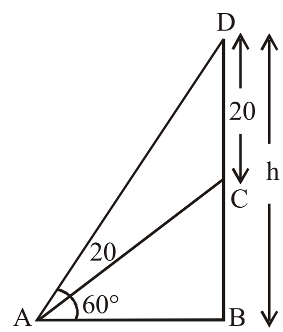
$\tan^2\theta = \sec^2\theta - 1 = 3 - 1 = 2$

$$\text{cosec}^2\theta = \frac{\sec^2\theta}{\tan^2\theta} = \frac{3}{2}$$

Now, $$\frac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta} = \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{7}{2}} = \frac{1}{7}$$

21. (b) In ΔABD,



$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{AB}$$

$$\Rightarrow AB = \frac{h}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{h}{3}\sqrt{3}$$

Now, in ΔABC

$AC^2 = AB^2 + BC^2$

$$\Rightarrow 20^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + (h - 20)^2$$

$\Rightarrow h^2 + 3h^2 - 120h = 0$

$\Rightarrow 4h^2 - 120h = 0$

$\Rightarrow h(h - 30) = 0$

$h = 0$ or 30

$h = 0$ not possible

$\Rightarrow h = 30$ ft

22. (a) $\sin\theta = \cos(2\theta - 45°)$

or, $\cos(90° - \theta) = \cos(2\theta - 45°)$

$\Rightarrow 90° - \theta = 2\theta - 45°$

$\Rightarrow \theta = 45°$

$\therefore \tan\theta = \tan 45° = 1$

23. (b) Given, $\sin 5\theta = \cos 4\theta = \sin(90° - 4\theta)$

$\Rightarrow 5\theta = 90° - 4\theta$

$\theta = 10°$

$2\sin 3\theta - \sqrt{3}\tan 3\theta$

$= 2\sin 30° - \sqrt{3}\tan 30°$

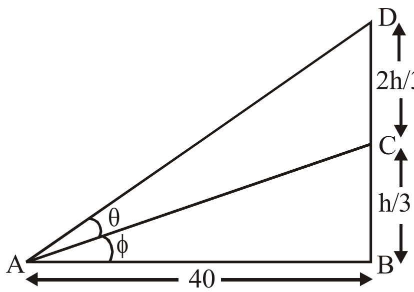
$$= 2 \times \frac{1}{2} - \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 - 1 = 0.$$

24. (c) Let h be the height of pole, upper portion CD subtend angle θ at A.

Then, $\tan\theta = \frac{1}{2}$

Let lower part BC subtend angle φ at A then

In Δ ABC,



$\tan\phi = \frac{BC}{AB} = \frac{h/3}{40} = \frac{h}{120}$

In Δ ABD,

$\tan(\theta+\phi) = \frac{BD}{AB}$

$\Rightarrow \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi} = \frac{h}{40}$

$\Rightarrow \frac{\frac{1}{2}+\frac{h}{120}}{1-\frac{h}{240}} = \frac{h}{40}$

$\Rightarrow \frac{2(60+h)}{(240-h)} = \frac{h}{40}$

$\Rightarrow 80(60+h) = 240h - h^2 \Rightarrow 4800 + 80h = 240h - h^2$

$\Rightarrow h^2 - 160h + 4800 = 0 \Rightarrow (h-120)(h-40) = 0$

$\Rightarrow h = 120$

[h = 40 is discarded, since h > 100 is given]

25. (b) Given, $\sec\theta + \tan\theta = x$ .....(i)

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

or $\sec\theta - \tan\theta = \frac{1}{\sec\theta+\tan\theta} = \frac{1}{x}$ .....(ii)

Adding (i) & (ii), we get

$2\sec\theta = x + \frac{1}{x} = \frac{x^2+1}{x}$

$\sec\theta = \frac{x^2+1}{2x}$

26. (d) Given identity

$2(\sin^6x + \cos^6x) + t(\sin^4x + \cos^4x) = -1$

$\Rightarrow 2[(\sin^2x + \cos^2x)^3 - 3\sin^2x\cos^2x(\sin^2x + \cos^2x)$
$+ t[(\sin^2x + \cos^2x)^2 - 2\sin^2x\cos^2x] = -1$

