## NETWORK ANALYSIS

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## INTRODUCTION

Analysis and Design:-continuous processes for Improvement of response ---- (Basis of Research and Development activities)

## Analysis



Pre requisites: (i) A.C single phase circuits - chapter 2-ELE 15/25
(ii) Elementary Calculus-Part B \& C - Mat 11
(iii)Differential Equations part C - MAT 21
(iv)Laplace Transform -part D - MAT 21
(v) Solutions of Simultaneous equations by Kramars Rule
(vi)Simple Matrix operations with real numbers

Co-requisite: usage of calculator (preferably CASIO fx 570ms or fx 991ms)

## Books for Reference:

(i)Engineering circuit Analysis----- Hayt, Kimmerily and Durbin for chapters 1,3,4,6,7
(ii)Network Analysis--- Van ValkenBerg- chapters 5,6,7
(iii)Network and Systems---- Roy Choudary - chapter 2

## V-I RELATIONS

| ELEMENT | VOLTAGE |  | CURRENT |  |
| :---: | :---: | :---: | :---: | :---: |
|  | t-Domain | Jw-Domain | t-Domain | Jw-Domain |
| RESISTANCE (R) | $\mathrm{v}=\mathrm{Ri}$ | V=RI | $\mathrm{i}=\mathrm{v} / \mathrm{R}$ | $\mathrm{I}=\mathrm{V} / \mathrm{R}$ |
| INDUCTANCE (L) | $\mathrm{v}=\mathrm{L}(\mathrm{di} / \mathrm{dt})$ | $\mathrm{V}=(\mathrm{JwL}) \mathrm{I}$ | $\mathrm{i}=(1 / \mathrm{L}) \int_{\mathrm{V} ~ d t}$ | $\mathrm{I}=\mathrm{V} / \mathrm{JwL}$ |
| CAPACITANCE <br> (C) | $\mathrm{V}=(1 / \mathrm{C}) \int(\mathrm{idt})$ | $\mathrm{V}=(\mathrm{J} / \mathrm{wC}) \mathrm{I}$ | $\mathrm{i}=\mathrm{C}(\mathrm{dv} / \mathrm{dt})$ | $\mathrm{I}=\mathrm{V} /(-\mathrm{J} / \mathrm{wC})$ |
| $\mathrm{X}_{\mathrm{L}}=\mathrm{wL} . \quad \mathrm{X}_{\mathrm{C}}=1 / \mathrm{wC}$ |  |  |  |  |

## BASIC LAWS



## CONNECTIONS

| SERIES | PARELLEL |
| :---: | :---: |
|  |  |
| $Z=\sum_{1}^{n} Z_{K}=Z_{1}+Z_{2}+Z_{3}---Z_{n}$ <br> Voltage Division $\begin{aligned} & \mathrm{V}_{\mathrm{i}}=\left(\mathrm{Z}_{\mathrm{i}} / \mathrm{Z}\right) \mathrm{V} \\ & \mathrm{I}=\mathrm{V} / \mathrm{Z}=\mathrm{V}_{1} / \mathrm{Z}_{1}=\mathrm{V}_{2} / \mathrm{Z}_{2}=. \end{aligned}$ | $Y=\sum_{1}^{n} Y_{K}=Y_{1}+Y_{2}+Y_{3}---Y_{n}$ <br> Current Division $\mathrm{I}_{\mathrm{I}}=\left(\mathrm{Y}_{\mathrm{i}} / \mathrm{Y}\right) \mathrm{I}$ $\mathrm{V}=\mathrm{I} / \mathrm{Y}=\mathrm{I}_{1} / \mathrm{Y}_{1}=\mathrm{I}_{2} / \mathrm{Y}_{2}=------------$ |

## Problems

1.Calculate the voltages $\mathrm{V}_{12}, \mathrm{~V}_{23}, \mathrm{~V}_{34}$ in the network shown in Fig, if $\mathrm{Va}=17.32+\mathrm{j} 10 \mathrm{~V}_{\mathrm{b}}=3080^{0}$ V and $\mathrm{V}_{\mathrm{C}=} 15-100 \mathrm{~V}$
with Calculator in complex and degree mode

$$
\begin{aligned}
& \mathrm{V}_{12}=-\mathrm{V}_{\mathrm{c}}+\mathrm{V}_{\mathrm{b}} \\
& =(0-15 L-100+30 L 80-1)=45-80^{0} \mathrm{~V} * \\
& \mathrm{~V}_{23}=\mathrm{V}_{\mathrm{a}}-\mathrm{V}_{\mathrm{b}}+\mathrm{V}_{\mathrm{c}}=\mathrm{V}_{\mathrm{a}}-\mathrm{V}_{12} \\
& =17.32+10 \mathrm{i}-45\left|80^{0}=35.61\right|-74.52^{0} \\
& \mathrm{~V}_{34}=\mathrm{V}_{\mathrm{b}}-\mathrm{V}_{\mathrm{a}}=3 Q 80-17.32-10 \mathrm{i}=23\left\llcorner 121.78^{0}\right.
\end{aligned}
$$


2. How is current of 10 A shared by 3 impedances $\mathrm{Z}_{1}=2-\mathrm{J} 5 \Omega \mathrm{Z}_{2}=6.708 \mathrm{I} 26.56$ and $\mathrm{Z}_{3}=3+\mathrm{J} 4$ all connected in parallel
Ans:

$$
\begin{aligned}
& \mathrm{Z}=\mathrm{Y}^{-1}=\left((2-5 \mathrm{i})^{-1}+(6.708 \mid 26.56)^{-1}+(3+4 \mathrm{i})^{-1}=3.06 \underline{\underline{9} .55^{0}}\right. \\
& \mathrm{V}=1 \mathrm{Z}=30.6 \underline{9.55^{0}} \quad \mathrm{I}_{1}=\mathrm{V} / \mathrm{Z}_{1}=\left(30.69 .55^{0}\right)-:(2-5 \mathrm{i})=5.6877 .75^{0} \\
& \mathrm{I}_{2}=\frac{\mathrm{V}}{\mathrm{Z}_{2}}=\left(30.6 \underline{9.55^{0}}\right) \quad \div(6.708 \underline{26.56})=4.56-17^{0} \\
& \left.\mathrm{I}_{3}=\frac{\mathrm{V}}{\mathrm{Z}}=\left(30.6 \underline{9.55^{0}}\right) \quad \div(3+4 \mathrm{i})=6.12 \right\rvert\,-43.6^{0}
\end{aligned}
$$

3. In the circuit determine what voltage must be applied across AB in order that a current of 10 A may flow in the capacitor


$$
\begin{aligned}
& \mathrm{V}_{\mathrm{AC}}=(7-8 \mathrm{i})(10)=106.3 \underline{\mathrm{~L}}-48.8^{0} \\
& \mathrm{I}_{1}=\underline{\mathrm{V}_{\mathrm{AC}}}=13.61 \underline{\mid-99^{0}} \\
& 5+6 \mathrm{i} \\
& \mathrm{I}=\underline{\mathrm{I}_{1}+\mathrm{I}_{2}}=10 \underline{0^{0}}+13.6 \underline{-99^{0}}=15.576 \mid-59.66^{0} \\
& \mathrm{~V}=\mathrm{V} 1+\mathrm{V} 2=106.3 \underline{-} 48.8+(15.576 \mid-59.66)(8+10 \mathrm{i})=289 \underline{\underline{-}} \underline{-22^{0}}
\end{aligned}
$$

## NETWORK ANALYSIS

Network is a system with interconnected electrical elements. Network and circuit are the same. The only difference being a circuit shall contain at least one closed path.

(a)Current controlled current source
(b) Voltage controlled current source
(c) Voltage controlled voltage source
(d) Current controlled voltage source

## M)

(Value of source Quantity is not affected in anyway by activities in the reminder of the circuit.)
(Source quantity is determined by a voltage or current existing at some other Location in the circuit)
These appear in the equivalent models for many electronic devices like transistors, OPAMPS and integrated circuits.


## TERMINOLOGY

## TYPES OF NETWORKS

## Linear and Nonlinear Networks:

A network is linear if the principle of superposition holds i.e if e1(t), r1 (t) and e2(t), $r 2(t)$ are excitation and response pairs then if excitation is $\mathrm{e} 1(\mathrm{t})+\mathrm{e} 2(\mathrm{t})$ then the response is $\mathrm{r} 1(\mathrm{t})$ $+\mathrm{r} 2(\mathrm{t})$.

The network not satisfying this condition is nonlinear Ex:- Linear - Resistors, Inductors, Capacitors.

Nonlinear - Semiconductors devices like transistors, saturated iron core inductor, capacitance of a p-n function.

## Passive and active Networks:

A Linear network is passive if (i) the energy delivered to the network is nonnegative for any excitation. (ii) no voltages and currents appear between any two terminals before any excitation is applied.

Example:- R,L and C.
Active network:- Networks containing devices having internal energy -Generators, amplifiers and oscillators.

## Unilateral \& Bilateral:

The circuit, in which voltage current relationship remains unaltered with the reversal of polarities of the source, is said to be bilateral.

Ex:- R, L \& C
If V-I relationships are different with the reversal of polarities of the source, the circuit is said to be unilateral.

Ex:- semiconductor diodes.

## Lumped \& Distributed:

Elements of a circuit, which are separated physically, are known as lumped elements.

Ex:- L \& C.
Elements, which are not separable for analytical purposes, are known as distributed elements.

Ex:- transmission lines having R, L, C all along their length.
In the former care Kirchhoff's laws hold good but in the latter case Maxwell’s laws are required for rigorous solution.

## Reciprocal:

A network is said to be reciprocal if when the locations of excitation and response are interchanged, the relationship between them remains the same.

## Source Transformation

In network analysis it may be required to transform a practical voltage source into its equivalent practical current source and vice versa. These are done as explained below

fig 1

fig 2

Consider a voltage source and a current source as shown in Figure 1 and 2. For the same load $\mathrm{Z}_{\mathrm{L}}$ across the terminals a \& b in both the circuits, the currents are

$$
\mathrm{I}_{\mathrm{L}}=\frac{\mathrm{E}_{\mathrm{S}}}{\mathrm{Z}_{\mathrm{s}}+\mathrm{Z}_{\mathrm{L}}} \quad \text { in fig } 1 \quad \text { and } \quad \mathrm{I}_{\mathrm{L}}=\frac{\mathrm{I}_{\mathrm{S}} . \mathrm{Z}_{\mathrm{P}}}{\mathrm{Zp}_{\mathrm{p}+\mathrm{ZL}}} \quad \text { in fig } 2
$$

For equivalence $\frac{\mathrm{E}_{\mathrm{S}}}{\mathrm{Z}_{\mathrm{S}+} \mathrm{Z}_{\mathrm{L}}}=\mathrm{I}_{\mathrm{S}} \cdot \frac{\mathrm{Z}_{\mathrm{P}}}{\mathrm{Z}_{\mathrm{P}+\mathrm{Z}_{\mathrm{L}}}}$
Therefore $\mathrm{E}_{\mathrm{S}}=\mathrm{I}_{\mathrm{S}} \mathrm{Z}_{\mathrm{P}}$ and $\mathrm{Z}_{\mathrm{S}}=\mathrm{Z}_{\mathrm{P}}$
Therefore

$$
I_{S}=\frac{E_{S}}{Z_{P}}=\frac{E_{S}}{Z_{S}}
$$

Transformation from a practical voltage source to a practical current source eliminates a node. Transformation from a practical current source to a current source eliminates a mesh.
A practical current source is in parallel with an impedance Zp is equivalent to a voltage source Es=Is Zp in series with Zp.

A practical voltage source Es in series with a impedance Zs is equivalent to a current source Es/Zs in parallel with Zs.

## SOURCE SHIFTING

Source shifting is occasionally used to simplify a network. This situation arises because of the fact than an ideal voltage source cannot be replaced by a current source. Like wise ideal current source cannot be replaced by a voltage source. But such a source transformation is still possible if the following techniques are fallowed.

(a) E shift operation

(b) I shift operation

## Sources with equivalent terminal characteristics


(i)Series voltage sources

(ii) Parallel voltage sources(ideal)

(iii) Parallel current sources

(v)Voltage source with parallel Z

(vii) V and I in Parallel

(iv)Series current sources(ideal)


(vi)Current source with series Z

(viii) V and I in Series

1. Any element in parallel with ideal voltage source (dependent or independent) is trivial
2. any element in series with ideal current source (dependent or independent) is trivial



Or


## Clues to Simplify the Network

(A network with too many trivial elements)


Fig:

Network of fig with trivial elements marked by $\qquad$


Fig

Network after removal of trivial elements


Fig

## Delta-star transformation

A set of star connected ( Y or T ) immittances can be replaced by an equivalent set of mesh ( $\Delta$ or $\pi$ ) connected immittances or vice versa. Such a transformation is often necessary to simplify passive networks, thus avoiding the need for any mesh or nodal analysis.

