

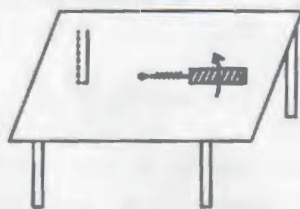
PART A – PHYSICS

1. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :

- (1) $\frac{Mg}{k}$
- (2) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{M} \right)$
- (3) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{2M} \right)$
- (4) $\frac{Mg}{k} \left(1 + \frac{LA\sigma}{M} \right)$

(Here k is spring constant)

2. A metallic rod of length ' l ' is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field ' B ' in the region, the e.m.f. induced across the ends of the rod is :



- (1) $\frac{2B\omega l^2}{2}$
- (2) $\frac{3B\omega l^2}{2}$
- (3) $\frac{4B\omega l^2}{2}$
- (4) $\frac{5B\omega l^2}{2}$

3. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M . If the maximum energy loss possible is given as

$$f \left(\frac{1}{2} m v^2 \right) \text{ then } f = \left(\frac{m}{M + m} \right).$$

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

- (1) Statement - I is true, Statement - II is true, Statement - II is a correct explanation of Statement - I.
- (2) Statement - I is true, Statement - II is true, Statement - II is not a correct explanation of Statement - I.
- (3) Statement - I is true, Statement - II is false.
- (4) Statement - I is false, Statement - II is true.

4. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then :

- (1) $[\epsilon_0] = [M^{-1} L^{-3} T^2 A]$
- (2) $[\epsilon_0] = [M^{-1} L^{-3} T^4 A^2]$
- (3) $[\epsilon_0] = [M^{-1} L^2 T^{-1} A^{-2}]$
- (4) $[\epsilon_0] = [M^{-1} L^2 T^{-1} A]$

5. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j}) \text{ m/s}$, where \hat{i} is along the ground and \hat{j} is along the vertical. If $g = 10 \text{ m/s}^2$, the equation of its trajectory is :
- (1) $y = x - 5x^2$
 - (2) $y = 2x - 5x^2$
 - (3) $4y = 2x - 5x^2$
 - (4) $4y = 2x - 25x^2$
6. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals :
- (1) 0.7
 - (2) 0.81
 - (3) 0.729
 - (4) 0.6
7. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :
- (1) $5C_1 = 3C_2$
 - (2) $3C_1 = 5C_2$
 - (3) $3C_1 + 5C_2 = 0$
 - (4) $9C_1 = 4C_2$
8. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3 \text{ kg/m}^3$ and $2.2 \times 10^{11} \text{ N/m}^2$ respectively ?
- (1) 188.5 Hz
 - (2) 178.2 Hz
 - (3) 200.5 Hz
 - (4) 770 Hz
9. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :
- (1) $9.1 \times 10^{-11} \text{ weber}$
 - (2) $6 \times 10^{-11} \text{ weber}$
 - (3) $3.3 \times 10^{-11} \text{ weber}$
 - (4) $6.6 \times 10^{-9} \text{ weber}$

10. Diameter of a plano - convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

- (1) 15 cm
- (2) 20 cm
- (3) 30 cm
- (4) 10 cm

11. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

- (1) $\frac{5GmM}{6R}$
- (2) $\frac{2GmM}{3R}$
- (3) $\frac{GmM}{2R}$
- (4) $\frac{GmM}{3R}$

12. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

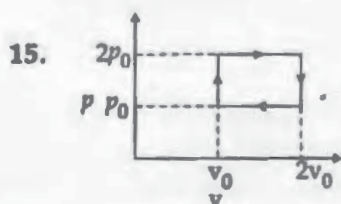
- (1) 10.62 MHz
- (2) 10.62 kHz
- (3) 5.31 MHz
- (4) 5.31 kHz

13. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is :

- (1) I_0
- (2) $I_0/2$
- (3) $I_0/4$
- (4) $I_0/8$

14. The supply voltage to a room is 120 V. The resistance of the lead wires is $6\ \Omega$. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

- (1) zero Volt
(2) 2.9 Volt
(3) 13.3 Volt
(4) 10.04 Volt



The above p - v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is :

