I Semester

1. Real Analysis.

Review of Number System: Dedekind's cut ; Topology of $\mathbb{R}$: Weirstrass Theorem, Heine Borel Theorem, connectedness ; Series and sequences, continuous and differentiable functions, Mean Value Theorems and their consequences, Maxima, Minima and curve tracing; Functions of bounded variation, Riemann integration, Riemann-Stieltjes integration in $\mathbb{R}$.

Sequences of functions, uniform convergence, Ascoli-Arzela Theorem.

Functions of several variables, continuity, differentiation - Directional derivatives and Fréchet derivatives, MeanValue Theorems, Maxima, Minima, Inverse and Implicit function theorem.

Riemann integration in $\mathbb{R}^2$, parametric hypersurfaces, tangent plane, normal and surface measure, surface integrals and Divergence Theorem.

References.

1. W. Rudin - Real Analysis.
2. T.M. Apostol - Mathematical Analysis.
3. S. Lang - Analysis.

2. Topology.

Axioms of set theory, partial order, Ordinality, Cardinality, Schroeder-Bernstein Theorem, axiom of choice and its equivalents.

Topological spaces, induced topology, product topology, separation axioms - T_0, T_1, T_2, T_3, T_4, Urysohn's lemma and Tietze extension theorem, Connectedness, Compactness, Tychnov theorem, Ist and IInd axiom of Countability, Metric spaces, Baire category theorem, Banach fixed point theorem.

References.

1. J.L. Kelly - General Topology

2. G.F. Simmons - Introduction to Topology and Modern Analysis
3. Munkres - Topology

3. Linear Algebra.

Vector spaces, Linear transformations and matrices, elementary Matricies, row and column operations.

Multilinear algebra and determinants, rank and nullity.

Eigenvalues and Cayley-Hamilton theorem.

Operator norms and spectral radius formula, Normal, Hermitian and unitary operators, and the spectral theorem, Jordan canonical form.

References.

1. K. Hoffman and R Kunze - Linear Algebra
2. G. Strang Linear Algebra and its applications.

4. Theory of ODE.

Methods of solving 1st and second order, linear ODE by variation of parameters and Wronskian. Solutions by series of some special 2nd order ODE.

1st order Non linear ODE, Cauchy-Picard theorem,

Two point boundary value problem and Strum-Liouville theory, Weyl-Titschmarsh theorem for unbounded interval - limit cycle, limit point cases.

Linear systems with constant coefficients, Fundamental Matrix, Linear Systems with periodic coefficients. Nonlinear Autonomous system, critical points. Phase plane analysis, stability, Periodic solutions.

References.

1. Coddington - Introduction to ODE
2. Coddington-Levinson - Theory of ODE
3. G. F. Simmons - Differential Equations with Applications and Historical Notes.

4. Lawrence Perko - Differential equations dynamical systems
5. Morris W. Hirsch and Smale - Differential equations, dynamical systems and linear algebra

II Semester

1. Functional Analysis.

Normed Linear Spaces, Banach spaces, Continuous lineal functional, dual spaces. Hahn - Banach theorem,

Applications of Baire Category Theorem: open mapping, closed graph and uniform boundedness theorems.

Weak, weak $*$ topologies, Banach- Alaoglu Theorem, reflexivity.

Hilbert spaces, Reisz representation theorem, Adjoint, Hermitian, Normal, unitary operators, Compact operators. Spectral theorem for Compact Hermitian operators

References.

1. B.M. Limaye- Functional Analysis
2.G. F. Simmons, Introduction to Topology Modern Analysis
3.A.E. Taylor - An Introduction to Functional Analysis.
4. Kreyzig - Functional Analysis.
5. Rudin - Functional Analysis.

2. Measure and integration.

Construction of Lebesgue measure on $\mathbb{R}$ using inner measure and outer measure, $\sigma-$algebras, abstract measures, measurable functions.

Lebesgue integration, Fatou, Monotone and dominated convergence theorems, product measures, Fubini theorem.