$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

and $(a+b)^2 = a^2 + b^2 + 2ab$

where $a = \sin^2x, b = \cos^2x]$

$\Rightarrow 2[1 - 3\sin^2x\cos^2x] + t[1 - 2\sin^2x\cos^2x] = -1$

$\Rightarrow 2 - 6\sin^2x\cos^2x + t - 2t\sin^2x\cos^2x = -1$

$= t(1 - 2\sin^2x\cos^2x) = -3(1 - 2\sin^2x\cos^2x)$

$\Rightarrow t = -3.$

27. (b) $(a\cos\theta - b\sin\theta)^2 + (a\cos\theta + b\sin\theta)^2$

$= a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta.\sin\theta.$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= a^2 \times 1 + b^2 \times 1$

$= a^2 + b^2.$

$\therefore (a\cos\theta - b\sin\theta)^2 + (a\cos\theta + b\sin\theta)^2 = a^2 + b^2.$

$\Rightarrow c^2 + (a\cos\theta + b\sin\theta)^2 = a^2 + b^2$

$\Rightarrow a\cos\theta + b\sin\theta = \pm\sqrt{a^2+b^2-c^2}$

28. (c) $\tan\theta\left[\frac{1}{\sec\theta-1} + \frac{1}{\sec\theta+1}\right]$

$= \tan\theta\left[\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta-1)(\sec\theta+1)}\right] = \tan\theta\left[\frac{2\sec\theta}{\sec^2\theta-1}\right]$

$= \tan\theta \times \frac{2\sec\theta}{\tan^2\theta} = \frac{2\sec\theta}{\tan\theta} = \frac{2}{\cos\theta\times\frac{\sin\theta}{\cos\theta}}$

$= \frac{2}{\sin\theta} = 2\,\text{cosec}\,\theta.$

29. (d) $(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2.$

$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta.\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta.\cos\theta = m^2 + n^2.$

or $a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2.$

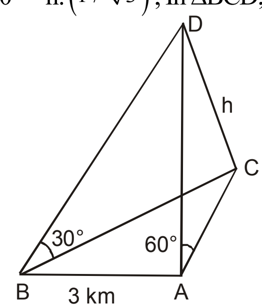
or $\boxed{a^2 + b^2 = m^2 + n^2}$

30. (d) In Δ ACD, we get $AC = h\cot 60° = h.(1/\sqrt{3})$, In ΔBCD,

$BC = h\cot 30° = h\sqrt{3}$.

Therefore, from right-angled triangle BAC, we have

$BC^2 = AB^2 + AC^2$



$\Rightarrow (h\sqrt{3})^2 = (3)^2 + \left(\frac{h}{\sqrt{3}}\right)^2$

$\Rightarrow 3h^2 = 9 + \frac{h^2}{3} \Rightarrow \frac{8}{3}h^2 = 9$

$\Rightarrow h^2 = \frac{27}{8}$

$\Rightarrow h = \frac{3\sqrt{3}}{2\sqrt{2}}$ km $= \frac{3\sqrt{6}}{4}$ km

31. (b) $\sin\alpha + \cos\beta = 2$

$\sin\alpha \le 1 : \cos\beta \le 2$

$\Rightarrow \alpha = 90° ; \beta = 0°$

$\therefore \sin\left(\frac{2\alpha+\beta}{3}\right) = \sin\left(\frac{180°}{3}\right)$

$= \sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos\dfrac{\alpha}{3} = \cos 30° = \dfrac{\sqrt{3}}{2}$

32. (c) $\cos^4\theta - \sin^4\theta = \dfrac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^4\theta)(\cos^2\theta - \sin^2\theta) = \dfrac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \dfrac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \dfrac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \dfrac{2}{3}$

33. (a) $\dfrac{\sin\alpha}{\cos(30° + \alpha)} = 1$

$\Rightarrow \dfrac{\sin\alpha}{\sin(90° - 30 - \alpha)} = 1$

$\Rightarrow \dfrac{\sin\alpha}{\sin(60° - \alpha)} = 1$

$\Rightarrow \sin\alpha = \sin(60° - \alpha)$
$\Rightarrow \alpha = 60° - \alpha$
$\Rightarrow 2\alpha = 60° \Rightarrow \alpha = 30°$
$\therefore \sin\alpha + \cos 2\alpha$
$= \sin 30° + \cos 60°$

