

IIT-JEE 2012

PAPER - 1

PART - III : MATHEMATICS

Section I : Single Correct Answer Type

This section contains 10 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1, -1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $(2, 1, 4)$ to the line QR, then the length of the line segment PS is

- (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Sol. **Ans. (A)**

Equation of QR is

$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$$

Let $P \equiv (2 + \lambda, 3 + 4\lambda, 5 + \lambda)$

$$10 + 5\lambda - 1 - 15\lambda - 5 - \lambda = 1$$

$$-7 - 11\lambda = 1$$

$$\Rightarrow \lambda = -\frac{2}{3}$$

$$\text{then } P \equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$$

Let $S = (2 + \mu, 3 + 4\mu, 5 + \mu)$

$$\vec{TS} = (\mu - 2)\hat{i} + (4\mu + 2)\hat{j} + (\mu + 1)\hat{k}$$

$$\vec{TS} \cdot (\hat{i} + 4\hat{j} + \hat{k}) = 0$$

$$(\mu - 2) + 16\mu + \mu + 1 = 0$$

$$\mu = -\frac{1}{2}$$

$$S = \left(\frac{3}{2}, 1, \frac{9}{2} \right)$$

$$PS = \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \frac{4}{9} + \left(\frac{13}{3} - \frac{9}{2}\right)^2} = \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}} = \sqrt{\frac{1}{18} + \frac{4}{9}} = \sqrt{\frac{9}{18}} = \frac{1}{\sqrt{2}}$$

42. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $\frac{-1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2 \right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2 \right\} + K$

(C) $\frac{-1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2 \right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2 \right\} + K$

Sol. **Ans (C)**

Put $\sec x + \tan x = t$

$(\sec x \tan x + \sec^2 x) dx = dt$

$\sec x \cdot t dx = dt$

$\sec x - \tan x = \frac{1}{t}$

$\sec x = \frac{t + \frac{1}{t}}{2}$

$\int \frac{\sec x \cdot dt}{t^{9/2} \cdot t} = \int \frac{1}{2} \frac{\left(t + \frac{1}{t}\right)}{t \cdot t^{9/2}} dt$

$= \frac{1}{2} \int \left(\frac{1}{t^{9/2}} + \frac{1}{t^{13/2}} \right) dt$

$= -\frac{1}{2} \left[\frac{2}{7t^{7/2}} + \frac{2}{11t^{11/2}} \right] + k$

$= -\frac{1}{t^{11/2}} \left[\frac{t^2}{7} + \frac{1}{11} \right] + k$

43. Let z be a complex number such that the imaginary part of z is non zero and $a = z^2 + z + 1$ is real. Then a cannot take the value

- (A) -1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

43. **Ans (D)**

Here $z^2 + z + 1 - a = 0$

$$\Rightarrow z = \frac{-1 \pm \sqrt{4a - 3}}{2}$$

Here $a \neq \frac{3}{4}$ otherwise z will be purely real.

44. Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$, then f is

- (A) differentiable both at $x = 0$ and at $x = 2$
 (B) differentiable at $x = 0$ but not differentiable at $x = 2$
 (C) not differentiable at $x = 0$ but differentiable at $x = 2$
 (D) differentiable neither at $x = 0$ nor at $x = 2$

Sol. **Ans (B)**

(i) for derivability at $x = 0$

$$\begin{aligned} \text{L.H.D. } = f'(0^-) &= \lim_{h \rightarrow 0^+} \frac{f(0-h) - f(0)}{-h} \\ &= \lim_{h \rightarrow 0^+} \frac{h^2 \cdot \left| \cos \left(-\frac{\pi}{h} \right) \right| - 0}{-h} \\ &= \lim_{h \rightarrow 0^+} -h \cdot \left| \cos \frac{\pi}{h} \right| = 0 \end{aligned}$$

$$\begin{aligned} \text{RHD } f'(0^+) &= \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{h^2 \cdot \left| \cos \left(\frac{\pi}{h} \right) \right| - 0}{h} = 0 \end{aligned}$$

So $f(x)$ is derivable at $x = 0$

(ii) check for derivability at $x = 2$

$$\begin{aligned} \text{RHD } = f'(2^+) &= \lim_{h \rightarrow 0^+} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0^+} \frac{(2+h)^2 \cdot \left| \cos \left(\frac{\pi}{2+h} \right) \right| - 0}{h} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0^+} \frac{(2+h)^2 \cdot \cos\left(\frac{\pi}{2+h}\right)}{h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2+h)^2 \cdot \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)}{h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2+h)^2 \cdot \sin\left(\frac{\pi h}{2(2+h)}\right)}{\left(\frac{\pi}{2(2+h)}\right)h} \cdot \frac{\pi}{2(2+h)} \\
 &= (2)^2 \cdot \frac{\pi}{2(2)} = \pi
 \end{aligned}$$

