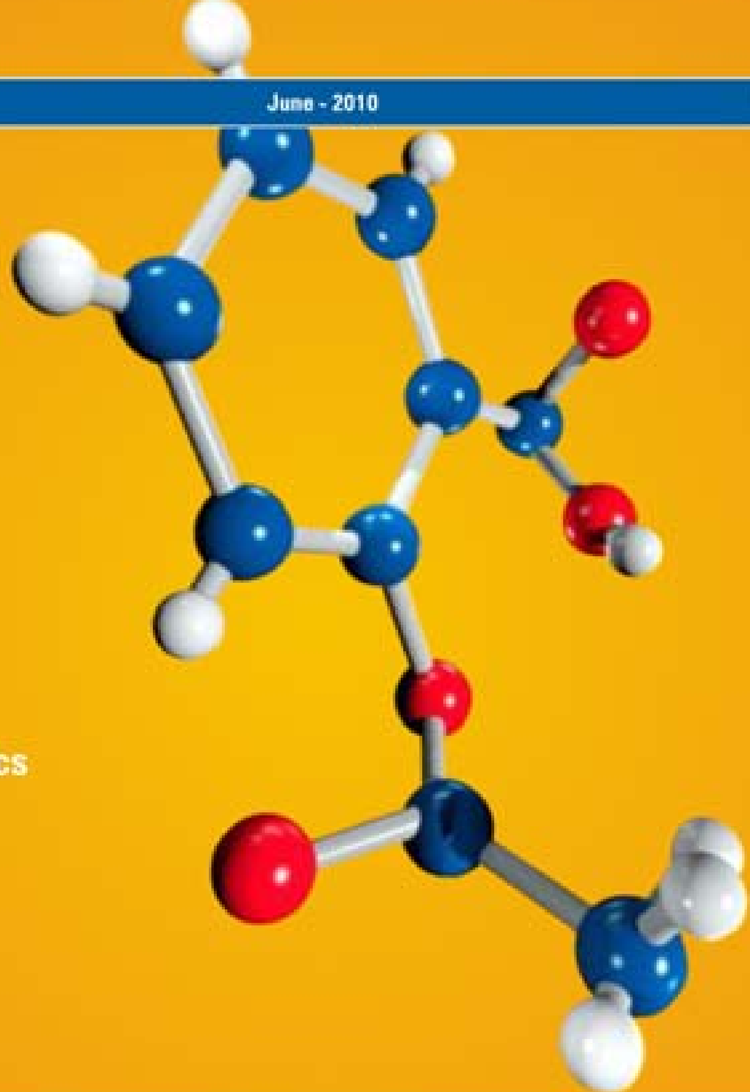
XtraEdge for IIT-JEE®

Volume 5, Issue-12 June - 2010 Price: Rs. 20/-



Physics Challenge

Catalyst Chemistry

Mathematics Challenge

IIT-JEE NEWS ARTICLE

IIT Kharagpur set to devise high-speed trains

IIT-Delhi to take in 850 more students in B-Tech this year

SOLUTION Physics

8 Challenging Problems

MY SUCCESS STORY

Mr. Srikanth Jagabathula

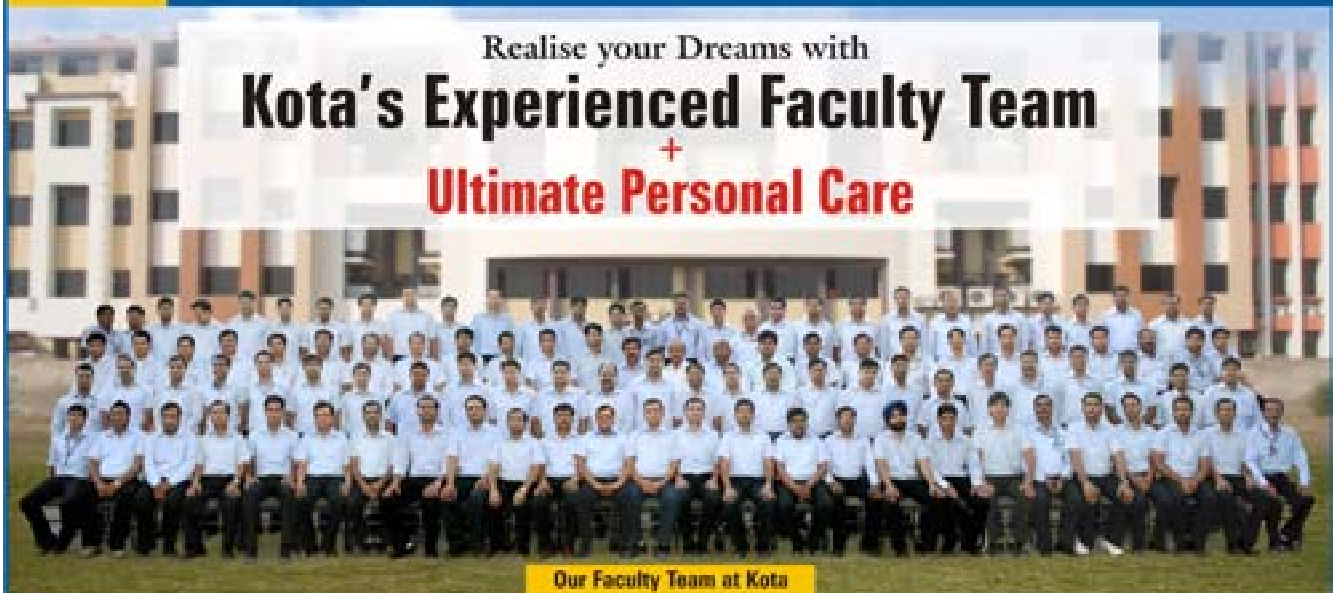
KEY CONCEPT

Nomenclature & IsomerismElectro Chemistry

IIT-JEE | AIEEE | Pre-Medical

Realise your Dreams with

Kota's Experienced Faculty Team
+
Ultimate Personal Care



Our Faculty Team at Kota

Admission Announcement for session 2010-2011

Direct Admission with Scholarship

Special Batch for IIT-JEE 2011

For EML Selected & Extra Meritorious Students

Eligibility	Scholarship
IIT-JEE 2010 qualified/IIT-JEE EML Selected	90%
NTSE (All Round) Qualifed/ XII PCM>= 95%	75%
XII PCM >= 90%/Old Career Point Student	50%
For other students Admission cum Scholarship Test on **12 June 2010**	

	IIT-JEE	AIEEE	Pre-Medical
Foundation Course [For 10th to 11th moving]	Admission cum Scholarship Test on **Phase- III (Round-1)** : 22 May'10 **Phase- III (Round-2)** : 30 May'10 **Phase- IV** : 12 June, 2010	Direct Admission, Class Starting **Phase - II** : 23 May, 2010 **Phase - III** : 17 June 2010	Nurture Course 2012 Direct Admission, Class Starting **Phase - II** : 28 June, 2010
Fresher Course [For 11th to 12th moving]	Direct Admission, Class Starting **Phase -III** : 17 June 2010	Direct Admission, Class Starting **Phase - III** : 17 June 2010	Enthuse Course 2011 Admission Closed
Target Course [For 12th appearing/pass]	Admission cum Scholarship Test on **Phase -II(Round-1)** : 12 June'10 **Phase -II (Round-2)** : 20 June'10 **Phase -III** : 27 June 2010	Direct Admission, Class Starting **Phase - II**: 23 May, 2010 **Phase - III**: 11 June, 2010 **Phase - IV**: 22 June, 2010	Target Course 2011 Direct Admission, Class Starting **Phase - I**: 30 May, 2010 **Phase - II**: 13 June, 2010
Synchro School [for 8th to 9th moving]	Admission Throught Entrance Test on : **Phase-III**: 6 June,2010 Classes will be held in association with renowned CBSE School at KOTA		Achiever Course 2011 [for Repeaters Students] Direct Admission **Phase - I**: 27 June, 2010

Application Form and Brochure for admission can be obtained (For Kota Center)

1. **Apply Offline:** ■ **In Person:** on cash payment of (a) Rs. 1000/- for IIT-JEE (b) Rs. 500/- for Pre-Medical/AIEEE/Synchro From our office OR from authorised form issue counters in your city (list on our website) ■ **By Post:** Send DD of (a) Rs. 1000/- for IIT-JEE (b) Rs. 500/- for Pre-Medical/ AIEEE/Synchro in favour of "Career Point" payable at KOTA (Rajasthan).
2. **Apply online:** Fill online Application Form available ONLINE on our website and submit it by post along with the requisite fee. (**Note:** Please, mention your Name; Father's Name, present class of studying, postal address and contect/mobile numbers on backside of the bank DD. Also keep the photocopy of DD with you for future references.)

SMS : type CP and send it to 56767 | National Helpline : 097995-77888

CAREER POINT

KOTA (H.O.) : CP Tower, Road No.1, IPIA, Kota (Raj.) Tel: 0744-3040000, 2430505 Fax : 3040050

Website: www.careerpointgroup.com | **E-mail:** info@careerpointgroup.com

XtraEdge for IIT-JEE

Volume - 5 Issue - 12

June, 2010 (Monthly Magazine)

Editorial / Mailing Office :

112-B, Shakti Nagar, Kota (Raj.) 324009

Tel. : 0744-2500492, 2500692, 3040000

e-mail : xtraedge@gmail.com

Editor :

Pramod Maheshwari

[B.Tech. IIT-Delhi]

Cover Design

Govind Saini, Om Gocher

Layout

Rajaram Gocher

Circulation & Advertisement

Ankesh Jain, Praveen Chandna

Ph (0744)- 3040007, 9001799502

Subscription

Suruchi Kabra Ph. 0744-2500492, 2500692

Unit Price Rs. 20/-

Special Subscription Rates

6 issues : Rs. 100 /- [One issue free]

12 issues : Rs. 200 /- [Two issues free]

24 issues : Rs. 400 /- [Four issues free]

Owned & Published by Pramod Maheshwari, 112, Shakti Nagar, Dadabari, Kota & Printed by Naval Maheshwari, Published & Printed at 112, Shakti Nagar, Dadabari, Kota.