$= \dfrac{1}{2} + \dfrac{1}{2} = 1$

34. (b) $2\sin^2\theta + 3\cos^2\theta$
$= 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta$
$= 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta$
$= 2 + \cos^2\theta$
$\therefore$ Minimum value of $\cos\theta = -1$
$\therefore$ Required minimum value $= 2 + 1 = 3$

35. (c) $\dfrac{1}{\text{cosec}^2 51°} + \sin^2 39° + \tan^2 51°$

$= -1\dfrac{1}{\sin^2 51°.\sec^2 39°}$

$= \sin^2 51° + \sin^2 39° + \tan^2(90° - 39°)$

$- \dfrac{1}{\sin^2(90° - 39°).\sec^2 39°}$

$= \cos^2 39° + \sin^2 39° + \cot^2 39° - \dfrac{1}{\cos^2 39°.\sec^2 39°}$

[$\therefore$ $\sin(90° - \theta) = \cos\theta$, $\tan(90° - \theta) = \cot\theta$]
$= 1 + \cot^2 39° - 1$
$= \text{cosec}^2 39° - 1 = x^2 - 1$

36. (b) When $\theta = 0°$
$\sin^2\theta + \cos^4\theta = 1$
When $\theta = 45°$,

$\sin^2\theta + \cos^4\theta = \dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4}$

When $\theta = 30°$

$\sin^2 + \cos^4\theta = \dfrac{1}{4} + \dfrac{9}{16} = \dfrac{13}{16}$

37. (c) $\tan 20 = \dfrac{1}{\tan 4\theta} = \cot 4\theta$

$\Rightarrow \tan 2\theta = \tan(90° - 4\theta)$
$\Rightarrow 2\theta = 90° - 4\theta$
$\Rightarrow 6\theta = 90° \Rightarrow \theta = 15°$
$\therefore \tan 3\theta = \tan 45° = 1$

38. (d) $\tan(\theta_1 + \theta_2) = \sqrt{3} = \tan 60°$
$\Rightarrow \theta_1 + \theta_2 = 60°$ and $\sec(\theta_1 - \theta_2)$

$= \dfrac{2}{\sqrt{3}} = \sec 30°$

$\Rightarrow \theta_1 - \theta_2 = 30°$
$\therefore \theta_1 = 45°$ and $\theta_2 = 15°$
$\therefore \sin 2\theta_1 + \tan 3\theta_2$
$= \sin 90° + \tan 45°$
$= 1 + 1 = 2$

39. (b) $\sec\theta = \dfrac{4x^2 + 1}{4x}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$= \sqrt{\left(\dfrac{4x^2 + 1}{4x}\right)^2 - 1}$

$= \sqrt{\dfrac{(4x^2 + 1)^2 - (4x)^2}{(4x)^2}}$

$= \dfrac{(2x + 1)(2x - 1)}{4x} = \dfrac{4x^2 - 1}{4x}$

$\therefore \sec\theta + \tan\theta = \dfrac{4x^2 + 1}{4x} + \dfrac{4x^2 - 1}{4x}$

$= \dfrac{4x^2 + 1 + 4x^2 - 1}{4x}$

$= \dfrac{8x^2}{4x} = 2x$

40. (a) $x = a\sec\theta.\cos\phi$; $y = b\sec\theta.\sin\phi$; $z = c\tan\theta$

$\therefore \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2}$

$= \dfrac{a^2\sec^2\theta\cos^2\phi}{a^2} + \dfrac{b^2\sec^2\theta\sin^2\phi}{b^2} - \dfrac{c^2\tan^2\theta}{c^2}$

$= \sec^2\theta.\cos^2\phi + \sec^2\theta.\sin^2\phi - \tan^2\theta$
$= \sec^2\theta(\cos^2\phi + \sin^2\phi) - \tan^2\theta$
$= \sec^2\theta - \tan^2\theta = 1$

41. (a) $\dfrac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=\dfrac{5}{3}$