LHD

$$\begin{aligned}
 &= \lim_{h \rightarrow 0^+} \frac{f(2-h) - f(2)}{-h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2-h)^2 \cdot \cos\left(\frac{\pi}{2-h}\right) - 0}{-h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2-h)^2 \cdot \left(-\cos\left(\frac{\pi}{2-h}\right)\right) - 0}{-h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2-h)^2 \cos\left(\frac{\pi}{2-h}\right)}{h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2-h)^2 \cdot \sin\left(\frac{\pi}{2} - \frac{\pi}{2-h}\right)}{h} \\
 &= \lim_{h \rightarrow 0^+} \frac{(2-h)^2 \cdot \sin\left(-\frac{\pi h}{2(2-h)}\right)}{\left(-\frac{\pi h}{2(2-h)}\right)} \cdot \frac{-\pi}{2(2-h)}
 \end{aligned}$$

$$= -\pi$$

So $f(x)$ is not derivable at $x = 2$

45. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

- (A) 75 (B) 150 (C) 210 (D) 243

Sol. Ans (B)

	B_1	B_2	B_3
Case-1:	1	1	3
Case-2:	2	2	1

$$\begin{aligned} \text{Ways of distribution} &= \frac{5!}{1!1!3!2!} \cdot 3! + \frac{5!}{2!2!1!2!} \cdot 3! \\ &= 150 \end{aligned}$$

46. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then

- (A) $a = 1, b = 4$ (B) $a = 1, b = -4$
 (C) $a = 2, b = -3$ (D) $a = 2, b = 3$

Sol. Ans (B)

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$$

$$\lim_{x \rightarrow \infty} \left(\frac{x^2(1-a) + x(1-a-b) + (1-b)}{x+1} \right) = 4$$

Limit is finite

It exists when $1 - a = 0 \Rightarrow a = 1$

then $\lim_{x \rightarrow \infty} \left(\frac{1 - a - b + \frac{1-b}{x}}{1 + \frac{1}{x}} \right) = 4$

$\therefore 1 - a - b = 4 \Rightarrow b = -4$

47. The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 1x^2 + 36x + 1$, is

- (A) one-one and onto (B) onto but not one-one
 (C) one-one but not onto (D) neither one-one nor onto

Sol. Ans (B)

$F : [0, 3] \rightarrow [1, 29]$

$f(x) = 2x^3 - 1x^2 + 36x + 1$

$f'(x) = 6x^2 - 2x + 36$

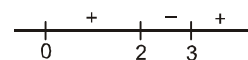
$= 6(x^2 - \frac{1}{3}x + 6)$

$= 6(x - 2) (x + 3)$

in given domain function has local maxima, it is many-one

Now at $x = 0 \quad f(0) = 1$

$x = 2 \quad f(2) = 16 - 4 + 72 + 1 = 85$



$$x = 3 \quad f(3) = 54 - 135 + 108 + 1 = 163 - 135 = 28$$

Has range = [1, 29]
Hence given function is onto

48. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is
- (A) $20(x^2 + y^2) - 36x + 45y = 0$
 - (B) $20(x^2 + y^2) + 36x - 45y = 0$
 - (C) $36(x^2 + y^2) - 20x + 45y = 0$
 - (D) $36(x^2 + y^2) + 20x - 45y = 0$

Sol. **Ans** (A)

Circle $x^2 + y^2 = 9$
line $4x - 5y = 20$

$$P \left(t, \frac{4t - 20}{5} \right)$$

equation of chord AB whose mid point is M (h, k)

$$T = S_1$$

$$\therefore hx + ky = h^2 + k^2 \quad \dots\dots(1)$$

equation of chord of contact AB with respect to P.

$$T = 0$$

$$tx + \left(\frac{4t - 20}{5} \right) y = 9 \quad \dots\dots(2)$$

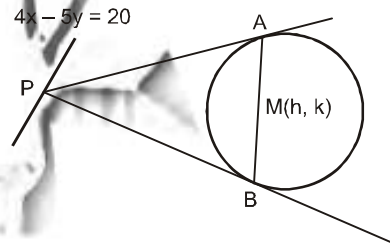
comparing equation (1) and (2)

$$\frac{h}{t} = \frac{5k}{4t - 20} = \frac{h^2 + k^2}{9}$$

on solving

$$45k = 36h - \frac{h^2 + k^2}{9}$$

$$\Rightarrow \text{Locus is } 20(x^2 + y^2) - 36x + 45y = 0$$



49. Let $P = [a_{ij}]$ be a 3×3 matrix, and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is
- (A) 2^{10}
 - (B) 2^{11}
 - (C) 2^{12}
 - (D) 2^{13}