Editor : Pramod Maheshwari

To Determine Your Priorities, Examine Your Goals

Editorial

Dear Students,

Everyone knows that setting goals will help you achieve more and adds excitement and meaning to life. But setting a goal is only the beginning. We often fail to follow through and our goals turn into unfulfilled daydreams. To eliminate that pitafall, here is a systematic, approach that will help you turn your goals into realities.

- **Decide** what you want to achieve. Determine exactly what you want. Be **specific**. Be sure your goal is measurable, so you can tell when you're making progress. Pick a target date for a achieving it. Be sure it is realistically achievable.
- Ask yourself why it is important for your to achieve this goal. How you will benefit from reaching this goal ? Knowing why you want something raises your level of motivation. The higher your motivation level, the more likely you are to act on your goal.
- Consider what obstacles, problems or personal shortcomings might block your progress. List every one you can think of some obstacles will be real, others may be only imaginary. You must conquer both.
- Examine the obstacles one at a time, and think about how you might solve each problem.
- List the people or organizations who could help you achieve your goal. Decide specifically what you will ask them to do.
- Consider what information you need that you don't have now. Where will you get it? What could you read? Who could you talk to? What seminars could you attend?
- Write out a detailed action plan for achieving your goal. What are the priorities involved. ? Which tasks must be done first? When will different actions take place?

Setting a goal is a good step, but it is only the beginning. It takes all seven steps to make sure you actually follow through, and by so doing achieve your goal.

I guarantee that you will succeed and will secure a good rank in your exams if you make a habit of never to postpone your work.

Forever presenting positive ideas to your success.

Yours truly

Pramod Maheshwari

Pramod Maheshwari,
B.Tech., IIT Delhi

Volume-5 Issue-12
June, 2010 (Monthly Magazine)

NEXT MONTHS ATTRACTIONS

- Much more IIT-JEE News.
- Know IIT-JEE With 15 Best Questions of IIT-JEE
- Challenging Problems in Physics,, Chemistry & Maths
- Key Concepts & Problem Solving strategy for IIT-JEE.
- Xtra Edge Test Series for JEE- 2011 & 2012

Book Your Copy Today
Subscription Form on Pg. no. 87,88

Success Tips for the Months

- If you can't make a mistake, you can't make anything.
- Sometimes a big step is safer; you can't cross a ditch in small jumps
- Self-confidence grows not from what you can do, but what you know you can do.
- Children focus on what they can't do. Adults focus on what they can do.
- The secret of confidence is to know your resources.
- You never need to feel fear if you don't want to do anything.
- You got to know when to hold 'em and know when to fold 'em…
- An ounce of success is worth a pound of positive thinking.
- To understand motivation, know the power of the Hunter.
- Defeat is advance payment for victory.

CONTENTS

IIT Kharagpur set to devise high-speed trains

As per a MoU signed recently between IIT Kharagpur and Railway Ministry, the former will carry out extensive research on developing cutting edge **technologies** , including high-speed trains and better security mechanisms for Indian Railways.

As per media reports, 12 areas have been identified by the institute and the ministry in which research will be initiated. A Centre for Railway Research (CRR) has been set up by the ministry at the institute, which comprises faculty of at least 10 departments to initiate research in the chosen areas. The railway ministry will invest an initial amount of Rs 120 crore to get the centre working.

The research would focus on areas such as heavy haul technology, vehicle dynamics, high-speed technologies, energy-efficient traction power supply systems, track research, use of artificial intelligence for predictive maintenance and management, material sciences for railway-related composites, including rubber, polymer and insulation materials, development of integrated/embedded processors for railway applications, applications for access control, security and safety, including biometrics, non-conventional drives and technology, including Maglev, LIM and remote sensing, and measurement of overhead equipment, tracks and signals, says a report published in The Times of India.

Speaking to media, Siddhartha Mukherjee, a faculty member of the electrical **engineering** department who is also a spokesperson for CRR, said, "We are looking at high-speed trains. This doesn't mean designing only the train and the engine, but also tracks that will support such trains. So, while a lot of stress would be laid on vehicle dynamics, an important portion of the research would focus on developing fracture-proof tracks and sensing equipment that would diagnose failures on time. Withstanding the load of high-speed trains is not easy and would mean extensive relaying of tracks."

Centre for Railway Research will offer PhD programmes in research areas related to the railways. It will also involve IIT BTech and MTech students in research projects and offer course electives related to railway technology.

IIT-Delhi to take in 850 more students in B-Tech this year

In order to accommodate the quota for backward classes, the **Indian Institute of Technology** (IIT)-Delhi will increase its students intake by 27 percent this year. In all, the institute will admit 850 students at the B-Tech level.

In an interview to IANS, IIT-Delhi director Surendra Prasad said, "We are adding 27 percent students more this time. With this, the government rule of increasing students' intake by 54 percent to accommodate 27 percent OBC quota has been done."

Authorities in IIT Delhi reveal that 300 more students have been enrolled in the premier institute in last three years to accommodate the OBC quota as mandated by the central government.

As per the order issued by central government, all educational **institutes** aided by government have been ordered to implement the OBC quota.

Ropar in tier-I city league by IIT

The faculty and the student community of IIT-Ropar are still at the budding stage. In the coming years they are all set to rule the academic affairs of this region. They also expect that the establishment of a esteemed institute like IIT will certainly bring Ropar in league of tier-I cities of the countries.

The institute is expanding at a great pace and it is expected that it would certainly be profitable for other colleges because the expert faculty through interactive sessions will impart knowledge to them, revealed by IIT director MK Surrapa. Being the only IIT in the region IIT-Ropar will prove to be a boon for this region. He disclosed that the procedure for recruitment of expert in all the streams has already begun and by the end of March IIT-Ropar will have more than 50 regular faculty members. The total strength of the college is 209 out of which 190 are boys.

British minister, Pat McFadden visited to the IIT for business, innovation and skills. He also interacted with the students and took a round of the institute. IIT being renowned institute, he said the collaboration of UK and IIT Ropar under research initiative will help in expansions of institutes and better research. He also added that the work between IIT and their universities will open great opportunities for the Indian students as well as British students. It'll also include student exchange programmes for better development.

Prof BK Dhindaw, dean academic and research, said, "We would be carrying forward the work that would be provided by varsities in UK,"

The student's view point is that after the establishment of IIT in Punjab, more multinational companies in the region would provide job opportunities.

IIT-JEE candidates to get performance cards now

Students appearing for the next Joint Entrance Examination (JEE) for admission to IITs will get performance cards specifying marks and the ranks secured by them in the test. However, as per the new provision, they cannot seek regrading or re-totalling.

For the first time, the JEE Board would issue performance cards which can be considered as certificates by many other institutions wanting to give admission to JEE candidates. The board will also put out the answers of the questions on its website to help students make assessment of their performance.

IIT Guwahati Director Prof Gautam Baruah said the board had urged for issuing such performance cards which would serve as certificates for the students.

"Many other institutes, which want to take JEE candidates, can give admission to students on the basis of these performance cards," Baruah said.

At present, the IITs are not issuing any scorecards to students. Certain institutes, which are giving admission to JEE candidates, have to get authenticated data from the JEE Board on the list of students and marks secured by them.

The answers will be put in the JEE website two weeks after the exam.

IIT JEE to be more transparent

The Central Information Commission (CIC) has taken a decision that the IIT Joint Entrance Examination process will be more transparent. The details of candidates such as registration number, name, parents' name, category, and marks secured should be open to public scrutiny.

Information Commissioner Shailesh Ghandhi overruled the objections by IIT Guwahati which organized the JEE 2009 said that these providing these details under the Right to Information Act does not constitute invasion of privacy.

The decision is being viewed as an important step in making the process transparent as disclosure of 2006 data had shown that formulas for calculating subject cut-offs did not tally. Irregularities were alleged in marks scored by wards of faculty members.

Court refuses to stay IIT-JEE 2010 result

New Delhi: The Delhi High Court declined to stay the declaration of this year's results for the Indian Institute Of Technology's (IIT) Joint Entrance Examination (JEE) but asked the institutes to explain as to why so many mistakes crept in during the all-India test.

A division bench of acting Chief Justice Madan B. Lokur and Justice Mukta Gupta declined to stay the declaration of IIT-JEE result but directed the IIT to explain by way of affidavit how the mistakes occurred.

"Demonstrate to us the software with which you set these papers and also how the papers are scrutinized," the court said when the counsel for IIT claimed that their system is foolproof and is up to standard.

The court directed the IIT to file an affidavit by June 2.

The court was hearing a public interest litigation (PIL) of a non-government organisation (NGO) that has sought a stay on the declaration of result of the IIT entrance exam held on April 11.

Raising the issue of errors in the instructions for examinees who took the IIT-JEE in Hindi, the NGO, Satya Foundation, filed the PIL.

Chetan Upadhyaya, secretary and counsel of Satya Foundation, submitted before the court a list of serious blunders in the IIT-JEE 2010 and said that instead of accepting the faults and re-conducting the examination, the Joint Admission Board was trying to cover up the issue with "corrective measures" which are "totally illogical and can't be digested by anybody".

"The IIT-JEE board evolved corrective measures on May 2 to ensure that genuine candidates were not affected by the examination errors. It formulated a point-by-point remedial action and posted the same on the IIT-JEE website," Upadhyaya argued.

However, after dismissal of Upadhyaya's petition, he said he will approach the Supreme Court.

Success Story

This article contains story of a person who get succeed after graduation from different IIT's

Srikanth Jagabathula

President of India gold medal winner.

Internet connectivity in rural areas at cheap rates? Well, this could be a reality if Srikanth's dream comes true.

Meet Srikanth Jagabathula, IIT's pride, the President of India gold medal winner for scoring the highest marks among all batches at Indian Institute of Technology-Bombay.

After an enviable stint at the IIT, Srikanth is all set to fly to the United States to pursue his studies at the prestigious Massachusetts Institute of Technology. After five years he plans to come back to Indian to start his own communications company.

As a kid he dreamt of becoming an engineer. Somehow he always thought an engineer's job would be very fascinating. He heard about the IIT when he was in the 7th standard. Since then IIT was his aim. After clearing his 10th class, he religiously worked towards cracking the IIT-Joint Entrance Examination. A rank of 38 at the IIT-JEE meant a smooth entry into IIT-Bombay. Srikanth, who hails from Hyderabad, was always a topper in school. Mathematics and physics were his favourite subjects, but he dreaded biology and chemistry.

