BITSAT

Birla Institute of Science and Technology, Pilani

Solved Paper 2013

Instructions

1. There are 150 questions in all. The number of questions in each part is as follows

Subject	No. of Questions
Part I (Physics)	1-40
Part II (Chemistry)	41-80
Part III	
(a) English Proficiency	81-95
(b) Logical Reasoning	96-105
Part IV (Mathematics)	106-150

2. All questions are multiple choice questions with four options, only one being correct.

3. Each correct answer fetches 3 marks while incorrect answer fetches 1 mark.

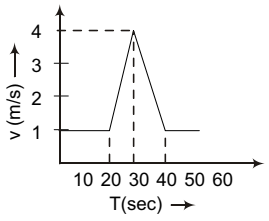
Part-1

Physics

1. In the relation $p = \frac{\alpha}{\beta} e^{-\frac{\alpha z}{k\theta}}$, p is the pressure, z the distance, k is Boltzmann constant and θ is the temperature, the dimensional formula of β will be

(a) $[M^0L^2T^0]$ (b) $[M\,L^2T^0]$ (c) $[M\,L^0T^{-1}]$ (d) $[M\,L^2T^{-1}]$

2. Velocity-time (v-t) graph for a moving objects is shown in the figure. Total displacement of the object during the time intervals when there is non-zero acceleration, is



(a) 60 m
(b) 50 m
(c) 30 m
(d) 40 m

3. Three weights w, $2w$ and $3w$ are connected to identical spring suspended from a rigid horizontal rod. The assembly of the rod and the weights falls freely. The positions of the weight from the rod are such that

(a) 3w will be farthest
(b) w will be farthest
(c) all will be at the same distance
(d) 2 w will be farthest

4. At the top of the trajectory of a projectile, the direction of its velocity and acceleration are

(a) perpendicular to each other
(b) parallel to each other
(c) inclined to each other at an angle of 45°
(d) ant-iparallel to each other

5. Consider the following statement, when jumping from some height, you should bend your knees as you come to rest instead of keeping your legs stiff. Which of the following relations can be useful in explaining the statement

(a) $D_1 = -D_2$
(b) $\Delta E = -\Delta(\text{PE} + \text{KE}) = 0$
(c) $Dt = m$
(d) $\Delta \propto \Delta F$

where symbols have their usual meanings.

6. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 9 (c) 1 : 3 : 5 (d) 1 : 5 : 3

7. Two rings of radius R and nR made of same material have the ratio of moment of inertia about an axis passing through centre is 1:8. The value of n is.

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $\frac{1}{2}$

8. There are two planets. The ratio of radius of the two planets is K but ratio of acceleration due to gravity of both planets is g. What will be the ratio of their escape velocity?

(a) $(Kg)^{1/2}$ (b) $(Kg)^{-1/2}$ (c) $(Kg)^2$ (d) $(Kg)^{-2}$

9. The extension in a string obeying Hooke's law v is x. The speed of sound in the stretched string is v. If the extension in the string is increased to 1.5 x, the speed of sound will be

(a) 1.22 v (b) 0.61 v (c) 1.50 v (d) 0.75 v

10. A ball whose density is 0.4×10^3 kg/m^3 falls into water from a height of 9 cm. To what depth does the ball sink?

(a) 9 cm (b) 6 cm
(c) 4.5 cm (d) 2.25 cm

11. A thermodynamical systems is changed from state (p_1, V_1) to (p_2, V_2) by two different process, the quantity which will remain same will be

(a) ΔQ (b) ΔW
(c) $\Delta Q + \Delta W$ (d) $\Delta Q - \Delta W$

12. The relative humidity on a day when partial pressure of water vapour is 0.012×10^5 Pa at 12°C is (Take vapour pressure of water at this temperature as 0.016×10^5 Pa)

(a) 70% (b) 40% (c) 75% (d) 25%

13. In the absence of intermolecular forces of attraction, the observed pressure P will be

(a) p (b) $< p$ (c) $> p$ (d) zero

14. In a second pendulum, mass of bob is 30g . If it is replaced by 90g mass, then its time period will be

(a) 1 s (b) 2 s
(c) 4 s (d) 3 s

15. A wave has velocity v in medium P and velocity $2v$ in medium Q. If the wave is incident in medium P at an angle of 30°, then the angle of refraction will be

(a) 30° (b) 45° (c) 60° (d) 90°

16. The equation of progressive wave is $y = 0.2 \sin 2\pi\left[\frac{t}{0.01} - \frac{x}{0.}\right]$, where x and y are in metre and t is in second. The velocity of propagation of the wave is

(a) 30 m/s (b) 40 m/s
(c) 300 m/s (d) 400 m/s

17. The displacement of a charge Q in the electric field is $\mathbf{E} = e_1\hat{\mathbf{i}} + e_2\hat{\mathbf{j}} + e_3\hat{\mathbf{k}}$ is $r = a\,\hat{\mathbf{i}} + b\hat{\mathbf{j}}$. The work done is

(a) $Q\,(ae_1 + be_2)$ (b) $Q\sqrt{(ae_1)^2 + (be_2)^2}$
(c) $Q\,(e_1 + e_2)\sqrt{a^2 + b^2}$ (d) $Q\,(\sqrt{e_1^2 + e_2^2})(a + b)$

18. An electric line of force in the xy-plane is given by equation $x^2 + y^2 = 1$. A particle with unit positive charge, initially at rest at the point $x = 1$, $y = 0$ in the xy-plane

(a) not move at all
(b) will move along straight line
(c) will move along the circular line of force
(d) information is insufficient to draw any conclusion

19. If a rod has resistance 4 Ω and if rod is turned as half circle, then the resistance along diameter is

(a) 1.56 Ω (b) 2.44 Ω (c) 4 Ω (d) 2 Ω

20. The relation between voltage sensitivity (σ_v) and current sensitivity (σ_i) of a moving coil galvanometer is (resistance of galvanometer is G)

(a) $\frac{\sigma_i}{G} = \sigma_v$ (b) $\frac{\sigma_v}{G} = \sigma_i$ (c) $\frac{G}{\sigma_v} = \sigma_i$ (d) $\frac{G}{\sigma_i} = \sigma_v$

21. A current carrying small loop behaves like a small magnet. If A be its area and M its magnetic moment, the current in the loop will be

(a) M/A (b) A/M (c) MA (d) Am^2

22. A magnet of magnetic moment 20 CGS units is freely suspended in a uniform magnetic field of intensity 0.3 CGS units. The moment of work done in deflecting it by an angle of 30° in CGS units is

(a) 6 (b) $3\sqrt{3}$ (c) $3(2-\sqrt{3})$ (d) 3

23. An inductor of 2H and a resistance of 10 Ω are connected in series with a battery of 5 V. The initial rate of change of current is

(a) 0.5 A/s (b) 2.0 A/s (c) 2.5 A/s (d) 0.25 A/s

24. When radiation is incident on a photoelectron emitter, the stopping potential is found to be 9V. If e/m for the electron is 1.8×10^{11} Ckg^{-1}

(a) 6×10^5 ms^{-1} (b) 8×10^5 ms^{-1}
(c) 1.8×10^6 ms^{-1} (d) 1.8×10^5 ms^{-1}

25. A and B are two radioactive substances whose half-lives are 1 and 2 years respectively. Initially 10 g of A and 1 g of B is taken. The time (approximate) after which they will have same quantity remaining is

(a) 6.62 year (b) 5 year (c) 3.2 year (d) 7 year

26. The optical path of a monochromatic light is same if it goes through 4.0 cm of glass of 4.5 cm of water. If the refractive index of glass is 1.53, the refractive index of the water is

(a) 1.30 (b) 1.36 (c) 1.42 (d) 1.46

27. The length, breadth and thickness of a block are given by $l=12$ cm, $b=6$ cm and $t=2.45$ cm. The volume of the block according to the idea of significant figure should be

(a) 2×10^2
(b) 2×10^2 cm^3
(c) 1.763×10^2 cm
(d) None of these

28. 10000 small balls, each weighing 1g, strike one square centimetre of area per second with a velocity 100 m/s in a normal directions and rebound with the same velocity. The value of pressure on the surface will be

(a) 2×10^3 N/m^2
(b) 2×10^5 N/m^2
(c) 10^7 N/m^2
(d) 2×10^7 N / m^2

29. Two springs have their force constant as k_1 and k_2 $(k_1 > k_2)$ when they are stretched by same force

(a) no work is done in case of both the springs
(b) equal work is done in case of both the springs
(c) more work is done in case of second spring
(d) more work is done in case of first spring

30. A mass m is moving with a constant velocity along a line parallel to x-axis. Its angular momentum with respect to origin on z-axis is

(a) zero
(b) remain constant
(c) goes on increasing
(d) goes on decreasing

31. At a given place where acceleration due to gravity is g m/s^2, a sphere of lead of density d kg/m^3 is gently released in a column of liquid of density ρ kg/m^3. If $d > \rho$, the sphere will

(a) fall vertically with an acceleration g m/s^2
(b) fall vertically with no acceleration
(c) fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$
(d) fall vertically with an acceleration $g\left(\frac{\rho}{d}\right)$

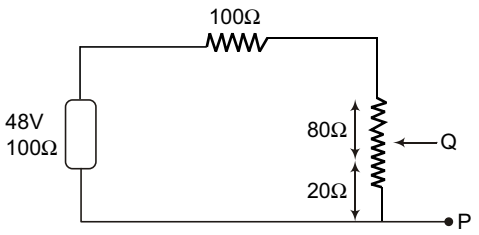
32. Amplitude of a wave is represented by $A=\frac{c}{a+b-c}$. Then resistance will occur when

(a) $b = -c/2$
(b) $b = 0$ and $a = c$
(c) $b = -a/2$
(d) None of the above

33. Capacitance of a capacitor made by a thin metal foil is 2μF. If the foil is folded with paper of thickness 0.15 mm, dielectric constant of paper is 2.5 and width of paper is 400 mm, the length of foil will be

(a) 0.34 m (b) 1.33 m
(c) 13.4 m (d) 33.9 m

34. In the circuit, the potential difference across PQ will be nearest to



(a) 9.6 V (b) 6.6 V (c) 4.8 V (d) 3.2 V

35. A rod of a certain metal is 1.0 m long and 0.6 cm in diameter. Its resistance is $3.0 \times 10^{-3}\,\Omega$. Another disc made of the same metal is 2.0 cm in diameter and 1.0 mm thick. What is the resistance between the round faces of the disc?

(a) $1.35 \times 10^{-8}\ \Omega$ (b) $2.70 \times 10^{-7}\Omega$
(c) $4.05 \times 10^{-6}\ \Omega$ (d) $8.10 \times 10^{-5}\ \Omega$

36. The cyclotron frequency of an electron grating in a magnetic field of 1 T is approximately

(a) 28 MHz (b) 280 MHz
(c) 2.8 GHz (d) 28 GHz

37. The transformation ratio in the step-up transformer is

(a) 1
(b) greater than one
(c) less than one
(d) the ratio greater or less than one depends on the other factors

38. Radiations of intensity 0.5 W/m^2 are striking a metal plate. The pressure on the plate is

(a) 0.166×10^{-8} N/m^2
(b) 0.332×10^{-8} N/m^2
(c) 0.111×10^{-8} N/m^2
(d) 0.083×10^{-8} N/m^2

39. If n represents the order of a half period zone the area of this zone is approximately proportional to n^m where m is equal to

(a) zero (b) half
(c) one (d) two

40. Monochromatic light of wavelength 3000$\mathring{A}$ is incident on a surface area 4 cm^2. If intensity of light is 150 mW/m^2, then rate at which photons strike the target is

(a) 3×10^{10} /s (b) 9×10^{13} /s
(c) 7×10^{15} /s (d) 6×10^{19} /s

Chemistry

41. The ratio of Fe_2O_3 and Al, in thermite is

(a) 1 : 3
(b) 1 : 2
(c) 3 : 1
(d) None of the above

42. A solid has a structure in which 'W' atom are located at the corners of a cubic lattice 'O' atom at the centre of edge and Na atoms at the centre of a cube. The formula for the compound is

(a) Na_2WO_3 (b) Na_2WO_2
(c) $NaWO_2$ (d) $NaWO_3$

43. Which one of the following substances is used in the laboratory for a fast drying of neutral gases?

(a) Phosphorus pentoxide
(b) Active charcol
(c) Anhydrous calcium chloride
(d) Na_3PO_4