For equivalence, the immittance measured between any two terminals under specified conditions must be the same in either case.

## $\Delta$ to Y transformation:

Consider three $\Delta$-connected impedances $\mathrm{Z}_{\mathrm{AB}}, \mathrm{Z}_{\mathrm{BC}}$ and $\mathrm{Z}_{\mathrm{CA}}$ across terminals $\mathrm{A}, \mathrm{B}$ and C . It is required to replace these by an equivalent set $\mathrm{Z}_{\mathrm{A}}, \mathrm{Z}_{\mathrm{B}}$ and $\mathrm{Z}_{\mathrm{C}}$ connected in star.



In $\Delta$, impedance measured between A and B with C open is

$$
\frac{\mathrm{Z}_{\mathrm{AB}}\left(\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}}
$$

With C open, in Y , impedance measured between A and B is $\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}$.

$$
\begin{equation*}
\text { For equivalence } \mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}=\frac{\mathrm{Z}_{\mathrm{AB}}\left(\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}} \tag{1}
\end{equation*}
$$

Similarly for impedance measured between $B$ and $C$ with $A$ open

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}=\frac{\mathrm{Z}_{\mathrm{BC}}\left(\mathrm{Z}_{\mathrm{CA}}+\mathrm{Z}_{\mathrm{AB}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}} \tag{2}
\end{equation*}
$$

For impedance measured between C and A with B open

$$
\begin{equation*}
\mathrm{Z}_{\mathrm{C}}+\mathrm{Z}_{\mathrm{A}}=\frac{\mathrm{Z}_{\mathrm{CA}}\left(\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}} \tag{3}
\end{equation*}
$$

Adding (1), (2) and (3)

$$
\begin{aligned}
& 2\left(\mathrm{Z}_{\mathrm{A}}+\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}\right)=\frac{2\left(\mathrm{Z}_{\mathrm{AB}} \mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{BC}} \mathrm{Z}_{\mathrm{CA}}+\mathrm{Z}_{\mathrm{CA}} \mathrm{Z}_{\mathrm{AB}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}} \\
& \mathrm{Z}_{\mathrm{A}}=\frac{\left(\mathrm{Z}_{\mathrm{AB}} \mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{BC}} \mathrm{Z}_{\mathrm{CA}}+\mathrm{Z}_{\mathrm{CA}} \mathrm{Z}_{\mathrm{AB}}\right)}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{BC}}+\mathrm{Z}_{\mathrm{CA}}}-\left(\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}\right)
\end{aligned}
$$

Substituting for $\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{C}}$ from (2)

$$
\begin{aligned}
& \mathrm{Z}_{\mathrm{A}}= \frac{\mathrm{Z}_{\mathrm{CA}} \mathrm{Z}_{\mathrm{AB}}}{\mathrm{Z}_{\mathrm{AB}}+\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{CA}}}= \\
& \frac{\mathrm{Z}_{\mathrm{CA}} \mathrm{Z}_{\mathrm{AB}}}{\sum \mathrm{Z}_{\mathrm{AB}}} \\
& \mathrm{Z}_{\mathrm{AB}} \mathrm{Z}_{\mathrm{BC}} \\
& \sum \mathrm{Z}_{\mathrm{AB}}
\end{aligned}
$$

Similarly by symmetry

$$
\mathrm{Z}_{\mathrm{B}}=
$$

$\qquad$

$$
\mathrm{Z}_{\mathrm{C}}=\frac{\mathrm{Z}_{\mathrm{BC}} \mathrm{Z}_{\mathrm{CA}}}{\sum \mathrm{Z}_{\mathrm{AB}}}
$$

$$
\text { If } \mathrm{Z}_{\mathrm{AB}}=\mathrm{Z}_{\mathrm{BC}}=\mathrm{Z}_{\mathrm{CA}}=\mathrm{Z}_{\Delta} \text { then } \mathrm{Z}_{\mathrm{A}}=\mathrm{Z}_{\mathrm{B}}=\mathrm{Z}_{\mathrm{C}}=\mathrm{Z}_{\mathrm{Y}} \frac{\mathrm{Z}_{\Delta}}{3} \text {. }
$$

## $Y$ to $\Delta$ transformation:

Consider three Y connected admittance $\mathrm{Y}_{\mathrm{a}}, \mathrm{Y}_{\mathrm{b}}$ and $\mathrm{Y}_{\mathrm{c}}$ across the terminals $\mathrm{A}, \mathrm{B}$ and C. It is required to replace them by a set of equivalent $\Delta$ admittances $\mathrm{Y}_{\mathrm{ab}}, \mathrm{Y}_{\mathrm{bc}}$ and $\mathrm{Y}_{\mathrm{ca}}$.
Admittance measured between A and B with B \& C shorted

$$
\text { In } \mathrm{Y} \quad \frac{\mathrm{Y}_{\mathrm{A}}\left(\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}\right)}{\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}}
$$

In $\Delta \quad Y_{\mathrm{AB}}+\mathrm{Y}_{\mathrm{CA}}$



For equivalence $Y_{A B}+Y_{C A}=\frac{Y_{A}\left(Y_{B}+Y_{C}\right)}{Y_{A}+Y_{B}+Y_{C}}$
Admittance between B and C with C \& A shorted

$$
\begin{equation*}
\mathrm{Y}_{\mathrm{BC}}+\mathrm{Y}_{\mathrm{AB}}=\frac{\mathrm{Y}_{\mathrm{B}}\left(\mathrm{Y}_{\mathrm{C}}+\mathrm{Y}_{\mathrm{A}}\right)}{\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}} \tag{2}
\end{equation*}
$$

Admittance between C and A with A \& B shorted

$$
\begin{equation*}
\mathrm{Y}_{\mathrm{CA}}+\mathrm{Y}_{\mathrm{BC}}=\frac{\mathrm{Y}_{\mathrm{C}}\left(\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}\right)}{\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}} \tag{3}
\end{equation*}
$$

Adding (1), (2) and (3) $\mathrm{Y}_{\mathrm{AB}}+\mathrm{Y}_{\mathrm{BC}}+\mathrm{Y}_{\mathrm{CA}}=\frac{\mathrm{Y}_{\mathrm{A}} \mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{B}} \mathrm{Y}_{\mathrm{C}}+\mathrm{Y}_{\mathrm{C}} \mathrm{Y}_{\mathrm{A}}}{\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}}$
$\mathrm{Y}_{\mathrm{AB}}=\frac{\Sigma \mathrm{Y}_{\mathrm{A}} \mathrm{Y}_{\mathrm{B}}}{\Sigma \mathrm{Y}_{\mathrm{A}}}-\left(\mathrm{Y}_{\mathrm{BC}}+\mathrm{Y}_{\mathrm{CA}}\right)$
substituting from (3)
$=\frac{Y_{A} Y_{B}}{Y_{A}+Y_{B}+Y_{C}}: Y_{B C}=\frac{Y_{B} Y_{C}}{Y_{A}+Y_{B}+Y_{C}}: Y_{C A}=\frac{Y_{A} Y_{B}}{Y_{A}+Y_{B}+Y_{C}}$
In terms of impedances,
$\mathrm{Z}_{\mathrm{AB}}=\frac{\mathrm{Y}_{\mathrm{A}}+\mathrm{Y}_{\mathrm{B}}+\mathrm{Y}_{\mathrm{C}}}{\mathrm{Y}_{\mathrm{A}} \mathrm{Y}_{\mathrm{B}}}=\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{B}} \mathrm{Z}_{\mathrm{C}}+\mathrm{Z}_{\mathrm{C}} \mathrm{Z}_{\mathrm{A}}}{\mathrm{Z}_{\mathrm{C}}}$
Similarly $\mathrm{Z}_{\mathrm{BC}}=\frac{\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{B}} \mathrm{Z}_{\mathrm{C}}+\mathrm{Z}_{\mathrm{C}} \mathrm{Z}_{\mathrm{A}}}{\mathrm{Z}_{\mathrm{A}}}$

$$
\begin{gathered}
\mathrm{Z}_{\mathrm{A}} \mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{B}} \mathrm{Z}_{\mathrm{C}}+\mathrm{Z}_{\mathrm{C}} \mathrm{Z}_{\mathrm{A}} \\
\mathrm{Z}_{\mathrm{B}}
\end{gathered}
$$

$$
\begin{aligned}
& \mathrm{Z}_{\mathrm{CA}}= \\
& \text { If } \mathrm{Z}_{\mathrm{A}}=\mathrm{Z}_{\mathrm{B}}=\mathrm{Z}_{\mathrm{C}}=\mathrm{Z}_{\mathrm{Y}} \text { then } \mathrm{Z}_{\mathrm{AB}}=\mathrm{Z}_{\mathrm{BC}} \\
& =\mathrm{Z}_{\mathrm{CA}}=\mathrm{Z}_{\Delta}=3 \mathrm{Z}_{\mathrm{Y}} \text {. }
\end{aligned}
$$

## NETWORK THEOREMS

Mesh current or node voltage methods are general methods which are applicable to any network. A number of simultaneous equations are to be set up. Solving these equations, the response in all the branches of the network may be attained. But in many cases, we require the response in one branch or in a small part of the network. In such cases, we can use network theorems, which are the aides to simplify the analysis. To reduce the amount of work involved by considerable amount, as compared to mesh or nodal analysis. Let us discuss some of them.

## SUPERPOSITION THEOREM

The response of a linear network with a number of excitations applied simultaneously is equal to the sum of the responses of the network when each excitation is applied individually replacing all other excitations by their internal impedances.

Here the excitation means an independent source. Initial voltage across a capacitor and the initial current in an inductor are also treated as independent sources.

This theorem is applicable only to linear responses and therefore power is not subject to superposition.

During replacing of sources, dependent sources are not to be replaced. Replacing an ideal voltage source is by short circuit and replacing an ideal current source is by open circuit.
"In any linear network containing a number of sources, the response (current in or voltage across an element) may be calculated by superposing all the individual responses caused by each independent source acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits". Initial capacitor voltages and initial inductor currents, if any, are to be treated as independent sources.

To prove this theorem consider the network shown in fig.


We consider only one-voltage sources and only one current sources for simplicity. It is required to calculate Ia with Is acting alone the circuit becomes

$$
\left.\begin{array}{cc}
\mathrm{I}_{\mathrm{S}} & \mathrm{Z}_{1} \\
\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3} \mathrm{Z}_{4}
\end{array}\right] \begin{gathered}
\mathrm{Z}_{3} \\
\mathrm{Z}_{3}+\mathrm{Z}_{4}
\end{gathered}
$$



$$
\begin{equation*}
=I_{S} \frac{Z_{1} Z_{3}}{\left(Z_{1}+Z_{2}+Z_{3}\right) Z_{4}+\left(Z_{1}+Z_{2}\right) Z_{3}} \tag{1}
\end{equation*}
$$

with Es acting alone

$$
\begin{gather*}
\mathrm{Ia}_{1}=\frac{\left.-\mathrm{E}_{5}+Z_{2}\right) \mathrm{Z}_{3}}{} \begin{array}{c}
\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}\right) \mathrm{Z}_{1}+\mathrm{Z}_{3}
\end{array} \\
=\frac{-\mathrm{E}_{\mathrm{S}}\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}\right)}{\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}\right) \mathrm{Z}_{4}+\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}\right) \mathrm{Z}_{3}} \tag{2}
\end{gather*}
$$

Next converting the current source to voltage source, the loop equations


$$
\begin{align*}
\mathrm{I}_{2} & =\frac{\left|\begin{array}{cc}
\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3} & \mathrm{I}_{\mathrm{S}} \mathrm{Z}_{1} \\
-\mathrm{Z}_{3} & -\mathrm{E}_{\mathrm{S}}
\end{array}\right|}{\left|\begin{array}{cc}
\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3} & -\mathrm{Z}_{3} \\
-\mathrm{Z}_{3} & \mathrm{Z}_{3}+\mathrm{Z}_{4}
\end{array}\right|} \\
& =\frac{\mathrm{I}_{\mathrm{S}} \mathrm{Z}_{1} \mathrm{Z}_{3}-\mathrm{E}_{\mathrm{S}}\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}\right)}{\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{3}\right) \mathrm{Z}_{4}+\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}\right) \mathrm{Z}_{3}} \tag{3}
\end{align*}
$$

From equation (1), (2) and (3) $\mathrm{Ia}_{1}+\mathrm{Ia}_{2}=\mathrm{I}_{2}=\mathrm{Ia}$
Hence proof

## Reciprocity Theorem :

In an initially relaxed linear network containing one independent source only. The ratio of the response to the excitation is invariant to an interchange of the position of the excitation and the response.
i.e if a single voltage source Ex in branch $X$ produces a current response $I_{y}$ the branch $Y$, then the removal of the voltage source from branch x and its insertion in branch Y will produce the current response $\mathrm{I}_{\mathrm{y}}$ in branch X .