Sol. **Ans** (D)

Given $P = [a_{ij}]_{3 \times 3}$ $b_{ij} = 2^{i+j} a_{ij}$

$$Q = [b_{ij}]_{3 \times 3}$$

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad |P| = 2$$

$$Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{bmatrix}$$

$$\text{Determinant of } Q = \begin{vmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{vmatrix}$$

$$= 4 \times 8 \times 16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{vmatrix}$$

$$= 4 \times 8 \times 16 \times 4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^2 \cdot 2^3 \cdot 2^4 \cdot 2^1 \cdot 2^2 \cdot 2^1$$

$$= 2^{13}$$

50. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes.

Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R.. The eccentricity of the ellipse E_2 is

- (A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Sol. **Ans (C)**

Let required ellipse is

$$E_2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

It passes through $(0, 4)$

$$0 + \frac{16}{b^2} = 1 \quad \Rightarrow \quad b^2 = 16$$

It also passes through $(2, 2)$

$$\frac{9}{a^2} + \frac{4}{b^2} = 1$$

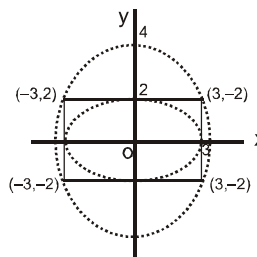
$$\frac{9}{a^2} + \frac{1}{4} = 1$$

$$\frac{9}{a^2} = \frac{3}{4} \quad \Rightarrow \quad a^2 = b^2 (1 - e^2)$$

$$\frac{12}{16} = 1 - e^2$$

$$e^2 = 1 - \frac{12}{16} = \frac{4}{16} = \frac{1}{4}$$

$$e = \frac{1}{2}$$



Section II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** are correct.

51. If $y(x)$ satisfies the differential equation $y' - y \tan x = x \sec x$, and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

(B) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$

(D) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

Sol. Ans (AD)

$$\frac{dy}{dx} - y \tan x = x \sec x,$$

$$y(0) = 0$$

$$\text{I.F.} = e^{-\int \tan x \, dx} = e^{-\log \sec x}$$

$$\text{I.F.} = \cos x$$

$$\cos x \cdot y = \int 2x \sec x \cdot \cos x \, dx$$

$$\cos x \cdot y = x^2 + c$$

$$c = 0$$

$$y = x^2 \sec x$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{16} \cdot \sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$$

$$y'\left(\frac{\pi}{4}\right) = \frac{\pi}{2} \cdot \sqrt{2} + \frac{\pi^2}{16} \cdot \sqrt{2}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} \cdot 2 = \frac{2\pi^2}{9}$$

$$y'\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{2} \cdot 2 + \frac{\pi^2}{9} \cdot 2 \cdot \sqrt{3}$$

$$\frac{4\pi}{3} + \frac{2\pi^2\sqrt{3}}{9}$$

52. A ship is fitted with three engines E_1 , E_2 and E_3 . The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1 , X_2 and X_3 denotes respectively the events that the engines E_1 , E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P[X_1^c | X] = \frac{3}{16}$

(B) $P[\text{Exactly two engines of the ship are functioning} | X] = \frac{7}{8}$

(C) $P[X | X_2] = \frac{5}{16}$

(D) $P[X | X_1] = \frac{7}{16}$

Sol. **Ans (BD)**

$$P(x_1) = \frac{1}{2}$$

$$P(x_2) = \frac{1}{4}$$

$$P(x_3) = \frac{1}{4}$$

$$P(x) = P(E_1 E_2 E_3) + P(\bar{E}_1 E_2 E_3) + P(E_1 \bar{E}_2 E_3) + P(E_1 E_2 \bar{E}_3)$$

$$= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}$$

$$P(x) = \frac{1}{4}$$

(A) $P\left(\frac{x_1^c}{x}\right) = \frac{P(x_1^c \cap x)}{P(x)}$

$$= \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4}}{\frac{1}{4}} = \frac{1}{8}$$