44. H_2O_2 used in rocket has the concentration

(a) 50% (b) 70% (c) 30% (d) 90%

45. The IUPAC name of the compound

$$\begin{array}{l} CH_2\text{—}CH\text{—}COOH \\ \quad\ |\quad\ \ | \\ \quad OH\quad NH_2 \end{array}$$

(a) 2-amino-3-hydroxy propanoic acid
(b) 1-hydroxy-2-amino propan-3-oic acid
(c) 1-amino-2-hydroxypropanoic acid
(d) 3-hydroxy-2-amino propanoic acid

46. The compound which gives the most stable carbonium ion on dehydration is

(a) $CH_3CH(CH_3)CH_2OH$
(b) $(CH_3)_3COH$
(c) $CH_2 = CH - CH_2CH_2OH$
(d) $CH_3CHOHCH_2$

47. The ionic conductance is least for

(a) Cs^+ (b) Rb^+ (c) K^+ (d) Na^+

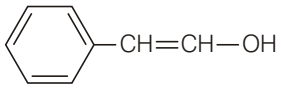
48. Setting of plaster of Paris involves
(a) oxidisation with atmospheric oxygen
(b) combination with atmospheric CO_2
(c) dehydration
(d) hydration to yield another hydrate

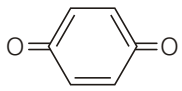
49. A solution of sucrose (molar mass = 342 g/mol) is prepared by dissolving 68.4 g of it per litre of solution, what is its osmotic pressure (R = 0.082 L atom $K^{-1}mol^{-1}$) at 273 K?
(a) 3.92 atm (b) 4.48 atm
(c) 5.92 atm (d) 29.4 atm

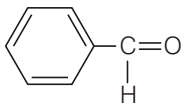
50. A 27°C one mole of an ideal gas is compressed isothermally and reversible from a pressure of 2 atm to 10 atm. The value of and ΔE and q are (R = 2 cal)
(a) 0, – 965.84 cal
(b) –965.84 cal, – 865.58 cal
(c) + 865.58 cal, – 865.58 cal
(d) + 965.84 cal, + 865.58 cal

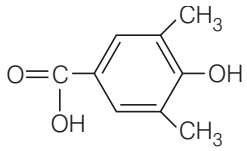
51. For a reaction equilibrium, $Na_2O_4(g) \rightleftharpoons 2NO_2(g)$, the concentration of $N_2O_4(g)$ and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol/L respectively. The value of k_c for the reaction is
(a) 3×10^{-3} mol/L (b) 3.3×10^{-3} mol/L
(c) 3×10^{-1} mol/L (d) 3.3×10^{-1} mol/L

52. Tautomerism is exhibited

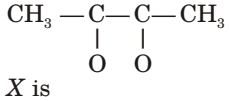
(a) 

(b) 

(c) 

(d) 

53. $CH_3 - C \equiv C - CH_3 \xrightarrow[(ii)\ Zn/H_2O]{(i)\ X}$



in the above reaction X is
(a) HNO_3 (b) O_2
(c) O_3 (d) $KMnO_4$

54. $C_7H_8 \xrightarrow{3\,Cl_2,\ heat} A \xrightarrow{Fe/Br} B \xrightarrow{Zn/Hcl} C$
Here, the compound C is
(a) 3-bromo 2, 4 = 6-trichlorotoluene
(b) *O*– bromotoluene
(c) *P*–bromotoluene
(d) *m*–bromotoluene

55. Alizarin belongs to the class of
(a) vat dyes
(b) mordant dyes
(c) basic dyes
(d) reactive dyes

56. 2,4-dichlorophenoxyacetic acid is used as
(a) fungicide (b) insecticide
(c) herbicide (d) moth repellant

57. Which glass has the highest percentage of lead
(a) soda glass (b) flint glass
(c) jena glass (d) pyrex glass

58. Which one of the following pentafluorides cannot be formed?
(a) PF_5 (b) AsF_5
(c) SbF_5 (d) BiF_5

59. Which out of the following compounds is called photographer's fixer?
(a) Na_2SO_3 (b) $Na_2S_2O_3.5H_2O$
(c) Na_2SO_4 (d) Na_2S

60. The isoelectronic pair is
(a) Cl_2O, ICl_2^-
(b) Cl_2^-O, ClO_2
(c) IF_2^+, I_2^-
(d) ClO_2^-, ClF_2^+

61. When radioactive minerals like clevelte, monazite and pitchblende are heated to 1273 K in vacuo, the noble gas obtained is
(a) Rn (b) Kr (c) He (d) Ne

62. Conjugate base of $H_2PO_4^-$ is
(a) H_3PO_4 (b) P_2O_5 (c) PO_4^{3-} (d) HPO_4^{2-}

63. Given standard electrode potential

$Fe^{2+} + 2e^- \rightarrow Fe; E^0 = -0.440V$

$Fe^{3+} + 3e^- \rightarrow Fe; E^0 = -0.036\ V$

Calculate the electrode potential of cell

(a) + 0.772 V (b) – 0.772 V
(c) + 0.569 V (d) – 0.569 V

64. For the reaction,

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

The rate of change of concentration for hydrogen is $0.3 \times 10^{-4}\ Ms^{-1}$

The rate of change of concentration ammonia is

(a) -0.2×10^{-4} (b) 0.2×10^{-4}
(c) 1×10^{-4} (d) 0.3×10^{-4}

65. The root mean square velocity of a gas is double when temperature is

(a) increased four times
(b) increased two times
(c) reduced to half
(d) reduced to one-fourth

66. The specific conductivity of 0.1 N KCl solution is 0.0129 ohm^{-1} cm^{-1}. The resistance of the solution in the cell is 100Ω. The cell constant of the cell will be

(a) 1.10 (b) 1.29
(c) 0.56 (d) 2.80

67. Which of the following most volatile compounds?

(a) HI (b) HCl
(c) HBr (d) HF

68. Which of the following transition metal ions will have definite value of magnetic moment?

(a) Sc^{3+} (b) Ti^{3+} (c) Cu^+ (d) Zn^{2+}

69. Cr has electronic configuration as

(a) $3s^2 3p^6 3d^4 4s^1$ (b) $3s^2 3p^6 d^5 4s^1$
(c) $3s^2 3p^6 3d^6$ (d) None of the above

70. Which of the following compound is expected to be coloured?

(a) Ag_2SO_4 (b) CuF_2
(c) MgF_2 (d) CuCl

71. The effective atomic number of Cr (atomic number = 24) in $[Cr(NH_3)_6]Cl_3$ is

(a) 35 (b) 27 (c) 33 (d) 36

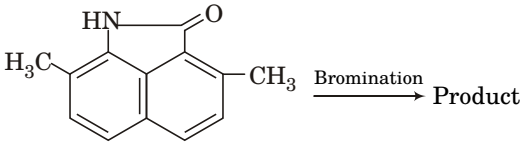
72. In Nessler's reagent for the direction of ammonia the active species is

(a) Hg_2Cl_2
(b) Mg^{2+}
(c) Hg_2I_2
(d) HgI_4^{2-}

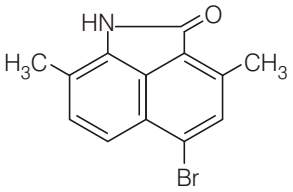
73. Which of the following ketones will not respond to iodoform test?

(a) Methyl isopropyl ketone
(b) Ethyl isopropyl ketone
(c) Dimethyl ketone
(d) 2-hexanone

74.



(a)



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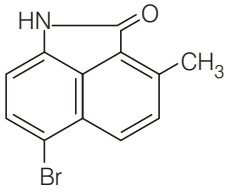

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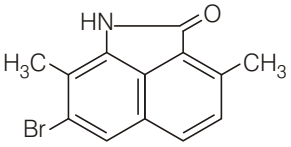

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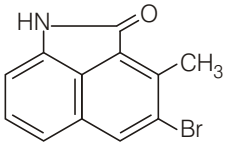
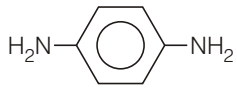
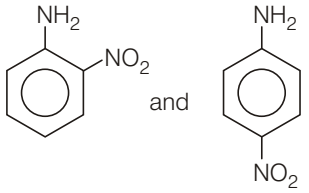
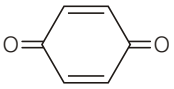
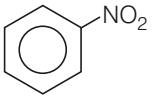


75. Aniline reacts with conc HNO_3 to give

(a) 

(b) 

(c) 

(d) 

76. Bakelite is a product of the reaction between

(a) formaldehyde and NaOH
(b) aniline and urea
(c) phenol and methanal
(d) phenol and chloroform

77. Cellulose is a polymer of

(a) glucose (b) fructose
(c) ribose (d) sucrose

78. Iodine value related to

(a) fats and oils (b) alcohols
(c) esters (d) hydrocarbon

79. In aqueous solution, amino acids mostly exit as

(a) $H_2N — CHR — COOH$
(b) $H_2N — CHR — COO^-$
(c) $H_3N — CHR — COOH$
(d) $H_3NCHR — COO^-$

80. Gibbs' free energy G, enthalpy H and entropy S are interrelated as in

(a) $G = H + TS$ (b) $G = H – TS$
(c) $G – TS = H$ (d) $G = S = H$

English Proficiency

Directions from (81 to 83) : Out of the four alternative choose one which expresses the right meaning of the given word :

81. Disherelled

(a) peculiar (b) fierce
(c) foolish (d) untidy

82. Will supersede the old law.

(a) enforce
(b) continue with respect from
(c) substitute
(d) repeat

83. Lenient

(a) tall (b) wise
(c) severe (d) not strict

Directions (84 to 86) Choose the word opposite in meaning to the given word :

84. Enormous

(a) soft (b) average (c) tiny (d) weak

85. EXODUS

(a) influx (b) home-coming
(c) return (d) restoration

86. Relinquish

(a) convert (b) condense
(c) longest (d) possess

Direction (87 to 89) A part of the sentence is underlined below are given alternatives to the underlined part at a, b and c which may improve the sentence. Choose the correct alternate. In case no improvement is needed, your answer is (d).

87. Each one of the students in Ms. Smith's art class must supply <u>their own</u> paint brushes paints, and water colour paper.

(a) his or her own.
(b) his own
(c) to each their own
(d) no change

88. In the past, hurricanes were given <u>women's names, how they</u> are given names of both men and women.

(a) women's names, how they are
(b) womens' names, how they are
(c) women's names; how they are
(d) no improvement

89. If the tribesmen did not dance, the spirits whom attended the feast would be angry and the animals, necessary for food and warmth, would stay away.

(a) who attended (b) whom have attended
(c) with whom attended (d) no improvement

Direction (90 to 102) : Sentences are given with blanks to be filled in with an appropriate and suitable word. Four alternatives are suggested for each question. Choose the correct alternative out of the four.

90. It sounds ____ that if a low carb diet can prevent a second heart attack from occuring, it should also be able to ____ the first occurrence.

(a) realistic _____ disintegrate
(b) logical _____ allay.
(c) fascinating _____ foment
(d) rational _____ dlienate

91. Rajeev failed in the examination because none of his answer were _____ to the questions asked.

(a) abusive
(b) revealing
(c) pestiment
(d) referential

92. There are _____ views on the issue of the giving bonus to the employees.

(a) independent
(b) divergent
(c) modest
(d) adverse

Directions (93 to 95) : Arrange the following sentences in correct pattern and mark at the correct combination.

93. 1. Passivity is not, of course, universal

2. In areas where there are no laws, or in frontier zones where all men go armed, the attitude of the peasantry may will be different.

3. to indeed it may be on the fringe of the unsubmissive.

4. However, for most of the soil bound peasants the problem is not whether to be normally passive or active, but when to pass from one state to another.

5. This depends on an assessment of the political situation.

(a) 54132
(b) 34125
(c) 54213
(d) 12345

94. 1. Then two astronomers-the German, Johannes Kepler, and the Italian, Galieo, Galiliei-started publicly to support the Copernicus theory; despite the fact. that the orbits it predicted did not quite match the ones observed.

2. This idea was that the sun was stationary at the centre and that the earth and the planets move in circular orbits around the sun.

3. A simple model was proposed in 1514 by a polish priest, Nicholas Copernicus.

4. Nearly a century passed before this idea was taken seriously.

(a) 3421
(b) 3241
(c) 2314
(d) 3142

95. (1) If you are used to having your stimulations come in from outside, your mind never developes its own habbits of thinking and reflecting.

(2) Marx thought that religion was the opiate because it soothed people's pain and suffering and prevented them from rising in rebellion.