A desire to top in everything he did from his schooldays helped him score high grades and he always lived up to his parents' expectations. Studies were always a priority for Srikanth and his efforts won him several awards.

From securing the first rank in the board exam in the 7th standard, securing the 8th rank in the Andhra Pradesh State Science Talent Exam when he was in the 8th standard to securing the President of India gold medal for topping across all the batches in 2005-06 in IIT-B, Srikanth's success is an inspiring story.

Any IIT-ians in the family? "No," he says, "my father has retired from the technical education department in Andhra Pradesh and my mother is a homemaker. They are really happy to see me achieve my goals though they are sad to see me go aboard for further studies."

His role models: Apple Computer founder Steve Jobs, Google founders Sergey Brin and Larry Page, Infosys founder N R Narayana Murthy and Nandan Nilekani. Srikanth is excited about his next stint at MIT and takes us through his journey from IIT to MIT.

My IIT experience

"IIT has been a dream come true for me. Initially, it was very difficult as I came from Hyderabad. It (Mumbai) was a new city and for the first time I was staying away from the family. But from the second year, everything was perfect and I made good friends here.

It is really a great learning experience. IIT helps you excel as it provides you with the best facilities and faculty.

My mantra for success

I have a desire to excel. I am very organised, am always on time, and have deep love for the subjects I learn. I have always done things I like to do, which is very important.

Interests

Reading books and participating in debates. I also like watching action-packed films and thrillers. On OBC quota.

The 50 per cent quota is not a good idea. The focus should be on improving basic education. Unless you get the basics right, there is no point introducing the quota system at the highest level. There are many backward regions in the country, so the focus should be towards a region-based development than a caste-based reservation system.

Next move

"After I joined IIT, I heard more about MIT from my seniors who studied there. I'm very happy that I got selected. I will be joining MIT soon to do my PhD. I'm very excited about it. I have got a scholarship which will waive the entire tuition fees and I will get a stipend of about $950 a month," he grins.

Advice to IIT aspirants

Don't get bogged down by pressure, many people are very talented, but it is important how you perform on the entrance day. You have to prepare well for two years, you must learn to cope with the pressure.

On India

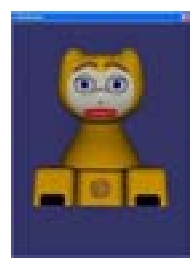
Earlier foreign jobs offered much better prospects. But now all the multinational corporations have set shop here. There are plenty of opportunities in India. So there is no need to actually go abroad. India is on a growth path and in 5 to 6 years there will be big changes in India.

Future plans

I would like to come back to India and start my own company. It will basically be a communications company. The idea is to develop products to provide cost-effective Net connectivity and communication facilities in rural India.

Now the cost is a big hindrance to Net penetration. My aim will be to remove the barriers and make communication effective and increase its reach.

THE BRAND NEW EMOTIONAL ROBOT



Most of the people always think, the robots are only the machines that didn't have any feeling. It's definitely impossible for the scientists to apply emotion on the robots.

Well, please don't be so sure about your thought yet, as the scientists at Georgia Tech had decided to test our ability to interpret a robot's emotion. The research group discovered that older adults showed some unexpected differences in the way they read a robot's face from the way younger adult did.

Jenay Beer, a graduate student in Georgia Tech's School of Psychology described that the home-based assistive robots have the potential to help older adults, as they can be used to keep the older adults independent longer. As a result, it reduces healthcare needs and provides everyday assistance to the elders.

Based on the previous research, the robot found out that older adults are less accurate in recognizing anger, fear and happiness. Furthermore, the older adults have problem recognizing the happy robot compared with their success in recognizing happy people.

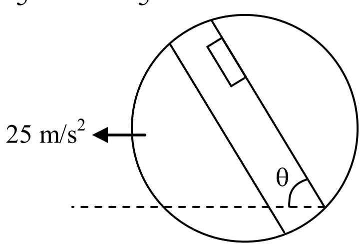
Another interesting fact about the experiment was the researchers discovered that neither the young nor old could easily distinguish the emotion disgust on the virtual iCat. It might be due to the difficulty in programming a robot to show the emotion!

KNOW IIT-JEE

By Previous Exam Questions

PHYSICS

1. A circular disc with a groove along its diameter is placed horizontally on a rough surface. A block of mass 1 kg is placed as shown. The co-efficient of friction between the block and all surfaces of groove and horizontal surface in contact is $\mu = 2/5$. The disc has an acceleration of 25 m/s^2 towards left. Find the acceleration of the block with respect to disc. Given $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. **[IIT-2006]**



Sol. Applying pseudo force ma and resolving it.

Applying $F_{net} = ma_x$ for x-direction

$ma\cos\theta - (f_1 + f_2) = ma_x$

$ma\cos\theta - \mu N_1 - \mu N_2 = ma_x$

$ma\cos\theta - \mu ma\sin\theta - \mu mg = ma_x$

$\Rightarrow a_x = a\cos\theta - \mu a\sin\theta - \mu g$

$= 25 \times \frac{4}{5} - \frac{2}{5} \times 25 \times \frac{3}{5} - \frac{2}{5} \times 10 = 10\ m/s^2$

2. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along + ve x-axis pass through the origin at t = 0. At this position the string snaps. Position of mass m_1 at time t is given by the equation $x_1(t) = v_0t - A(1 - \cos\omega t)$. Calculate **[IIT-2003]**

(a) position of the particle m_2 as a function of time

(b) ℓ_0 in terms of A.

Sol. The string snaps and the spring force comes into play. The spring force being an internal force for the two mass-spring system will not be able to change the velocity of centre of mass. This means the location of center of mass at time t will be V_0t

Now, $x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = V_0t$

$\Rightarrow m_1[V_0t - A(1 - \cos\omega t)] + m_2x_2 = V_0t\, m_1 + V_0tm_2$

$\Rightarrow m_2x_2 = V_0tm_1 + V_0tm_2 - V_0tm_1 + m_1A(1 - \cos\omega t)$

$\Rightarrow m_2x_2 = V_0tm_2 + m_1A(1 - \cos\omega t)$

$\Rightarrow x_2 = V_0t + \frac{m_1}{m_2}A(1 - \cos\omega t)$

(b) Given that $s_1 = V_0t - A(1 - \cos\omega t)$

$\therefore \frac{dx_1}{dt} = V_0 - A\omega\sin\omega t$

$\therefore \frac{d^2x_1}{dt^2} = -A\omega^2\cos\omega t$ ...(i)

This is the acceleration of mass m_1. When the spring comes to its natural length instantaneously,

then $\frac{d^2x_1}{dt^2} = 0$ and $x_2 - x_1 = \ell_0$

$\therefore \left[V_0t + \frac{m_1}{m_2}A(1 - \cos\omega t)\right] - [V_0t - A(1 - \cos\omega t)] = \ell_0$

$\left(\frac{m_1}{m_2} + 1\right)A(1 - \cos\omega t) = \ell_0$

Also when $\frac{d^2x_1}{dt^2} = 0$

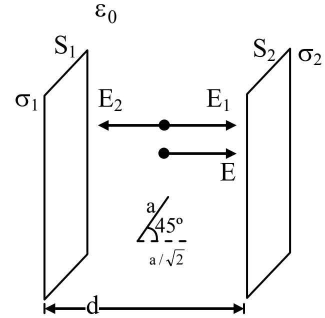
$\cos\omega t = 0$ from (i)

$\therefore \ell_0 = \left(\frac{m_1}{m_2} + 1\right)A$

3. Two uniformly charged large plane sheets S_1 and S_2 having charge densities σ_1 and $\sigma_2 (\sigma_1 > \sigma_2)$ are placed at a distance d parallel to each other. A charge q_0 is moved along a line of length $a (a < d)$ at an angle 45° with the normal to S_1. Calculate the work done by the electric field. **[IIT-2004]**

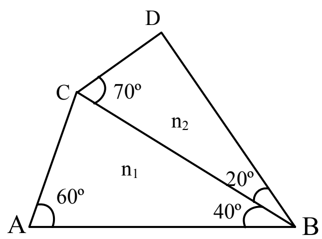
Sol. $E_1 = \frac{\sigma_1}{\varepsilon_0}$; $E_2 = \frac{\sigma_2}{\varepsilon_0}$

$E = E_1 - E_2 = \frac{\sigma_1 - \sigma_2}{\varepsilon_0}$



$W = q_0 E \times \frac{a}{\sqrt{2}}$ $\quad\therefore\ W = \frac{q_0(\sigma_1 - \sigma_2)a}{\sqrt{2}\varepsilon_0}$

4. A prism of refractive index $\mathbf{n_1}$ and another prism of refractive index $\mathbf{n_2}$ are stuck together without a gap as shown in figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light, according to $\mathbf{n_1} = 1.20 + \frac{10.8\times10^4}{\lambda^2}$ and $\mathbf{n_2} = 1.45 + \frac{1.80\times10^4}{\lambda^2}$, λ where is in nm.

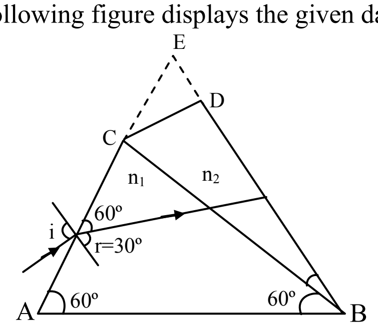


(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

[IIT-1998]

Sol. The following figure displays the given data.