Similarly if the single current source Ix between nodes $X$ and $X^{\prime}$ produces the voltage response Vy between nodes $Y$ and $Y^{\prime}$ then the removal of the current source from $X$ and $X^{\prime}$ and its insertion between Y and $\mathrm{Y}^{\prime}$ will produce the voltage response Vy between the nodes X and $\mathrm{X}^{\prime}$.

Between the excitation and the response, one is voltage and other is current. It should be noted that after the source and response are interchanged, the current and the voltages in other parts of the network will not remain the same.

## Proof:



Consider a network as shown in which the excitation is E and the response is I in $\mathrm{Z4}$. The reading of the ammeter is

$$
\mathrm{I}_{1}=\frac{\mathrm{E}}{\mathrm{Z}_{1}+\frac{\mathrm{Z}_{3}\left(\mathrm{Z}_{2}+\mathrm{Z}_{4}\right)}{\mathrm{Z}_{2}+\mathrm{Z}_{3}+\mathrm{Z}_{4}}} \quad \cdot \frac{\mathrm{Z}_{3}}{\mathrm{Z}_{2}+\mathrm{Z}_{3}+\mathrm{Z}_{4}}
$$

$$
I_{1}=\frac{E Z_{3}}{Z_{1}\left(Z_{2}+Z_{3}+Z_{4}\right)+Z_{3}\left(Z_{2}+Z_{4}\right)}
$$

Next interchange the source and ammeter.
E


Now the reading of the Ammeter is :

$$
I_{2}=\frac{E}{\left(Z_{2}+Z_{4}\right)+\frac{Z_{1} Z_{3}}{Z_{1}+Z_{3}}} \quad \cdot \frac{Z_{3}}{Z_{1}+Z_{3}}
$$

$$
\mathrm{I}_{2}=\frac{\mathrm{E} \mathrm{Z}_{3}}{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{3}+\mathrm{Z}_{4}\right)+\mathrm{Z}_{3}\left(\mathrm{Z}_{2}+\mathrm{Z}_{4}\right)}
$$

From (1) \& (2)

$$
\mathbf{I}_{1}=\mathbf{I}_{2}
$$

It can be similarly be shown for a network with current sources by writing node equations.

## Transfer Impedance :

The transfer impedance between any two pairs of terminals of a linear passive network is the ratio of the voltage applied at one pair of terminals to the resulting current at the other pair of terminals .

With this definition the reciprocity theorem can be stated as :
"Only one value of transfer impedance is associated with two pairs of terminals of a linear passive network " .

w.r.t figs shown $\frac{\mathrm{E} 1}{\mathrm{I}_{2}}=\frac{\mathrm{E} 2}{\mathrm{I}_{1}}=\mathrm{ZT}$

If $\mathrm{E} 1=\mathrm{E} 2$ then $\mathrm{I} 1=\mathrm{I} 2$.

## Thevinin's and Norton's Theorems:

If we are interested in the solution of the current or voltage of a small part of the network, it is convenient from the computational point of view to simplify the network, except that small part in question, by a simple equivalent. This is achieved by Thevinin's Theorem or Norton's theorem.

## Thevinin's Theorem :

If two linear networks one M with passive elements and sources and the other N with passive elements only and there is no magnetic coupling between M and N , are connected together at terminals A and B, then with respect to terminals A and B, the network M can
be replaced by an equivalent network comprising a single voltage source in series with a single impedance. The single voltage source is the open circuit voltage across the terminals A and B and single series impedance is the impedance of the network M as viewed from A and B with independent voltage sources short circuited and independent current sources open circuited. Dependent sources if any are to be retained.

Arrange the networks M and N such that N is the part of the network where response is required.


To prove this theorem, consider the circuit shown in Fig.


Suppose the required response is the current IL in ZL. Connected between A and B. According to Thevinin's theorem the following steps are involved to calculate IL

Step 1:
Remove ZL and measure the open circuit voltage across AB. This is also called as Thevinin's voltage and is denoted as VTH


$$
\begin{aligned}
& V_{T H}=V_{A B}=E_{1}-\frac{\mathrm{E}_{1}-\mathrm{I}_{\mathrm{s}} \mathrm{Z}_{\mathrm{S}}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}} \mathrm{Z}_{1}+\mathrm{E}_{2} \\
& \mathrm{~V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{AB}}=\frac{\left(\mathrm{E}_{1}+\mathrm{E}_{2}\right)\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}\right)-\left(\mathrm{E}_{1}-\mathrm{I}_{\mathrm{S}} \mathrm{Z}_{\mathrm{s}}\right) \mathrm{Z}_{1}}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}}
\end{aligned}
$$

## Step 2:

To obtain the single impedance as viewed from A and B, replace the network in Fig. replacing the sources. This single impedance is called Thevinin's Impedance and is denoted by $\mathrm{Z}_{\mathrm{TH}}$

$\mathrm{Z}_{\mathrm{TH}}=\frac{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}}$
Step 3 :

Write the thevinin's network and re introduce ZL


Then the current in $\mathrm{Z}_{\mathrm{L}}$ is

$$
\mathrm{I}_{\mathrm{L}}=\frac{\mathrm{V}_{\mathrm{TH}}}{\mathrm{Z}_{\mathrm{TH}}+\mathrm{Z}_{\mathrm{L}}}
$$

$$
\begin{gathered}
=\frac{\frac{\left(E_{1}+E_{2}\right)\left(Z_{1}+Z_{2}+Z_{s}\right)-\left(E_{1}-I_{s} Z_{s}\right) Z_{1}}{Z_{1}+Z_{2}+Z_{s}}}{\frac{Z_{1}\left(Z_{2}+Z_{s}\right)}{Z_{1}+Z_{2}+Z_{s}}+Z_{L}} \\
=\frac{\left(E_{1}+E_{2}\right)\left(Z_{1}+Z_{2}+Z_{s}\right)-\left(E_{1}-I_{s} Z_{s}\right) Z_{1}}{Z_{1}\left(Z_{2}+Z_{s}\right)+Z_{2}\left(Z_{1}+Z_{2}+Z_{s}\right)}
\end{gathered}
$$

To verify the correctness of this, write loop equations for the network to find the current in ZL
$\left|\begin{array}{ll}\left(\mathrm{E}_{1}+\mathrm{E}_{2}\right) & \mathrm{Z}_{1} \\ \left(\mathrm{E}_{1}-\mathrm{I}_{\mathrm{S}} \mathrm{Z}_{\mathrm{s}}\right) & \mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}} \\ \hline \mathrm{Z}_{1}+\mathrm{Z}_{\mathrm{L}} & \mathrm{Z}_{1} \\ \mathrm{Z}_{1} & \mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}\end{array}\right|$

$$
\begin{aligned}
& =\frac{\left(E_{1}+E_{2}\right)\left(Z_{1}+Z_{2}+Z_{s}\right)-\left(E_{1}-I_{s} Z_{s}\right) Z_{1}}{\left(Z_{1}+Z_{L}\right)\left(Z_{1}+Z_{2}+Z_{s}\right)-Z_{1}^{2}} \\
& =\frac{\left(E_{1}+E_{2}\right)\left(Z_{1}+Z_{2}+Z_{s}\right)-\left(E_{1}-I_{s} Z_{s}\right) Z_{1}}{Z_{1}\left(Z_{2}+Z_{s}\right)+Z 2\left(Z_{1}+Z_{2}+Z_{s}\right)}
\end{aligned}
$$

Norton's Theorem :-


The Thevinins equivalent consists of a voltage source and a series impedance . If the circuit is transformed to its equivalent current source, we get Nortons equivalent. Thus Norton's theorem is the dual of the Thevinin's theorem.

If two linear networks, one M with passive elements and sources and the other N with passive elements only and with no magnetic coupling between $M$ and $N$, are connected together at terminals A and B, Then with respect to terminals A and B, the network M can be replaced by a single current source in parallel with a single impedance. The single current source is the short circuit current in AB and the single impedance is the impedance of the network $M$ as viewed from $A$ and $B$ with independent sources being replaced by their internal impedances

The proof of the Norton's theorem is simple
Consider the same network that is considered for the Thevinin's Theorem and for the same response.

Step 1: Short the terminals A and B and measure the short circuit current in AB, this is Norton's


$$
\mathrm{I}_{\mathrm{N}}=\mathrm{I}_{\mathrm{sc}}=\frac{\mathrm{E}_{1}+\mathrm{E}_{2}}{\mathrm{Z}_{1}}+\frac{\mathrm{E}_{2}+\mathrm{I}_{\mathrm{s}} \mathrm{Z}_{\mathrm{S}}}{\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}}
$$

$$
=\left(\frac{\left.\mathrm{E}_{1}+\mathrm{E}_{2}\right)\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}\right)+\left(\mathrm{E}_{2}+\mathrm{I}_{\mathrm{S}} \mathrm{Z}_{\mathrm{S}}\right)}{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{S}}\right)} \mathrm{Z}_{1}\right.
$$

Step 2: This is the same as in the case of thevnin's theorem
Step 3: write the Nortons equivalent and reintroduce $\mathrm{Z}_{\mathrm{L}}$


Then the current in $\mathrm{Z}_{\mathrm{L}}$ is

$$
\begin{aligned}
& \begin{array}{l}
\mathrm{I}_{\mathrm{L}}=\mathrm{I}_{\mathrm{N}} \cdot \mathrm{Z}_{\mathrm{n}} \\
\mathrm{Z}_{\mathrm{n}}+\mathrm{Z}_{\mathrm{L}} \\
=
\end{array} \\
&=\frac{\left(\mathrm{E}_{1}+\mathrm{E}_{2}\right)\left(\mathrm{Z}_{2}+\mathrm{Zs}^{2}+\left(\mathrm{E}_{2}+\mathrm{I}_{\mathrm{s}} \mathrm{Z}_{\mathrm{s}}\right) \mathrm{Z}_{1}\right.}{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)} \frac{\cdot \mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}} \\
&=\frac{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)}{\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}} \mathrm{Z}_{\mathrm{L}} \\
&= \frac{\left(\mathrm{E}_{1}+\mathrm{E}_{2}\right)\left(\mathrm{Z}_{2}+\mathrm{Zs}\right)+\left(\mathrm{E}_{2}+\mathrm{I}_{\mathrm{s}} \mathrm{Z}_{\mathrm{s}}\right) \mathrm{Z}_{1}}{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)+\mathrm{Z}_{\mathrm{L}}\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)} \\
&= \frac{\left(\mathrm{E}_{1}+\mathrm{E}_{2}\right)\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Zs}\right)-\left(\mathrm{E}_{1}-\mathrm{I}_{\mathrm{s}} \mathrm{Z}_{\mathrm{s}}\right) \mathrm{Z}_{1}}{\mathrm{Z}_{1}\left(\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)+\mathrm{Z}_{\mathrm{L}}\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}+\mathrm{Z}_{\mathrm{s}}\right)}
\end{aligned}
$$

Verification is to be done as in Thevinin's Theorem
Determination of Thevinin's or Norton's equivalent when dependent sources are present
Since

$$
\mathrm{I}_{\mathrm{L}}=\frac{\mathrm{V}_{\mathrm{TH}}}{\mathrm{Z}_{\mathrm{TH}}+\mathrm{Z}_{\mathrm{L}}} \frac{=\mathrm{I}_{\mathrm{N}} \cdot \mathrm{Z}_{\mathrm{TH}}}{\mathrm{Z}_{\mathrm{TH}}+\mathrm{Z}_{\mathrm{L}}}
$$

$\mathrm{Z}_{\mathrm{TH}}$ can also be determined as $\mathrm{Z}_{\mathrm{TH}} \quad=\frac{\mathrm{V}_{\mathrm{TH}}}{\mathrm{I}_{\mathrm{N}}}=\frac{\text { o.c voltage across AB }}{\text { s.c current in } \mathrm{AB}}$
When network contains both dependent and independent sources. It is convenient to determine $\mathrm{Z}_{\text {TH }}$ by finding both the open circuit voltage and short circuit current

If the network contains only dependent sources both $\mathrm{V}_{T H}$ and $\mathrm{I}_{\mathrm{N}}$ are zero in the absence of independent sources. Then apply a constant voltage source (or resultant source) and the ratio of voltage to current gives the $\mathrm{Z}_{\mathrm{TH}}$. However there cannot be an independent source ie, $\mathrm{V}_{\mathrm{TH}}$ or $\mathrm{I}_{\mathrm{N}}$ in the equivalent network.