- (1) $p_0 v_0$
(2) $\left(\frac{13}{2}\right) p_0 v_0$
(3) $\left(\frac{11}{2}\right) p_0 v_0$
(4) $4 p_0 v_0$

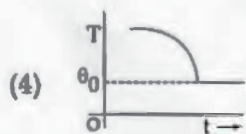
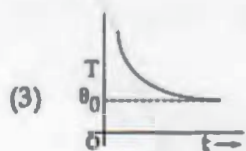
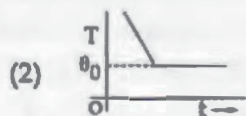
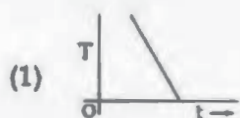
16. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

- (1) $\frac{r\omega_0}{4}$
(2) $\frac{r\omega_0}{3}$
(3) $\frac{r\omega_0}{2}$
(4) $r\omega_0$

17. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and the cylinder have equal cross sectional area A . When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency :

- (1) $\frac{1}{2\pi} \frac{A\gamma P_0}{V_0 M}$
(2) $\frac{1}{2\pi} \frac{V_0 M P_0}{A^2 \gamma}$
(3) $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{M V_0}}$
(4) $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A \gamma P_0}}$

18. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 , the graph between the temperature T of the metal and time t will be closest to :



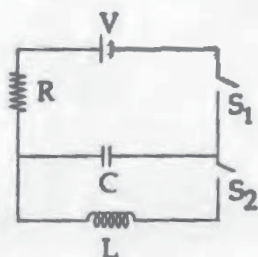
19. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement - I : Higher the range, greater is the resistance of ammeter.

Statement - II : To increase the range of ammeter, additional shunt needs to be used across it.

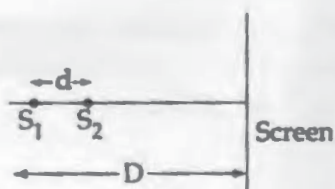
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- (4) Statement - I is false, Statement - II is true.

20. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct ?



- (1) Work done by the battery is half of the energy dissipated in the resistor
- (2) At $t = \tau$, $q = CV/2$
- (3) At $t = 2\tau$, $q = CV(1 - e^{-2})$
- (4) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

21. Two coherent point sources S_1 and S_2 are separated by a small distance ' d ' as shown. The fringes obtained on the screen will be :

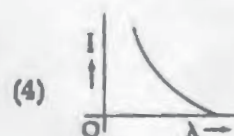
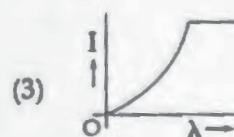
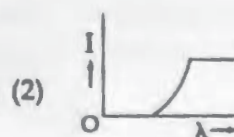
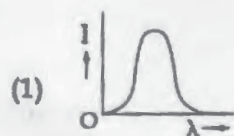


- (1) points
- (2) straight lines
- (3) semi - circles
- (4) concentric circles

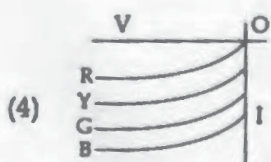
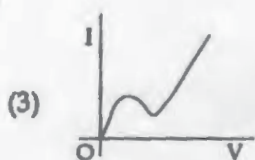
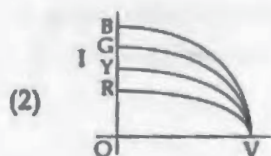
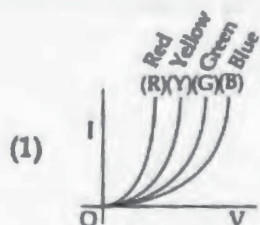
22. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

- (1) 3 V/m
- (2) 6 V/m
- (3) 9 V/m
- (4) 12 V/m

23. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows :



24. The I - V characteristic of an LED is :



25. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible ? The surface tension is T , density of liquid is ρ and L is its latent heat of vaporization.