(a) The rays of wavelength λ_0 incident at any angle on the interface BC will pass through without bending, provided the refractive indices n_1 and n_2 have the same value for the wavelength λ_0. Equating the expressions of n_1 and n_2, we get

$$1.20 + \frac{1.80\times10^{-4}}{\lambda_0^2} = 1.45 + \frac{1.80\times10^{-4}}{\lambda_0^2}$$

(where λ_0 is in nm)

or $\frac{1}{\lambda_0^2}(10.8 \times 10^4 - 1.80 \times 10^4) = 1.45 - 1.20$

or $\lambda_0 = \left(\frac{9.0\times10^4}{0.25}\right)^{1/2} = 600$ nm

(b) For the wavelength 600 nm, the combination of prism acts as a single prism shaped like an isosceles triangle (ABE). At the minimum deviation, the ray inside the prism will be parallel to the base. Hence, the angle of refraction on the face AC will be r = 30º

Now using the expression $n = \frac{\sin i}{\sin r}$

we get $\sin i = n \sin r = n \sin 30º = \frac{n}{2}$ ..(1)

The value of n at 600 nm is

$$n = 1.20 + \frac{10.8\times10^4}{(600)^2} = 1.50 \qquad ...(2)$$

From (1) and (2)

The angle of incidence is $i = \sin^{-1}\left(\frac{3}{4}\right)$

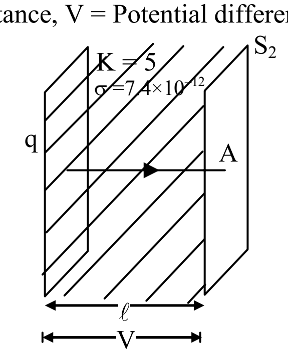
5. A leaky parallel plate capacitor is filled completely with a material having dielectric constant k = 5 and electrical conductivity $\sigma = 7.4 \times 10^{-12}\ \Omega^{-1}m^{-1}$. If the charge on the plane at instant t = 0 is q = 8.85 mC, then calculate the leakage current at the instant t = 12 s. **[IIT-1997]**

Sol. $q_0 = 8.85 \times 10^{-6}$C at t = 0

q = q at t = 12 sec

Now, $I = \frac{V}{R} = \frac{AV}{\rho\ell}$ ...(i)

R = Resistance, V = Potential difference at t sec.



$$\Rightarrow -\frac{dq}{dt} = \frac{AV}{\rho\ell}$$

$$\Rightarrow -\frac{dq}{dt} = \frac{A}{\rho\ell}\frac{q}{C} \qquad (\because q = CV)$$

$$\Rightarrow -\frac{dq}{q} = \frac{A\ell dt}{\rho\ell K\varepsilon_0 A} \qquad \left(\because C = \frac{K\varepsilon_0 A}{\ell}\right)$$

$$\Rightarrow -\frac{dq}{dt} = \frac{\sigma}{K\varepsilon_0}dt \qquad \left(\because \sigma = \frac{1}{\rho}\right)$$

$$\frac{\sigma}{K\varepsilon_0} = \frac{7.4\times10^{-12}}{5\times8.85\times10^{-12}} = 0.1672$$

$$\Rightarrow \frac{dq}{q} = -0.1672\ dt$$

On integrating

$$\int_{q_0}^{q}\frac{dq}{q} = -0.1672\int_0^t dt$$

$$\log_e\frac{q}{q_0} = -0.1672\ t \Rightarrow q = q_0e^{-0.1672\ t}$$

When t = 12 sec

$$q = \frac{q_0}{e^{0.1672t}} = \frac{8.85\times10^{-6}}{e^{0.1672t\times1L}} = \frac{8.85}{7.439}\times10^{-6}$$

$= 1.1896 \times 10^{-6}C$

From (i)

$$\therefore\ I = \frac{\sigma A}{\ell}\times\frac{q\ell}{K\varepsilon_0 A} = \frac{\sigma}{K\varepsilon_0}\times q$$

$= 0.1672 \times 1.1896 \times 10^{-6}$

$I = 0.199\ \mu A$

CHEMISTRY

6. How many grams of silver could be plated out on a serving tray by electrolysis of solution containing silver in +1 oxidation state for a period of 8.0 hours at a current of 8.46 amperes ? What is the area of the tray if the thickness of the silver plating is 0.00254 cm? Density of silver is 10.5 g cm^{-3}. **[IIT-1997]**

Sol. Given that,

$t = 8.0$ hrs $= 8 \times 3600$ s

$I = 8.46$ A

Thickness = 0.00254 cm

Density = 10.5 g cm^{-3}

$M = 108$ g mol^{-1}

$p = 1$ (for Ag)

$F = 96500$ C mol^{-1}

We know, according to Faraday's first law of electrolysis,

$$m = \frac{MIt}{Fp} = \frac{108\times8.46\times8\times3600}{96500\times1}$$

= 272.684 g

Also, m = Density × Volume

= Density × Area × Thickness

$$\therefore\ \text{Area} = \frac{m}{\text{Density}\times\text{Thickness}}$$

$$= \frac{272.684}{10.5\times0.00254}\ \mathbf{cm^2 = 10224.37\ cm^2}$$

7. At room temperature, the following reactions proceed nearly to completion :

$2NO + O_2 \rightarrow 2NO_2 \rightarrow N_2O_4$

The dimer, N_2O_4, solidified at 262 K. A 250 ml flask and a 100 ml flask are separated by a stopcock. At 300 K, the nitric oxide in the larger flask exerts a pressure of 1.053 atm and the smaller one contains oxygen at 0.789 atm. The gases are mixed by opening the stopcock and after the end of the reaction the flasks are cooled to 200 K. Neglecting the vapour pressure of the dimer, find out the pressure and composition of the gas remaining at 220 K. (Assume the gases to behave ideally) **[IIT-1992]**

Sol. According to the gas equation,

$PV = nRT$

or $n = \frac{PV}{RT}$

At room temperature,

For NO, P = 1.053 atm, V = 250 ml = 0.250 L

$\therefore$ Number of moles of NO $= \frac{1.053\times0.250}{0.0821\times300}$

= 0.01069 mol

For O_2, P = 0.789 atm, V = 100 ml = 0.1L

$\therefore$ Number of moles of $O_2 = \frac{0.789\times0.1}{0.0821\times300}$

= 0.00320 mol

According to the given reaction,

$2NO + O_2 \rightarrow 2NO_2 \rightarrow N_2O_4$

Composition of gas after completion of reaction,

Number of moles of O_2 = 0

1 mol of O_2 react with = 2 mol of NO

$\therefore$ 0.00320 mol of O_2 react with = 2 × 0.00320

= 0.0064 mol of NO

Number of moles of NO left = 0.01069 – 0.0064

= 0.00429 mol

Also, 1 mol of O_2 yields = 1 mol of N_2O_4

$\therefore$ Number of moles of N_2O_4 formed = 0.00320 mol

N_2O_4 condenses on cooling,

$\therefore$ 0.350 L (0.1 + 0.250) contains only 0.00429 mol of NO

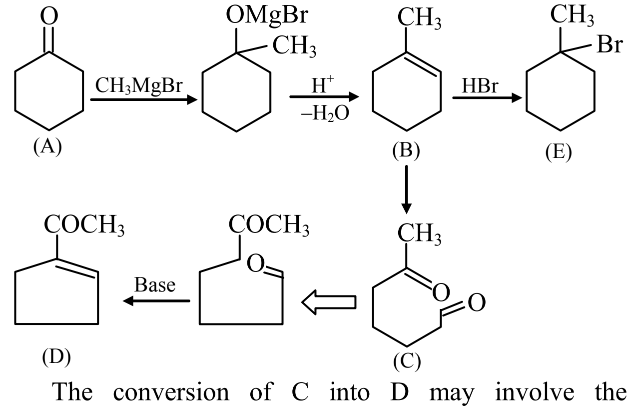
At T = 220 K,

Pressure of the gas,

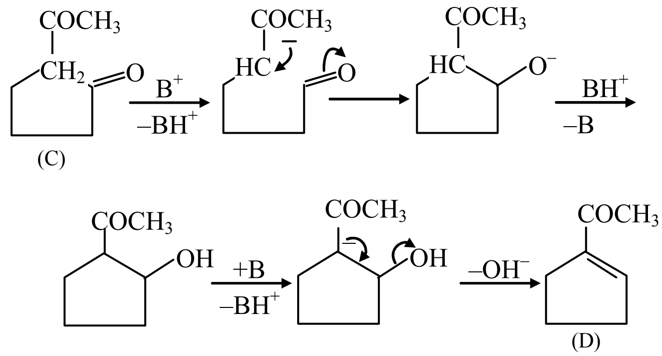
$$P = \frac{nRT}{V} = \frac{0.00429\times0.0821\times220}{0.350} = 0.221\ \text{atm}$$

8. An organic compound A, $C_6H_{10}O$, on reaction with CH_3MgBr followed by acid treatment gives compound B. The compound B on ozonolysis gives compound C, which in presence of a base gives 1-acetyl cyclopentene D. The compound B on reaction with HBr gives compound E. Write the structures of A, B, C and E. Show how D is formed from C. **[IIT-2000]**

Sol. The given reactions are as follows.

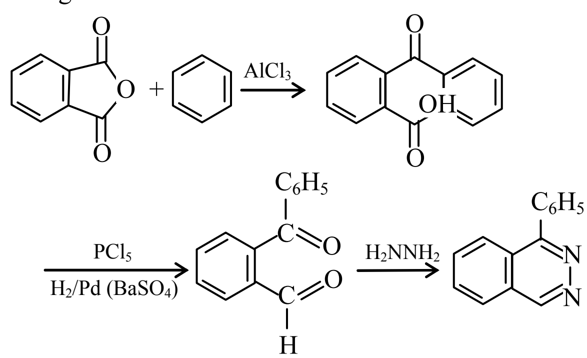


The conversion of C into D may involve the following mechanism.

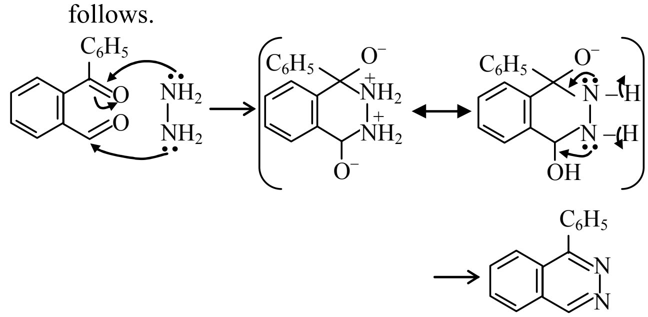


9. An organic compound A, $C_8H_4O_3$, in dry benzene in the presence of anhydrous $AlCl_3$ gives compound B. The compound B on treatment with PCl_5 followed by reaction with $H_2/Pd(BaSO_4)$ gives compound C, which on reaction with hydrazine gives a cyclised compound $D(C_{14}H_{10}N_2)$. Identify A, B, C and D. Explain the formation of D from C. **[IIT-2000]**

Sol. The given reactions are as follows.