Maximum Transfer Theorem:-
When a linear network containing sources and passive elements is connected at terminals A and B to a passive linear network, maximum power is transferred to the passive network when its impedance becomes the complex conjugate of the Thevinin's impedance of the source containing network as viewed form the terminals A and B.

Fig represents a network with sources replaced by its Thevinin's equivalent of source of $\mathrm{E}_{\mathrm{TH}}$ volts and impedance $\mathrm{Z}_{\mathrm{s}}$, connected to a passive network of impedance z at terminals A \& B . With $\mathrm{Z}_{\mathrm{s}}=$ Rs +JXs and $\mathrm{z}=\mathrm{R}+\mathrm{JX}$, The proof of the theorem is as follows
Current in the circuit is

$$
\mathrm{I}=\frac{\mathrm{E}_{T H}}{\sqrt{(\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2}}}
$$


power delivered to the load is $\mathrm{P}=\mathrm{I}^{2} \mathrm{R}$

$$
\begin{equation*}
=\frac{\mathrm{E}^{2} \mathrm{Th}}{(\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2}} \cdot \mathrm{R} \quad \square \tag{2}
\end{equation*}
$$

As $P=\int(R, X)$ and since $P$ is maximum when $d P=0$
We have $\mathrm{dP}=\frac{\delta \mathrm{P}}{\delta \mathrm{R}} \cdot \mathrm{dR}+\frac{\delta \mathrm{P}}{\delta \mathrm{X}} \cdot \mathrm{dX}$
power is maximum when $\frac{\delta \mathrm{P}}{\delta \mathrm{R}}=0$ and $\frac{\delta \mathrm{P}}{\delta \mathrm{X}}=0$ simultaneously
$\frac{\delta \mathrm{P}}{\delta \mathrm{R}}=\frac{(\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2}-\mathrm{R}\{2(\mathrm{Rs}+\mathrm{R})\}}{\mathrm{D}^{2}}=0$
ie, $(\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2}-2 \mathrm{R}\{2(\mathrm{Rs}+\mathrm{R})\} \quad=0$ $\qquad$

```
\(\frac{\delta \mathrm{P}}{\delta \mathrm{X}}=\frac{-\mathrm{R}\{2(\mathrm{Xs}+\mathrm{X})\}}{\mathrm{D}^{2}}=0\)
ie \(2 R(X s+X)=0\)
From (5) we have \(\mathrm{X}=-\mathrm{Xs}\)
Substituting in \((4)(R s+R)^{2}=2 R(R s+R)\), ie, \(R s+R=2 R\) ie, \(\mathrm{R}=\mathrm{Rs}\)
```

Alternatively as $\mathrm{P}=$ $\qquad$

$$
\begin{align*}
& (\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2} \\
= & \frac{\mathrm{E}^{2} \mathrm{Z} \operatorname{Cos} \Theta}{(\mathrm{Rs}+\mathrm{ZCos} \Theta)^{2}+(\mathrm{Xs}+\mathrm{ZSin} \Theta)^{2}} \\
= & \frac{\mathrm{E}^{2} \mathrm{Z} \operatorname{Cos} \Theta}{\mathrm{Zs}^{2}+\mathrm{Z}^{2}+2 \mathrm{ZZsCos}(\Theta-\Theta \mathrm{s})} \tag{7}
\end{align*}
$$

ie $\mathrm{P}=\mathrm{f}(\mathrm{Z}, \mathrm{B})$

$$
\mathrm{dP}=\frac{\delta \mathrm{P}}{\delta Z} \cdot \mathrm{dZ}+\frac{\delta \mathrm{P}}{\delta \Theta} \cdot \mathrm{~d} \boldsymbol{\theta}=0
$$

for Pmax

$$
\begin{equation*}
\frac{\delta \mathrm{P}}{\delta \mathrm{Z}}=0=\left\{\mathrm{Z}_{\mathrm{s}}^{2}+\mathrm{Z}^{2}+2 \mathrm{Z} \mathrm{Z} \mathrm{Z}_{\mathrm{s}} \operatorname{Cos}(\Theta-\Theta \mathrm{s})\right\} \operatorname{Cos} \Theta-\mathrm{Z} \operatorname{Cos} \Theta\left\{2 \mathrm{Z}+2 \mathrm{Z}_{\mathrm{s}} \operatorname{Cos}(\Theta-\Theta \mathrm{s})\right\} \tag{8}
\end{equation*}
$$

ie $\mathrm{Z}_{\mathrm{s}}{ }^{2}+\mathrm{Z}^{2}=2 \mathrm{Z}^{2}+2 \mathrm{Z} \mathrm{Z}_{\mathrm{s}} \operatorname{Cos}(\Theta-\Theta \mathrm{s})$. Or $|\mathrm{Z}|=\left|\mathrm{Z}_{\mathrm{s}}\right|$
then with
$\delta \mathrm{P}=0=\left\{\mathrm{Z}_{\mathrm{s}}{ }^{2}+\mathrm{Z}^{2}+2 \mathrm{Z} \mathrm{Z}_{\mathrm{s}} \operatorname{Cos}(\Theta-\Theta \mathrm{s})\right\} \mathrm{Z}(-\operatorname{Sin} \Theta)-\mathrm{Z} \operatorname{Cos} \Theta\left\{\mathrm{ZS}^{2}+\mathrm{Z}^{2} 2 \mathrm{Z} \mathrm{Z} \mathrm{Z}_{\mathrm{s}} \operatorname{Sin}\left(\boldsymbol{\Theta}-\boldsymbol{\theta}_{\mathrm{s}}\right)\right\}$ $\delta \Theta$

$$
\begin{align*}
\left(\mathrm{Z}_{\mathrm{s}}^{2}+\mathrm{Z}^{2}\right) \operatorname{Sin} \Theta & =2 \mathrm{Z} \mathrm{Z} \mathrm{Z}_{\mathrm{s}}\left\{\operatorname{Cos} \Theta \operatorname{Sin}\left(\Theta-\Theta_{\mathrm{s}}\right)-\operatorname{Sin} \Theta \operatorname{Cos}\left(\Theta-\Theta_{\mathrm{s}}\right)\right\} \\
& =-2 \mathrm{Z} \mathrm{Z}_{\mathrm{s}} \operatorname{Sin} \Theta_{\mathrm{s}} \tag{9}
\end{align*}
$$

Substituting (8) in (9)

$$
\begin{array}{r}
2 \mathrm{Z}_{\mathrm{s}} \operatorname{Sin} \boldsymbol{\theta}=-2 \mathrm{Z}_{\mathrm{s}}^{2} \operatorname{Sin} \boldsymbol{\theta}_{\mathrm{s}} \\
\boldsymbol{\theta}=-\boldsymbol{\theta}_{\mathrm{s}} \\
\mathrm{Z} \bigsqcup \boldsymbol{\theta}=\mathrm{Zs} \underline{-\boldsymbol{\theta}_{\mathrm{s}}}
\end{array}
$$

Efficiency of Power Transfer:
With Rs $=\mathrm{R}_{\mathrm{L}}$ and $\mathrm{Xs}=-\mathrm{X}_{\mathrm{L}}$ Substituting in (1)

$$
P_{\text {Lmax }}=\frac{E^{2} T H R}{(2 R)^{2}}=\frac{E^{2} T H}{4 R}
$$

and the power supplied is $\mathrm{Ps}=\frac{\mathrm{E}_{\mathrm{TH}}^{2} 2 \mathrm{R}}{(2 \mathrm{R})^{2}}=\frac{\mathrm{E}^{2}{ }_{\mathrm{TH}}}{2 \mathrm{R}}$

This means to transmit maximum power to the load $50 \%$ power generated is the loss. Such a low efficiency cannot be permitted in power systems involving large blocks of power where $\mathrm{R}_{\mathrm{L}}$ is very
large compared to Rs. Therefore constant voltage power systems are not designed to operate on the basis of maximum power transfer.

However in communication systems the power to be handled is small as these systems are low current circuits. Thus impedance matching is considerable factor in communication networks.

However between $R \& X$ if either $R$ or $X$ is restricted and between $Z$ and $\Theta$ if either $|Z|$ or $\Theta$ is restricted the conditions for Max P is stated as follows

Case (i) :- R of Z is varied keeping X constant with R only Variable, conditions for max power transfer is $(R s+R)^{2}+(X s+X)^{2}-2 R\left(R_{s}+R\right)=0$

$$
\begin{aligned}
& \mathrm{Rs}^{2}+\mathrm{R}^{2}+2 \mathrm{RsR}+(\mathrm{Xs}+\mathrm{X})^{2}-2 \mathrm{RsR}-2 \mathrm{R}^{2}=0 \\
& \mathrm{R}^{2}=\mathrm{Rs}^{2}+(\mathrm{Xs}+\mathrm{X})^{2} \\
& \mathrm{R}=\sqrt{\mathrm{Rs}^{2}+(\mathrm{Xs}+\mathrm{X})^{2}}
\end{aligned}
$$

Case (ii):- If Z contains only R ie, $\mathrm{x}=0$ then from the eqn derived above

$$
\mathrm{R}=|\mathrm{Zs}| \cdot \sqrt{\mathrm{Rs}^{2}+\mathrm{Xs}^{2}}
$$

Case (iii):- If $|\mathrm{Z}|$ is varied keeping $\theta$ constant then from (8) $\quad|\mathrm{Z}|=|\mathrm{Zs}|$
Case (iv):- If $|\mathrm{Z}|$ is constant but $\theta$ is varied
Then from eqn (9) $\left(Z^{2}+Z s^{2}\right) \operatorname{Sin} \theta=-2 Z Z s \operatorname{Sin} \theta s$

$$
\operatorname{Sin} \theta=\frac{-2 Z Z s}{\left(Z^{2}+Z s^{2}\right)} \operatorname{Sin} \theta s
$$

Then power transfer to load may be calculated by substituting for R and X for specified condition. For example
For case(ii) Pmax is given by

$$
\begin{aligned}
& \text { Pmax }=\frac{E^{2} R}{(\mathrm{Rs}+\mathrm{R})^{2}+(\mathrm{Xs}+\mathrm{X})^{2}} \\
&= \frac{\mathrm{E}^{2} \mathrm{Zs}}{(\mathrm{Rs}+\mathrm{Zs})^{2}+\mathrm{Xs}^{2}}=\frac{\mathrm{E}^{2} \mathrm{Zs}}{\mathrm{Rs}^{2}+2 \mathrm{RsZs}^{2}+\mathrm{Zs}^{2}+\mathrm{Xs}^{2}} \\
&=\frac{\mathrm{E}^{2}}{2(\mathrm{Zs}+\mathrm{Rs})} \quad\left(\text { ie } \mathrm{Rs}^{2}+\mathrm{Xs}^{2}=\mathrm{Zs}^{2}\right)
\end{aligned}
$$

Millman's Theorem:
Certain simple combinations of potential and current source equivalents are of use because they offer simplification in solutions of more extensive networks in which combinations occur. Millman's Theorem says that "if a number of voltage sources with internal impedances are connected in parallel across two terminals, then the entire combination can be replaced by a single voltage source in series with single impedance".

The single voltage is the ratio

## Sum of the product of individual voltage sources and their series admittances <br> Sum of all series admittances

and the single series impedance is the reciprocal of sum of all series admittances.


Let $\mathrm{E}_{1}$, E2. $\qquad$ . $\mathrm{E}_{\mathrm{n}}$ be the voltage sources and $\mathrm{Z}_{1}, \mathrm{Z} 2$. $\qquad$ $\mathrm{Z}_{\mathrm{n}}$ are their respective impedances. All these are connected between A \& B with $\mathrm{Y}=1 / \mathrm{Z}$, according to Millman's Theorem, the single voltage source that replaces all these between A \& B is

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{AB}} \stackrel{\mathrm{n}}{\mathrm{~K}=1} \sum_{\mathrm{K}} \mathrm{E}_{\mathrm{K}} \mathrm{Y}_{\mathrm{K}} \\
& \text { n } \\
& \sum_{\mathrm{K}=1} \mathrm{Y}_{\mathrm{K}}
\end{aligned}
$$

And

The single impedance is

$$
\mathrm{Z}=\frac{1}{\sum_{\mathrm{K}=1}^{\mathrm{n}} \mathrm{Y}_{\mathrm{K}}}
$$

Proof: Transform each voltage into its equivalent current source. Then the circuit is as in Fig.