(3) If Karl Marx was alive today, he would say that television is the opiate of the people.

(4) Television and similar entertainments are even more of an opiate because of their addictive tendencies.

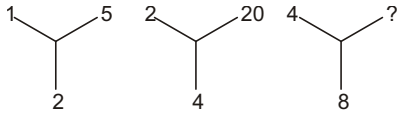
(a) 2134 (b) 1423
(c) 2431 (d) 3241

Logical Reasoning

96. In a certain code TELEPHONE is written as LETHPEENO. How ALIGATORE written in that code ?

(a) ROTAGILAE (b) ROTAGAILE
(c) ILATAGERO (d) ROTEGIAEL

97. Find the missing number from the given responses.

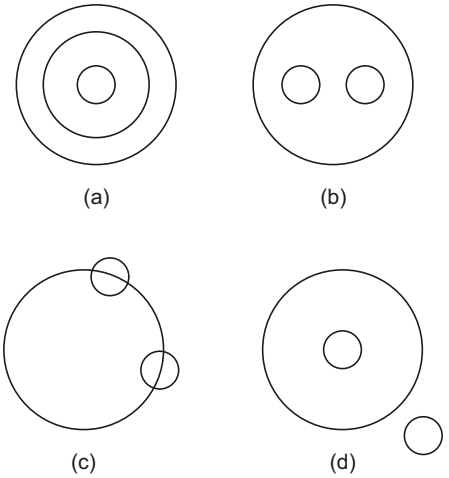


(a) 78 (b) 80 (c) 36 (d) 35

98. A P, R, X, S and Z are sitting in a row. S and Z are in the centre, and A and P are at the ends, R is sitting on the left of A. Then who is sitting on the right of P ?

(a) A (b) S (c) X (d) Z

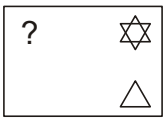
99. Which of the following represents mammal, man and snake.

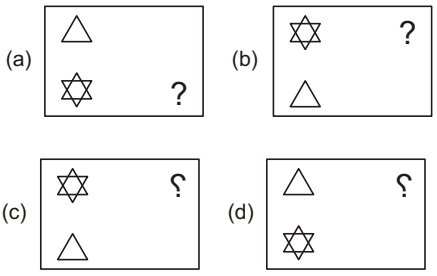


100. 2, 3, 13, 5, 7, 74, 11, 13, ?

(a) 88 (b) 185 (c) 99 (d) 290

101. What is the mirror image of





102. If $\alpha > \beta$, $\gamma > \delta$ and $\beta > y$ then which of the following conclusion is definitely wrong ?

(a) $\alpha > \delta$ (b) $\alpha > \gamma$ (c) $\delta > \alpha$ (d) $\beta > \delta$

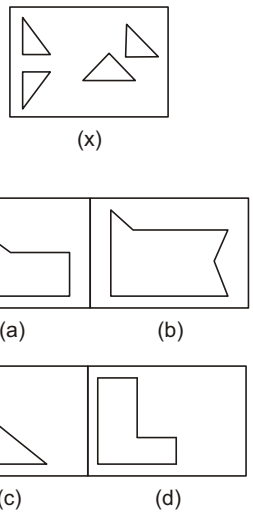
103. Introducing X, Y said "She is my father's sister-in-laws son's wife." How is X related to Y ?

(a) sister-in-law (b) aunty
(c) cousin (d) none of these

104. If 9 th of the month falls on the day preceding sunday then what day will be 1st of the month fall ?

(a) friday (b) saturday
(c) sunday (d) monday

105. Find out, which of the figures (a), (b), (c) and d can be formed from the pieces given in figure (x).



Mathematics

106. If three lines whose equations are $y=m_1x+c_1$, $y=m_2x+c_2$ and $y=m_3+c_3$ are concurrent, then $m_1(c_2-c_1)+m_2(c_3-c_1)+m_3(c_1-c_2)$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

107. The equation of circle which passes through the origin and cuts off intercepts 5 and 6 from the poisitive parts of the axis respetively is $\left(x-\frac{5}{2}\right)^2+(y-3)^2=\lambda$, where λ is

(a) $\frac{61}{4}$ (b) $\frac{6}{4}$ (c) $\frac{1}{4}$ (d) 0

108. Centre of the circle whose radius is 3 and which touches internally the circle $x^3+y^2-4x-6y-12=0$ at the point $(-1,-1)$ is

(a) $\left(\frac{7}{5},-\frac{4}{5}\right)$ (b) $\left(\frac{4}{5},\frac{7}{5}\right)$ (c) $\left(\frac{3}{5},\frac{4}{5}\right)$ (d) $\left(\frac{7}{5},\frac{3}{5}\right)$

109. If $\frac{x^2}{f(4a)}+\frac{y^2}{f(a^2-5)}$ respresents and ellipse with major axis as y-axis and f is a decreasing function, then

(a) $a\in(-\infty,1)$ (b) $a\in(5,\infty)$
(c) $a\in(1,4)$ (d) $a\in(-1,5)$

110. If the normal at the end of latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through $(0,-b)$ then e^4+e^2 (where e is eccentricity) equals

(a) 1 (b) $\sqrt{2}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}+1}{2}$

111. If the eccentricity of the hyperbola $x^2-y^2\sec^2\alpha=5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha+y^2=25$, then a value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

112. Let n be a fixed positive integer such that $\sin\frac{\pi}{2n}+\cos\frac{\pi}{2n}=\frac{\sqrt{n}}{2}$, then

(a) $n=4$ (b) n = 5
(c) $n=6$ (d) none of these

113. In a quadrilateral if

$$\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)+\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)=2$$

then $\sum\cos\frac{A}{2}.\cos\frac{B}{2}$ is equal to

(a) 0 (b) 6 (c) 3 (d) 2

114. if $\pi<\alpha<3\frac{\pi}{2}$, then the value of the expression $\sqrt{(4\sin^4\alpha+\sin^2 2\alpha)}+4\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$ is equal to

(a) $2+4Sin\alpha$ (b) $2-4Sin\alpha$
(c) 2 (d) none of these

115. The value of

$$\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$$ is equal to

(a) $\frac{\pi}{4}$ (2) $\frac{5\pi}{12}$
(c) $\frac{3\pi}{4}$ (d) $\frac{13\pi}{12}$

116. In a triangle $ABC, 2a^2+4b^2+c^2=4ab+2ac$, then the numerical value of $\cos B$ is equal to

(a) 0 (b) $\frac{3}{8}$
(c) $\frac{5}{8}$ (d) $\frac{7}{8}$

117. If $(\bar{a}\times\bar{b})^2+(\bar{a}-\bar{b})^2=144$ and $|\bar{a}^{-1}|=4$, then $|\bar{b}|$ is equal to

(a) 16 (b) 8 (c) 3 (d) 12

118. If $\bar{a}$ and $\bar{b}$ are two vectors, such that $\bar{a}\cdot\bar{b}<0$ and $|\bar{a}\cdot\bar{b}|=|\bar{a}\times\bar{b}|$, then angle between vectors $\bar{a}$ and $\bar{b}$ is

(a) π (b) $\frac{7\pi}{4}$
(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

119. If $\lim_{x\to0}\frac{x^a\sin^b x}{\sin(x^c)}$, a,b,c, $\in R\sim\{0\}$ exists and has non-zero value, then

(a) a,b,c are in A.P (b) a,b,c are in G.P
(c) a,b,c are in H.P (d) None of these

15. If the derivative of the functions $f(x)=\begin{Bmatrix} bx^2+ax+4; x\geq -1, \\ ax^2+b \quad ; x<-1 \end{Bmatrix}$ is everywhere continuous then

(a) $a=2, b=3$ (b) $a=3, b=2$
(c) $a=-2, b=-3$ (d) $a=-3, b=-2$

121. If $y=\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$, $\frac{\pi}{2}<x<\pi$, then $\frac{dy}{dx}$ equals

(a) $\frac{-1}{2}$ (b) -1 (c) $\frac{1}{2}$ (d) 1

122. If $f(x)=|x|$ then $f'(x)$, where $x\neq 0$ is equal to

(a) -1 (b) 0 (c) 1 (d) $\frac{|x|}{x}$

123. If $x=2\cos 2t-\cos 2t$, $y=2\sin t-\sin 2t$, then $\frac{d^2y}{dx^2}$ at $t=\frac{\pi}{2}$ is

(a) $\frac{-5}{2}$ (b) $\frac{-3}{2}$
(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

124. If $f(x)=x+tanx$ and f is inverse of g, then $g'(x)$ is equal to

(a) $\frac{1}{1+(g(x)-x)^2}$ (b) $\frac{1}{1-(g(x)-x)^2}$
(c) $\frac{1}{2+(g(x)-x^{2)}}$ (d) $\frac{1}{2-(g(x)-x)^2}$

125. A spherical baloon is pumped at the rate of to inch^3/min, the rate of increase of its radius is 15 inch is

(a) $\frac{1}{30\pi}$ inch/min (b) $\frac{1}{60\pi}$ Inch/min
(c) $\frac{1}{90\pi}$ inch/min (d) $\frac{1}{120\pi}$ inch/min

126. The value of 'a' in order $f(x)=\sqrt{3}\sin x-\cos x-2ax+b$ decreases for all real values of x, is given by

(a) $a<1$ (b) $a\geq 1$
(c) $a\geq\sqrt{2}$ (c) $a<\sqrt{2}$

127. In the function $f(x)=ax^3+bx^2+11x-6$ satisfies conditon of Rolle's theorem in [1,3] and $f'\left(2+\frac{1}{\sqrt{3}}\right)=0$ then value of a and b are respectively

(a) 1,–6 (b) –1,6 (c) –2,1 (d) $-1,\frac{1}{2}$

128. The value of the integral $\int\frac{dx}{x^n(1+x^n)^{1/n}}$, $n\in N$ is

(a) $\frac{1}{(1-n)}\left(1+\frac{1}{x^n}\right)^{1-\frac{1}{n}}+C$

(b) $\frac{1}{(1+n)}\left(1-\frac{1}{x^n}\right)^{1+\frac{1}{n}}+C$

(c) $-\frac{1}{(1+n)}\left(1-\frac{1}{x^n}\right)^{1-\frac{1}{n}}+C$

(d) $-\frac{1}{(1+n)}\left(1+\frac{1}{x^n}\right)^{1+\frac{1}{n}}+C$

129. Let $f(x)$ be a continous function such that $f(a-x)+f(x)=0$ for all $x\in[0,a]$ then $\int_0^a\frac{dx}{1+e^{f(x)}}$ is equal to.

(a) a (b) $\frac{a}{2}$
(c) $f(a)$ (d) $\frac{1}{2}f(a)$

130. $\lim\limits_{n\to\infty}\frac{(n!)^{1/n}}{n}$ equals

(a) e (b) e^{-1}
(c) 1 (d) none of these

131. The value of the integral $\int_0^1\frac{x^\alpha-1}{\log x}dx$, is

(a) $\log\alpha$ (b) $2\log(\alpha+1)$
(c) $3\log\alpha$ (d) none of these

132. If $y=f(x)$ passing through (1, 2) satisfies the differential equation $y(1+xy)dx-x\,dy=0$, then

(a) $f(x)=\frac{2x}{2-x^2}$ (b) $f(x)=\frac{x+1}{x^2+1}$
(c) $f(x)=\frac{x-1}{4-x^2}$ (d) $f(x)=\frac{4x}{x^2+1}$

133. If $f(x)=\begin{cases}\frac{(1-\sin^3 x)}{3\cos^2 x}, & x<\frac{\pi}{2}\\ a, & x<\frac{\pi}{2}\\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & x<\frac{\pi}{2}\end{cases}$, is continous at $x=\frac{\pi}{2}$, then the value of $\left(\frac{b}{a}\right)^{5/3}$ is

(a) 1 (b) 8 (c) 32 (d) 54

134. If the roots of the equation $ax^2 + bx + c = 0$, are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then value of $(a+b+c)^2$ is

(a) $2b^2 - ac$ (b) $b^2 - 2ac$
(c) $b^2 - 4ac$ (d) $4b^2 - 2ac$

135. If the equation $ax^2 + 2bx - 3c = 0$ has non real roots and $(3c/4) < (a+b)$; then C is always

(a) <0 (b) >0
(c) ≥ 0 (d) Zero

136. The root of the equation
$2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$, where $i = \sqrt{-1}$, which has eater modulus is