Lebesgue decomposition, Radon-Nikodym theorem, L^p-spaces, duality, Lebesgue's differentiaon theorem, absolutely continuous functions, monotone functions.

Convolution and Fourier transforms.

References.

1. Rudin - Real and complex analysis
2. Royden - Real Analysis
3. E. Hewitt and K. Stromberg - Real and Abstract Analysis.

3. Elementary PDE.

First order PDE-Solutions by characteristics.

Classification of 2nd order pde's, Fundamental solutions and Green functions for Laplace, heat and wave equations, Explicit solution formulas, Harmonic functions, Mean value property, Maximum principles, uniqueness of solutions.

Solutions by other methods: separation of variables, similarity methods, transform methods, power series method, Cauchy- Kowalewsky theorem, Holmgren's uniqueness theorem

References.

1. L. C. Evans, PDE, Berkely Math Lecture Notes
2. Fritz John - PDE
3. Phoolan Prasad and R. Ravindran, PDE
4. R. Courant and D. Hilbert, Methods of Mathematical Physics (Vol. 1)
5. G. B. Folland, Introduction to PDE

4. Numerical Analysis

Round off Errors and Computer Arithmetic.
Interpolation: Lagrange Interpolation, Divided Differences, Hermite Interpolation, Splines.
Numerical Differentiation, Richardson Extrapolation.

Numerical Integration: Trapezoidal, Simpsons, Newton-Cotes, Gauss quadrature, Romberg integration, Multiple integrals.

Solutions of Linear Algebraic equations: Direct Methods, Gauss Elimination, Pivoting, Matrix factorizations.

Iterative Methods: Matrix norms, Jacobi and Gauss-Siedel methods, Relaxation methods.

Computation of Eigenvalues and EigenVectors: Power Method, Householder's method, QR algorithm.

Numerical solutions of Nonlinear Algebraic equations: Bisection, Secant and Newton's method. Zeroes of polynomials, Horner and Muller methods. Equations in higher dimensions.

Ordinary Differential Equations, Initial Value Problems: Euler method, Higher Order methods of the Runge-Kutta type. Multi-step methods, Adams-Bashforth, Adams-Moulton Methods, Systems of ODE's.

Ordinary Differential Equations, Boundary value Problems: Shooting methods, Finite differences, Rayleigh-Ritz methods.
Fast Fourier transforms.

References

1. Burden, R.L and Faires, D.J: Numerical Analysis.
2. Blum, E.K: Numerical Analysis and Computation, Theory and Practice.
3. Conte, S.D and De Boor, c: Elementary Numerical Analysis, An algorithmic approach.
4. Young, D.M and Gregory, R.T: Survey of Numerical Mathematics, Vols 1 and 2.
5. Quateroni, A, Sacco, R and Saleri, F: Numerical Mathematics.
6. Quateroni, A and Saleri, F: Scientific Computing with MATLAB.

5. Algebra

Groups. Groups, Subgroups, Homomorphisms, Normal subgroups, Quotient groups, Isomorphism theorems. Symmetric groups, Alternating and dihedral groups. Structure of finitely generated abelian groups. Group actions and its applications, Sylow theorems, Solvable groups.

Rings. Rings and homomorphisms, Ideals, Isomorphism theorems, Prime ideals and maximal ideals. Jacobson radical and Nil-radical. Chinese reminder theorem. Polynomial rings and power series rings. Division algorithm. Roots and multiplicities, Resultant and discriminant. Elementary symmetric functions and the main theorem on symmetric functions. Proof of fundamental theorem of algebra by using symmetric functions. Factorization in polynomial rings. Eisenstein criterion. Unique factorization domains.

Modules. Modules, Homomorphisms and exact sequences. Free modules. Rank of a free module (over commulative rings). Hom and tensor products. Chain conditions on modules. Noetherian rings and Hilbert basis theorem. Structure theorem for modules over PID's.