$\Rightarrow 5\sec\theta - 5\tan\theta = 3\sec\theta + 3\tan\theta$

$\Rightarrow 2\sec\theta = 8\tan\theta$

$\Rightarrow \dfrac{\tan\theta}{\sec\theta}=\dfrac{2}{8}=\dfrac{1}{4}$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta}\times\cos\theta=\dfrac{1}{4}$

$\Rightarrow \sin\theta=\dfrac{1}{4}$

42. (b) $\sec^2\theta+\tan^2\theta=7$

$1+\tan^2+\tan^2\theta=7$

$(\because\ 1+\tan^2\theta=\sec^2\theta)$

$\tan^2\theta=\dfrac{6}{2}=3$

$\tan\theta=\pm\sqrt{3}$

$\tan\theta=\sqrt{3}$ or $\tan\theta=-\sqrt{3}$

As $0\le\theta\le\pi/2$

$\therefore\ \theta=\tan^{-1}\sqrt{3}$

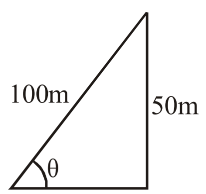
$\theta=\dfrac{\pi}{3}$

43. (c) $\sin^2x+2\tan^2\theta-2\sec^2x+\cos^2x$

$\sin^2x+\cos^2x-2(\sec^2x-\tan^2x)$

$1-2(1)=-1$

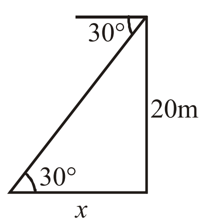
44. (d)



$\sin\theta=\dfrac{50\text{m}}{100\text{m}}=\dfrac{1}{2}$

$\theta=30°$

45. (b) $\tan 30°=\dfrac{20}{x}$



$\dfrac{1}{\sqrt{3}}=\dfrac{20}{x}$

$x=20\sqrt{3}\text{m}$

46. (a) $\dfrac{a^2}{x^2}-\dfrac{b^2}{y^2}=\dfrac{a^2}{a^2\sin^2\theta}-\dfrac{b^2}{b^2\tan^2\theta}$

$\Rightarrow \operatorname{cosec}^2\theta-\cot^2\theta=1$

47. (b) $2y\cos\theta=x\sin\theta$

$\Rightarrow \sin\theta=\dfrac{2y}{x}\cos\theta$

And $2x\sec\theta-y\operatorname{cosec}\theta=3$

$\Rightarrow 2x\sec\theta-\dfrac{y}{\sin\theta}=3$

$\Rightarrow \dfrac{2x}{\cos\theta}-\dfrac{yx}{2y\cos\theta}=3$

$\Rightarrow 3\cos\theta=\dfrac{3}{2}x\Rightarrow\cos\theta=\dfrac{x}{2}$

Now $\sin^2\theta+\cos^2\theta=1$

$\Rightarrow y^2+\dfrac{x^2}{4}=1$

$\Rightarrow 4y^2+x^2=4$

48. (b) $\sec^2\theta-\tan^2\theta=1$

$(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$

$\sqrt{3}(\sec\theta-\tan\theta)=1\Rightarrow\sec\theta-\tan\theta=\dfrac{1}{\sqrt{3}}$ ...(1)

$\sec\theta+\tan\theta=\sqrt{3}$ (Given) ...(2)

Adding eqns. (1) and (2)

$2\sec\theta=\sqrt{3}+\dfrac{1}{\sqrt{3}}\Rightarrow 2\sec\theta=\dfrac{4}{\sqrt{3}}\Rightarrow\sec\theta=\dfrac{2}{\sqrt{3}}$

$\therefore \cos\theta=\dfrac{\sqrt{3}}{2}$ $\left[\because \sec\theta=\dfrac{1}{\cos\theta}\right]$

Therefore, $\sin\theta=\sqrt{1-\cos^2\theta}$

$\Rightarrow \sqrt{1-\dfrac{3}{4}}=\dfrac{1}{2}$

49. (a) $63°\ 14'\left(\dfrac{51}{60}\right)'$ [1 minute = 60 seconds]