(a) $(3-5i)/2$ (b) $(5-3i)/2$
(c) $(3+i)/2$ (d) $(1+3i)/2$

137. The number of solutions of the equation $Z^2 + |Z|^2 = 0$ Where $Z \in C$ is

(a) one (b) Two
(c) Three (d) Infinitely many

138. The sum of n terms of the series
$1.4 + 3.04 + 5.004 + 7.0004 + ...$is

(a) $n^2 + \frac{4}{9}(1 + \frac{1}{10^n})$ (b) $n^2 + \frac{4}{9}(1 - \frac{1}{10^n})$
(c) $n + \frac{4}{9}(1 - \frac{1}{10^n})$ (d) none of these

139. If a,b,c are in H.P, b,c,d are in G.P and c,d,e are in A.P then $\frac{ab^2}{(2a-b)^2}$ is equal to

(a) b (b) a (c) e (d) d

140. Find the number of 5 card combination out of a deck of 52 cards if three is exactly on ace in each combination

(a) 778315 (b) 778325
(c) 778320 (d) 778328

141. Four boys picked up 30 mangoes. In how many ways can they divide them, if all mangoes be indentical?

(a) ${}^{33}C_4$
(b) ${}^{33}C_2$
(c) 5456
(d) None of these

142. If the coefficient of second, third and fourth terms in the expansion if $(1+x)^{2n}$ are in A.P, the $2n^2 - 3n$ is equal to

(a) -7 (b) 7
(c) 6 (d) –6

143. The value of determinant
$\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$ is

(a) independent of α
(b) independent of β
(c) independent of α and β
(d) none of these

144. The roots of the equation
$\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ are

(a) 1,2 (b) –1,2 (c) 1,–2 (d) –1,–2

145. If $\begin{vmatrix} a & a^3 & a^4-1 \\ b & b^3 & b^4-1 \\ c & c^3 & c^4-1 \end{vmatrix} = 0$ and a,b,c are all distinct then abc (ab + bc + ca) is equal to

(a) a+b+c (b) abc
(c) 0 (d) none of these

146. The values of α, for which the system of equation
$x + y + z = 1$
$x + 2y + 4z = \alpha$
$x + 4y + 10z = \alpha$ is consistent, are

(a) 1,–2 (b) –1,2
(c) 1,2 (d) None of these

147. The points (3,3) (h,0) and (0,K) are collinear, if $\frac{1}{h} + \frac{1}{K}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) 2

148. If coordinaters of vertices of triangle are (3,–5) and (–7,4) and its centroid is (2,–1). The coordinates of third vertex are

(a) (10,2) (b) (–10,2) (c) (–10,2) (d) (10,–2)

149. the height of the chimney when it is found that on walking towards it 50m in the horizontal line through its base, the angle of elevation of its top changes from 30° to 60° is

(a) 25m (b) $25\sqrt{2}$
(c) $25\sqrt{3}$ (d) None of these

150. The focus ot parabola $y^2 - x - zy + 2 + = 0$ is

(a) $\frac{1}{4}$ (b) $\left(\frac{5}{4}, 1\right)$ (c) $\left(1, \frac{5}{4}\right)$ (d) $\left(\frac{5}{4}, 0\right)$

Answer Sheet

Physics

1. (a)	2. (b)	3. (c)	4. (a)	5. (c)	6. (c)	7. (a)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (c)	14. (b)	15. (d)	16. (a)	17. (a)	18. (c)	19. (c)	20.(a)
21. (a)	22. (c)	23. (c)	24. (c)	25. (a)	26. (b)	27. (b)	28. (d)	29. (c)	30. (b)
31.(c)	32. (b)	33. (d)	34. (d)	35. (b)	36 (d)	37. (b)	38. (a)	39. (a)	40. (b)

Chemistry

41. (c)	42. (d)	43. (c)	44. (d)	45. (a)	46. (a)	47. (d)	48. (d)	49. (d)	50. (a)
51. (a)	52. (a)	53. (c)	54. (d)	55. (b)	56. (b)	57. (b)	58. (d)	59. (b)	60. (d)
61 (c)	62. (d)	63 (a)	64. (b)	65. (a)	66. (a)	67. (b)	68. (b)	69. (b)	70. (b)
71. (c)	72. (d)	73. (b)	74. (b)	75. (c)	76. (c)	77.(a)	78. (a)	79. (d)	80. (b)

English Procficiency & Logical Reasoning

81. (d)	82. (c)	83. (d)	84. (c)	85. (a)	86. (d)	87. (a)	88. (a)	89. (a)	90. (b)
91. (a)	92. (b)	93. (d)	94. (a)	95. (d)	96. (c)	97. (b)	98. (c)	99. (d)	100. (d)
101. (b)	102. (c)	103. (d)	104. (a)	105. (c)					

Mathematics

106. (b)	107. (a)	108. (b)	109. (d)	110. (a)	111. (b)	112. (c)	113. (a)	114. (c)	115. (c)
116. (d)	117. (c)	118. (d)	119. (d)	120. (d)	121. (a)	122. (d)	123. (b)	124. (c)	125. (c)
126. (b)	127. (a)	128. (d)	129. (b)	130. (b)	131. (d)	132. (a)	133. (c)	134. (c)	135. (a)
136. (a)	137. (d)	138. (b)	139. (b)	140. (c)	141. (c)	142. (a)	143. (a)	144. (b)	145. (a)
146. (c)	147. (b)	148. (d)	149. (c)	150. (b)					

Hints & Solutions

Physics

1. In the given equation, $\frac{\alpha z}{K\theta}$ should be dimensionless

$$\therefore \quad \alpha = \frac{K\theta}{z}$$

$$\Rightarrow \quad [a] = \frac{[\mathrm{M\,L^2T^{-2}\,K^{-1}}]\times[K]}{[\mathrm{L}]} = [\mathrm{MLT^{-2}}]$$

and $$P = \frac{\alpha}{\beta}$$

$$\Rightarrow \quad [a] = \left[\frac{a}{r}\right] = \frac{[\mathrm{MLT^{-2}}]}{[\mathrm{ML^{-1}T^{-2}}]} = [\mathrm{M^0L^2T^0}]$$

2. Between time interval 20 s to 40 s, there is non-zero acceleration and retardation. Hence, distance travelled during this interval

= Area between time interval 20 s to 40 s

$$= \frac{1}{2}\times 20\times 3 + 20\times 1 = 30 + 20 = 50\ \mathrm{m}$$

3. For w, $2w$ and $3w$ apparent weight will be zero because the system is falling freely. So the distances of the weights from the rod will be same.

4. Direction of velocity is always tangent to the path, so at the top of trajectory it is horizontal direction and acceleration due to gravity is always in vertically downward direction.

Hence, $\mathbf{v}$ and $\mathbf{g}$ are perpendicular to each other.

5. $\mathbf{F}\,\Delta t = m\Delta\mathbf{v}$

$$\Rightarrow \quad \mathbf{F} = \frac{m\Delta\mathbf{v}}{t}$$

By doing so time of change in momentum increases and impulsive force on knees decreases.

6. When ball is released from the top of a tower, then the ratio of distances covered by the ball in first, second and third second is

$$H_{\mathrm{I}} : H_{\mathrm{II}} : H_{\mathrm{III}} = 1 : 3 : 5$$

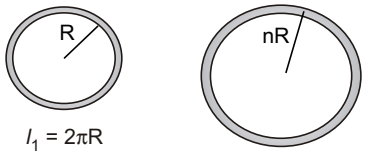
[because $h_n \propto (2n-1)$]

$\therefore$ Ratio of work done

$$mgh_{\mathrm{I}} : mgh_{\mathrm{II}} : mgh_{\mathrm{III}}$$

$$= 1 : 3 : 5$$

7.



$I_1 = 2\pi R$ $\qquad$ $I_1 = 2\pi nR$

Ratio of moment of inertia of the rings

$$\frac{I_1}{I_2} = \left(\frac{M_1}{M_2}\right)\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{\lambda L_1}{\lambda L_2}\right)^2\left(\frac{R_1}{R_2}\right)^2$$

$$= \left(\frac{2\pi R}{2\pi nR}\right)\left(\frac{R}{nR}\right)^2$$

[λ = linear density of wire = constant]

$$\Rightarrow \frac{L_1}{L_2} + \frac{1}{n_3} + \frac{1}{8} \qquad \text{(given)}$$

$$\therefore \quad n^3 = 8$$

$$\Rightarrow \quad n = 2$$

8. $v = \sqrt{2gR}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2} \times \frac{R_1}{R_2}} = \sqrt{g \times k} = (kg)^{1/2}$$

9. Speed of sound in stretched string

$$v = \sqrt{\frac{T}{\mu}} \qquad \ldots(i)$$

where T is the tension in the string and μ is mass per unit length.

According to Hooke's law, $F \propto \times$

$$\therefore \quad T \propto \times \qquad \ldots(ii)$$

From Eqs. (i) and (ii)

$$v \propto \sqrt{\times}$$

$$\therefore \quad v' = \sqrt{1.5\,V} = 1.22\,V$$

10. The velocity of ball before entering the water surface

$$v = \sqrt{2gh} = \sqrt{2g \times 9}$$

When ball enters into water, due to upthrust of water, the velocity of ball decreases (or retarded)

The retardation,

$$a = \frac{\text{apparent weight}}{\text{mass of ball}}$$

$$= \frac{V(\rho - \sigma)g}{V\rho} = \frac{(\rho - \sigma)g}{\rho}$$

$$= \left(\frac{(0.4 - 1)}{0.4}\right)g = -\frac{3}{2}g$$

If h be the depth upto which ball sin x, then

$$0 - v^2 = 2 \times \left(-\frac{3}{2}g\right) \times h$$

$$\Rightarrow \quad 2g \times 9 = 3gh \qquad \therefore \; h = 6 \text{ cm}$$

11. For all processes, change in internal energy $\Delta U = (-\Delta Q - \Delta W)$ does not change. It depends only on initial and final states.

12. Relative humidity at a given temperature (R)

$$= \frac{\text{Partial pressure of water pressure}}{\text{Vapour pressure of water}}$$

$$= \frac{0.012 \times 10^5}{0.016 \times 10^5} = 0.75 = 75\%$$

13. In the absence of intermolecular forces, there will be no stickiness of molecules. Hence, pressure will increase.

14. Time period is independent of mass of bob of pendulum.

15. $v = \dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2}$

$$\Rightarrow \quad \sin r = \sin 30° \times \frac{2v}{v}$$

$$\Rightarrow \quad \sin r = \frac{1}{2} \times 2 \times 1$$

$$\Rightarrow \quad r = 90°$$

16. $v = \dfrac{\text{coefficient of } t}{\text{coefficient of } x} = \dfrac{2\pi/0.01}{2\pi/0.3} = 30 \text{ m/s}$

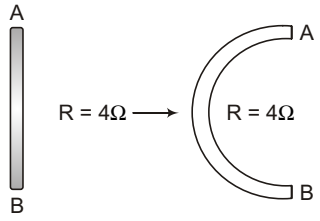
17. By using $W = Q(\mathbf{E}\,\Delta\mathbf{r})$

$$\Rightarrow \quad W = [Q(e_1\hat{\mathbf{i}} + e_2\hat{\mathbf{j}} + e_3\hat{\mathbf{k}}).(a\hat{\mathbf{i}} + a\hat{\mathbf{j}})]$$

$$= Q(e_1 a + e_2 b)$$

18. Charge will move along the circular line of force because $x^2 + y^2 = 1$ is the equation of circle in xy-plane.

19.