Fields. Field extensions and elementary Galois theory.

References.

1. Fraleigh, J.B., A First course in Abstract Algebra
2. Artin, M., Algebra, Prentice-Hall, 1994
3. Hungerford, T.W., Algebra, Springer-Verlag (GTM), 1974.
4. Herstein, I.N., Topics in Algebra, John Wiley and Sons, 1995
5. Gallian, J.A., Contemporary Abstract Algebra, Fourth edition, Narosa, 1992.

III Semester

1. **Probability Theory.**

    1. Probability spaces, probabilistic language: event, sample point, expectation, variance, moments, etc. Random variables and their distributions. Important examples: binomial, Poisson, Hypergeometric, Gaussian, Cauchy, exponential, gamma and beta distributions.

    2. Independence of events, of classes of events, of random variables. Kolmogorov's 0-1 law. The Borel-Cantelli lemma. Elementary conditional probability. Bayes formula. Simple examples of Markov chains. Markov inequality. The weak and strong laws of large numbers.

    3. Moment generating functions and characteristic functions. Uniqueness theorem. Inversion theorem. Application to the dependence of random variables, to the existence of moments.

4. Convergence of probability measures. Tightness.

5. Convergence in distribution. Levy's continuity theorem. The central limit theorem for independent identically distributed summands.

6. Conditional expectation and conditional probability, Basic therorems. Regular conditioanal probability.

7. $\mathbb{R}^2-$valued Gaussian random variables.

8. Markov chains with countable state space. Examples. Transience and recurrence. Stationary distributions. Continuous parameter Markov chains, Poisson process.

9. Martingales with discrete parameter. Inequalities. Convergence theorems. Optional stopping theorem. Applications in particular Markov chains.

References.

1. W. FELLER, An Introduction to Probability and its applications vol 1.
2. L. BREIMAN, Probability
3. J. NEVEU, Mathematical Foundations of Probability theory
4. P. BILLINGSLEY, Probability and Measure
5. P. BILLINGSLEY, Convergence of probability measures
6. K. L. CHUNG, Probability Theory
7. K. L. CHUNG, Markov Chains.

2. Advanced PDE I.

Distribution Theory, Sobolev Spaces, embedding theorems, Rellich's Lemma, Trace Theorems,

Second order elliptic equations:- Formulation of Dirichlet, Neumann and Oblique derivative problems, Weak formulation, Lax - Milgram Lemma, existence and regularity upto the boundary, Maximum principle, elementary variational inequality.

Linear evolution equations, existence of weak solutions, energy methods.

References.

1. Barros-Neto- An introduction to the Introduction to the Theory of Distributions
2. Adams- Sobolev Spaces,
3. Kesavan - Topics in Functional Analysis and Applications
4. Evans - Partial Differential Equations
5. H.Brezis - Analyse Fonctionnelle

3. Complex Analysis.

Complex numbers, Complex differentiation, Cauchy-Riemann equations . Complex integration, Homotopy version of Cauchy theorem, Fundamental theorem of algebra, power series expansion.

Maximum Modulus and Residue theorems, Singularities and meromorphic functions, Laurent series, Rouché theorem, Herwitz-theorem, Weirstrass and Mittag- Leffler theorem.

Conformal maps, Schwarz theorem, Montel's theorem, Schwarz-Christoffel formula, Riemann mapping theorem.

References.

1. Conway- Functions of one Complex variable
2. Rudin - Real and Complex Analysis
3. Ahlfors - Complex Analysis

4. Differential Geometry.

Curves and Surfaces. Definition, Curvature of plane curve, Frenet Formulas, Vector Fields, Orientation, Gaussian curvature, mean curvature.

Differentiable manifold. Definition of differentiable manifold, tangent space, derivatives of maps, vector fields, integral curves, orientation, tensor fields, Lie bracket, sub-manifold, connection.