The formation of D from C may be explained as follows.



10. An aqueous solution of salt A gives a white crystalline precipitate B with NaCl solution. The filtrate gives a black precipitate C when H_2S is passed through it. Compound B dissolves in hot water and the solution gives yellow precipitate D on treatment with potassium iodide and cooling. The compound A does not give any gas with dilute HCl but liberates a raddish brown gas on heating. Identify the compounds A to D giving the involved equation. **[IIT-1976]**

Sol. The given information's are as follows.

(a) Aqueous solution of A $\xrightarrow{NaCl}$ White precipitate of B

Filter → Residue $\xrightarrow{\text{hot water}}$ Soluble $\xrightarrow{KI}$ Yellow precipitate (D)

Filter → Filtrate $\xrightarrow{H_2S}$ Black precipitate (C)

(b) No gas $\xleftarrow{HCl}$ Salt $\xrightarrow{heat}$ Brown coloured gas

From the information given in part (a), it may be concluded that the compound A is a lead salt. B is lead chloride as it is soluble in hot water. Yellow precipitate D is due to PbI_2. Black precipitate C is due to PbS. Lead chloride being spraingly soluble in water, a little of it remains in the filtrate which is subsequently precipitated as PbS.

From the information given in part (b), it may be concluded that anion associated with lead (II) is nitrate because lead nitrate dissociates on heating as

$$2Pb(NO_3)_2 \rightarrow 2PbO + \underset{\text{Brown coloured gas}}{4NO_2} + O_2$$

Hence, A is $Pb(NO_3)_2$, B is $PbCl_2$, C is PbS and D is PbI_2.

MATHEMATICS

11. Find the centre and radius of the circle formed by all the points represented by z = x + iy satisfying the relation $\left|\frac{z-\alpha}{z-\beta}\right| = k (k \neq 1)$, where α and β are constant complex numbers given by $\alpha = \alpha_1 + i\alpha_2$, $\beta = \beta_1 + i\beta_2$. **[IIT-2004]**

Sol. As we know; $|z|^2 = z.\bar{z}$

$$\Rightarrow \frac{|z-\alpha|^2}{|z-\beta|^2} = k^2$$

$$\Rightarrow (z-\alpha)(\bar{z}-\bar{\alpha}) = k^2(z-\beta)(\bar{z}-\bar{\beta})$$

$$|z|^2 - \alpha\bar{z} - \bar{\alpha}z + |\alpha|^2 = k^2(|z|^2 - \beta\bar{z} - \bar{\beta}z + |\beta|^2)$$

or $|z|^2(1-k^2) - (\alpha - k^2\beta)\bar{z} - (\bar{\alpha} - \bar{\beta}k^2)z + (|\alpha|^2 - k^2|\beta|^2) = 0$

$$\Rightarrow |z|^2 - \frac{(\alpha - k^2\beta)}{(1-k^2)}\bar{z} - \frac{(\bar{\alpha} - \bar{\beta}k^2)}{(1-k^2)}z + \frac{|\alpha|^2 - k^2|\beta|^2}{(1-k^2)} = 0 \quad \text{...(i)}$$

On comparing with equation of circle,

$$|z|^2 + a\bar{z} + \bar{\alpha}z + b = 0$$

whose centre is (–a) and radius $= \sqrt{|a|^2 - b}$

∴ centre for (i)

$$= \frac{\alpha - k^2\beta}{1-k^2} \text{ and radius}$$

$$= \sqrt{\left(\frac{\alpha - k^2\beta}{1-k^2}\right)\left(\frac{\overline{\alpha} - k^2\overline{\beta}}{1-k^2}\right) - \frac{\alpha\overline{\alpha} - k^2\beta\overline{\beta}}{1-k^2}}$$

$$\text{radius} = \left|\frac{k(\alpha-\beta)}{1-k^2}\right|$$

12. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit, then find the probability that B hits the target and C does not. **[IIT-2003]**

Sol. Here,

$P(A)$ = probability that A will hit B $= \frac{2}{3}$

$P(B)$ = probability that B will hit A $= \frac{1}{2}$

$P(C)$ = probability that C will hit A $= \frac{1}{3}$

$P(E)$ = probability that A will be hit

$\Rightarrow$ $P(E) = 1 - P(\overline{B}).P(\overline{C})$

$$= 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$$

Probability if A is hit by B and not by C.

$\Rightarrow$ $P(B \cap \overline{C} / E)$

$$\Rightarrow \frac{P(B).P(C)}{P(E)} = \frac{\frac{1}{2}.\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$$

13. Find the equation of the normal to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ **[IIT-1993]**

Sol. $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ (given)

Let $y = u + v$, where $u = (1+x)^y$, $v = \sin^{-1}(\sin^2 x)$.

Differentiating

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad .....(1)$$

Now, $u = (1 + x)$

take logarithm of both sides

$\log_e u = \log_e (1+x)^y$

$\Rightarrow$ $\log_e u = y \log_e (1+x)$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{1+x} + \frac{dy}{dx}.\{\log_e(1+x)\}$$

$$\Rightarrow \frac{du}{dx} = (1+x)^y\left[\frac{y}{1+x} + \frac{dy}{dx}\log_e(1+x)\right] \quad .....(2)$$

Again, $v = \sin^{-1}\sin^2 x$

$\Rightarrow$ $\sin v = \sin^2 x$

$$\Rightarrow \cos v \frac{dv}{dx} = 2.\sin x \cos x$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\cos v}[2\sin x \cos x]$$

$$\Rightarrow = \frac{2\sin x\cos x}{\sqrt{1-\sin^2 v}} = \frac{2\sin x \cos x}{\sqrt{1-\sin^4 x}} \quad .....(3)$$

Put these values in equation (1)

$$\frac{dy}{dx} = (1+x)^y\left[\frac{y}{1+x} + \frac{dy}{dx}\log_e(1+x)\right] + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(1+x)^{y-1} + 2\sin x\cos x/\sqrt{1-\sin^4 x}}{1-(1+x)^y\ln(1+x)}$$

At $x = 0$

$y = (1+0)^y + \sin^{-1}\sin(0) = 1$

$$\Rightarrow \frac{dy}{dx} = \frac{1(1+0)^{1-1} + 2\sin 0.\cos 0/\sqrt{(1-\sin^4 0)}}{1-(1+0)^1\ln(1+0)}$$

$$\Rightarrow \frac{dy}{dx} = 1$$

Again the slope of the normal is

$$m = -\frac{1}{dy/dx} = -1$$

Thus, the required equation of the normal is

$y - 1 = (-1)(x - 0)$

i.e., $y + x - 1 = 0$.

14. Determine the equation of the curve passing through the origin in the from $y = f(x)$, which satisfies the differential equation $\frac{dy}{dx} = \sin(10x + 6y)$ **[IIT-1996]**

Sol. $\frac{dy}{dx} = \sin(10x+6y)$

Let $10x + 6y = t$ (given) .....(1)

$$\Rightarrow 10 + 6\frac{dy}{dx} = \left(\frac{dt}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$$

Now the given differential equation becomes

$$\sin t = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$$

$$\Rightarrow 6\sin t = \frac{dt}{dx} - 10$$

$$\Rightarrow \frac{dt}{dx} = 6\sin t + 10$$

$$\Rightarrow \frac{dt}{6\sin t + 10} = dx \text{ apply variable separable}$$

Integrating both the sides, we get

$$\int \frac{dt}{6\sin t + 10} = \int dx$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{dt}{3\sin t + 5} = x + c \qquad .....(2)$$

Let $\quad I_1 = \int \frac{dt}{3\sin t + 5}$

Put tan t/2 = u

$$\Rightarrow \quad \frac{1}{2}\sec^2 t/2 \, dt = du$$

$$\Rightarrow \quad dt = \frac{2du}{\sec^2 t/2}$$

$$\Rightarrow \quad dt = \frac{2du}{1 + \tan^2 t/2}$$

$$\Rightarrow \quad dt = \frac{2du}{1 + u^2}$$

Also, $\quad I_1 = \int \frac{dt}{3\sin t + 5} = \int \frac{dt}{3\left(\frac{2\tan t/2}{1 + \tan^2 t/2}\right) + 5}$

$$= \int \frac{(1 + \tan^2 t/2)dt}{(6\tan\frac{t}{2} + 5 + 5\tan^2\frac{t}{2})}$$

$$= \int \frac{2(1 + u^2)du}{(1 + u^2)(5u^2 + 6u + 5)}$$

$$= \frac{2}{5}\int \frac{du}{u^2 + (6/5)u + 1}$$

$$= \frac{2}{5}\int \frac{du}{u^2 + \frac{6}{5}u + \frac{9}{25} - \frac{9}{25} + 1}$$

$$= \frac{2}{5}\int \frac{du}{\left(u + \frac{3}{5}\right)^2 + \frac{16}{25}}$$

$$= \frac{2}{5}\int \frac{du}{\left(u + \frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \frac{2}{5} \cdot \frac{5}{4}\tan^{-1}\left(\frac{u + 3/5}{4/5}\right)$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5u + 3}{4}\right]$$

$$= \frac{1}{2}\tan^{-1}\left[\frac{5\tan t/2 + 3}{4}\right]$$

Putting this in (2)

Now $\quad \frac{1}{2} I_1 = x + c$

$$\Rightarrow \quad \frac{1}{4}\tan^{-1}\left[\frac{5\tan\frac{t}{2} + 3}{4}\right] = x + c$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{5\tan\frac{t}{2} + 3}{4}\right] = 4x + 4c$$

$$\Rightarrow \quad \frac{1}{4}[5\tan(5x + 3y) + 3] = \tan(4x + 4c)$$

$\Rightarrow$ 5tan (5x + 3y) + 3 = 4 tan (4x + 4c)

When x = 0, y = 0 we get

5 tan 0 + 3 = 4 tan (4c)

$$\Rightarrow \quad \frac{3}{4} = \tan 4c$$

$$\Rightarrow \quad 4c = \tan^{-1}\frac{3}{4}$$

Then, 5 tan (5x + 3y) + 3 = 4 tan $(4x + \tan^{-1} 3/4)$

$$\Rightarrow \quad \tan(5x + 3y) = \frac{4}{5}\tan(4x + \tan^{-1} 3/4) - \frac{3}{5}$$

$$\Rightarrow \quad 5x + 3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x + \tan^{-1} 3/4\} - \frac{3}{5}\right]$$

$$\Rightarrow \quad 3y = \tan^{-1}\left[\frac{4}{5}\{\tan(4x + \tan^{-1} 3/4\} - \frac{3}{5}\right] - 5x$$

$$\Rightarrow \quad y = \frac{1}{3}\tan^{-1}\left[\frac{4}{5}\{\tan(4x + \tan^{-1} 3/4\} - \frac{3}{5}\right] - \frac{5x}{3}$$

15. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Prove that the tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ arc at right angles.