$\Rightarrow 63°\left[14+\dfrac{17}{20}\right]'\Rightarrow 63°\left[\dfrac{297}{20}\right]'\Rightarrow 63°+\dfrac{297}{20\times60}$

[1 degree = 60 minutes]

$\Rightarrow \left(\dfrac{75897}{1200}\right)^{\circ}\Rightarrow\dfrac{75897}{1200}\times\dfrac{\pi}{180}\text{ radian}\Rightarrow\left(\dfrac{2811}{8000}\pi\right)^{c}$

50. (d) $\dfrac{\cos^4\alpha}{\cos^2\beta}+\dfrac{\sin^4\alpha}{\sin^2\beta}=1$

$\Rightarrow \cos^4\alpha\sin^2\beta+\sin^4\alpha\cos^2\beta=\cos^2\beta\sin^2\beta$

$\Rightarrow \cos^4\alpha(1-\cos^2\beta)+\cos^2\beta(1-\cos^2\alpha)^2=\cos^2\beta(1-\cos^2\beta)$

$\Rightarrow \cos^4\alpha-\cos^4\alpha\cos^2\beta+\cos^2\beta-2\cos^2\alpha\cos^2\beta+\cos^4\alpha\cos^2\beta=\cos^2\beta-\cos^4\beta$

$\Rightarrow \cos^4\alpha-2\cos^2\alpha\cos^2\beta+\cos^4\beta=0$

$\Rightarrow (\cos^2\alpha-\cos^2\beta)^2=0$

$\Rightarrow \cos^2\alpha=\cos^2\beta$

$\Rightarrow \sin^2\alpha=\sin^2\beta$

Then, $\dfrac{\cos^4\beta}{\cos^2\alpha}+\dfrac{\sin^4\beta}{\sin^2\alpha}$

$$\Rightarrow \frac{\cos^2\beta\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\beta\ \sin^2\alpha}{\sin^2\alpha}$$

$\Rightarrow \cos^2\beta + \sin^2\beta = 1$

51. (a) $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$

Dividing Numerator and Denominator by cos θ

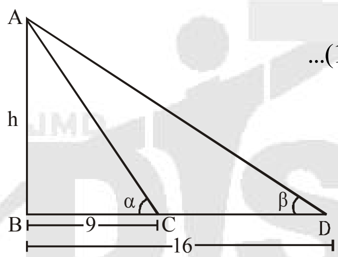
$$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\cos\theta}-\frac{1}{\cos\theta}} \Rightarrow \frac{\tan\theta-1+\sec\theta}{\tan\theta+1-\sec\theta}$$

$$\Rightarrow \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$\Rightarrow \frac{(\tan\theta+\sec\theta)[1-\sec\theta+\tan\theta]}{\tan\theta-\sec\theta+1} \Rightarrow \tan\theta+\sec\theta$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} \quad \Rightarrow \frac{1+\sin\theta}{\cos\theta}$$

52. (b) In ΔABC



$$\tan\alpha = \frac{h}{9} \qquad ...(1)$$

In ΔABD

$$\tan\beta = \frac{h}{16}$$

$\alpha+\beta = 90°$ (given)

$\beta = 90-\alpha$

since $\tan\beta = \dfrac{h}{16}$

$$\tan(90-\alpha) = \frac{h}{16} \Rightarrow \cot\alpha = \frac{h}{16} \text{ or } \tan\alpha = \frac{16}{h} \qquad ...(2)$$

From eqn. (1) and (2)

$$\frac{h}{9}=\frac{16}{h} \Rightarrow h^2 = 16\times 9 \Rightarrow h = 12 \text{ feet.}$$

53. (a) If $\sin^2\alpha = \cos^3\alpha$

$$\tan^2\alpha = \cos\alpha \qquad ...(1)$$

Now consider, $\cot^6\alpha - \cot^2\alpha$

$$= \frac{1}{\tan^6\alpha}-\frac{1}{\tan^2\alpha} \quad \text{Since } \cot\alpha = \frac{1}{\tan\alpha}$$