Riemannian Geometry. Riemannian metric, Levi-Civita connection, Parallel Transport, geodesic, exponential map, isometry, conformal maps, volume-element, curvature tensor, Ricci curvature, scalar curvature, sectional curvature.

Differential forms. Definition, integration of forms, Poincaré's Lemma, Stokes theorem, Gauss-Bonnet theorem.

References.

1. M.P. do Carmo: Differential Geometry of Curves and Surfaces.
2. M. P. do Carmo: Differential Forms and Applications.
3. M. P. do Carmo: Riemannian Geometry
4. S. Kumaresan: A course in Differential Geometry and Lie Groups.
5. Thorpe, J.A: Elementary Topics in Differential Geometry.
6. Spivak, M: A comprensive Introduction to Differential Geometry I-V.

IV Semester

1. Advaned PDE II.

Review of harmonic functions, Extension of maximum principles to 2nd order elliptic equations, existence via sub-super solutions.

Apriori estimates of Schauder, existence via fixed point method.

Existence via variational methods: direct minimization and constrained minimization.

Evolution eqautions: existence via semigroup theory,

Well posedness of Cauchy Problem for strictly hyperbolic systems, Initial and Initial-Boundary Value problems for symmetric hyperbolic systems, and linear second order systems,

Nonlinear Hyperbolic systems- Asymptotic solutions of oscillatory initial value problems (Lax Theory)

Scalar conservation laws (Hopf-Lax formula). Systems of hyperbolic conservation laws-2x2 Lax theory. General Case - Introduction to Glimm's theory.

References.

1. Evans, PDE
2.Q.Han and F-H. Lin, Elliptic PDE's
3. J. Smoller - Shock waves and Reaction Diffusion Equations.

2. Mechanics

The aim of this course is to show how certain ODE, PDE arise as models starting from certain basic principles. These principles will be formulated in mathematical terms. Apart from this, we will derive some properties of these models.

Classical Mechanics. Various formulations: Lagrangian, Hamiltonian, Hamilton-Jacobi, Principle of stationary action, Legendre transform, Noether's theorem

Continuum Mechanics. Conservation equations, strain and constraint tensors, constitutive laws (solid and fluid), frame indifference, isotropy, elasticity system, Stokes, Navier-Stokes and Euler systems, Equilibrium states, Maxwell system

Quantum Mechanics. Schroedinger equation

References. 1. V.I Arnold, Mathematical Methods in Classical Mechanics
2. P. G. Ciarlet, Three-dimensional elasticity
3. G. Duvaut, Mechanics of continuous media
4. G. Duvaut & J. L. Lions, Inequalities in Physics & Mechanics
5. R. Dautray & J. L. Lions, Mathematical Analysis and Numerical Methods for Science and Technology
6. A. Chorin & Marsden, A Mathematical Introduction to Fluid Mechanics.
7. J. Marsden & T. Hughes, Mathematical Foundations of Elasticity
8. E. Zeidler, Nonlinear Functional Analysis and its Applications IV.
10. T. Frankel, Geometry of Physiscs
11. H. Goldstein, Classical Mechanics

3. Computational Methods.

1. Review of Basic Numerical Analysis.

2. Finite Differences for Linear Equations.

Linear Hyperbolic equations, Finite differences, Theoretical concepts of Stability and consisitency, order of accuracy, upwind, Lax Fredrichs and Lax-Wendroff schemes.

Linear Parabolic equations-explicit and implicit schemes, Crank Nicholson Method, Introduction to multi dimensional problems.

Linear Elliptic equations-Finite difference schemes.

3. Finite Difference schemes for Nonlinear Equations.

One dimensional scalar conservation laws, Review of basic theory, Solutions of the Riemann problem and entropy conditions. First order schemes like Lax Fredrichs, Godunov, Enquist Osher and Roe's scheme. Convergence results, entropy consistency and Numerical viscosity.

Introduction to higher order schemes-Lax Wendroff scheme, Upwind schemes of Van Leer, ENO schemes, Central schemes, Relaxation methods. Introduction to finite volume methods.