[IIT-1997]

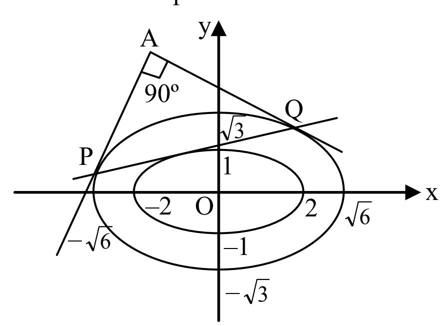
Sol. $x^2 + 4y^2 = 4$ (given)

$$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{1} = 1 \qquad .....(1)$$

Equation of any tangent to the ellipse on (1) can be written as

$$\frac{x}{2}\cos\theta + y\sin\theta = 1 \qquad .....(2)$$

Equation of second ellipse



$x^2 + 2y^2 = 6$ (given)

$$\Rightarrow \qquad \frac{x^2}{6} + \frac{y^2}{3} = 1 \qquad .....(3)$$

Suppose the tangents at P and Q meet in A(h, k).

$$\frac{hx}{6} + \frac{ky}{3} = 1 \qquad .....(4)$$

But (4) and (2) represent the same straight line, so comparing (4) and (2)

$$\frac{h/6}{\cos\theta/2} = \frac{k/3}{\sin\theta} = \frac{1}{1}$$

$$\Rightarrow \qquad h = 3\cos\theta \quad \text{and} \quad k = 3\sin\theta$$

Equation of the chord of contact of the tangents through A(h, k) is

Therefore, coordinates of A are (3cosθ.3sinθ)

Now, the joint equation of the tangents at A is given by $T^2 = SS_1$

i.e., $$\left(\frac{hx}{6} + \frac{ky}{3} - 1\right)^2 = \left(\frac{x^2}{6} + \frac{y^2}{3} - 1\right)\left(\frac{h^2}{6} + \frac{k^2}{3} - 1\right) .....(5)$$

In equation (5)

$$\text{coefficient of } x^2 = \frac{h^2}{36} - \frac{1}{6}\left(\frac{h^2}{6} + \frac{k^2}{3} - 1\right)$$

$$= \frac{h^2}{36} - \frac{h^2}{36} - \frac{k^2}{18} + \frac{1}{6}$$

$$= \frac{1}{6} - \frac{k^2}{18}$$

$$\text{coefficient of } y^2 = \frac{k^2}{9} - \frac{1}{3}\left(\frac{h^2}{6} + \frac{k^2}{3} - 1\right)$$

$$= \frac{k^2}{9} - \frac{h^2}{18} - \frac{k^2}{9} + \frac{1}{3} = -\frac{h^2}{18} + \frac{1}{3}$$

Again coefficient of x^2 + coefficient of y^2

$$= -\frac{1}{18}(h^2 + k^2) + \frac{1}{6} + \frac{1}{3}$$

$$= -\frac{1}{18}(9\cos^2\theta + 9\sin^2\theta) + \frac{1}{2}$$

$$= -\frac{9}{18} + \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{1}{2} = 0$$

which shows that two lines represent by (5) are at right angles to each other.

Science Facts

1. The storm cock or male mistlethrush sings as a thunderstorm approaches.
2. A typical double mattress contains as many as two million house dust mites.
3. The average human will grow 590 miles (949.5 km) of hair in their lifetime.
4. About 51% of incoming solar radiation is absorbed by the earth's surface and 14% absorbed by the atmosphere.
5. Only 2% of male red deer are seriously injured in their antler-rattling contests.
6. The United States recycles 25 percent of its annual 180 million tons of household rubbish.
7. The largest sapphire weighed 2,302 carats. It was found in Australia circa 1935, and was carved into the shape of the head of President Abraham Lincoln.
8. When the pharaohs' tombs were opened in Egypt early in the nineteenth century, not only human mummies were found but also those of sacred animals such as cats and ibises
9. Flies are one of the major success's of the insect world, and the 120 000+ species are divided into three sub-orders, the Nematocera, the Brachycera, and the Cyclorrhapha, and these in turn are divided into about 100 families.
10. The angle between Earth's equatorial and orbital planes varies by a few degrees every 40,000 years.
11. The San Francisco earthquake and fire of 18th April 1906 caused the deaths of around 700 people, obliterated 500 city blocks and caused $500 million of damage.
12. The slogan on New Hampshire licence plates is 'Live Free or Die'. These licence plates are manufactured by prisoners in the state prison in Concord.
13. Venus is the hottest planet with a temperature of 480 °C.

Physics Challenging Problems

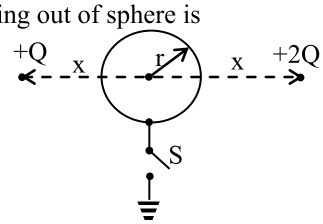
Set # 2

This section is designed to give IIT JEE aspirants a thorough grinding & exposure to variety of possible twists and turns of problems in physics that would be very helpful in facing IIT JEE. Each and every problem is well thought of in order to strengthen the concepts and we hope that this section would prove a rich resource for practicing challenging problems and enhancing the preparation level of IIT JEE aspirants.

By : Dev Sharma
Director Academics, Jodhpur Branch

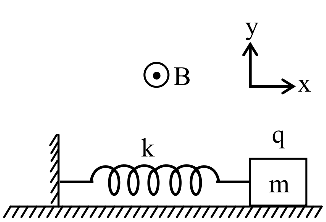
Solutions will be published in next issue

1 Two particles having positive charges +Q and +2Q are fixed at equal distance x from centre of an conducting sphere having zero net charge and radius r as shown. Initially the switch S is open. After the switch S is closed, the net charge flowing out of sphere is



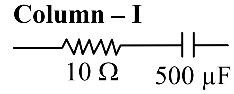
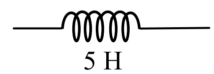
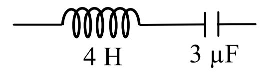
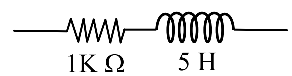
(A) $\frac{Qr}{x}$ (B) $\frac{2Qr}{x}$ (C) $\frac{3Qr}{x}$ (D) $\frac{6Qr}{x}$

2 A spring of spring constant 'K' is fixed at one end has a small block of mass m and charge q is attached at the other end. The block rests over a smooth horizontal surface. A uniform and constant magnetic field B exists normal to the plane of paper as shown in figure. An electric field $\vec{E} = E_0\hat{i}$ (E_0 is a positive constant) is switched on at t = 0 sec. The block moves on horizontal surface without ever lifting off the surface. Then the normal reaction acting on the block is

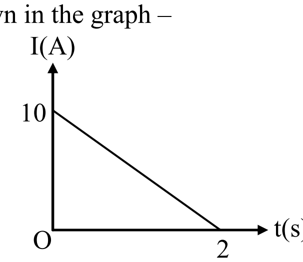


(A) maximum at extreme position and minimum at mean position
(B) maximum at mean position and minimum at extreme position
(C) uniform throughout the motion
(D) both maximum and minimum at mean position

3 Four different circuit components are given in each situation of column – I and all the components are connected across an ac source of same angular frequency $\omega = 200\,\text{rad/sec}$. The information of phase difference between the current and source voltage in each situation of column – I is given in column – II.

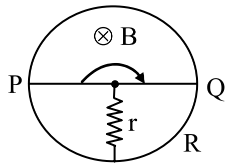
Column – I	Column – II
(A) 	(P) the magnitude of required phase difference is $\pi/2$
(B) 	(Q) the magnitude of phase difference is $\pi/4$
(C) 	(R) the current leads in phase to source voltage
(D) 	(S) the current lags in phase to source voltage
	(T) the magnitude of required phase

4 A variable current flows through a 1Ω resistor for 2 seconds. Time dependence of the current is shown in the graph –



(A) Total charge flown through the resistor is 10C
(B) Average current through the resistor is 5A
(C) Total heat produced in the resistor is 50J
(D) Maximum power during the flow of current is 100W

Q. 5 In the figure shown, 'R' is a fixed conducting fixed ring of negligible resistance and radius 'a'. PQ is a uniform rod of resistance r. It is hinged at the centre of the ring and rotated about this point in clockwise direction with a uniform angular velocity ω. There is a uniform magnetic field of strength 'B' pointing inwards, 'r' is a stationary resistance, then-



(A) Current through 'r' is zero

(B) Current through 'r' is $\dfrac{2B\omega a^2}{5r}$

(C) Direction of current in external 'r' is from centre to circumference

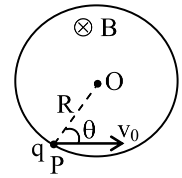
(D) Direction of current in external 'r' is from circumference to centre

Q. 6 In column – I condition on velocity, force and acceleration of a particle is given. Resultant motion is described in column – II.