Substituting for $\tan^2\alpha$ with cos α from (1) above equation will be

$$= \frac{1}{\cos^3\alpha}-\frac{1}{\cos\alpha} = \frac{1-\cos^2\alpha}{\cos^3\alpha} = \frac{\sin^2\alpha}{\cos^3\alpha} = \frac{\tan^2\alpha}{\cos\alpha} = 1$$

54. (b) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

$$\Rightarrow \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$\Rightarrow \left(\frac{\sin\theta+\cos\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta} = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

55. (b) $\dfrac{\sin 53°}{\cos 37°} \div \dfrac{\cot 65°}{\tan 25°}$

$$\frac{\sin 53°}{\cos 37°}\times\frac{\tan 25°}{\cot 65°} \Rightarrow \frac{\sin 53°}{\cos(90°-53°)}\times\frac{\tan 25°}{\cot(90°-25°)}$$

$$\Rightarrow \frac{\sin 53°}{\sin 53°}\times\frac{\tan 25°}{\tan 25°} = 1$$

[∵ cos (90° – θ) = sin θ and cot (90° – θ) = tan θ]

56. (c) $$\frac{\cos 60°+\sin 60°}{\cos 60°-\sin 60°} = \frac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{\frac{1}{2}-\frac{\sqrt{3}}{2}} = \frac{1+\sqrt{3}}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$\Rightarrow \frac{(1+\sqrt{3})^2}{1^2-(\sqrt{3})^2} = \frac{1+3+2\sqrt{3}}{1-3} = \frac{4+2\sqrt{3}}{-2}$$

$$\Rightarrow \frac{-2(2+\sqrt{3})}{2} = -(2+\sqrt{3})$$

57. (d) $$\frac{\cot 5°.\cot 10°.\cot 15°.\cot 60°.\cot 75°.\cot 80°.\cot 85°}{(\cos^2 20+\cos^2 70°)+2}$$

$$\Rightarrow \frac{\cot(90°-85°).\cot(90°-80°).\cot(90°-75°).\cot 60°.\cot 75°.\cot 80°\cot 85°}{(\cos^2(90°-70°)+\cos^2 70°)+2}$$

$$\Rightarrow \frac{\cot 60°}{(1+2)} = \frac{\frac{1}{\sqrt{3}}}{3} = \frac{1}{3\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{9}$$

58. (d) Let angles are $2x$, $5x$ and $3x$.

$2x + 5x + 3x = 180°$

(sum of interior angle of triangles is 180°)

$10x = 18°$

$x = 18°$

∴ Least angle in degree $= 2x = 2 \times 18 = 36°$

$$\text{In radian} = \frac{\pi}{180°}\times 36° = \frac{\pi}{5}$$

59. (d) $x = a\cos\theta - b\sin\theta$

$y = b\cos\theta + a\sin\theta$

$x^2 + y^2 = (a\cos\theta - b\sin\theta)^2 + (b\cos\theta + a\sin\theta)^2$

⇒ $a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta + b^2\cos^2\theta + a^2\sin^2\theta + 2ab\cos\theta\sin\theta$

⇒ $(a^2 + b^2)\cos^2\theta + (a^2 + b^2)\sin^2\theta$

⇒ $a^2 + b^2(\cos^2\theta + \sin^2\theta)$

⇒ $a^2 + b^2.(1) \Rightarrow a^2 + b^2$

60. (a) $\tan\alpha + \cot\alpha = 2$

$$\tan\alpha + \frac{1}{\tan\alpha} = 2 \Rightarrow \tan^2\alpha + 1 = 2\tan\alpha$$

⇒ $\tan^2\alpha - 2\tan\alpha + 1 = 0$

⇒ $\tan^2\alpha - \tan\alpha - \tan\alpha + 1 = 0$

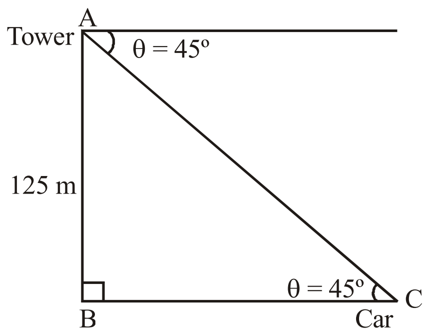
⇒ $\tan\alpha(\tan\alpha - 1) - 1(\tan\alpha - 1) = 0$