With $\mathrm{Y}=1 / \mathrm{Z}$ the circuit is simplified as $\mathrm{E}_{1} \mathrm{Y} 1+\mathrm{E}_{2} \mathrm{Y} 2+\ldots \ldots \ldots . . \mathrm{EnYn}=\Sigma \mathrm{E}_{\mathrm{K}} \mathrm{Y}_{\mathrm{K}}$


Which is a single current source in series with a single admittance
Retransforming this into the equivalent voltage source


The theorem can be stated as "If a number of current sources with their parallel admittances are connected in series between terminals A and B, then they can be replaced by a single current source in parallel with a single admittance. The single current source is the ratio

## Sum of products of individual current sources and their impedances <br> Sum of all shunt impedances

And the single shunt admittance is the reciprocal of the sum of all shunt impedances.
Let $\mathrm{I}_{1}, \mathrm{I}_{2}$, $\qquad$ . $\mathrm{I}_{\mathrm{n}}$ be the n number of current sources and $\mathrm{Y}_{1}, \mathrm{Y}$ $\qquad$ Yn be their respective shunt admittances connected in series between A \& B. Then according to Millman's

Theorem they can be replaced by single current $\mathrm{I}_{\mathrm{AB}}$ in parallel with a single admittance $\mathrm{Y}_{\mathrm{AB}}$ where $\mathrm{I}_{\mathrm{AB}}=\Sigma \mathrm{I}_{\mathrm{K}} \mathrm{Z}_{\mathrm{K}}$

$$
\Sigma Z_{\mathrm{K}}
$$

And $\quad \mathrm{Y}_{\mathrm{AB}}=\frac{1}{\Sigma \mathrm{Z}_{\mathrm{K}}}$


Transform each current source into its equivalent voltage source to get the circuit as in fig
B

$=$

$$
=\frac{1}{\Sigma \mathbf{Z}_{\mathbf{k}}}
$$

## TWO PORT PARAMETERS



PORT:- Pair of terminals at which an electrical signal enters or leaves a network.
One port network:- Network having only one port.
Ex: Domestic appliances, Motor, Generator, Thevinin's or Norton networks
Two port network:- Network having an input port and an output port.
Ex:Amplifiers,Transistors, communication circuits, Power transmission \& distribution lines Filters,attenuators ,transformers etc

Multi port network:-Network having more than two ports.
Ex: PowerTransmission lines, DistributionsLines,Communication lines.
Two port networks act as building blocks of electrical or electronic circuits such as electronic systems, communication circuits, control systems and transmission \& distribution systems. A one port or two port network can be connected with another two port network either in cascade, series or in parallel. In Thevinins or Nortons networks , we are not interested in the detailed working of a major part of the network. Similarly it is not necessary to know the inner working of the two port network but by measuring the voltages and currents at input and at output port, the network can be characterized with a set of parameters to predict how a two port network interact with other

## networks.Often the circuit between the two ports is highly complex The two port parameters provide a shorthand method for analyzing the input-output properties of two ports without having to deal directly with the highly complex circuit internal to the two port.

These networks are linear and passive and may contain controlled sources but not independent sources.inside..

While defining two port parameters we put the condition that one of the ports is either open circuited or short circuited.

In these networks there are four variables $V_{1}, I_{1}$ and $V_{2}, I_{2}$. Two of them are expressed in terms of the other two, to define two port parameters.

## Four important Parameters

| Sl. <br> No. | Parameters | Dependent <br> Variable | Independent <br> Variable | Equations |
| :--- | :--- | :--- | :--- | :--- |
| 1. | z Parameters | $\mathrm{V}_{1}, \mathrm{~V}_{2}$ | $\mathrm{I}_{1}, \mathrm{I}_{2}$ | $\left[\begin{array}{l}V_{1} \\ V_{2}\end{array}\right]=\left[\begin{array}{ll}z_{11} & z_{12} \\ z_{21} & z_{22}\end{array}\right]\left[\begin{array}{l}I_{1} \\ I_{2}\end{array}\right]$ |
| 2. | y parameters | $\mathrm{I}_{1}, \mathrm{I}_{2}$ | $\mathrm{~V}_{1}, \mathrm{~V}_{2}$ | $\left[\begin{array}{l}I_{1} \\ I_{2}\end{array}\right]=\left[\begin{array}{ll}y_{11} & y_{12} \\ y_{21} & y_{22}\end{array}\right]\left[\begin{array}{l}V_{1} \\ V_{2}\end{array}\right]$ |
| 3. | h parameters | $\mathrm{V}_{1}, \mathrm{I}_{2}$ | $\mathrm{I}_{1}, \mathrm{~V}_{2}$ | $\left[\begin{array}{l}\mathrm{V}_{1} \\ \mathrm{I}_{2}\end{array}\right]=\left[\begin{array}{ll}\mathrm{h}_{11} & \mathrm{~h}_{12} \\ \mathrm{~h}_{21} & \mathrm{~h}_{22}\end{array}\right]\left[\begin{array}{l}\mathrm{I}_{1} \\ \mathrm{~V}_{2}\end{array}\right]$ |
| 4. | t parameters | $\mathrm{V}_{1}, \mathrm{I}_{1}$ | $\mathrm{~V}_{2}, \mathrm{I}_{2}$ | $\left[\begin{array}{l}V_{1} \\ I_{1}\end{array}\right]=\left[\begin{array}{ll}A & B \\ C & D\end{array}\right]\left[\begin{array}{l}V_{2} \\ I_{2}\end{array}\right]$ |

DEFINITIONS
(1) Z parameters (open circuit impedance parameters)
$\left.\mathrm{V}_{1}=\mathrm{z}_{11} \mathrm{I}_{1}+\mathrm{z}_{12} \mathrm{I}_{2} \quad \mathrm{z}_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{\mathrm{I}_{2}=0} \quad z_{12}=\frac{V_{1}}{I_{2}} \right\rvert\, \mathrm{I}_{1}=0$
$\mathrm{V}_{2}=\mathrm{Z}_{21} \mathrm{I}_{1}+\mathrm{Z}_{22} \mathrm{I}_{2}$
$Z_{21}=\left.\frac{V_{2}}{I_{1}}\right|_{I_{2}=0}$
$Z_{22}=\left.\frac{V_{2}}{I_{2}}\right|_{I_{1}=0}$

For $\mathrm{z}_{11}$ and $\mathrm{z}_{21}$ - output port opened $\}$
Hence the name open circuit impedance parameters

Equivalent networks in terms of controlled sources ;
Network (i)


Network (ii) By writing

$$
\begin{aligned}
& V_{1}=\left(\mathrm{z}_{11}-\mathrm{z}_{12}\right) \mathrm{I}_{1}+\mathrm{z}_{12}\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right) \\
& \mathrm{V}_{2}=\left(\mathrm{z}_{21}-\mathrm{z}_{12}\right) \mathrm{I}_{1}+\left(\mathrm{z}_{22}-\mathrm{z}_{12}\right) \mathrm{I}_{2}+\mathrm{z}_{12}\left(\mathrm{I}_{1}+\mathrm{I}_{2}\right)
\end{aligned}
$$



The z parameters simplify the problem of obtaining the characteristics of two 2 port networks connected in series
(2) y parameters

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{y}_{11} \mathrm{~V}_{1}+\mathrm{y}_{12} \mathrm{~V}_{2} \\
& \left.y_{1_{2}}=\frac{I_{1}}{\mathbb{W}_{1}} \right\rvert\, \\
& y_{11_{1}}=\frac{I_{1}}{D_{2}} \\
& \left.\mathrm{I}_{2}=\mathrm{y}_{21} \mathrm{~V}_{1}+\mathrm{y}_{22} \mathrm{~V}_{2} \quad y_{\mathbf{1}_{2}}=\frac{I_{2}}{\mathbb{W}_{1}} \right\rvert\, \\
& y_{2_{1}}=\frac{I_{2}}{=D_{2}}
\end{aligned}
$$

For $y_{11}$ and $y_{21}$ - port 2 is shorted $\mathrm{Z}_{12}$ and $\mathrm{z}_{22}$ - port 1 is shorted

Hence they are
called short circuit admittance parameters

Equivalent networks in terms of controlled sources

(ii) by writing

$$
\begin{aligned}
& I_{1}=\left(y_{11}+y_{12}\right) V_{1}-y_{12}\left(V_{1}+V_{2}\right) \\
& I_{2}=\left(y_{21}-y_{12}\right) V_{1}+\left(y_{22}+y_{12}\right) V_{2}-y_{12}\left(V_{2}-V_{1}\right)
\end{aligned}
$$



The y parameters are very useful to know the characteristics of two 2 port Networks connected in parallel

## Hybrid parameters:-

$$
\begin{array}{lll}
\mathrm{V}_{1}=\mathrm{h}_{11} \mathrm{I}_{1}+\mathrm{h}_{12} \mathrm{~V}_{2} & \left.h_{11}=\frac{V_{1}}{I_{1}} \right\rvert\, \mathrm{V}_{2}=0 & \left.h_{12}=\frac{V_{1}}{V_{2}} \right\rvert\, \mathrm{I}_{1}=0 \\
\mathrm{I}_{2}=\mathrm{h}_{21} \mathrm{I}_{1}+\mathrm{h}_{22} \mathrm{~V}_{2} & \left.h_{22}=\frac{I_{2}}{I_{1}} \right\rvert\, \mathrm{V}_{2}=0 & \left.h_{22}=\frac{I_{2}}{V_{2}} \right\rvert\, \mathrm{I}_{1}=0
\end{array}
$$

Equivalent Network in terms of controlled sources;


Parameter values for bipolar junction transistors are commonly quoted In terms of $h$ parameters

## Transmission or ABCD parameters

$$
\begin{array}{lll}
\mathrm{V}_{1}=\mathrm{AV}_{2}-\mathrm{BI}_{2} & \left.A=\frac{V_{1}}{V_{2}} \right\rvert\, \mathrm{I}_{2}=0 & \left.B=\frac{V_{1}}{-I_{2}} \right\rvert\, \mathrm{V}_{2}=0 \\
\mathrm{I}_{1}=\mathrm{CV}_{2}-\mathrm{DI}_{2} & \left.C=\frac{I_{1}}{V_{2}} \right\rvert\, \mathrm{I}_{2}=0 & \left.D=\frac{I_{1}}{-I_{2}} \right\rvert\, \mathrm{V}_{2}=0
\end{array}
$$

As the name indicates the major use of these parameters arise in transmission Line analysis and when two 2 ports are connected in cascade

## Relationship between two port parameters:-

Relationship between different two port parameters can be obtained as follows. From the given set of two port parameters, rearrange the equations collecting terms of dependent variables of new set of parameters to the left. Then form matrix equations and from matrix manipulations obtain the new set in terms of the given set.
(i) Relationship between z and y parameters for x parameters

$$
\begin{aligned}
{[\mathrm{V}] } & =[\mathrm{z}][\mathrm{I}] \\
{[I] } & =[\mathrm{z}]^{-1}[\mathrm{~V}] \\
\text { then } \left.\begin{array}{l}
I_{1} \\
I_{2}
\end{array}\right] & =\left[\begin{array}{ll}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{array}\right]^{-1}\left[\begin{array}{c}
V_{1} \\
V_{2}
\end{array}\right] \\
& =\frac{1}{\Delta_{z}}\left[\begin{array}{cc}
z_{22} & -z_{12} \\
-z_{21} & z_{11}
\end{array}\right]\left[\begin{array}{l}
V_{1} \\
V_{2}
\end{array}\right] \quad \text { where } \Delta_{z}=\mathrm{z}_{11} \mathrm{z}_{22}-\mathrm{z}_{12} \mathrm{z}_{21} \\
{\left[\begin{array}{ll}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{array}\right] } & =\left[\begin{array}{cc}
\frac{z_{22}}{\Delta_{z}} & \frac{-z_{12}}{\Delta_{z}} \\
\frac{-z_{21}}{\Delta_{z}} & \frac{z_{11}}{\Delta_{z}}
\end{array}\right] \\
\text { similarly }\left[\begin{array}{ll}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{array}\right] & =\frac{1}{\Delta_{y}}\left[\begin{array}{cc}
y_{22} & -y_{12} \\
-y_{21} & y_{22}
\end{array}\right]
\end{aligned}
$$