(B) $P(\text{exactly two} / x) = \frac{P(\text{exactly two} \cap x)}{P(x)} = \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}}{\frac{1}{4}} = \frac{7}{8}$

$$(C) P(x / x_2) = \frac{P(x \cap x_2)}{P(x_2)} = \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}}{\frac{1}{4}} = \frac{5}{8}$$

$$(D) P(x / x_1) = \frac{P(x \cap x_1)}{P(x_1)} = \frac{\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}}{\frac{1}{2}} = \frac{7}{16}$$

53. Let $\theta, \phi \in [0, 2\pi]$ be such that $2\cos\theta(1 - \sin\phi) = \sin^2\theta \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2} \right) \cos\phi - 1$, $\tan(\pi - \theta) > 0$ and

$-1 < \sin\theta < -\frac{\sqrt{3}}{2}$. Then ϕ cannot satisfy

- (A) $0 < \phi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \phi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \phi < 2\pi$

Sol. Ans (ACD)

As $\tan(2\pi - \theta) > 0$, $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, $\theta \in [0, 2\pi]$

$$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2} \right) \cos\phi - 1$

$$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta \cos\phi - 1$$

$$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$$

$$\text{As } \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3} \right) \Rightarrow 2\cos\theta + 1 \in (1, 2)$$

$$\Rightarrow 1 < 2\sin(\theta + \phi) < 2$$

$$\Rightarrow \frac{1}{2} < \sin(\theta + \phi) < 1$$

As $\theta + \phi \in [0, 4\pi]$

$$\Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6} \right) \text{ or } \theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6} \right)$$

$$\Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta \text{ or } \frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta$$

$$\Rightarrow \phi \in \left(-\frac{3\pi}{2}, -\frac{2\pi}{3} \right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6} \right)$$

$$\left(\because \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3} \right) \right)$$

54. If S be the area of the region enclosed by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

- (A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$ (C) $S \leq \frac{1}{4} \left(1 + \frac{1}{\sqrt{e}} \right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}} \left(1 - \frac{1}{\sqrt{2}} \right)$

Sol. Ans (ABD)

$$I = \int_0^1 e^{-x^2} dx$$

$$-x^2 \leq 0$$

$$e^{-x^2} \leq 1$$

$$\int_0^1 e^{-x^2} dx \leq 1$$

$$x^2 \leq x \Rightarrow -x^2 \geq -x, \Rightarrow e^{-x^2} \geq e^{-x}$$

$$\Rightarrow I \geq \int_0^1 e^{-x} dx$$

$$\geq -(e^{-x})_0^1$$

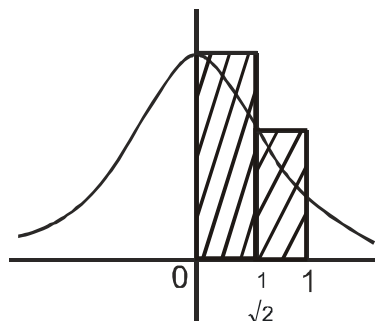
$$\geq -\left(\frac{1}{e} - 1\right)$$

$$I \geq 1 - \frac{1}{e} \Rightarrow \text{(B) is correct}$$

$$\text{Since if } I \geq 1 - \frac{1}{e} \Rightarrow I > \frac{1}{e} \Rightarrow \text{(A) is correct}$$

$$I < \frac{1}{\sqrt{2}} \times 1 \Rightarrow \frac{1}{\sqrt{e}} \times 1 < 1 - \frac{1}{\sqrt{2}}$$

So Ans. D



55. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, parallel to the straight line $2x - y = 1$. The points of contacts of the tangents on the hyperbola are

- (A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $(3\sqrt{3}, -2\sqrt{2})$ (D) $(-3\sqrt{3}, 2\sqrt{2})$

Sol. Ans (AB)

Slope of tangents = 2

$$\text{Equation of tangents } y = 2x \pm \sqrt{9 \cdot 4 - 4}$$

$$\Rightarrow y = 2x \pm \sqrt{32}$$

$$\Rightarrow 2x - y \pm 4\sqrt{2} = 0 \quad \dots(i)$$

Let point of contact be (x_1, y_1)

then equation (i) will be identical to the equation

$$\frac{xx_1}{9} - \frac{yy_1}{4} - 1 = 0$$

$$\therefore \frac{x_1/9}{2} = \frac{y_1/4}{1} = \frac{-1}{\pm 4\sqrt{2}}$$

$$\Rightarrow (x_1, y_1) = \left(-\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \text{ and } \left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

Section III : Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

- 56.** Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 4y = 0$ and the given parabola. The area of the triangle SPQ is.