(1) $\rho L/T$

(2) $\sqrt{T/\rho L}$

(3) $T/\rho L$

(4) $2T/\rho L$

26. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to :

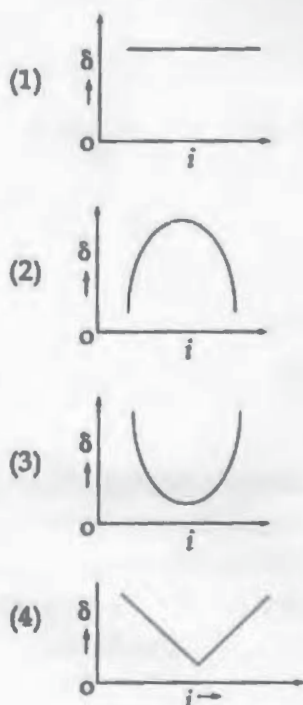
(1) $\frac{1}{n}$

(2) $\frac{1}{n^2}$

(3) $\frac{1}{n^{3/2}}$

(4) $\frac{1}{n^3}$

27. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by :



28. Two charges, each equal to q , are kept at $x = -a$ and $x = a$ on the x -axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along the y -axis, the net force acting on the particle is proportional to :

- (1) y
 (2) $-y$
 (3) $\frac{1}{y}$
 (4) $-\frac{1}{y}$

29. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to

(Horizontal component of earth's magnetic induction is $3.6 \times 10^{-5} \text{ Wb/m}^2$)

- (1) $3.6 \times 10^{-5} \text{ Wb/m}^2$
 (2) $2.56 \times 10^{-4} \text{ Wb/m}^2$
 (3) $3.50 \times 10^{-4} \text{ Wb/m}^2$
 (4) $5.80 \times 10^{-4} \text{ Wb/m}^2$

30. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is :



- (1) $\frac{Q}{8\pi\epsilon_0 L}$
 (2) $\frac{3Q}{4\pi\epsilon_0 L}$
 (3) $\frac{Q}{4\pi\epsilon_0 L \ln 2}$
 (4) $\frac{Q \ln 2}{4\pi\epsilon_0 L}$

PART B – CHEMISTRY

31. Which of the following complex species is not expected to exhibit optical isomerism ?

- (1) $[\text{Co(en)}_3]^{3+}$
- (2) $[\text{Co(en)}_2 \text{Cl}_2]^+$
- (3) $[\text{Co}(\text{NH}_3)_3 \text{Cl}_3]$
- (4) $[\text{Co(en)}(\text{NH}_3)_2 \text{Cl}_2]^+$

32. Which one of the following molecules is expected to exhibit diamagnetic behaviour ?

- (1) C_2
- (2) N_2
- (3) O_2
- (4) S_2

33. A solution of (–)-1-chloro-1-phenylethane in toluene racemises slowly in the presence of a small amount of SbCl_5 due to the formation of :

- (1) carbanion
- (2) carbene
- (3) carbocation
- (4) free radical

34. Given

$$E^0_{\text{Cr}^{3+}/\text{Cr}} = -0.74 \text{ V}; E^0_{\text{MnO}_4^-/\text{Mn}^{2+}} = 1.51 \text{ V}$$

$$E^0_{\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}} = 1.33 \text{ V}; E^0_{\text{Cl}/\text{Cl}^-} = 1.36 \text{ V}$$

Based on the data given above, strongest oxidising agent will be :

- (1) Cl^-
- (2) Cr^{3+}
- (3) Mn^{2+}
- (4) MnO_4^-

35. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C . As it does so, it absorbs 208 J of heat. The values of q and w for the process will be :

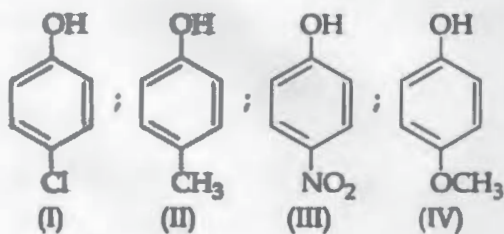
$$(R = 8.314 \text{ J/mol K}) (\ln 7.5 = 2.01)$$