(ii) Relationship between [ y ] and [h]

$$
\begin{array}{ll}
\text { From } & I_{1}=y_{11} V_{1}+y_{12} V_{2} \\
& I_{2}=y_{21} V_{1}+y_{22} V_{2}
\end{array}
$$

$$
\text { Rearranging } \quad y_{11} V_{1}=I_{1}-y_{12} V_{2}
$$

$$
\begin{aligned}
y_{21} V_{1}-I_{2} & =-y_{22} V_{2} \\
{\left[\begin{array}{cc}
y_{11} & 0 \\
y_{21} & -1
\end{array}\right]\left[\begin{array}{l}
V_{1} \\
I_{2}
\end{array}\right] } & =\left[\begin{array}{ll}
1 & -y_{12} \\
0 & -y_{22}
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
V_{2}
\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
\therefore\left[\begin{array}{l}
V_{1} \\
I_{2}
\end{array}\right] & =\left[\begin{array}{ll}
y_{11} & 0 \\
y_{21} & -1
\end{array}\right]^{-1}\left[\begin{array}{cc}
1 & -y_{12} \\
0 & -y_{22}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
V_{2}
\end{array}\right] \\
& =\frac{-1}{y_{11}}\left[\begin{array}{cc}
-1 & 0 \\
-y_{21} & y_{11}
\end{array}\right]\left[\begin{array}{ll}
1 & -y_{12} \\
0 & -y_{22}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
V_{2}
\end{array}\right] \\
& =\frac{-1}{y_{11}}\left[\begin{array}{cc}
-1 & y_{12} \\
-y_{21} & y_{12} y_{21}-y_{11} y_{22}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
V_{2}
\end{array}\right] \\
{\left[\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}\right] } & =\left[\begin{array}{ll}
\frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\
\frac{y_{21}}{y_{11}} & \frac{\Delta_{y}}{y_{11}}
\end{array}\right]\left[\begin{array}{c}
I_{1} \\
V_{2}
\end{array}\right]
\end{aligned}
$$

(iii) To Express T-parameters in terms of h-Parameters:

Equations for T-parameters,
Equations for h-parameters,
$\left.\begin{array}{l}\mathrm{V}_{1}=\mathrm{AV}_{2}-\mathrm{BI}_{2} \\ \mathrm{I}_{1}=\mathrm{CV}_{2}-\mathrm{DI}_{2}\end{array}\right\} \longrightarrow(1), ~(1) ~$

$$
\left.\begin{array}{r}
\mathrm{V}_{1}=\mathrm{h}_{11} \mathrm{I}_{1}+\mathrm{h}_{12} \mathrm{~V}_{2}  \tag{2}\\
\mathrm{I}_{2}=\mathrm{h}_{21} \mathrm{I}_{1}+\mathrm{h}_{22} \mathrm{~V}_{2}
\end{array}\right\}
$$

Re arranging Equation (2) $\mathrm{V}_{1}-\mathrm{h}_{11} \mathrm{I}_{1}=\mathrm{h}_{12} \mathrm{~V}_{2}$

$$
-\mathrm{h}_{21} \mathrm{I}_{1}=\mathrm{h}_{22} \mathrm{~V}_{2}-\mathrm{I}_{2}
$$

$$
\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{I}_{1}
\end{array}\right]=\left[\begin{array}{cc}
1 & -\mathrm{h}_{11} \\
0 & -\mathrm{h}_{21}
\end{array}\right]^{-1}\left[\begin{array}{cc}
\mathrm{h}_{12} & 0 \\
\mathrm{~h}_{22} & -1
\end{array}\right]\left[\begin{array}{l}
\mathrm{V}_{2} \\
\mathrm{I}_{2}
\end{array}\right]
$$

For which $[\mathrm{T}]=-\frac{1}{\mathrm{~h}_{21}}\left[\begin{array}{cc}-\mathrm{h}_{21} & \mathrm{~h}_{11} \\ 0 & 1\end{array}\right]\left[\begin{array}{cc}\mathrm{h}_{12} & 0 \\ \mathrm{~h}_{22} & -1\end{array}\right]$

$$
=-\frac{1}{\mathrm{~h}_{21}}\left[\begin{array}{cc}
\mathrm{h}_{11} \mathrm{~h}_{22}-\mathrm{h}_{21} \mathrm{~h}_{12} & \mathrm{~h}_{11} \\
\mathrm{~h}_{22} & 1
\end{array}\right]=\left[\begin{array}{cc}
-\frac{\Delta_{\mathrm{h}}}{\mathrm{~h}_{21}} & -\frac{\mathrm{h}_{11}}{\mathrm{~h}_{21}} \\
-\frac{\mathrm{h}_{22}}{\mathrm{~h}_{21}} & -\frac{1}{\mathrm{~h}_{21}}
\end{array}\right]
$$

By a similar procedure, the relationship between any two sets of parameters can be established. The following table gives such relationships:

|  | $\mathbf{Y}$ | $\mathbf{z}$ | $\mathbf{H}$ | $\mathbf{T}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{[ y ]}$ | $y_{11}$ | $y_{12}$ | $\frac{z_{22}}{\Delta_{z}}$ | $\frac{-z_{12}}{\Delta_{z}}$ | $\frac{1}{h_{11}}$ |
|  | $\frac{-h_{12}}{h_{11}}$ | $\frac{D}{B}$ | $\frac{-\Delta_{t}}{B}$ |  |  |
| $[\mathbf{y}]$ | $y_{22}$ | $\frac{-z_{21}}{\Delta_{z}}$ | $\frac{z_{11}}{\Delta_{z}}$ | $\frac{h_{21}}{h_{11}}$ | $\frac{\Delta_{h}}{h_{11}}$ |
|  | $\frac{y_{22}}{\Delta_{y}}$ | $\frac{-y_{12}}{\Delta_{y}}$ | $z_{11}$ | $z_{12}$ | $\frac{-1}{B}$ |
| $z_{21}$ | $\frac{A}{B}$ |  |  |  |  |


|  | $\frac{-y_{21}}{\Delta_{y}}$ | $\frac{y_{11}}{\Delta_{y}}$ |  |  | $\frac{-h_{21}}{h_{22}}$ | $\frac{1}{h_{22}}$ | $\frac{1}{C}$ | $\frac{D}{C}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [h] | $\frac{1}{y_{11}}$ | $\frac{-y_{12}}{y_{11}}$ | $\frac{\Delta_{z}}{z_{22}}$ | $\frac{z_{12}}{z_{22}}$ |  | $h_{11}$ | $h_{12}$ | $\frac{B}{D}$ |
|  | $\frac{y_{21}}{D}$ | $\frac{\Delta_{y}}{y_{11}}$ | $\frac{-z_{21}}{y_{11}}$ | $\frac{1}{z_{22}}$ | $h_{21}$ | $h_{22}$ | $\frac{-1}{D}$ | $\frac{C}{D}$ |
| [t] | $\frac{-y_{22}}{y_{21}}$ | $\frac{-1}{y_{21}}$ | $\frac{z_{11}}{z_{21}}$ | $\frac{\Delta_{z}}{z_{21}}$ | $\frac{-\Delta_{h}}{h_{21}}$ | $\frac{-h_{11}}{h_{21}}$ | $A$ | $B$ |
| $\frac{-\Delta_{y}}{y_{21}}$ | $\frac{-y_{11}}{y_{21}}$ | $\frac{1}{z_{21}}$ | $\frac{z_{22}}{z_{21}}$ | $\frac{-h_{22}}{h_{21}}$ | $\frac{-1}{h_{21}}$ | $C$ | $D$ |  |

COMPUTATIONS OF TWO PORT PARAMETERS:
A. By direct method i.e. using definitions

For z parameters, open output port $\left(\mathrm{I}_{2}=0\right)$ find $\mathrm{V}_{1} \& V_{2}$ in terms of $\mathrm{I}_{1}$ by equations Calculate $\mathrm{Z}_{11}=\mathrm{V}_{1} / \mathrm{I}_{1} \& \mathrm{Z}_{21}=\mathrm{V}_{2} / \mathrm{I}_{1}$.
Open input port ( $\mathrm{I}_{1}=0$ ) find $\mathrm{V}_{1} \& \mathrm{~V}_{2}$ in terms of $\mathrm{I}_{2}$. Calculate $\mathrm{Z}_{12}=\mathrm{V}_{1} / \mathrm{I}_{2} \& \mathrm{Z}_{22}=\mathrm{V}_{2} / \mathrm{I}_{2}$ Similar procedure may be followed for $y$ parameters by short circuiting the ports
$\mathrm{h} \& \mathrm{t}$ parameters may be obtained by a combination of the above procedures.
B. z and y parameters:By node \& mesh equations in standard form

For a reciprocal network (passive without controlled sources) with only two current
Sources at input and output nodes,the node equations are
$\mathrm{I}_{1}=\mathrm{Y}_{11} \mathrm{~V}_{1}+\mathrm{Y}_{12} \mathrm{~V}_{2}+\mathrm{Y}_{1} 3 \mathrm{~V}_{3}+-------+\mathrm{Y}_{1 \mathrm{n}} \mathrm{V}_{\mathrm{n}}$
$\mathrm{I}_{2}=\mathrm{Y}_{21} \mathrm{~V}_{1}+\mathrm{Y}_{22} \mathrm{~V}_{2}+\mathrm{Y}_{23} \mathrm{~V}_{3}+-------\mathrm{Y}_{2 \mathrm{n}} \mathrm{V}_{\mathrm{n}}$
$0=Y_{31} V_{1}+Y_{32} V_{2}+Y_{33} V_{3}+-------+Y_{3 n} V_{n}$
$0=Y_{n 1} V_{1}+Y_{n 2} V_{2}+Y_{n 3} V_{3}-------+Y_{n n} V_{n}$
then $V_{1}=\frac{\Delta_{11}}{\Delta} \mathrm{I}_{1}+\frac{\Delta_{21}}{\Delta} \mathrm{I}_{2} \quad$ where $\Delta$ is the determin ent of the Y matrix.

$$
\mathrm{V}_{2}=\frac{\Delta_{12}}{\Delta} \mathrm{I}_{1}+\frac{\Delta_{22}}{\Delta} \mathrm{I}_{2} \quad \Delta_{1 \mathrm{j}} \text { cofactor of } \mathrm{Y}_{1 \mathrm{j}} \text { of } \Delta
$$

Comparing these with the z parameter equations.
wehave $\quad \mathrm{Z}_{11}=\frac{\Delta_{11}}{\Delta} \quad \mathrm{Z}_{22}=\frac{\Delta_{22}}{\Delta} \quad \mathrm{Z}_{12}=\frac{\Delta_{21}}{\Delta} \quad \mathrm{Z}_{21}=\frac{\Delta_{12}}{\Delta}$

Similarly for such networks, the loop equations with voltage sources only at port 1 and 2

$$
\begin{aligned}
& V_{1}=Z_{11} I_{1}+Z_{12} I_{2}+\ldots \ldots \ldots .+Z_{1 m} I_{m} \\
& V_{2}=Z_{21} I_{1}+Z_{22} I_{2}+\ldots \ldots \ldots+Z_{2 m} I_{m} \\
& O=--------\cdots----- \\
& O=Z_{m 1} I_{1}+Z_{m 2} I_{2}+\ldots \ldots \ldots . .+Z_{m m} I_{m}
\end{aligned}
$$

Then

$$
\begin{aligned}
& I_{1}=\frac{D_{11}}{D} V_{1}+\frac{D_{21}}{D} V_{2} \\
& I_{2}=\frac{D_{12}}{D} V_{1}+\frac{D_{22}}{D} V_{2}
\end{aligned}
$$

where D is the determinant of the Z matrix and $\mathrm{D}_{\mathrm{ij}}$ is the co-factor of the element $\mathrm{Z}_{\mathrm{ij}}$ of Z matrix .comparing these with [y] equations

$$
\text { Thus we have } y_{11}=\frac{D_{11}}{D} \quad y_{22}=\frac{D_{22}}{D} \quad y_{12}=\frac{D_{12}}{D} \quad y_{22}=\frac{D_{22}}{D}
$$

Alternative methods

For z parameters the mesh equations are

$$
\begin{aligned}
& V_{1}=Z_{11} I_{1}+Z_{12} I_{2}+\ldots \ldots \ldots .+Z_{1 m} I_{m} \\
& V_{2}=Z_{21} I_{1}+Z_{22} I_{2}+\ldots \ldots \ldots+Z_{2 m} I_{m} \\
& O=-------\cdots------ \\
& O=Z_{m 1} I_{1}+Z_{m 2} I_{2}+\ldots \ldots \ldots \ldots+Z_{m m} I_{m}
\end{aligned}
$$

By matrix partitioning the above equations can be written as

$$
\left[\begin{array}{c}
V_{1} \\
V_{2} \\
\hdashline 0 \\
- \\
0
\end{array}\right]\left[\begin{array}{llll|}
Z_{11} & Z_{12} & -- & Z_{1 n} \\
Z_{21} & Z_{22} & -- & Z_{2 n} \\
\hdashline Z_{31} & Z_{32} & - & Z_{3 n} \\
- & \vdots & - & - \\
Z_{n 1} & Z_{1 n} & -- & Z_{n n}
\end{array}\right]\left[\begin{array}{l}
I_{1} \\
I_{2} \\
I_{3} \\
- \\
I_{n}
\end{array}\right]
$$