$\vec{u}$ = instantaneous velocity –

Column – I	Column – II
(A) $\vec{u} \times \vec{F} = 0$ and $\vec{F}$ = constant	(P) path will be circular path
(B) $\vec{u} \cdot \vec{F} = 0$ and $\vec{F}$ = constant	(Q) speed will increase
(C) $\vec{v} \cdot \vec{F} = 0$ all the time and $\lvert\vec{F}\rvert$ = constant and the particle always remains in one plane	(R) path will be straight line
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and acceleration at all time $\vec{a} = 6\hat{i} - 9\hat{j}$	(S) path will be parabolic

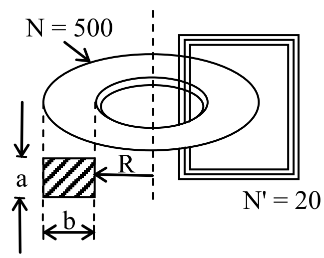
Q. 7 A particle of charge q and mass m is projected with a velocity v_0 towards a circular region having uniform magnetic field B perpendicular and into the plane of paper from point P as shown in the figure. R is the radius and O is the centre of the circular region. If the line OP makes angle θ with the direction of v_0, then the value of v_0 so that particle passes through O is



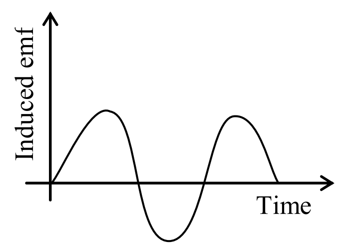
(A) $\dfrac{qBR}{m\sin\theta}$ (B) $\dfrac{qBR}{2m\sin\theta}$

(C) $\dfrac{2qBR}{m\sin\theta}$ (D) $\dfrac{3qBR}{2m\sin\theta}$

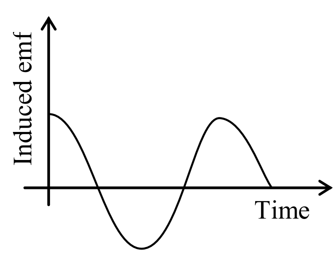
Q. 8 A toroid having a rectangular cross section (a = 2.00 cm by b = 3.00 cm) and inner radius r = 4.00 cm consists of 500 turns of wire that carries a current $I = I_{max}\sin\omega t$ with $I_{max} = 50.0\text{A}$ and a frequency $f = \dfrac{\omega}{2\pi} = 60.0\text{Hz}$. A coil that consists of 20 turns of wire links with the toroid as shown in figure –



(A)



(B)



(C) Max value of induced emf is 0.422 V

(D) Max value of induced emf is 0.122 V

8 Solution

Physics Challenging Problems

Set # 1

Questions were Published in May Issue

1. **[D]**

After closing both switch potential difference across C_1 and C_2 will be different

$V_{C_1} = V; V_{C_2} = 0$

2. **[B,C]**

$$E_0 z^2\left(1-\frac{1}{9}\right) - E_0 z^2\left(\frac{1}{4}-\frac{1}{9}\right) = 3E_0$$

$z = 2$

$$\frac{\lambda_1}{\lambda_2} = 3$$

$$KE_1 = E_0\left(1-\frac{1}{9}\right) - \phi$$

$$KE_2 = E_0 z^2\left(1-\frac{1}{4}\right) - \phi$$

$$KE \alpha \frac{1}{\lambda^2} = 8.5\text{eV}$$

3. **[B,D]**

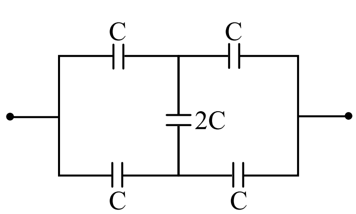
$e_{AB} = \left(\frac{dB}{dt}\right)$ area of ΔAOB, using (E.M.I.)

$$= (4)\frac{1}{2}\times\left(4\times\frac{\sqrt{3}}{2}\times 2\right)\times 2$$

Total e of loop

$$= 3\times\left(4\times\frac{1}{2}\times 4\times\frac{\sqrt{3}}{2}\times 2\right)\times 2 = 2\times 24\sqrt{3} = 48\sqrt{3}\text{Volt}$$

4. **[1]**



5. (A) $\rightarrow$ T,R,S (B) $\rightarrow$ R

(C) $\rightarrow$ Q,R (D) $\rightarrow$ P,T

6. **[B]**

$V_{total} = V_{solid} + V_{shell}$

$$= \frac{KQ}{2a^3}\left[3a^2 - \frac{a^2}{4}\right] + \left(\frac{-KQ}{b}\right)$$

$$= KQ\left[\frac{11}{8a} - \frac{1}{b}\right]$$

7. **[C]**

$V_{surface}$ at inner solid sphere $= \frac{KQ}{a} - \frac{KQ}{b}$

$V_{inner\ surface}$ of shell $= 0$

$$\Delta V = KQ\left(\frac{1}{a}-\frac{1}{b}\right); W = KqQ\left(\frac{1}{a}-\frac{1}{b}\right)$$

8. **[B]**

If outer ball is grounded.

No change is charge distribution will occurs at outer shell is already at zero potential

HOW HEAVY IS THE EARTH ?

Now despite science coming forward in leaps and bounds the simple question of how much the Earth weighs isn't as straightforward as you might think. Certainly placing the earth onto a massive set of weighing scales isn't an option. So how do we measure the total weight of the earth ?

Students' Forum

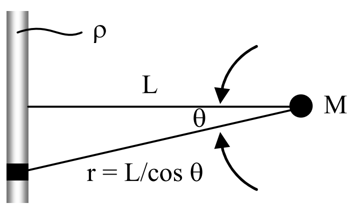
Expert's Solution for Question asked by IIT-JEE Aspirants

1. Find the gravitational force between a point like mass M and an infinitely long, thin rod, of mass density ρ, which is at a distance L from the mass M.

Sol. Considering figure, let us divide the rod into very small mass elements, each one of magnitude dm, which can be expressed as :

$$dm = \frac{r d\theta}{\cos\theta}\rho = \frac{L d\theta}{\cos^2\theta}\rho$$

Note : We are using a polar co-ordinate system. Hence, have $d\ell = \frac{r\, d\theta}{\cos\theta}$ on the rod.



Since the rod is symmetric, the components of the force element parallel to the rod will cancel each other, and the total force will be the sum of the force elements perpendicular to the rod. So, for a mass element dm at distance r from M, we have:

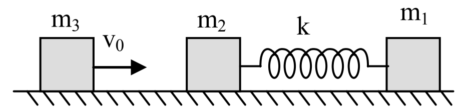
$$F_{\perp(dm)} = \frac{GM(dm)}{\left(\frac{L}{\cos\theta}\right)^2}\cos\theta = \frac{MG\rho}{L}\cos\theta\, d\theta$$

Total force is :

$$F = \frac{MG\rho}{L}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{2MG\rho}{L}\int_{0}^{\pi/2}\cos\theta d\theta = \frac{2MG\rho}{L}$$

2. Two masses m_1 ad m_2 are attached to the two ends of a spring of force constant k. The system lies horizontally on a perfectly smooth surface. A third mass, m_3, is thrown with a velocity of v_0, horizontally onto the plane to hit mass m_2. The two masses m_2 and m_3 stick together at the moment of collision. The sticking process occurs almost immediately, so that the length of the spring does not change. It is given that $m_1 = 2m$, $m_2 = m$ and $m_3 = \frac{1}{2}m$.



(i) What is the velocity of the center of mass before and after the collision ?

(ii) How much kinetic energy is lost during the collision ?

(iii) What are the velocities of the masses m_1 and $m_2 + m_3$ immediately after the collision ?

(iv) What is the maximum that the spring shrinks ?

Sol. **(i)** There are no external forces acting on the system; therefore, its total linear momentum is conserved. This means that $\vec{v}_{CM}$ is conserved throughout the collision process, or

$$\vec{v}_{CM}^{\,before} = \frac{\Sigma_i m_i \vec{v}_i}{\Sigma_i m_i} = \frac{m_3 v_0}{m_1 + m_2 + m_3}\hat{x} = \vec{v}_{CM}^{\,after}$$

Since this problem is unidimensional, we omit the vector notation from now on.

(ii) Let us write the expressions for the kinetic energy:

$$E_{before} = E_k^{(m_3)} = \frac{1}{2} m_3 v_0^2$$

$$E_{after} = E_k^{(m_2+m_3)} = \frac{1}{2}(m_2 + m_3)\, v_{2+3}^2$$

Notice that since the time that elapses when masses m_2 and m_3 stick together is short, mass m_1 stays at rest during the process. We calculate v_{2+3} using the law of conservation of linear momentum.

$$m_3 v_0 = (m_2 + m_3)v_{2+3} \rightarrow v_{2+3} = \frac{m_3}{m_2 + m_3} v_0$$

Substituting the value of v_{2+3} into E_{after}, we find :

$$E_{after} = \frac{1}{2}\frac{m_3^2}{m_2 + m_3} v_0^2$$

Therefore, the loss of kinetic energy, ΔE, is :

$$\Delta E = E_{before} - E_{after}$$

$$= \frac{1}{2}\left(m_3 - \frac{m_3^2}{m_2 + m_3}\right)v_0^2$$

$$= \frac{1}{2}\frac{m_2 m_3}{m_2 + m_3} v_0^2$$

(iii) Velocities in the centre of mass frame are determined by :

$$v' = v - v_{CM}$$

where v is the velocity in the laboratory frame. Hence,

$$v'_{2+3} = v_{2+3} - v_{CM} = \frac{m_1 m_3}{(m_2 + m_3)(m_1 + m_2 + m_3)}$$

and since $v_1 = 0$, we have

$$v'_1 = v_1 - v_{CM} = -\frac{m_3}{m_2 + m_3} v_0$$

(iv) We use the law of conservation of energy

$$E_{after} = \frac{1}{2}(m_2 + m_3) v_{2+3}^2$$

$$E'_{after} = \frac{1}{2}(m_1 + m_2 + m_3) v_{CM}^2 + \frac{1}{2} k(\Delta x)^2_{max}$$

to determine the maximal shrinking of the spring. E'_{after} is the energy in the state of maximal shrinking. Note that in the state of maximal shrinking, the three masses move at the same velocity as the center of mass. Using the equality $E'_{after} = E'_{after}$, and substituting v_{CM} and v_{2+3}, we obtain :

$$(\Delta x)_{max} = m_3 v_0 \sqrt{\frac{m_1}{k(m_1 + m_3)(m_1 + m_2 + m_3)}}$$

3. Two identical buggies move one after the other due to inertia (without friction) with the same velocity $\vec{V}_0$. A man of mass m rides the rear buggy. At a certain moment the man jumps into the front buggy with a velocity $\vec{u}$ relative to his buggy knowing that the mass of each buggy is equal to M. Find the velocity with which the buggies will move after that.