- (1) $q = +208 \text{ J}, w = -208 \text{ J}$
- (2) $q = -208 \text{ J}, w = -208 \text{ J}$
- (3) $q = -208 \text{ J}, w = +208 \text{ J}$
- (4) $q = +208 \text{ J}, w = +208 \text{ J}$

36. The molarity of a solution obtained by mixing 750 mL of 0.5(M)HCl with 250 mL of 2(M)HCl will be :

- (1) 0.875 M
- (2) 1.00 M
- (3) 1.75 M
- (4) 0.975 M

37. Arrange the following compounds in order of decreasing acidity :



- (1) II > IV > I > III
 (2) I > II > III > IV
 (3) III > I > II > IV
 (4) IV > III > I > II
38. For gaseous state, if most probable speed is denoted by C^* , average speed by \bar{C} and mean square speed by C , then for a large number of molecules the ratios of these speeds are :
- (1) $C^* : \bar{C} : C = 1.225 : 1.128 : 1$
 (2) $C^* : \bar{C} : C = 1.128 : 1.225 : 1$
 (3) $C^* : \bar{C} : C = 1 : 1.128 : 1.225$
 (4) $C^* : \bar{C} : C = 1 : 1.225 : 1.128$
39. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be :
 ($R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ and $\log 2 = 0.301$)
- (1) 53.6 kJ mol^{-1}
 (2) 48.6 kJ mol^{-1}
 (3) 58.5 kJ mol^{-1}
 (4) 60.5 kJ mol^{-1}

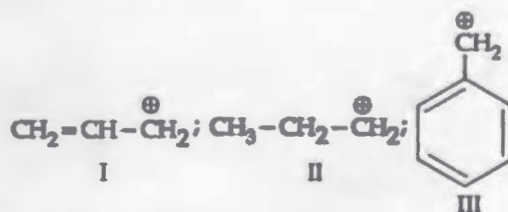
40. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is :

- (1) 2
 (2) 5
 (3) 4
 (4) 6

41. Which of the following arrangements does not represent the correct order of the property stated against it ?

- (1) $\text{V}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+} < \text{Fe}^{2+}$: paramagnetic behaviour
 (2) $\text{Ni}^{2+} < \text{Co}^{2+} < \text{Fe}^{2+} < \text{Mn}^{2+}$: ionic size
 (3) $\text{Co}^{3+} < \text{Fe}^{3+} < \text{Cr}^{3+} < \text{Sc}^{3+}$: stability in aqueous solution
 (4) $\text{Sc} < \text{Ti} < \text{Cr} < \text{Mn}$: number of oxidation states

42. The order of stability of the following carbocations :



is :

- (1) III > II > I
 (2) II > III > I
 (3) I > II > III
 (4) III > I > II

43. Consider the following reaction :



The values of x , y and z in the reaction are, respectively :

- (1) 5, 2 and 16
- (2) 2, 5 and 8
- (3) 2, 5 and 16
- (4) 5, 2 and 8

44. Which of the following is the wrong statement ?

- (1) ONCl and ONO^- are not isoelectronic.
- (2) O_3 molecule is bent.
- (3) Ozone is violet-black in solid state.
- (4) Ozone is diamagnetic gas.

45. A gaseous hydrocarbon gives upon combustion 0.72 g. of water and 3.08 g. of CO_2 . The empirical formula of the hydrocarbon is :

- (1) C_2H_4
- (2) C_3H_4
- (3) C_6H_5
- (4) C_7H_8

46. In which of the following pairs of molecules/ions, both the species are not likely to exist ?

- (1) H_2^+ , He_2^{2-}
- (2) H_2^- , He_2^{2-}
- (3) H_2^{2+} , He_2
- (4) H_2^- , He_2^{2+}

47. Which of the following exists as covalent crystals in the solid state ?

- (1) Iodine
- (2) Silicon
- (3) Sulphur
- (4) Phosphorus

48. Synthesis of each molecule of glucose in photosynthesis involves :

- (1) 18 molecules of ATP
- (2) 10 molecules of ATP
- (3) 8 molecules of ATP
- (4) 6 molecules of ATP

49. The coagulating power of electrolytes having ions Na^+ , Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order :

- (1) $\text{Al}^{3+} < \text{Ba}^{2+} < \text{Na}^+$
- (2) $\text{Na}^+ < \text{Ba}^{2+} < \text{Al}^{3+}$
- (3) $\text{Ba}^{2+} < \text{Na}^+ < \text{Al}^{3+}$
- (4) $\text{Al}^{3+} < \text{Na}^+ < \text{Ba}^{2+}$

50. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar ?