$\cdots\left[\begin{array}{c}\mathrm{V}_{1} \\ \mathrm{~V}_{2} \\ \hdashline \mathrm{O} \\ - \\ \mathrm{O}\end{array}\right]=-\left[\begin{array}{ll}\mathrm{M} & \mathrm{N} \\ \vdots & \\ \vdots & -\cdots \\ \vdots & \\ \vdots & - \\ \vdots & \mathrm{Q}\end{array}\right]\left[\begin{array}{l}\mathrm{I}_{1} \\ \mathrm{I}_{2} \\ \bar{I}_{3} \\ - \\ \mathrm{I}_{2}\end{array}\right]$

$$
\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{~V}_{2}
\end{array}\right]=\left[\mathrm{M}-\mathrm{NQ}^{-1} \mathrm{P}\left[\begin{array}{l}
\mathrm{I}_{1} \\
\mathrm{I}_{2}
\end{array}\right]\right.
$$

Similarly for Y parameters

$$
\left[\begin{array}{c}
\mathrm{I}_{1} \\
\mathrm{I}_{2} \\
\hdashline 0 \\
- \\
0
\end{array}\right]=\left[\begin{array}{llll}
\mathrm{Y}_{11} & \mathrm{Y}_{\mathrm{n} 2}^{1} & -- & \mathrm{Y}_{1 \mathrm{n}} \\
\mathrm{Y}_{21} & \mathrm{Y}_{22}^{1} & -- & \mathrm{Y}_{2 \mathrm{n}} \\
\hdashline \overline{\mathrm{Y}}_{31} & \mathrm{Y}_{32}^{2} & -- & -\overline{\mathrm{Y}}_{3 \mathrm{n}}^{-} \\
- & - & - & - \\
\mathrm{Y}_{\mathrm{n} 1} & \mathrm{Y}_{\mathrm{p} 2} & -- & \mathrm{Y}_{\mathrm{nn}}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{1} \\
\mathrm{~V}_{2} \\
\mathrm{~V}_{3} \\
- \\
\mathrm{V}_{\mathrm{n}}
\end{array}\right]
$$

$$
\left[\begin{array}{c}
\mathrm{I}_{1} \\
\mathrm{I}_{2} \\
\hdashline \mathrm{O} \\
- \\
\mathrm{O}
\end{array}\right]=\left[\begin{array}{cc}
\mathrm{M} & \mathrm{~N} \\
\vdots & \\
\hdashline \vdots & \\
\vdots & - \\
\vdots & - \\
\mathrm{P} & \mathrm{Q}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{1} \\
\mathrm{~V}_{2} \\
-\mathrm{V}_{3} \\
- \\
\mathrm{V}_{2}
\end{array}\right]
$$

$$
\left[\begin{array}{c}
\mathrm{I}_{1} \\
\mathrm{I}_{2}
\end{array}\right]=\left[\mathrm{M}-\mathrm{NQ}^{-1} \mathrm{P}\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{~V}_{2}
\end{array}\right]\right.
$$

C. By reducing the network (containing passive elements only) to single T or D by T-D transformations

If the network is reduced to a T network as shown


Then

$$
\begin{aligned}
& \mathrm{V}_{1}=\left(\mathrm{Z}_{1}+\mathrm{Z}_{3}\right) \mathrm{I}_{1}+\mathrm{Z}_{3} \mathrm{I}_{2} \\
& \mathrm{~V}_{2}=\mathrm{Z}_{3} \mathrm{I}_{1}+\left(\mathrm{Z}_{2}+\mathrm{Z}_{3}\right) \mathrm{I}_{2}
\end{aligned}
$$

from which

$$
\begin{aligned}
& \mathrm{Z}_{11}=\mathrm{Z}_{1}+\mathrm{Z}_{3} \quad \mathrm{Z}_{22}=\mathrm{Z}_{2}+\mathrm{Z}_{3} \\
& \mathrm{Z}_{12}=\mathrm{Z}_{21}=\mathrm{Z}_{13}
\end{aligned}
$$

If the network is brought to $\Pi$ network as shown


Then
$\mathrm{I}_{1}=\left(\mathrm{Y}_{1}+\mathrm{Y}_{3}\right) \mathrm{V}_{1}-\mathrm{Y}_{3} \mathrm{~V}_{2}$
$I_{2}=-Y_{3} V_{1}+\left(Y_{2}+Y_{3}\right) V_{2}$
from which

$$
\begin{array}{ll}
\mathrm{y}_{11}=\mathrm{Y}_{1}+\mathrm{Y}_{3} \quad \mathrm{y}_{22}=\mathrm{Y}_{2}+\mathrm{Y}_{3} \\
\mathrm{y}_{12}=\mathrm{y}_{21}=-\mathrm{Y}_{3}
\end{array}
$$

## t-PARAMETERS FOR T $\& \pi$ NETWORKS

## For a $T$ network the mesh equations are:



$$
\begin{aligned}
& \mathrm{V}_{1}=\left(\mathrm{Z}_{1}+\mathrm{Z}_{2}\right) \mathrm{I}_{1}+\mathrm{I}_{2} \mathrm{Z}_{3} \\
& \mathrm{~V}_{2}=\mathrm{Z}_{3} \mathrm{I}_{1}+\left(\mathrm{Z}_{2}+\mathrm{Z}_{3}\right) \mathrm{I}_{2}
\end{aligned}
$$

Re arranging

$$
\begin{aligned}
& \mathrm{V}_{1}-\left(\mathrm{Z}_{1}+\mathrm{Z}_{3}\right) \mathrm{I}_{1}=\mathrm{I}_{2} \mathrm{Z}_{3} \\
& -\mathrm{Z}_{3} \mathrm{I}_{1}=\left(\mathrm{Z}_{2}+\mathrm{Z}_{3}\right) \mathrm{I}_{2}-\mathrm{V}_{2}
\end{aligned}
$$

In matrix form

$$
\begin{aligned}
& {\left[\begin{array}{c}
\mathrm{V}_{1} \\
\mathrm{I}_{1}
\end{array}\right]=\left[\begin{array}{cc}
1 & -\left(\mathrm{Z}_{1}+\mathrm{Z}_{3}\right) \\
0 & -\mathrm{Z}_{3}
\end{array}\right]^{-1}\left[\begin{array}{cc}
0 & \mathrm{Z}_{3} \\
-1 & \mathrm{Z}_{2}+\mathrm{Z}_{3}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right]} \\
& =-\frac{1}{\mathrm{Z}_{3}}\left[\begin{array}{cc}
-\mathrm{Z}_{3} & \mathrm{Z}_{1}+\mathrm{Z}_{3} \\
0 & 1
\end{array}\right]\left[\begin{array}{cc}
0 & \mathrm{Z}_{3} \\
-1 & \mathrm{Z}_{2}+\mathrm{Z}_{3}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right] \\
& =\frac{1}{\mathrm{Z}_{3}}\left[\begin{array}{cc}
\left(\mathrm{Z}_{1}+\mathrm{Z}_{3}\right) & \mathrm{Z}_{1} \mathrm{Z}_{2}+\mathrm{Z}_{2} \mathrm{Z}_{3}+\mathrm{Z}_{3} \mathrm{Z}_{1} \\
1 & \mathrm{Z}_{2}+\mathrm{Z}_{3}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right]
\end{aligned}
$$

$$
\left[\begin{array}{ll}
\mathrm{A} & \mathrm{~B} \\
\mathrm{C} & \mathrm{D}
\end{array}\right]=\frac{1}{\mathrm{Z}_{3}}\left[\begin{array}{cc}
\left(\mathrm{Z}_{1}+\mathrm{Z}_{3}\right) & \mathrm{Z}_{1} \mathrm{Z}_{2}+\mathrm{Z}_{2} \mathrm{Z}_{3}+\mathrm{Z}_{3} \mathrm{Z}_{1} \\
1 & \mathrm{Z}_{2}+\mathrm{Z}_{3}
\end{array}\right]
$$

For the $\pi$ network shown, the equations are:


$$
\begin{aligned}
& \mathrm{I}_{1}=\left(\mathrm{Y}_{1}+\mathrm{Y}_{3}\right) \mathrm{V}_{1}-\mathrm{Y}_{3} \mathrm{~V}_{2} \\
& \mathrm{I}_{2}=-\mathrm{V}_{1} \mathrm{Y}_{3}+\left(\mathrm{Y}_{2}+\mathrm{Y}_{3}\right) \mathrm{V}_{2}
\end{aligned}
$$

Re arranging

$$
\begin{aligned}
& \left(V_{1}+Y_{3}\right) V_{1}-I_{1}=V_{2} Y_{3} \\
& Y_{3} V_{1}=\left(Y_{2}+Y_{3}\right) V_{2}-I_{2}
\end{aligned}
$$

In matrix form

$$
\begin{aligned}
& {\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{I}_{1}
\end{array}\right]=\left[\begin{array}{cc}
\mathrm{Y}_{1}+\mathrm{Y}_{3} & 1 \\
\mathrm{Y}_{3} & 0
\end{array}\right]^{-1}\left[\begin{array}{cc}
0 & -\mathrm{Y}_{3} \\
\mathrm{Y}_{2}+\mathrm{Y}_{3} & -1
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right]} \\
& =\frac{1}{\mathrm{Y}_{3}}\left[\begin{array}{cc}
0 & -1 \\
-\mathrm{Y}_{3} & \mathrm{Y}_{1}+\mathrm{Y}_{3}
\end{array}\right]\left[\begin{array}{cc}
0 & -\mathrm{Y}_{3} \\
\mathrm{Y}_{2}+\mathrm{Y}_{3} & -1
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right] \\
& =\frac{1}{\mathrm{Y}_{3}}\left[\begin{array}{cc}
\left(\mathrm{Y}_{2}+\mathrm{Y}_{3}\right) & 1 \\
\mathrm{Y}_{1} \mathrm{Y}_{2}+\mathrm{Y}_{2} \mathrm{Y}_{3}+\mathrm{Y}_{3} \mathrm{Y}_{1} & \left(\mathrm{Y}_{1}+\mathrm{Y}_{3}\right)
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-\mathrm{I}_{2}
\end{array}\right]
\end{aligned}
$$

$$
\left[\begin{array}{ll}
\mathrm{A} & \mathrm{~B} \\
\mathrm{C} & \mathrm{D}
\end{array}\right]=\frac{1}{\mathrm{Y}_{3}}\left[\begin{array}{cc}
\left(\mathrm{Y}_{2}+\mathrm{Y}_{3}\right) & 1 \\
\mathrm{Y}_{1} \mathrm{Y}_{2}+\mathrm{Y}_{2} \mathrm{Y}_{3}+\mathrm{Y}_{3} \mathrm{Y}_{1} & \left(\mathrm{Y}_{1}+\mathrm{Y}_{3}\right)
\end{array}\right]
$$

D. The above methods are based on the assumptions that the network does not contain controlled sources. However irrespective of the presence of the controlled sources., network equations may be written and then by any elimination process variables other than $V_{1}, V_{2}, I_{1} \& I_{2}$ are eliminated. Then resulting two equations are brought to the Required form of two port parameters by manipulation.

## SYMMETRICAL CONDITIONS

A two port is said to be symmetrical if the ports can be interchanged without changing the port voltage and currents..
i.e. if $\left.\frac{V_{1}}{\mathrm{I}_{1}}\right|_{\mathrm{I}_{2}=0}=\left.\frac{\mathrm{V}_{2}}{\mathrm{I}_{2}}\right|_{\mathrm{I}_{1}=0} \quad \therefore \quad \mathrm{z}_{11}=\mathrm{z}_{22}$

By using the relationship between z and other parameters we can obtain the conditions for Symmetry in terms of other parameters.