20. $\sigma_t = \dfrac{\theta}{i} = \dfrac{\theta}{i\,G}.G = \sigma_v\,G \Rightarrow \dfrac{\sigma_i}{G}$

21. $M = iA \Rightarrow i = \dfrac{M}{A}$

22. Work done, $W = MB(1 - \cos\theta)$

$$= 20 \times 0.3(1 - \cos 30°)$$

$$= 6 \times \left(1 - \frac{\sqrt{3}}{2}\right) = 3\left(2 - \sqrt{3}\right)$$

23. $i = i_0\left[1 - e^{-\frac{Rt}{L}}\right]$

$$\Rightarrow \frac{di}{dt} = \frac{d}{dt}i_0 - \frac{d}{dt}\left[i_0 e^{-\frac{Rt}{L}}\right] = 0 + -\frac{i_0 R}{L}e^{-\frac{Rt}{L}}$$

Initially $t = 0$

$$\Rightarrow \frac{di}{dt} = \frac{i_0 \times R}{L} = \frac{E}{L} = \frac{5}{2} = 2.5\,\text{A/s}$$

24. $\frac{1}{2}mv_{max}^2 = eV_0$

$$\Rightarrow v_{max} = \sqrt{2\left(\frac{e}{m}\right)V_0} = \sqrt{2 \times 1.8 \times 10^{11} \times 9}$$

$$= 18 \times 10^{15}\ \text{m/s} = 1.8 \times 10^6\ \text{m/s}$$

25. $N = N_0\left(\frac{1}{2}\right)^{t_{1/2}}$

$$\Rightarrow N_A = 10\left(\frac{1}{2}\right)^{t/1} \text{ and } N_8 = 1\left(\frac{1}{2}\right)^{t/2}$$

Given, $N_A = N_B$

$$\Rightarrow 10\left(\frac{1}{2}\right)^{t} = \left(\frac{1}{2}\right)^{t/2} \Rightarrow 10 = \left(\frac{1}{2}\right)^{-t/2}$$

$$\Rightarrow \quad 10 = 2^{t/2}$$

Taking log on both the sides

$$\log_{10} 10 = \frac{t}{2}\log_{10} 2 \Rightarrow 1 = \frac{t}{2} \times 0.3010$$

26. Optical path, μx = constant

i.e., $\quad \mu_1 x_1 = \mu_2 x_2$

$$\Rightarrow \quad 1.53 \times 4 = \mu_2 \times 4.5$$

$$\Rightarrow \quad \mu_2 = \frac{1.53 \times 4}{4.5} = 1.36$$

27. Volume, $V = l \times b \times t = 12 \times 6 \times 2.45$

$= 176.4\ \text{cm}^3$ or $V = 1.764 \times 10^2\ \text{cm}^3$

Since, the minimum number of significant figure is one in breadth, hence volume will also contain only one significant figure. Hence,

$$V = 2 \times 10^2\ \text{cm}^3$$

28. $P = \frac{F}{A} = \frac{n\{mv - (-mv)\}}{A} = \frac{2mnv}{A}$

$$= \frac{2 \times 10^{-3} \times 10^4 \times 10^2}{10^{-4}} 2 \times 10^7\ \text{N/m}^2$$

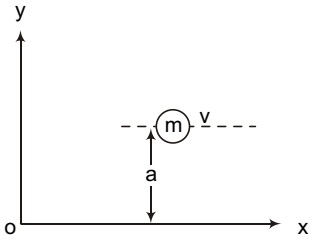
29. $W = \frac{F^2}{2K}$

If both springs are stretched by same force, then $W \propto \frac{1}{K}$.

As $K_1 > K_1$ therefore, $W_1 < W_2$

i.e., more work is done in case of second spring.

30. Angular moment of particle w.r.t. origin = linear momentum x perpendicular distance on line of action of linear momentum from origin.



$= mv \times a = mva$ = constant

31. Apparent weight = actual weight – upthrust

$vdg' = Vdg - V\rho g$

$$\Rightarrow \quad g' = \left(\frac{d - \rho}{d}\right)g$$

32. $A = \frac{c}{a + b - c}$: when $b = 0, a = c$

Amplitude A $\rightarrow \infty$. This correspondence to resonance.

33. If length of the foil is l then

$$C = \frac{K\varepsilon_0 (l \times b)}{d}$$

$$\Rightarrow \quad 2 \times 10^{-6} = \frac{2.5 \times 8.85 \times 10^{-12}\ (l \times 400 \times 10^{-3})}{0.15 \times 10^{-3}}$$

$$l = 33.9\ \text{m}$$

$$l = 33.9\ \text{m}$$

34. Potential difference across PQ *i.e.*, potential difference across the resistance of 20Ω, which is $V = IX\ 20$

and $\quad i = \frac{48}{(100 + 100 + 80 + 20)} = 0.16\,\text{A}$

35. Resistance of the material of the rod

$$\rho = \frac{RA}{l} = \frac{3 \times 10^{-3} \times \pi(0.3 \times 10^{-2})^2}{1}$$

$$= 27 \times 10^{-9}\ \pi\,\Omega\ \text{m}$$

Resistance of disc,

$$R = \frac{\text{Resistivity of rod} \times \text{thickness}}{\text{Area of cross - secion}}$$

$$= 27 \times 10^{-9}\,\pi \times \frac{10^{-3}}{\pi(1 \times 10^{-2})^2}$$

$$= 2.7 \times 10^{-7}\ \Omega$$

36. Cyclotron frequency, $v = \frac{Bq}{2\pi m}$

$$\Rightarrow \quad v = \frac{1 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$$

$$= 2.79 \times 10^{10}\ \text{Hz} \approx 28\ \text{GHz}$$

37. Transformation ratio, $k = \frac{N_S}{N_P} = \frac{V_S}{V_P}$

For step-up transformer,

$N_S > N_P$, *i.e.*, $V_S > V_P$, hence $k > 1$.

38. Intensity or power per unit area of the radiations,

$$P = pv$$

$$\Rightarrow \quad p = \frac{P}{v} = \frac{0.5}{3 \times 10^8} = .0166 \times 10^{-8} \text{ N/m}^2$$

39. Area of half period zone is independent of order of zone. Therefore, m is equal to zero in n^m.

40. $\frac{n}{t} = \frac{IA\lambda}{hc}$

$$= \frac{150 \times 10^{-3} \times 4 \times 10^{-3} \times 3 \times 10^{-7}}{6.6 \times 10^{-34} \times 3 \times 10^8}$$

$$= 9 \times 10^{13} \text{ s.}$$

Chemistry

41. Thermite is the mixture of Fe_2O_3 and Al. Due to great affinity of aluminium towards oxygen, it readily combines with oxygen. Hence, Goldsmith used Al to reduce metal oxides in extraction. In thermite, the ratio of Fe_2O_3 and Al is taken in 3:1 by weight.

$$Fe_2O_3 + \quad 2Al \rightarrow 2Fe + Al_2O_3$$

$$(2 \times 56 + 3 \times 16 = 160) \quad (2 \times 27 = 54)$$

42. In a unit cell, W atoms at the corner $= \frac{1}{8} \times 8 = 1$

O–atoms at the centre of edge $= \frac{1}{4} \times 12 = 3$

Na atoms at the centre of the cube = 1

W : O : Na = 1:3:1

Hence, formula = $NaWO_3$

43. Anhydrous $CaCl_2$ is used for fast drying of neutral gases.

44. H_2O_2 is used as an oxidant for rocket fuel and has 90% concentration to be used in rockets.

45. $\overset{3}{C}H_2(OH)-\overset{2}{C}H(NH_2)-\overset{1}{C}OOH$

2-amino-3-hydroxy propanoic acid

46. $\underset{\text{tertiary alcohol}}{(CH_3)_3C-OH} \xrightarrow[-H_2O]{+H^+} \underset{\substack{3^\circ \text{carbocation} \\ \text{(more stable)}}}{(CH_3)_3\overset{+}{C}}$

$$CH_2 = CH - CH_2 - CH_2OH \xrightarrow[-H_2O]{+H^+}$$

$$CH_2 = CH - CH_2 - \underset{\substack{1^\circ \text{ carbocation} \\ \text{(less stable)}}}{\overset{+}{C}H_2}$$

Increasing order of stability of carbocation

1° carbocation $< 2^\circ$ carbocation $< 3^\circ$ carbocation

47. Due to small size of Na^+, it is heavily hydrated and become large molecule.

Ionic conductance increase down the group in alkali metals.

Order of Ionic conductance

$$Na^+ < K^+ < Rb^+ < Cs^+$$

48. On hydration of plaster of Paris, converts into gypsum.

$$\underset{\text{Plaster of Paris}}{CaSO_4 \cdot \frac{1}{2}H_2O} + \frac{3}{2}H_2O \rightarrow \underset{\text{Gypsum}}{CaSO_4 \cdot 2H_2O}$$

49. Osmatic pressure $(\pi) = CRT$

Here, C = concentration of solution

$$C = \frac{n}{V}$$

$$\therefore \quad n = \frac{w}{m} = \frac{\text{weight in gram of substance}}{\text{mol. weight of substance}}$$

$V = 1$ Litre

$$C = \frac{68.4}{342}$$

$$\pi = \frac{68.4}{342} \times 0.082 \times 273$$

$$= 4.48$$

50. Isothermally (at constant temperature) and reversible work

$$W = 2.303 nRT \log \frac{p_2}{p_1}$$

$$= 2.303 \times 1 \times 2 \times 300 \log \frac{10}{2}$$

$$= 2.303 \times 600 \times \log 5$$

$$= 965.84$$

At constant temperature, $\Delta E = 0$

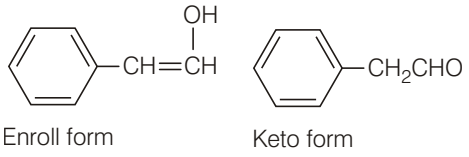
$\Delta E = q + W, q = -W = -965.84$ cal

51. According law of active mass

$$Kc = \frac{[NO_2]^2}{[N_2O_4]} = \frac{[1.2 \times 10^{-2}]^2}{4.8 \times 10^{-2}}$$

$$= 0.3 \times 10^{-2} = 3 \times 10^{-3}$$

52. **Tautomerism** It is functional isomerism in which the isomers are readily interchangeable and maintain a dynamic equilibrium with each other.

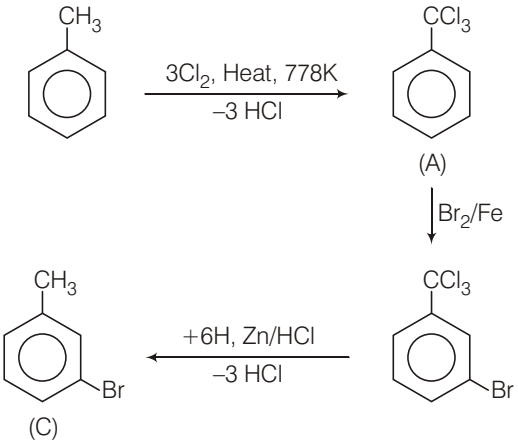


Enroll form Keto form

53. $CH_3-C\equiv C-CH_3 \xrightarrow{O_3} CH_3-\underset{\underset{O}{\diagdown\diagup}}{\overset{O-O}{C-C}}-CH_3$ (ozonide)

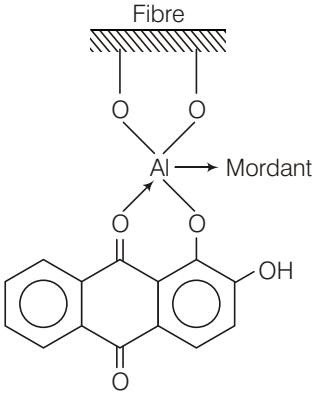
$\xrightarrow[-H_2O_2]{Zn/H_2O} CH_3-\underset{O-O}{\overset{\|}{C}}-\overset{\|}{C}$

54.



m-bromotoluene

55. Alizarin is mordant dye. Alizarin gives a bright red colour with aluminium and a blue colour with barium.



56. 2, 4-D or 2, 4-dichlorophenoxy acetic acid is used as a herbicides.

57. Flint glass or lead glass has composition of $K_2O.PbO.6SiO_2$. It is used in making electric bulb and optical instruments.

58. The + 5 oxidation state of Bi is unstable due to inert pair effect. Thus, BiF_5 cannot be formed.

59. $Na_2S_2O_3.5H_2O$ (hypo). It is called photographer's fixer because it removed the excess AgBr in the form of soluble silver complex.

60. Cl_2O = 42 electrons

ICl_2 = 87 electrons
Cl_2 = 35 electrons
IF_2^+ = 70 electrons
I_3^- = 160 electrons
ClO_2 = 33 electrons
ClO_2^- = 34 electrons
ClF_2^+ = 34 electrons

ClO_2^- and ClF_2^+ contain 34 electrons each hence they are isoelectronic.

61. These radioactive minerals have entrapped He atoms, produced from α-particle, which they give on heating in vacuo.

62. $\underset{\text{acid}}{H_2PO_4^-} + H_2O \rightarrow H_3O^+ + \underset{\text{conjugated base}}{HPO_4^{2-}}$

H_2PO_4 gives HPO_4^{2-} (conjugated base) in aqueous solutions. It acts as proton donor.