- (1) $\text{Ca} < \text{S} < \text{Ba} < \text{Se} < \text{Ar}$
- (2) $\text{S} < \text{Se} < \text{Ca} < \text{Ba} < \text{Ar}$
- (3) $\text{Ba} < \text{Ca} < \text{Se} < \text{S} < \text{Ar}$
- (4) $\text{Ca} < \text{Ba} < \text{S} < \text{Se} < \text{Ar}$

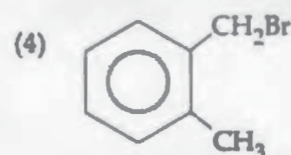
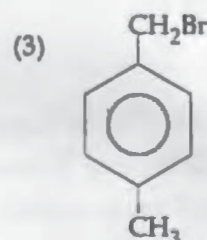
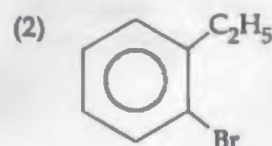
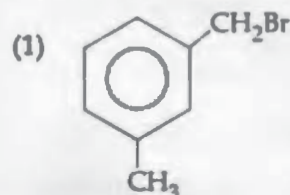
51. Energy of an electron is given by

$E = -2.178 \times 10^{-18} \text{J} \left(\frac{Z^2}{n^2} \right)$. Wavelength of light required to excite an electron in an hydrogen atom from level $n=1$ to $n=2$ will be :

($h = 6.62 \times 10^{-34} \text{Js}$ and $c = 3.0 \times 10^8 \text{ms}^{-1}$)

- (1) $1.214 \times 10^{-7} \text{m}$
- (2) $2.816 \times 10^{-7} \text{m}$
- (3) $6.500 \times 10^{-7} \text{m}$
- (4) $8.500 \times 10^{-7} \text{m}$

52. Compound (A), $\text{C}_8\text{H}_9\text{Br}$, gives a white precipitate when warmed with alcoholic AgNO_3 . Oxidation of (A) gives an acid (B), $\text{C}_8\text{H}_6\text{O}_4$. (B) easily forms anhydride on heating. Identify the compound (A).



53. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E_{M^{3+}/M^{2+}}^0$ value ?

- (1) $\text{Cr}(Z=24)$ 7, 8, 14
- (2) $\text{Mn}(Z=25)$ 7, 8, 15
- (3) $\text{Fe}(Z=26)$ 16
- (4) $\text{Co}(Z=27)$ 17

54. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2 ?
- (1) 0.1 L
 - (2) 0.9 L
 - (3) 2.0 L
 - (4) 9.0 L
55. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be :
- (1) -2.55 eV
 - (2) -5.1 eV
 - (3) -10.2 eV
 - (4) +2.55 eV
56. An organic compound A upon reacting with NH_3 gives B. On heating, B gives C. C in presence of KOH reacts with Br_2 to give $\text{CH}_3\text{CH}_2\text{NH}_2$. A is :
- (1) CH_3COOH
 - (2) $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$
 - (3) $\text{CH}_3-\underset{\text{CH}_3}{\underset{|}{\text{CH}}}-\text{COOH}$
 - (4) $\text{CH}_3\text{CH}_2\text{COOH}$
57. Stability of the species Li_2 , Li_2^- and Li_2^+ increases in the order of :
- (1) $\text{Li}_2 < \text{Li}_2^+ < \text{Li}_2^-$
 - (2) $\text{Li}_2^- < \text{Li}_2^+ < \text{Li}_2$
 - (3) $\text{Li}_2 < \text{Li}_2^- < \text{Li}_2^+$
 - (4) $\text{Li}_2^- < \text{Li}_2 < \text{Li}_2^+$
58. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism :
- (1) secondary alcohol by $\text{S}_{\text{N}}1$
 - (2) tertiary alcohol by $\text{S}_{\text{N}}1$
 - (3) secondary alcohol by $\text{S}_{\text{N}}2$
 - (4) tertiary alcohol by $\text{S}_{\text{N}}2$
59. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was :
- (1) Methylisocyanate
 - (2) Methylamine
 - (3) Ammonia
 - (4) Phosgene
60. Experimentally it was found that a metal oxide has formula $\text{M}_{0.98}\text{O}$. Metal M is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be :
- (1) 7.01 %
 - (2) 4.08 %
 - (3) 6.05 %
 - (4) 5.08 %