As $\mathrm{z}_{11}=\mathrm{z}_{22}$, in terms of y we have $\mathrm{y}_{11}=\mathrm{z}_{12} / \mathrm{dz} \& \mathrm{y}_{22}=\mathrm{z}_{1} / \mathrm{dz}, \quad \therefore \mathbf{y}_{11}=\mathbf{y}_{22}$.
In terms of $h$ parameters as $\mathrm{z}_{11}=\Delta \mathrm{h} / \mathrm{h}_{22} \& \mathrm{z}_{22}=1 / \mathrm{h}_{22}$ we have $\Delta \mathbf{h}=\mathbf{h}_{11} \mathbf{h}_{22}-\mathbf{h}_{12} \mathbf{h}_{21}=\mathbf{1}$.
In terms of t parameters as $\mathrm{z}_{1}=\mathrm{A} / \mathrm{C} \& \mathrm{z}_{22}=\mathrm{D} / \mathrm{C}$ the condition is $\mathbf{A}=\mathbf{D}$

## Reciprocity condition in terms of two port parameters



Fig 1


Fig 2

For the two networks shown for
Fig 1

$$
\mathrm{V}_{1}=\mathrm{V} \quad \mathrm{I}_{2}=-\mathrm{I}_{\mathrm{a}}
$$

$$
\mathrm{V}_{2}=0
$$

Fig 2
$\mathrm{V}_{2}=\mathrm{V}$
$\mathrm{I}_{1}=-\mathrm{I}_{\mathrm{b}}$
$\mathrm{V}_{1}=0$
Condition for reciprocity is $\mathrm{I}_{\mathrm{a}}=\mathrm{I}_{\mathrm{b}}$
From z parameters

$$
\begin{array}{lr}
\mathrm{V}_{1}=\mathrm{z}_{11} \mathrm{I}_{1}+\mathrm{z}_{12} \mathrm{I}_{2} & \text { wehavefrom fig(1) } \\
\mathrm{V}_{2}=\mathrm{z}_{21} \mathrm{I}_{1}+\mathrm{z}_{22} \mathrm{I}_{2} & \mathrm{~V}=\mathrm{z}_{11} \mathrm{I}_{1}-\mathrm{z}_{12} \mathrm{I}_{\mathrm{a}} \\
\mathrm{O}=\mathrm{z}_{21} \mathrm{I}_{1}-\mathrm{z}_{22} \mathrm{I}_{\mathrm{a}}
\end{array}
$$

From fig(2)

$$
\begin{gathered}
\mathrm{O}=\mathrm{z}_{11} \mathrm{I}_{\mathrm{b}}+\mathrm{z}_{12} \mathrm{I}_{2} \\
\mathrm{~V}=-\mathrm{z}_{21} \mathrm{I}_{\mathrm{b}}+\mathrm{z}_{12} \mathrm{I}_{2}
\end{gathered} \quad \mathrm{I}_{\mathrm{b}}=\frac{-\mathrm{z}_{21} \mathrm{~V}}{-\Delta \mathrm{z}} \quad \text { then for } \mathrm{I}_{\mathrm{a}}=\mathrm{I}_{\mathrm{b}}
$$

For reciprocity with $\mathrm{z}_{12}=\mathrm{z}_{21}$,
In terms of y parameters $\mathrm{z}_{12}=-\mathrm{y}_{12} / \Delta \mathrm{y}$ \& $\mathrm{z}_{21}=-\mathrm{y}_{21} / \Delta \mathrm{y}$ condition is $\quad \mathbf{y}_{12}=\mathbf{y}_{21}$ In terms of $h$ parameters $\mathrm{z}_{12}=\mathrm{h}_{12} / \mathrm{h}_{22} \& \mathrm{z}_{21}=-\mathrm{h}_{21} / \mathrm{h}_{22}$ the condition is $\mathbf{h}_{12}=-\mathbf{h}_{21}$ In terms of $t$ parameters $\mathrm{z}_{12}=\Delta \mathrm{t} / \mathrm{C} \& \mathrm{z}_{21}=1 / \mathrm{C}$ the condition is $\quad \Delta \mathbf{t}=\mathbf{A D}-\mathbf{B C}=\mathbf{1}$

| Parameters | Condition for |  |
| :--- | :--- | :--- |
|  | Reciprocity | Symmetry |
| $\mathbf{z}$ | $\mathbf{z}_{12}=\mathbf{Z}_{22}$ | $\mathbf{z}_{11}=\mathbf{Z}_{22}$ |
| $\mathbf{y}$ | $\mathbf{y}_{12}=\mathbf{y}_{22}$ | $\mathbf{y}_{11}=\mathbf{y}_{22}$ |
| $\mathbf{h}$ | $\mathbf{h}_{12}=-\mathbf{h}_{21}$ | $\mathbf{h}_{11} \cdot \mathbf{h}_{22}-$ <br> $\mathbf{h}_{12} \cdot \mathbf{h}_{21}=\mathbf{1}$ |
| $\mathbf{t}$ | AD-BC=1 | A=D |

CASCADE CONNECTION:-


In the network shown 2 two port networks are connected in cascade

For $\mathbf{N}_{a}, \quad[t]=\left[\begin{array}{ll}A_{a} & B_{a} \\ C_{a} & D_{a}\end{array}\right]$ for $\mathbf{N}_{b}, \quad[t]=\left[\begin{array}{cc}A_{b} & B_{b} \\ C_{b} & D_{b}\end{array}\right]$
For the resultant network $\mathbf{N} \quad[\mathrm{t}]=\left[\begin{array}{ll}A & B \\ C & D\end{array}\right]$

## From the cascaded network we have

$$
\begin{array}{ll}
V_{1 a}=A_{a} V_{2 a}-B_{a} I_{2 a} & \text { for network } N_{a} \\
I_{1 a}=C_{a} V_{2 a}-D_{a} I_{2 a} & \\
V_{1 b}=A_{b} V_{2 b}-B_{b} I_{2 b} & \text { for network } N_{b} \\
I_{1 b}=C_{b} V_{2 b}-D_{b} I_{2 b} & \\
V_{1}=A V_{2}-B I_{2} & \text { for network } N \\
I_{1}=C V_{2}-D I_{2} &
\end{array}
$$

From the network

$$
\begin{aligned}
& \mathrm{I}_{1}=\mathrm{I}_{1 \mathrm{a}} \quad \mathrm{I}_{2 \mathrm{a}}=-\mathrm{I}_{1 \mathrm{~b}} \quad \mathrm{I}_{2}=\mathrm{I}_{2 \mathrm{~b}} \\
& \mathrm{~V}_{1}=\mathrm{V}_{1 \mathrm{a}} \quad \mathrm{~V}_{2 \mathrm{a}}=\mathrm{V}_{1 \mathrm{~b}} \quad \mathrm{~V}_{2}=\mathrm{V}_{2 \mathrm{~b}} \\
& {\left[\begin{array}{c}
\mathrm{V}_{1 \mathrm{a}} \\
\mathrm{I}_{1 \mathrm{a}}
\end{array}\right]=\left[\begin{array}{ll}
\mathrm{A}_{\mathrm{a}} & \mathrm{~B}_{\mathrm{a}} \\
\mathrm{C}_{\mathrm{a}} & \mathrm{D}_{\mathrm{a}}
\end{array}\right]\left[\begin{array}{c}
-\mathrm{V}_{2 \mathrm{a}} \\
-\mathrm{I}_{2 \mathrm{a}}
\end{array}\right] \text { and }\left[\begin{array}{c}
\mathrm{V}_{1 \mathrm{~b}} \\
\mathrm{I}_{1 \mathrm{~b}}
\end{array}\right]=\left[\begin{array}{cc}
\mathrm{A}_{\mathrm{b}} & \mathrm{~B}_{\mathrm{b}} \\
\mathrm{C}_{\mathrm{b}} & \mathrm{D}_{\mathrm{b}}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2 \mathrm{~b}} \\
-\mathrm{I}_{2 \mathrm{~b}}
\end{array}\right]} \\
& \text { or }\left[\begin{array}{l}
V_{1 a} \\
I_{1 a}
\end{array}\right]=\left[\begin{array}{cc}
A_{a} & B_{a} \\
C_{a} & D_{a}
\end{array}\right]\left[\begin{array}{l}
V_{1 b} \\
I_{1 b}
\end{array}\right]=\left[\begin{array}{cc}
A_{a} & B_{a} \\
C_{a} & D_{a}
\end{array}\right]\left[\begin{array}{cc}
A_{b} & B_{b} \\
C_{b} & D_{b}
\end{array}\right]\left[\begin{array}{c}
V_{2 b} \\
-I_{2 b}
\end{array}\right] \\
& =\left[\begin{array}{ll}
A_{a} & B_{a} \\
C_{a} & D_{a}
\end{array}\right]\left[\begin{array}{ll}
A_{b} & B_{b} \\
C_{b} & D_{b}
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{2} \\
-I_{2}
\end{array}\right] \\
& \therefore\left[\begin{array}{ll}
\mathrm{A} & \mathrm{~B} \\
\mathrm{C} & \mathrm{D}
\end{array}\right]=\left[\begin{array}{ll}
\mathrm{A}_{\mathrm{a}} & \mathrm{~B}_{\mathrm{a}} \\
\mathrm{C}_{\mathrm{a}} & \mathrm{D}_{\mathrm{a}}
\end{array}\right]\left[\begin{array}{ll}
\mathrm{A}_{\mathrm{b}} & \mathrm{~B}_{\mathrm{b}} \\
\mathrm{C}_{\mathrm{b}} & \mathrm{D}_{\mathrm{b}}
\end{array}\right] \\
& {[\mathrm{T}]=\left[\mathrm{T}_{\mathrm{a}}\right\rfloor\left[\mathrm{T}_{\mathrm{b}}\right]}
\end{aligned}
$$

## Writing the network from equations:

## 1) To write the network from mesh equations:

With the equations in matrix form $[\mathrm{E}]=[\mathrm{Z}][\mathrm{I}]$ and all meshes in clockwise direction, draw the graph of the network keeping in mind that there is a branch between mesh J and mesh K if Zjk exists and the number of meshes is equal to the number of I's. If Zjk is zero,there is no branch common to meshes J and K.

For example, if the network contains 3 meshes with mutual Z exists among all the three, the graph is of the form as shown in fig 1 .

On the other hand if there is no mutual Z between first and third meshes the graph is of the form as shown in fig 2


Fig. 1


Fig. 2

With this information
Insert in each mutual line, the respective mutual Z.(negative of Zjk )
Insert in non-mutual line the sum $\mathrm{Z}_{\mathrm{KK}}+\mathrm{Z}_{\mathrm{Kj}}$ for the $\mathrm{K}^{\text {th }}$ mesh.
Insert $\mathrm{E}_{\mathrm{K}}$ in the non mutual line of mesh K . This is not unique since $\mathrm{E}_{\mathrm{K}}$ can be split into many E's and may be placed in many branches of the $\mathrm{K}^{\text {th }}$ mesh or loop.

Thus the network is obtained.

## Problem

For the equations shown draw the network

$$
\left[\begin{array}{ccc}
35-\text { J } 16.12 & -10+\text { J31.83 } & -20-\text { J15.71 } \\
-10+\text { J 31.83 } & 35-\text { J } 25.55 & -20 \\
-20-\text { J15.71 } & -20 & 50+\text { J15.71 }
\end{array}\right]\left[\begin{array}{l}
\mathrm{I}_{1} \\
\mathrm{I}_{2} \\
\mathrm{I}_{3}
\end{array}\right]=\left[\begin{array}{c}
200 \frac{0^{\circ}}{0} \\
0
\end{array}\right]
$$

Step-1


Step-2


Step-3


## 2) To write the network from node equations:

The number of equations indicate the number of independent nodes and non diagonal terms indicate -ve of the mutual admittances between the respective nodes
. Zero in the non diagonal terms indicate no branh between the respective nodes.With this information draw the graph of the network including reference node.

Insert in each mutual branch the respective mutual admittance (-ve of the non diagonal term)

Insert in non mutual line the sum $\mathrm{Y}_{\mathrm{KK}}+\mathrm{K}_{\mathrm{Kj}}$ for the $\mathrm{K}^{\mathrm{th}}$ node.
Insert $I_{K}^{\prime}$ in the non mutual line of the node $K$. This is not unique since $I_{K}$ can be split into may I's distributed in some of the other branches connected to node K .

Thus obtain the network..
For example, if the network contains 3 independent nodes with mutual Y among all the three, the graph is of the form shown in fig 1

On the other hand if mutual Y exists between two nodes only then the graph is of the form shown in fig 2


Fig. 1


Fig. 2

In case the network contains, mixed sources and controlled sources, super meshes and super nodes are carefully identified.

## Problem

For the equation shown draw the network

$$
\left[\begin{array}{ccc}
1 & -\mathrm{J} 1 & \mathrm{~J} 1 \\
-\mathrm{J} 1 & \mathrm{~J} 2 & -\mathrm{J} 1 \\
\mathrm{~J} 1 & -\mathrm{J} 1 & 1
\end{array}\right]\left[\begin{array}{l}
\mathrm{V}_{1} \\
\mathrm{~V}_{2} \\
\mathrm{~V}_{3}
\end{array}\right]=\left[\begin{array}{c}
1 \\
-1 \\
\mathrm{~J} 1
\end{array}\right]
$$



Step-3