Convection-Reaction-Diffusion equations,Extension to the above methods.Splitting schemes for multi dimensional problems.

Hamilton Jacobi equations-Level set formulation with application in Geometric Motion,Image processing and Control theory, Introduction to first and higher order schemes.

4. Finite element methods for linear equations.

Review of Elliptic equations,weak formulation and Lax Milgram lemma. Galerkin approximation, basis functions, energy methods and error estimates, Cea's estimate and Babuska Brezzi theorem.

Finite elements for Parabolic equations-Galerkin approximation and error estimates. A posteriori error estimates for Elliptic and Parabolic equations.

h_p elements, Least square finite element methods.

5. Spectral Methods.

Fast Fourier Transformation, Introduction to Fourier, spectral and pseudo spectral methods.

6. Introduction to Optimization.

Differential Calculus - Revision

Convexity, lower semi continuity
Existence, uniqueness of optimizers.
Optimality conditions: unconstrained, constrained problems. Lagrange multipliers (equality constraints),
Kuhn -Tucker Conditions, Variational inequality (inequality constrants).

Algorithms. descent method. Conjugate Gradient method.

References.

For Chapter 1:

1. Burden, Faires and Raynolds, Numerical analysis.
2. Young and Gregory- Survey of Numerical Mathematics, Vol 1 & 2.

For Chapter 2:

1. Strikwerda - Finite Difference Schemes and PDEs.
2. Sod - Numerical Methods in Fluid Dynamics.
3.Iserles- First course in the numerical Analysis of Differential Equations

For Chapter 3:

1. Godlewski and Raviart - 1. Hyperbolic systems of conservation laws
2. Numerical approximations of hyperbolic systems of conservation laws.
2. LeVeque - Numerical Methods for conservation laws.
3. Sethian - Level set methods & fast marching methods
level set Methods, evolving interfaces in geometry, fluid mechanics and computer vision & Material Science.
4. Hundsdorfer and Verver- Numerical solution of time dependent advection diffusion reaction equations.

For chapter 4:

1. Brenner and Scott- Mathematical theory of FEM
2. Thomee- Galerkin FEMs for parabolic problems
3. Johnson - Numerical solution of PDE by the FEM
4. Ciarlet - FEM for elliptic problems
5. Mercier - Lectures on topics in Finite Element solutions of Elliptic problems.
6. Ern & Guermond - Finite Elements: Theory and Practice

For chapter 5:

1. Gottlieb - Numerical Analysis of Spectral Methods
2. Boyd - Chebyshev & Fourier Spectral Methods
3. Trefethen - Spectal methods

For chapter 6:

1. Jean Cea - Lectures on Optimization-Theory and Algorithms
2. Magnus Hestense - Conjugate Direction Methods in Optimization
3. Philippe G. Ciarlet - Introduction to numerical linear algebra and optimization

⋆ Implementation of algorithms on computers is an integrable part of this course.

Appendix

The students from Int. Ph.D. programme who successfully complete the M.Sc programme are expected to register for Ph.D. during the sixth semester.

During the fifth semester they will choose some optional courses from among the following.

1. Nonlinear Analysis and Applications to PDE/ODE
2. Homogenization
3. Conservation laws and Hamilton Jacobi Equations

4. Control Theory
5. Microlocal Analysis
6. Appropriate courses by Visiting Professors
7. Self Study and other seminars.

Apart from above optional subjects there will be a compulsory course on Differential Geomety and Algebraic Topology.

Ph.D. Courses

Students who join for Ph.D. after finishing M.Sc elsewhere where will take the following courses before they register for the Ph.D.

1. Measure and Integration
2. Function Analysis
3. Elementary ODE & PDE
4. Advanced PDE
5. Algebraic topology
6. Mechanics
7. Computational Mathematics

The above courses would be compulsory and apart from these the student is expected to choose some of the special courses that will be offered. These special courses are the same as listed for Integrated Ph.D. programme.