Sol. Ans (4)

Focus is $(a, 0) = (2, 0)$

$P = (0, 0)$

$$A = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 2 & 0 & 1 \\ \frac{\alpha^2}{8} & \alpha & 1 \end{vmatrix}$$

$$= \frac{1}{2} (2\alpha) = \alpha \text{ we need y coordinate of Q.}$$

$(2t^2, 4t)$ satisfies circle

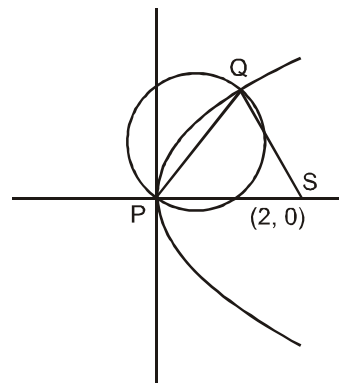
$$4t^4 + 16t^2 - 4t^2 - 16t = 0$$

$$t^3 + 3t - 4 = 0$$

$$(t - 1)(t^2 + t + 4) = 0$$

$$t = 1$$

so Ans. 4



57. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ $p(3) = 2$, then $p'(0)$ is

Sol. **Ans (9)**

$$p' = \lambda(x - 1)(x - 3) = \lambda(x^2 - 4x + 3)$$

$$p(x) = \lambda(x^3/3 - 2x^2 + 3x) + \mu$$

$$p(1) = 6$$

$$6 = \lambda(1/3 - 2 + 3) + \mu$$

$$6 = \lambda(1/3 + 1) + \mu$$

$$18 = 4\lambda + 3\mu \quad \dots(i)$$

$$p(3) = 2$$

$$2 = \lambda(27/3 - 2 \cdot 9 + 9) + \mu$$

$$2 = \mu$$

$$\mu = 2 \Rightarrow \lambda = 3$$

$$p'(x) = 3(x - 1)(x - 3)$$

$$p'(0) = 3(-1)(-3)$$

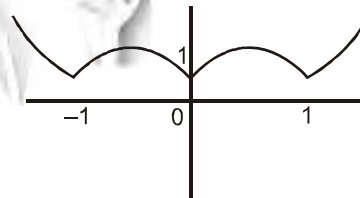
$$= 9$$

58. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Sol. **Ans (5)**

$$f(x) = |x| + |x^2 - 1|$$

$$f(x) = \begin{cases} -x + x^2 - 1 & x < -1 \\ -x - x^2 + 1 & -1 \leq x \leq 0 \\ x - x^2 + 1 & 0 < x < 1 \\ x + x^2 - 1 & x \geq 1 \end{cases}$$



$$f(x) = \begin{cases} x^2 - x - 1 & x < -1 \\ -x^2 - x + 1 & -1 \leq x \leq 0 \\ -x^2 + x + 1 & 0 < x < 1 \\ x^2 + x - 1 & x \geq 1 \end{cases}$$

59. The value of $6 + \log_3 \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots \right)$ is

Sol. **Ans (4)**

$$\text{Let } \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots = t$$

$$\sqrt{4 - \frac{1}{3\sqrt{2}}} t = t$$

$$4 - \frac{1}{3\sqrt{2}}t = t^2 \Rightarrow$$

$$t^2 + \frac{1}{3\sqrt{2}}t - 4 = 0 \Rightarrow 3\sqrt{2}t^2 + t - 12\sqrt{2} = 0$$

$$t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$$

$$t = \frac{16}{6\sqrt{2}}, \frac{-18}{6\sqrt{2}}$$

$$t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}} \text{ and } \frac{-3}{\sqrt{2}} \text{ is rejected}$$

$$\text{so } 6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}} \right) = 6 + \log_{3/2} \left(\frac{4}{9} \right) = 6 + \log_{3/2} \left(\left(\frac{2}{3} \right)^2 \right) = 6 - 2 = 4$$

60. If \vec{a} , \vec{b} and \vec{c} are unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|2\vec{a} + 5\vec{b} + 5\vec{c}|$ is

Sol. **Ans (3)**

$$6 - 2\vec{a}\vec{b} - 2\vec{b}\vec{c} - 2\vec{c}\vec{a} = 9$$

$$(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) = \frac{-3}{2}$$

$$|\vec{a} + \vec{b} + \vec{c}|^2 \geq 0$$

$$3 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \geq 0$$

$$\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a} \geq \frac{-3}{2}$$

$$\text{Since } \vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a} = \frac{-3}{2}$$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = 0 \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$$

$$\Rightarrow |2\vec{a} + 5(-\vec{a})| = |3\vec{a}| \Rightarrow 3$$