63. $\Delta G^\circ = -nFE^\circ$

$Fe^{2+} + 2e^- \rightarrow Fe$

$\Delta G^\circ = -2 \times F \times (-0.440\ V) = 0.880F$ ...(i)

$Fe^{3+} + 3e^- \rightarrow Fe$

$\Delta G^\circ = -3 \times F \times (-0.036) = 0.108F$ ...(ii)

$= 0.108\ F$

On subtracting Eq. (i) and (ii)

$Fe^{3+} + e^- \rightarrow Fe^{2+}$

$\Delta G^0 = 0.108F - 0.880F = -0.772F$

$E^0 = \dfrac{\Delta G^0}{nF} = \dfrac{-0.772F}{1 \times F} = +0.772\ V$

64. $N_2 + 3H_2 \rightleftharpoons 2NH_3$

$\dfrac{d[H_2]}{dt} = -0.3 \times 10^{-4}\ Ms^{-1}$

$\text{Rate} = -\dfrac{1}{3}\dfrac{d[H_2]}{dt} = +\dfrac{1}{2}\dfrac{d[NH_3]}{dt}$

$= \dfrac{d[NH_2]}{dt} = -\dfrac{2}{3}\dfrac{d[H_2]}{dt}$

$= -\dfrac{2}{3} \times (-0.3 \times 10^{-4})$

$= 0.2 \times 10^{-4}$

65. $v_{rms} + \sqrt{\frac{3RT}{M}}$

$v_{rms} \propto \sqrt{T}$

$\frac{v_{rms}}{v_{rms}} = \sqrt{\frac{T}{T'}}$

$\frac{1}{2} = \sqrt{\frac{T}{T'}}$

$T' = 4T$

66. The specific conductivity

$(k) = \frac{1}{R}$ cell constant $= k \times R$

$= 0.0129 \times 100 = 1.29$

67. Boiling point of HF is highest due to H-bonding. For other halogen acids boiling point increase in the order HCl<HBr< HI. Therefore, most volatile (with lower boiling point) is HCl.

68. Value of magnetic moment depends upon number of unpaired electrons. All except $Ti^{3+}|3d^1|$ have either fully filled *d*-subshell (*i.e.*, Zn^{2+}, Cu^{+}) or empty *d*-subshell (*i.e.* Sc^{3+}). As such only Ti^{3+} has a net value of magnetic moment.

Magnetic moment of $Ti^{3+} = \sqrt{n(n+2)}$BM

$= \sqrt{1(1+2)}$BM

$= \sqrt{3} = 1.73$BM

69. $Cr(24) = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^5, 4s^1$

70. Ag_2SO_4 contain $Ag^{+}(4d^{10})$and is colourless. CuF_2 contains Cu^{2+} $(3d^9)$ and is coloured due to presence of one unpaired electron is *d*-orbital of Cu^{2+}.MgF_2 contains Mg^{2+} and is colourless n/2 CuCl contains $Cu^{+}(3d^{10})$ and is colourless.

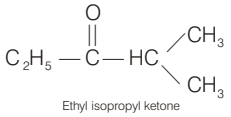
71. Effective atomic number = Electrons in Cr^{3+} + electrons from $6NH_3$ ligands $= 21 + 6 \times 2 = 33$

72. Nessler's reagent gives brown ppt. of iodide of Milion base with ammonium salt

$$[HgI_4]^{2-} + NH_4Cl + 4OH^- \rightarrow \underset{\text{Iodide of milion base (brown ppt)}}{NH_2HgOHgI}$$

$I^- + Cl^- \rightarrow NH_2O$

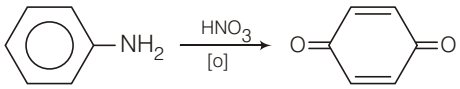
73. All the ketones except ethyl isopropyl ketone gives iodoform test in this question



Ethyl isopropyl ketone

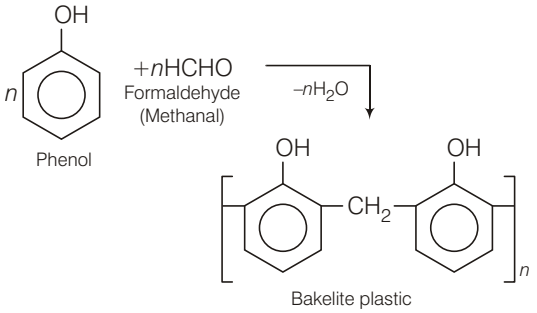
74. —NH— is the stronger electron releasing group than —CH_3 group, therefore bromination will take place at *p*-position with respect —NH— group.

75.



Benzoquinone

76.

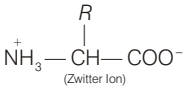


Bakelite plastic

77. Cellulose is a polymer of glucose $-\beta-D(+)$ glucose units are attached to each other by C1 to C4 bonds through β–glycosidic linkage in structure of cellulose.

78. Iodine value related to oils and fats. Iodine value measures the drying quality of an oil. More the unsaturation better is the drying quality of an oil. When on oil is treated with I_2. It adds to double bond. Iodine value is defined as the number of centigrams of I_2 that can be taken by 1g of the oil.

79. In aqueous solution, amino acids mostly exit as Zwitter ions.



(Zwitter Ion)

80. Gibbs' free energy *G*, enthalpy *H* and entropy *S* are interrelated as in

$G = H - TS$

Solutions for Logical Reasoning

96. T E L E P H O N E
L E T H P E E N O

Similarly

A L I G A T O R E
I L A T A G E R O

option (c)

97. $2^2 + 1^2 = 5$

$4^2 + 2^2 = 20$

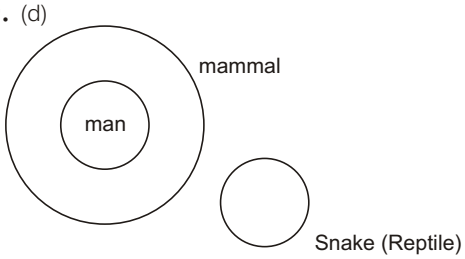
$8^2 + 4^2 = 80$

option (b)

98. Arrangment is PXZRA

option (c)

99. (d)



100. $2^2 + 3^2 = 13$

$5^2 + 7^2 = 74$

$11^2 + 13^2 = 290$

option (d)

101. (b)

102. Given $\alpha > \beta, \beta > \gamma, \gamma > \delta$

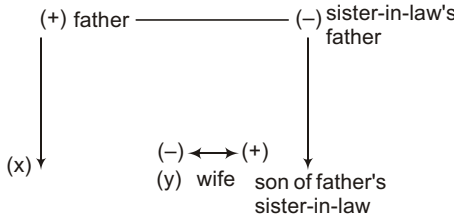
$\Rightarrow \quad \alpha > \beta > \gamma > \delta$

$\Rightarrow \quad \alpha > \delta$

$\Rightarrow \quad \delta > \alpha$

So, (c) is false

103. (d)



Hence option (d).

104. (a) '9th of the month = saturday.

Since day name repeats after 7 days.

$\therefore$ 9 – 7 = 2nd of the given month is also saturday.

Then, 1st of the given month would be friday option (a)

105. (c) Option (c). It is option from the pictures.

Mathematics

106. (b) Given equation of lines are

$$y = m_1x + c_1 \quad \ldots(i)$$

$$y = m_2x + c_2 \quad \ldots(ii)$$

$$y = m_3x + c_3 \quad \ldots(iii)$$

On solving Eqs. (i) and (ii), we get

$$x = \frac{c_2 - c_1}{m_1 - m_2} \text{ and } y = \frac{m_1c_2 - m_2c_1}{m_1 - m_2}$$

On putting the values of x and y in Eq. (iii),

We get

$$\frac{m_1c_2 - m_2c_1}{m_1 - m_2} = m_3\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + \frac{c_3}{1}$$

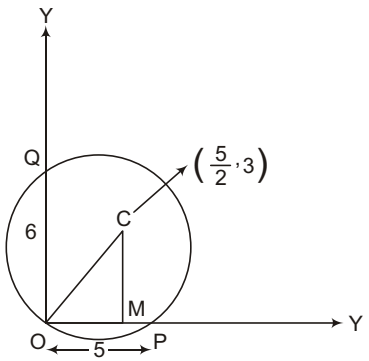
$$\Rightarrow \frac{m_1c_2 - m_2c_1}{m_1 - m_2} = \frac{m_3c_2 - m_3c_1 - m_1c_3 - m_2c_3}{m_1 - m_2}$$

$$\Rightarrow m_1c_2 - m_2c_1 = m_3c_2 - m_3c_1 + m_1c_3 - m_2c_3$$

$$\Rightarrow m_1c_2 - m_2c_1 - m_3c_2 + m_3c_1 - m_1c_3 + m_2c_3 = 0$$

$$\Rightarrow m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

107. (a) from figure, we have



$OP = 5, OQ = 6$

and $OM = \frac{5}{2}, CM = 3$

$\therefore$ In $\Delta OMC, OC^2 = OM^2 + MC^2$

$\Rightarrow \quad OC^2 = OM^2 + MC^2$

$\Rightarrow \quad OC^2 = \left(\frac{5}{2}\right)^2 + (3)^2$

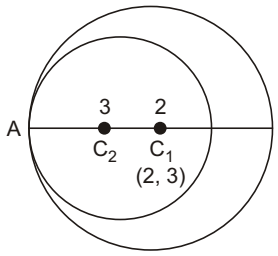
$\Rightarrow \quad OC = \frac{\sqrt{61}}{2}$

Thus, the required circle has its centre $\left(\frac{5}{2}, 3\right)$ and radius $\frac{\sqrt{61}}{2}$.

Hence, its equation is $\left(x - \frac{5}{2}\right)^2 + (y-3)^2 = \left(\frac{61}{4}\right)$

Hence, $\lambda = \frac{61}{4}$

108. (b) The given circle is



$$x^2 + y^2 - 4x - 6y - 12 = 0 \quad \ldots(i)$$

Whose centre is $c_1(2,3)$ and radius $r_1 = c_1A = 5$

If $c_2(h,k)$ is the centre of the circle of radius 3 which touches the circle (i) internally at the point $A(-1,1)$, then $C_2A = 3$ and $c_1c_2 = c_1A - c_2A = 5 - 3 = 2$

Thus, $c_2(h,k)$ divide c_1A in the ratio 2:3 internally

$\therefore h = \frac{2(-1)+3.2}{2+3} = \frac{4}{2}$ and $K = \frac{2(-1)+3.3}{2+3} = \frac{7}{2}$

Hence, required centre is $\left(\frac{4}{5}, \frac{7}{5}\right)$.

109. (d) Since y-axis is major axis.

$\Rightarrow \quad f(4a) < f(a^2 - 5)$

$\Rightarrow \quad 4a > a^2 - 5 \qquad (\because f \text{ is decreasing})$

$\Rightarrow \quad a^2 - 4a - 5 < 0$

$\Rightarrow \quad a \in (-1, 5)$

110. (a) Normal at the extremity of latus rectum in the first quadrant $\left(ae, \frac{b^2}{a}\right)$ is

$$\frac{x - ae}{ae/a^2} = \frac{y - b^2/a}{b^2/ab^2}$$

As it passes through $(0, -b)$

$$\frac{-ae}{ae/a^2} = \frac{-b - b^2/a}{1/a}$$

$\Rightarrow \quad -a^2 = -ab - b^2$

$\Rightarrow \quad a^2 - b^2 = ab$

$\Rightarrow \quad a^2e^2 = ab$

$\Rightarrow \quad e^2 = b/a$

$\therefore e^4 = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^4 + e^2 = 1.$

111. (b) The given condition gives :

$$\frac{5 + 5\cos^2\alpha}{5} = 3.\frac{25 - 25\cos^2\alpha}{25}$$

leading then, we get

$$\cos^2\alpha = \frac{1}{2}$$

or $\quad \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$

112. (c) $\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

$\Rightarrow \quad \frac{\sqrt{n}}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$

So, for $n > 1$, $\frac{\sqrt{n}}{2\sqrt{2}} = \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) > \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

or $\quad n > 4$

Since, $\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$ for all $n > 2$, we get

$$\frac{\sqrt{n}}{2\sqrt{2}} < 1 \text{ or } n < 8.$$

So, that $4 < n < 8$. By actual verification we find that only $n = 6$. satisfies the given relation.