$(\tan\alpha - 1)(\tan\alpha - 1) = 0$

$\therefore \quad \tan\alpha = 1$

Now, $\tan^7\alpha + \cot^7\alpha \Rightarrow (\tan\alpha)^7 + \dfrac{1}{(\tan\alpha)^7} = 1 + 1 = 2$

61. (a)



In ΔABC

$\tan\theta = \dfrac{AB}{BC} \Rightarrow \tan 45° = \dfrac{125}{BC} \Rightarrow 1 = \dfrac{125}{BC}$

$BC = 125\,m$

Hence, car is 125 m from the tower.

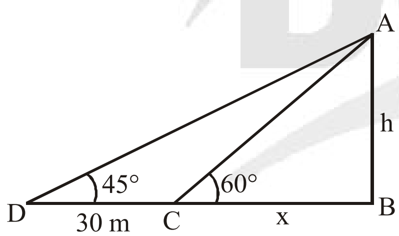
62. (c) $\dfrac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta\cos\theta)}{(\cos\theta + \sin\theta)}$

$+ \dfrac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$

$= 2\cos^2\theta + 2\sin^2\theta - \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 2$

63. (a)



In ΔABC, $\tan 60° = \dfrac{h}{x}$

$x = \dfrac{h}{\sqrt{3}}$ ...(1)

In ΔABD, $\tan 45° = \dfrac{h}{30 + x}$

$1 = \dfrac{h}{30 + x}$ or $h = 30 + x$

Putting value of x from (1)

$h = 30 + \dfrac{h}{\sqrt{3}}$

or $h\dfrac{(\sqrt{3} - 1)}{\sqrt{3}} = 30 \Rightarrow h = 15\,(3 + \sqrt{3})$ m

64. (d) $\sin 17° = \dfrac{x}{y}$

$\cos 17° = \sqrt{1 - \dfrac{x^2}{y^2}} = \dfrac{\sqrt{y^2 - x^2}}{y}$

$\sec 17° - \sin 73°$

$= \sec 17° - \cos 17°$

$= \dfrac{y}{\sqrt{y^2 - x^2}} - \dfrac{\sqrt{y^2 - x^2}}{y}$

$= \dfrac{y^2 - y^2 + x^2}{y\sqrt{y^2 - x^2}} = \dfrac{x^2}{y\sqrt{y^2 - x^2}}$

65. (c) $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$

$\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} = \sqrt{3}$

$\dfrac{1 + \cos\theta}{\sin\theta} = \sqrt{3}$

$\dfrac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \sqrt{3}$

$\cot\dfrac{\theta}{2} = \sqrt{3}$

$\tan\dfrac{\theta}{2} = \dfrac{1}{\sqrt{3}};\ \dfrac{\theta}{2} = 30°;\ \theta = 60°$

$\operatorname{cosec}\theta = \operatorname{cosec} 60° = \dfrac{2}{\sqrt{3}}$

66. (c) $\cos\alpha + \sec\alpha = \sqrt{3}$

taking cube both sides

$\cos^3\alpha + \sec^3\alpha + 3\cos\alpha\sec\alpha(\cos\alpha + \sec\alpha) = 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha + 3\sqrt{3} = 3\sqrt{3}$

$\cos^3\alpha + \sec^3\alpha = 0$

67. (a) $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$\sin\theta = (\sqrt{2} - 1)\cos\theta$

$\cot\theta = \dfrac{1}{\sqrt{2} - 1}$

$\cot\theta = \dfrac{1}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1} = \sqrt{2} + 1$

68. (d) $(\sin^2 1° + \sin^2 89°) + (\sin^2 2° + \sin^2 88°) + \ldots$

$+ (\sin^2 44° + \sin^2 48°) + \sin^2 45°$

$= (\sin^2 1° + \cos^2 1°) + (\sin^2 2° + \cos^2 2°) + \ldots$

$+ (\sin^2 44° + \cos^2 44°) + \sin^2 45°$

$= 1 + 1 + \ldots + 1\ (44\text{ times}) + \dfrac{1}{2}$

$= 44\dfrac{1}{2}$