PART C – MATHEMATICS

61. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is :

- (1) $\frac{3}{2}$
- (2) $\frac{5}{2}$
- (3) $\frac{7}{2}$
- (4) $\frac{9}{2}$

62. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is :

- (1) 2500
- (2) 3000
- (3) 3500
- (4) 4500

63. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is :

- (1) 256
- (2) 220
- (3) 219
- (4) 211

64. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have :

- (1) any value.
- (2) exactly one value.
- (3) exactly two values.
- (4) exactly three values.

65. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC , then the length of the median through A is :

- (1) $\sqrt{18}$
- (2) $\sqrt{72}$
- (3) $\sqrt{33}$
- (4) $\sqrt{45}$



$$5\hat{i} - 2\hat{j} + 4\hat{k} - 3\hat{i} - 4\hat{k} \\ \hat{i}(-2-3) - 2\hat{j} \\ 21 - 2\hat{j}$$

66. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in $[0, 1]$

- (1) lies between 1 and 2.
- (2) lies between 2 and 3.
- (3) lies between -1 and 0 .
- (4) does not exist.

$$k - 4.2 \cdot k > 0 \Rightarrow 9 - 8k > 0 \\ 9 - 4.2 \cdot k > 0 \Rightarrow k > \frac{9}{8}$$

$$\hat{i} - \hat{j} + 4\hat{k}$$

67. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,, is :

(1) $\frac{7}{81} (179 - 10^{-20})$

(2) $\frac{7}{9} (99 - 10^{-20})$

(3) $\frac{7}{81} (179 + 10^{-20})$

(4) $\frac{7}{9} (99 + 10^{-20})$

68. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is :

(1) $y = x + \sqrt{3}$

(2) $\sqrt{3}y = x - \sqrt{3}$

(3) $y = \sqrt{3}x - \sqrt{3}$

(4) $\sqrt{3}y = x - 1$

69. The number of values of k, for which the system of equations :

$$(k+1)x + 8y = 4k$$

$$kx + (k+3)y = 3k - 1$$

has no solution, is :

(1) infinite

(2) 1

(3) 2

(4) 3

70. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, have a common root, then $a : b : c$ is :

(1) 1 : 2 : 3

(2) 3 : 2 : 1

(3) 1 : 3 : 2

(4) 3 : 1 : 2

71. The circle passing through (1, -2) and touching the axis of x at (3, 0) also passes through the point :

(1) (-5, 2)

(2) (2, -5)

(3) (5, -2)

(4) (-2, 5)

72. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then :

(1) $x = y = z$

(2) $2x = 3y = 6z$

(3) $6x = 3y = 2z$

(4) $6x = 4y = 3z$

Q. 67

$$2y = x + 2$$

$$2 \tan^{-1} y = \tan^{-1} x + \tan^{-1} z$$

$$2y = x + z$$

73. Consider :

Statement - I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement - II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

- (1) Statement - I is true; Statement - II is true; Statement - II is a correct explanation for Statement - I.
- (2) Statement - I is true; Statement - II is true; Statement - II is not a correct explanation for Statement - I.
- (3) Statement - I is true; Statement - II is false.
- (4) Statement - I is false; Statement - II is true.