113. (a) Given,

$$2\left[\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A+B}{2}\right) + \sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)\right]$$

$$= 2 \times 2$$

$\Rightarrow \quad \sin A + \sin B + \sin C + \sin D = 4$

$\Rightarrow \quad \sin A = \sin B = \sin C = \sin D = 1$

$\Rightarrow \quad \angle A = \angle B = \angle C = \angle A = 90°$

Now,

$$\Sigma\cos\frac{A}{2}.\cos\frac{B}{2} = \Sigma\cos 45°.\cos 45° = 6\left(\frac{1}{\sqrt{2}}\right)^2 = 3.$$

114. (c) $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$

$\therefore \sin\alpha < 0 + \cos\alpha < 0, \tan\alpha > 0$

$$\sqrt{4\sin^4\alpha + \sin^2 2\alpha} + 4\cos^2\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)$$

$$= \sqrt{4\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)} + 4\cos^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= -2\sin\alpha + 2\left(1 + \cos\left(\frac{\pi}{2} - \alpha\right)\right)$$

$$= -2\sin\alpha + 2 + 2\sin\alpha = 2$$

115. (c) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3x}{4}$$

116. (d) $\because\ 2a^2 + 4b^2 + c^2 = 4ab + 2ac$

$\Rightarrow\ (a - 2b)^2 + (a - c)^2 = 0$

Which is possible only when

$$a - 2b = 0 \text{ and } a - c = 0$$

or $$\frac{a}{1} = \frac{b}{\left(\frac{1}{2}\right)} = \frac{c}{1} = \lambda \text{ (say)}$$

$\therefore$ $$a = \lambda,\ b = \frac{\lambda}{2},\ c = \lambda$$

$\therefore$ $$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{\lambda^2 + \lambda^2 - \frac{\lambda^2}{4}}{2\lambda^2}$$

$$= 1 - \frac{1}{8} = \frac{7}{8}.$$

117. (c) Since,

$$(\bar{a} \times \bar{b})^2 + (\bar{a}.\bar{b})^2 = 144$$

If angle between $\bar{a}$ and $\bar{b}$ is Q,

then $|\bar{a}|^2|\bar{b}|^2\sin^2\theta + |\bar{a}|^2|\bar{b}|^2\cos^2\theta = 144$

$\Rightarrow\ |\bar{a}|^2|\bar{b}|^2 = 144$

$\Rightarrow\ |\bar{a}|^2|\bar{b}|^2 = 12$

$\Rightarrow\ 4|\bar{b}| = 12$

$\Rightarrow\ |\bar{b}| = 3.$

118. (d) $|\bar{a}.\bar{b}| = |\bar{a} \times \bar{b}|$

$|\bar{a}||\bar{b}||\cos\theta| = |\bar{a}||\bar{b}||\sin\theta|$

(where θ is angle between $\bar{a}$ and $\bar{b}$)

$\Rightarrow\ |\cos\theta| = |\sin\theta|$

$\Rightarrow\ \theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ (as $0 \le \theta \le \pi$)

But $\bar{a}.\bar{b} < 0$

$\therefore\ \theta = \frac{3\pi}{4}$

119. (d) $$\lim_{x\to 0} x^a \cdot \frac{\left(\frac{\sin x}{x}\right)^b}{\left(\frac{\sin(x^c)}{x^c}\right)} \cdot \frac{x^b}{x^c}$$

$$= \lim_{x\to 0} x^{a+b-c} \frac{\left(\frac{\sin x}{x}\right)^b}{\left(\frac{\sin(x)^c}{x^c}\right)}$$

The above limit non-zero value only when

$$a + b - c = 0$$

120. (d) We have $f(x) = \begin{cases} ax^2 + b, x < -1 \\ bx^2 + ax + 4, x \ge -1 \end{cases}$

$\therefore$ $$f'(x) = \begin{cases} 2ax,\ x, x < -1 \\ 2bx + a,\ x \ge -1 \end{cases}$$

Since, $f(x)$ is differentiable at $x = -1$, therefore it is continuous at $x = -1$ and hence,

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$\Rightarrow\ a + b = b - a + 4$

$\Rightarrow\ a = 2$

and also, $\lim_{x\to -1^-} f'(x) = \lim_{x\to -1^+} f'(x)$

$\Rightarrow\ -2a = -2b + a$

$\Rightarrow\ 3a = 2b$

$\Rightarrow\ b = 3$ ($\because a = 2$)

Hence, $a = 2, b = 3$.

121. (a) $$y = \tan^{-1}\sqrt{\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{1 + \cos\left(\frac{\pi}{2} + x\right)}}$$

$$= \tan^{-1}\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| \quad \ldots(i)$$

Now, $$\frac{\pi}{2} < x < \pi$$

$\therefore$ $$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

or $$\frac{\pi}{2} < \frac{\pi}{4} + \frac{x}{2} < \frac{3\pi}{4}$$

$\therefore\ \left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ ($\because$ in II quadrant)

$$= \tan\left\{\pi - \left(\frac{\pi}{4} + \frac{x}{2}\right)\right\}$$

from Eq. (i);

$$y = \tan^{-1}\tan\left\{\pi - \left(\frac{\pi}{4} + \frac{\pi}{2}\right)\right\}$$

$$= \pi - \left(\frac{\pi}{4} + \frac{\pi}{2}\right)$$

$$= \frac{3\pi}{4} - \frac{x}{2}$$

($\because$ principle value of $\tan^{-1}$ is $-\frac{\pi}{2}$ to $\frac{\pi}{2}$)

$$\frac{dy}{dx} = -\frac{1}{2}$$

122. (d) $\because f(x) = \begin{cases} x, x > 0 \\ -x, x < 0 \end{cases}$

$$\because \quad f'(x) = \begin{cases} 1, x > 0, i.e., \dfrac{|x|}{x}, x > 0 \\ -1, x < 0, i.e., \dfrac{|x|}{x}, x < 0 \end{cases}$$

$$= \frac{|x|}{x}, x \neq 0$$

123. (b) $\because \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2\cos t - 2\cos 2t}{-2\sin t + 2\sin 2t}$

$$= \frac{\cos t - \cos 2t}{\sin 2t - \sin t}$$

$$= \frac{2\sin\left(\frac{3t}{2}\right)\sin\left(\frac{t}{2}\right)}{2\cos\left(\frac{3t}{2}\right)\sin\left(\frac{t}{2}\right)} = \tan\left(\frac{3t}{2}\right)$$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\tan\frac{3t}{2}\right)$$

$$= \frac{d}{dt}\tan\left(\frac{3t}{2}\right).\frac{dt}{dx}$$

$$= \frac{3}{2}\sec^2\left(\frac{3t}{2}\right).\frac{1}{(-2\sin t + 2\sin 2t)}$$

$$\therefore \quad \left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{2}} = \frac{3}{2}\sec^2\left(\frac{3\pi}{4}\right).\frac{1}{(-2+0)}$$

$$= \frac{-3}{4}(-\sqrt{2})^2 = -\frac{3}{2}$$

124. (c) Let $y = f(x) \Rightarrow x = f^{-1}(y)$

then $f(x) = x + \tan x$

$\Rightarrow \quad y = f^{-1}(y) + \tan(f^{-1}(y))$

$\Rightarrow \quad y = g(y) + \tan(g(y))$

or $\quad x - g(x) + \tan(g(x)) \quad \ldots(i)$

differentiating on both sides, then we get:

$$1 = g'(x) + \sec^2(g(x)).g'(x)$$

$$\therefore\ g'(x) = \frac{1}{1+\sec^2(g(x))} = \frac{1}{1+1+\tan^2(g(x))}$$

$$= \frac{1}{2+(x-g(x))^2} = \frac{1}{2+(g(x)-x)^2} \text{ [from (i)]}$$

125. (c) Let V be the volume and 'r' the radius of the baloon at any time t.

$$\therefore V = \frac{4}{3}\pi r^3$$

$$\therefore \quad \frac{dv}{dt} = \frac{4}{3}\pi(3r^2).\frac{dr}{dt}$$

$$\Rightarrow \quad 10 = \frac{4\pi}{3}.3.(15)^2\frac{dr}{dt}$$

$$\therefore \quad \frac{dr}{dt} = \frac{1}{90\pi} \text{ inch/min}$$

126. (b) Given $f'(x) < 0\ \forall\ x \in R$

$$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0\ \forall\ x \in R$$

$$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a\ \forall\ x \in R$$

$$\Rightarrow \quad \sin\left(x + \frac{\pi}{3}\right) < a$$

$$\Rightarrow \quad a \geq 1 \qquad \left[\because \sin\left(x + \frac{\pi}{3}\right) \leq 1\right]$$

127. (a) $\because f(x) = f(3)$

$$\Rightarrow \quad a + b + 11 - 6 = 27a + 9b + 33 - 6$$

$$\Rightarrow \quad 13a + 4b = -11 \quad \ldots(i)$$

and $\quad f'(x) = 3ax^2 + 2bx + 11$

$$\Rightarrow f'\left(2 + \frac{1}{\sqrt{3}}\right) = 3a\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11$$

$$= 0$$

$$\Rightarrow 3a\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11 \quad \ldots(ii)$$

from eq. (i) and (ii), we get

$$a = 1, b = -6.$$

128. (d) $\displaystyle\int \frac{dx}{x^n(1+x^n)^{1/n}} = \int \frac{dx}{x^n.x\left(\frac{1}{x^n}+1\right)^{1/n}}$

$$= \int \frac{dx}{x^{n+1}\left(\frac{1}{x^n}+1\right)^{1/n}}$$

Put, $\quad \dfrac{1}{x^n} + 1 = t$

$$\Rightarrow \quad -\frac{n}{x^{n+1}}dx = dt$$

$$\Rightarrow \quad -\frac{1}{n}\int \frac{dt}{t^{1/n}} = -\frac{1}{n}\int t^{1-/n}dt = \frac{-1}{n}.\frac{t^{1-\frac{1}{n}}}{\left(1-\frac{1}{n}\right)} + c$$

$$= \frac{1}{(1-n)}\left(1+\frac{1}{x^n}\right)^{1-\frac{1}{n}} + c$$

129. (b) Let $I = \int_0^a \dfrac{dx}{1+e\,f(x)} \quad \ldots(i)$

$$= \int_0^a \frac{dx}{1+e^{f(a-x)}} \qquad \text{(by property)}$$

$$= \int_0^a \frac{dx}{1+e^{-f(x)}}$$

$$\{\text{given, } f(x) + f(a-x) = 0\}$$

$$= \int_0^a \frac{e^{f(x)}}{1+e^{f(x)}}dx \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), then;

$$2I = \int_0^a dx = a$$

$$\Rightarrow \quad I = \frac{a}{2}.$$

130. (b)Let $P = \lim_{n\to\infty} \frac{(n!)^{1/n}}{n}$

$$= \lim_{n\to\infty}\left\{\frac{n!}{n^n}\right\}^{1/n}$$

$$= \lim_{n\to\infty}\left\{\frac{1.2.3\ldots n}{n.n.n\ldots n}\right\}^{1/n}$$

$$\therefore \log P = \lim_{n\to\infty}\frac{1}{n}\left[\log\left(\frac{1}{n}\right)+\log\left(\frac{2}{n}\right)+\log\left(\frac{3}{n}\right)+\ldots+\log\left(\frac{n}{n}\right)\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

$$= \int_0^1 \log x dx$$

$$= [\log x.\, x - x]_0^1$$

$$= 0 - 1 = -1$$

$$\therefore \quad P = e^{-1}$$

131. (d) Let $I(\alpha) = \int_0^1 \frac{x^\alpha - 1}{\log x}dx$ ...(i)

$$\therefore \quad I'(\alpha) = \int_0^1 \frac{x^\alpha \log x}{\log x}dx = \int_0^1 x^\alpha .dx$$

$$= \left[\frac{x^{\alpha+1}}{\alpha+1}\right]_0^1 = \frac{1}{(\alpha+1)}$$

$$\therefore \quad I(\alpha) = \int \frac{dx}{\alpha+1}$$

$$= \log|\alpha+1| + c$$

Put $\alpha = 0$, then

$$I(0) = \log 1 + c = 0 \qquad \text{[from (i)]}$$

$$\Rightarrow \quad 0 + c = 0, \qquad \therefore c = 0$$

Hence; $I(\alpha) = \log|\alpha+1|$.

132. (a) Since, $y = f(x)$

and given differential equation

$$y(1+xy)dx - xdy = 0$$

$$\Rightarrow \quad (ydx - xdy) + xy^2 2dx = 0$$

or $$\frac{ydx - xdy}{y^2} + xdx = 0$$

$$\Rightarrow \quad d\left(\frac{x}{y}\right) + x\,dx = 0$$

on integrating, then

$$\frac{x}{y} + \frac{x^2}{2} = c$$

Which passes through (1, 2)

$$\therefore \quad c = 1$$

then, curve is

$$\frac{x}{y} + \frac{x^2}{2} = 1$$

or $$y = \frac{2x}{2 - x^2}$$

$$\therefore \quad f(x) = \frac{2x}{2 - x^2}$$

133. (c) LHL = V · F = RHL

$$\Rightarrow \quad \lim_{x\to\frac{\pi^-}{2}} f(x) = f\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi^+}{1}} f(x)$$

$$\Rightarrow \quad \lim_{x\to 0} f\left(\frac{\pi}{2} - h\right) = a = \lim_{h\to 0} f\left(\frac{\pi}{2} + h\right)$$

$$\Rightarrow \quad \lim_{h\to 0}\frac{(1-\cos^2 h)}{3\sin^2 h} = a = \lim_{h\to 0}\frac{b(1-\cos)h}{4h^2}$$

$$\Rightarrow \quad \lim_{x\to 0}\frac{(1-\cosh)(1+\cosh+\cos^2 h)}{3(1-\cosh)(1+\cosh)}$$

$$= a = \lim_{x\to 0}\frac{b(1-\cosh)}{4h^2}$$

$$\Rightarrow \quad \frac{3}{3(2)} = a = \frac{b}{8} \text{ or } \frac{b}{a} = 8;$$

$$\therefore \quad \left(\frac{b}{a}\right)^{\frac{5}{2}} = (8)^{\frac{5}{2}} = 32$$

134. (c) By hypothesis, $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$

and $$\frac{\alpha}{\alpha-1}\cdot\frac{\alpha+1}{\alpha} = -\frac{c}{a}$$

$$\Rightarrow \quad \frac{2\alpha^2 - 1}{\alpha^2 - \alpha} - \frac{-b}{a} \text{ and } \alpha = \frac{c+a}{c-a}$$

$$\Rightarrow \quad (c+a)^2 + 4ac = -2b(c+a)$$

$$\Rightarrow \quad (c+a)^2 + 2b(c+a) + b^2 = b^2 - 4ac$$

$$\Rightarrow \quad (a+b+c)^2 = b^2 - 4ac$$

135. (a) Let $f(x) = ax^2 + 2bx - 3c$

Since, $f(x) = 0$ has non-real roots, $f(x)$ will have the same sign for values of x.

Given, $\frac{3c}{4} < a + b \Rightarrow 4a + 4b - 3c < 0$

$$\Rightarrow \quad f(2) > 0$$

$$\Rightarrow \quad f(0) > \Rightarrow c < 0$$

136. (a) Roots $= \frac{4(2-i) \pm \sqrt{16(2-i)^2 + 8(1+i)(5+3i)}}{4(1+i)}$

$$= \frac{-i}{i+1} \quad \text{or} \quad \frac{4-i}{1+i}$$

$$= \frac{-1-i}{2} \text{ or } = \frac{3-5i}{2}$$

$\therefore \quad \left|\frac{3-5i}{2}\right| = \sqrt{\left(\frac{9}{4}+\frac{25}{4}\right)} = \sqrt{\frac{17}{2}}$

and $\left|\frac{-1-i}{2}\right| = \sqrt{\left(\frac{1}{4}+\frac{1}{4}\right)} = \frac{1}{\sqrt{2}}$

Here, $\sqrt{\frac{17}{2}} > \frac{1}{\sqrt{2}}$

Hence, root is (3 – 5 i)/2

137. (d) $z^2 + |z|^2 = 0$

$\Rightarrow \quad z^2 + z\bar{z} = 0$

$\Rightarrow \quad z(z + \bar{z}) = 0$

or $\quad z \cdot 2\,\text{Re}(z) = 0$

$\therefore \quad z = 0$ and $\text{Re}(z) = 0$

if $\quad z = a + ib$

$\therefore \quad$ Solutions are $z = 0, ib \quad (b \in R)$

138. (b) $1.4 + 3.04 + 5.004 + ...$

$= (1 + 3 + 5 + ...) + (0.4 + 0.04 + 0.004 + ...)$

$= \frac{n}{2}[2 + (n-1)] + \left[\frac{4}{10} + \frac{4}{10^2} + ... n\right]$

$= n^2 + \frac{4}{10}\left\{\frac{1-\left(\frac{1}{10}\right)^n}{1-\left(\frac{1}{10}\right)}\right\} = n^2 + \frac{4}{9}\left(1 - \frac{1}{10^n}\right)$

139. (b) $\because a, b, c$ are in H.P.

$\therefore \quad b = \frac{2ac}{a+c}$ ...(i)

b, c, d are in G.P.

$c^2 = bd$ ...(ii)

and c, d, e are in A.P.

$\therefore \quad d = \frac{c+e}{2}$ ...(iii)

from eq. (i), $ab + bc = 2ac$

$\Rightarrow \quad c = \frac{ab}{2a-b}$ ...(iv)

from eqs. (iii) and (iv), we get

$d = \frac{1}{2}\left(\frac{ab}{2a-b} + e\right)$ ...(v)

on putting the value of c and d from Eq. (iv) and (v) in Eq (ii), we get:

$\frac{a^2b^2}{(2a-b)^2} = \frac{b}{2}\left\{\frac{ab}{2a-b} + e\right\}$

$\therefore \quad \frac{2a^2b}{(2a-b)^2} - \frac{ab}{(2a-b)} = e$

$\Rightarrow \quad e = \frac{2a^2 - 2a^2b + ab^2}{(2a-b)^2}$

Hence, $\quad e = \frac{ab^2}{(2a-b)^2}$

140. (c) Number of ways selecting one ace from 4 aces $= {}^4C_1$

Number of ways selecting 4 cards out of 48 cards $= {}^{48}C_4$

Hence by fundamental principle of counting, 5 cards out of 52 card with exactly one ace can be selected in ${}^4C_1 \times {}^{48}C_4$ ways

$= 4 \times \frac{48 \times 47 \times 46 \times 45}{24}$

$\left\{\because {}^nC_1 = n \text{ and } {}^nC_4 = \frac{n(n-1)(n-2)(n-3)}{24}\right\}$

$= \frac{48 \times 47 \times 46 \times 45}{6}$

$= 48 \times 47 \times 46 \times 45$

$= 778320$ ways

141. (c) Clearly, 30 mangoes can be distributed among 4 boys such that each boy can receive any number of mangoes.

Hence, total number of ways $= {}^{30+4-1}C_{4-1} = {}^{33}C_3$

$= \frac{33 \cdot 32 \cdot 31}{1 \cdot 2 \cdot 3}$

$= 5456$

142. (a) The general term of $(1 + x)^{2n}$ is

$T_{r+1} = {}^{2n}C_r x^r$

$T_2 = {}^{2n}C_r x^2, T_3 = {}^{2n}C_2 x^3, T_4 = {}^{2n}C_3 x^4$

Since, coefficients are in A.P.

$\Rightarrow \quad {}^{2n}C_1, {}^{2n}C_2, {}^{2n}C_3$ are in A.P.

$\Rightarrow \quad 2 \times {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow \quad 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2}$

$\Rightarrow \quad 2 = \frac{2}{(2n-2+1)} + \frac{(2n-3+1)}{3}$

$\Rightarrow \quad 2 = \frac{2}{2n-1} + \frac{2n-2}{3}$

$\Rightarrow \quad 2n^2 - 9n + 7 = 0$

$\therefore \quad 2n^2 - 9n = -7$

143. (a) $\begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1 \end{vmatrix}$

$= \begin{vmatrix} \cos\alpha & -\sin\alpha & 1 \\ \sin\alpha & \cos\alpha & 1 \\ 0 & 0 & 1 + \sin\beta - \cos\beta \end{vmatrix}$

(applying $R_3 \to R_3 - R_1\cos\beta + R_2\sin\beta$)

$= (1 + \sin\beta - \cos\beta)(\cos^2\alpha + \sin^2\alpha)$

$= 1 + \sin\beta - \cos\beta$

Which is independent of α

144. (b) $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x+1 & 1 & 1 \\ x+1 & x-1 & 1 \\ x+1 & 1 & x-1 \end{vmatrix} = 0 \quad [C_1 \to C_1 + C_2 + C_3]$

$\Rightarrow \quad (x-1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$\Rightarrow \quad (x-1)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-2 & 0 \\ 0 & 0 & x-2 \end{vmatrix} = 0 \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$\Rightarrow \quad (x-1)(x-2)^2 = 0 \Rightarrow x = -1, 2$

145. (a) Given, $\begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} + \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$

$\Rightarrow \quad abc \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} - 1 \begin{vmatrix} a & a^3 & -1 \\ b & b^3 & -1 \\ c & c^3 & -1 \end{vmatrix} = 0$

$\Rightarrow \quad abc\,(a-b)(b-c)(c-a)(ab+bc+ca)$
$\quad - (a-b)(b-c)(c-a)(a+b+c) = 0$

$\Rightarrow \quad abc\,(ab+bc+ca) - (a+b+c) = 0$

$\Rightarrow \quad abc\,(ab+bc+ca) = (a+b+c)$

146. (c) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

for consistent, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{vmatrix} = 0$

$\Rightarrow \quad 1(20-16) - 1(10-4) + 1(4-2) = 0$

$\Rightarrow \quad 4 - 6 + 2 = 0$

$\Rightarrow \quad 0 = 0$ and $(\text{adj}A)\,B = 0$

$\Rightarrow \quad \begin{bmatrix} 4 & -6 & 2 \\ -6 & 9 & -3 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} 4 & -6\alpha & 2\alpha^2 \\ -6 & +9\alpha & -3\alpha^2 \\ 2 & -3\alpha & +\alpha^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \quad 2\alpha^2 - 6\alpha + 4 = 0, -3\alpha^2 + 9\alpha - 6 = 0$

and $\quad \alpha^2 - 3\alpha + 2 = 0$

Now $\quad 2\alpha^2 + 6\alpha + 4 = 0$

$\Rightarrow \quad (2\alpha - 2)(\alpha - 2) = 0$

$\Rightarrow \quad \alpha = 1, 2$

Similarly, from other equations we also get the same value

147. (b) Given that, $P \equiv (3, 3)$, $Q \equiv (h, 0)$ and $R \equiv (0, K)$.

Now, these points are collinear, if area of $\Delta PQR = 0$

$\Rightarrow \quad \frac{1}{2}[3(0-k) + h(k-3) \text{ to } (3-0)] = 0$

$\Rightarrow \quad \frac{1}{2}(-3k + hk - 3h) = 0$

$\Rightarrow \quad 3k + 3h = hk \qquad \Rightarrow \frac{1}{h} + \frac{1}{k} = \frac{1}{3}$

148. (d) Let (x, y) be the coordinates of third vertex, then

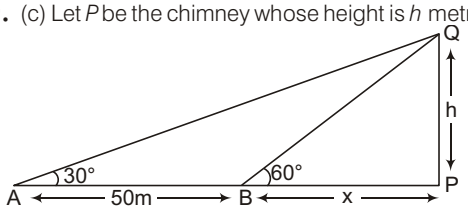
$\frac{x+3-7}{3} = 2$ and $\frac{y-5+4}{3} = 1$

$\Rightarrow \quad x - 4 = 6$ and $y - 1 = -3$

$\Rightarrow \quad x = 10$ and $y = -2$

Thus, the coordinates of the vertex are $(10, -2)$

149. (c) Let P be the chimney whose height is h metres.



In ΔBPQ

$\tan 60° = \frac{PQ}{BP}$

$\Rightarrow \quad \sqrt{3}\,\frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}\,x \qquad \ldots(i)$

and in ΔAPQ, $\tan 30° = \frac{PQ}{AP}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50+x} \quad \Rightarrow \quad 50 + x = h\sqrt{3}$

$\Rightarrow \quad 50 + x = 3x$ (from Eq. (i))

$\Rightarrow \quad x = 25$

$\therefore$ Height of the chimney is $25\sqrt{3}$m.

150. (b) We have, $y^2 - 2y = x - 2$

$\Rightarrow \quad (y-1)^2 = 1 \cdot (x-1) \Rightarrow y^2 = x$

Where, $y = y - 1$ and $x = x - 1$

Here, $\quad a = \frac{1}{4}$ focus is $x = a$, $y = 0$

$\Rightarrow \quad x - 1 = \frac{1}{4}$, $y - 1 = 0$

$\Rightarrow$ Focus of parabola is $\left(\frac{5}{4}, 1\right)$