74. If $\int f(x) dx = \Psi(x)$, then $\int x^5 f(x^3) dx$ is equal to :

- (1) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx] + C$
- (2) $\frac{1}{3} x^3 \Psi(x^3) - 3 \int x^3 \Psi(x^3) dx + C$
- (3) $\frac{1}{3} x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx + C$
- (4) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^3 \Psi(x^3) dx] + C$

75. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to :

- (1) $-\frac{1}{4}$
- (2) $\frac{1}{2}$
- (3) 1
- (4) 2

76. **Statement - I :**

The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement - II :

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx.$$

- (1) Statement - I is true ; Statement - II is true; Statement - II is a correct explanation for Statement - I.
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- (1) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx] + C$
- (2) $\frac{1}{3} x^3 \Psi(x^3) - 3 \int x^3 \Psi(x^3) dx + C$
- (3) $\frac{1}{3} x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx + C$
- (4) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^3 \Psi(x^3) dx] + C$

75. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to :

- (1) $-\frac{1}{4}$
- (2) $\frac{1}{2}$
- (3) 1
- (4) 2

76. **Statement - I :**

The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement - II :

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx.$$

- (1) Statement - I is true ; Statement - II is true; Statement - II is a correct explanation for Statement - I.
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- (4) Statement - I is false ; Statement - II is true.

83. If z is a complex number of unit modulus and argument θ , then $\arg \left(\frac{1+z}{1+\bar{z}} \right)$ equals :

- (1) $-\theta$
- (2) $\frac{\pi}{2} - \theta$
- (3) θ
- (4) $\pi - \theta$

84. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to :

- (1) $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$
- (2) $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$
- (3) $\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$
- (4) $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$

85. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to :

- (1) 4
- (2) 11
- (3) 5
- (4) 0

86. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt$, $x \in \mathbb{R}$, which are parallel to the line $y = 2x$, are equal to :

- (1) ± 1
- (2) ± 2
- (3) ± 3
- (4) ± 4

Handwritten notes for Q86:

$$y = \int_0^x |t| dt = \begin{cases} \frac{1}{2}x^2 & x \geq 0 \\ -\frac{1}{2}x^2 & x < 0 \end{cases}$$

For $x > 0$, $y = \frac{1}{2}x^2$. Tangent at (x, y) is $y - \frac{1}{2}x^2 = x(x - x_0)$. For it to be parallel to $y = 2x$, slope must be 2. $\frac{dy}{dx} = x = 2 \Rightarrow x = 2$. Then $y = 2$. Tangent line is $y - 2 = 2(x - 2) \Rightarrow y = 2x - 2$. Intercept on x-axis is 1. Similarly for $x < 0$, intercept is -1. So intercepts are ± 1 .

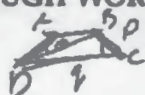
87. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement - I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement - II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ ($m \neq 0$) is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

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- (2) Statement - I is true; Statement - II is true; Statement - II is not a correct explanation for Statement - I.
- (3) Statement - I is true; Statement - II is false.
- (4) Statement - I is false; Statement - II is true.

Handwritten note: $\arg(1+z) - \arg(1+\bar{z})$



Handwritten note: $\arg z = \dots$

88. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is equal to :

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{2}$

(3) 1

☒ (4) $\sqrt{2}$

89. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as :

(1) $\sin A \cos A + 1$

☒ (2) $\sec A \operatorname{cosec} A + 1$

☒ (3) $\tan A + \cot A$

(4) $\sec A + \operatorname{cosec} A$

90. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) mean

(2) median

(3) mode

(4) variance

- o o -

SPACE FOR ROUGH WORK

$$\frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1 - \tan A}$$

$$\frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A - \tan^2 A} = \frac{\tan^2 A}{\tan A - 1} - \frac{1}{\tan A (\tan A - 1)}$$

$$\frac{\tan^2 A}{\tan A - 1} + \frac{\cot^2 A}{\tan A - \cot^2 A} + 0 = \frac{1}{\tan A - 1} \left[\tan^2 A - \frac{1}{\tan A} \right]$$

$$= \frac{\tan A}{\tan A - \cot A} \left[\frac{\sin^2 A}{\cos^2 A} - \frac{\cos A}{\sin A} \right]$$