Sol. Conserving momentum for the rear buggy.

$$(M + m)\vec{V}_0 = m[\vec{u} + \vec{V}_r] + M\vec{V}_r \qquad ...(1)$$

where $\vec{V}_r$ = Velocity of rear buggy finally.

The velocity of the man with respect to ground

$$\vec{V}_{man\,ground} = \vec{V}_{man\,buggy} + \vec{V}_{buggy\,ground} = \vec{u} + \vec{V}_r$$

Solving (1), we get

$$\vec{V}_r = \vec{V}_0 - \frac{m\vec{u}}{M + m}$$

Conserving momentum for front buggy

$$M\vec{V}_0 + m[\vec{u} + \vec{V}_r] = (M + m)\vec{V}_f$$

where $\vec{V}_f$ = Velocity of front buggy.

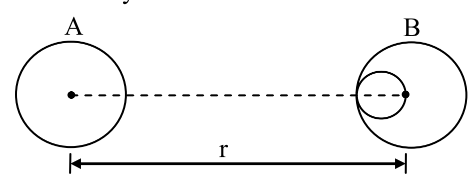
Finally as, $\vec{V}_r = \vec{V}_0 - \dfrac{m\vec{u}}{M + m}$

Hence, $M\vec{V}_0 + m\vec{u} + m\vec{V}_0 - \dfrac{m^2\vec{u}}{M + m} = (M + m)\vec{V}_f$

or, $(M + m)\vec{V}_0 + \dfrac{Mm\vec{u}}{M + m} = (M + m)\vec{V}_f$

$$\therefore \quad \vec{V}_f = \vec{V}_0 + \frac{Mm\vec{u}}{(M + m)^2}$$

4. Distance between centres of two spheres A and B, each of radius R is r as shown in Figure. Sphere B has a spherical cavity of radius



R/2 such that distance of centre of cavity is (r – R/2) from the centre of sphere A and R/2 from the centre of sphere B. Di-electric constant of material of each sphere is K = 1 and material of each sphere has a uniform charge density ρ per unit volume. Calculate interaction energy of the two spheres.

Sol. Interaction energy between two spheres having uniformly distributed charges q_1 and q_2 is given by

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

Where r is distance between their centres.

Since the right sphere has a cavity, therefore it does not have uniformly distributed charge. This sphere can be assumed to be result of superposition of two solid spheres, one of radius R having a positive charge density ρ and a solid sphere of radius R/2 and having a charge density (– ρ).

Required interaction energy will be the sum of the following two energies:

(i) Interaction energy U_1 of left sphere and a solid sphere of radius R and having charge density ρ. Separation between centres of these spheres is r.

$$\therefore \quad U_1 = \frac{1}{4\pi\varepsilon_0} \frac{\left(\frac{4}{3}\pi r^3 \rho\right)\left(\frac{4}{3}\pi R^3 \rho\right)}{r} = \frac{4\pi\rho^3 R^6}{9\varepsilon_0 r}$$

(ii) Interaction energy U_2 of left sphere and a solid sphere of radius R/2, having charge density (– ρ). Separation between centre of these spheres is (r – R/2)

$$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi r^3\rho\right).\left\{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3(-\rho)\right\}}{\left(r-\frac{R}{2}\right)} = \frac{\pi\rho^2 R^6}{9\varepsilon_0(2r-R)}$$

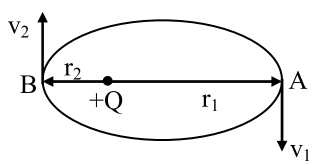
Hence, required interaction energy,

$U = U_1 + U_2$

$$= \frac{\pi\rho^2 R^6(7r-4R)}{9\rho_0 r(2r-R)}$$ **Ans.**

5. A particle of mass m having negative charge q moves along an ellipse around a fixed positive charge Q so that its maximum and minimum distances from fixed charge are equal to r_1 and r_2 respectively. Calculate angular momentum L of this particle.

Sol. During movement, moving particle experiences an electrical force which is always directed towards fixed particle. Therefore, moment of this force about fixed particle remains always zero. Hence, angular momentum L of moving particle about fixed particle remains constant.



At instant of maximum and minimum distance, velocity vectors of moving particle are perpendicular to respective radii vectors as shown in Figure. Let at these particle be υ_1 and υ_2 respectively.

$\therefore$ Angular momentum, $L = m\upsilon_1 r_1 = m\upsilon_2 r_2$

or $\upsilon_1 = \frac{L}{mr_1}$ and $\upsilon_2 = \frac{L}{mr_2}$

Total energy of the system of two particles when moving particle is at A is

E_1 = kinetic energy of moving particle at A + electrical energy of the system

$$= \frac{1}{2}m\upsilon_1^2 + \frac{1}{4\pi\varepsilon_0}\frac{Q(-q)}{r_1}$$

Similarly, total energy at B is E_2

$$= \frac{1}{2}m\upsilon_1^2 + \frac{1}{4\pi\varepsilon_0}\frac{Q(-q)}{r_2}$$

But there is no external force on the system of two particles, therefore, total energy remains constant.

$\therefore \quad E_1 = E_2$

or $$\frac{1}{2}m\left(\frac{L}{mr_1}\right)^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} = \frac{1}{2}m\left(\frac{L}{mr_2}\right)^2 - \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2}$$

or $$L = \sqrt{\frac{mr_1r_2Qq}{2\pi\varepsilon_0(r_1+r_2)}}$$ **Ans.**

Learn How to Study

Designate a specific time for studying.

Cramming is not a reliable way to retain information.

- Take understandable lecture and textbook notes.
- Become a good listener in class and review your notes daily.
- Organize your class materials prior to each study session.
- Concentrate on the class material and keep in mind course objectives.
- Ask for help early if needed (such as your professor or teaching assistants).
- Tutoring assistance can provide a positive impact on your academic success.
- Form study groups and partners. Keep everyone committed to studying, not small talk.

Roommate Tips

- Communication is a must! Listen and learn from each other.
- Talk to each other early about problems that may arise. Don't let problems build up.
- Living with someone requires increased sensitivity to the needs of others.
- Respect each other's privacy and personal space.
- Agree upon a set period of "quiet time" for study without distractions.

If you have time to whine and complain about something then you have the time to do something about it.

Once our personal connection to what is wrong becomes clear, then we have to choose: we can go on as before, recognizing our dishonesty and living with it the best we can, or we can begin the effort to change the way we think and live.

Once you choose hope, anything's possible.

Electrostatics-2

KEY CONCEPTS & PROBLEM SOLVING STRATEGY

Electric Potential Energy:

If a point charge q_1 is present in an electric field where potential is V, by definition

$V = (U/q_1)$

i.e., $U = q_1V$

And if the field is produced by a point charge q_2 which is at a distance r_{12} from q_1,

$$U = q_1\left[\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{12}}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}} \qquad \ldots(1)$$

So in case of discrete distribution of charges

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \ldots\ldots\right) = \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\sum_{i\neq j}\frac{q_iq_j}{r_{ij}}$$

Here it is worth noting

(a) Electrical potential energy is not localised but is distributed all over the field

(b) If the electric potential energy of a system in one configuration is U_1 and in the other U_F, work done in changing the configuration will be

$W_{IF} = -U_{IF} = -(U_1 - U_F) = U_F - U_1$

And as potential energy at infinity is zero, work done in assembling or disassembling a given charge distribution will be respectively,

$W = U_F$ [as $U_1 = 0$] and $W = -U_1$ [as $U_F = 0$]

(Assembling) (Disassembling)

Motion of a Charged Particle in an Electric Field

In case of motion of a charged particle in an electric field :

(a) As by definition of electric intensity $\vec{E}$, $\vec{F} = q\vec{E}$, a point charge always experiences a force either at rest or in motion.

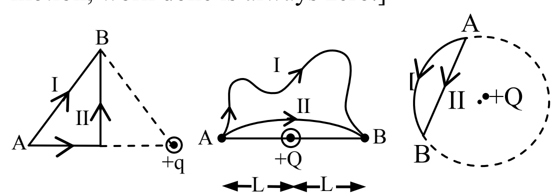
(b) The direction of force is parallel to the field if the charge is positive and opposite to the field if charge is negative.

(A) (B)

(c) Electric field is conservative in an electric field work is path independent and work done in moving a point charge q between two fixed points having potential difference V is equal to,

$$W_{AB} = -U_{AB} = q(V_B - V_A)qV \qquad \ldots(1)$$

And hence in moving a charged particle in an electric field work is always done unless the points are at same potential as shown in figure. [However, in magnetic field as force is always perpendicular to motion, work done is always zero.]



(A) (B) (C)

(d) When a charged particle is accelerated by an electric field (uniform or non-uniform) by Work energy theorem, i.e., $\Delta KE = W$, we have

$$\frac{1}{2}m\upsilon^2 - \frac{1}{2}mu^2 = qV \text{ [as from Eq. (1) W = qV]}$$

or $$\upsilon = \sqrt{\left[u^2 + \frac{2qV}{m}\right]} \qquad \ldots(2)$$

And if the charged particle is initially at rest, i.e., u = 0

$$\upsilon = \sqrt{\frac{2qV}{m}} \qquad \ldots(3)$$

And if the field is uniform, i.e., $E = (V/d)$

$$\upsilon = \sqrt{\frac{2qEd}{m}} \qquad \ldots(4)$$

(e) In case of motion of a charged particle in a uniform electric field if force of gravity does not exist (or is balanced by some other force

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \text{constant [as } \vec{F} = q\vec{E}\text{]} \qquad \ldots(5)$$

So equation of motion are valid. Now there are two possibilities:

(i) If the particle is initially at rest

From Eq. $\upsilon = u + at$, we get

$$\upsilon = at = \frac{qE}{m}t \quad \left[\text{as } u = 0 \text{ and } a = \frac{qE}{m}\right] \qquad \ldots(6)$$

And from Eq. $s = ut + \frac{1}{2}at^2$

$$s = \frac{1}{2}at^2 = \frac{1}{2}\frac{qE}{m}t^2 \qquad \ldots(7)$$

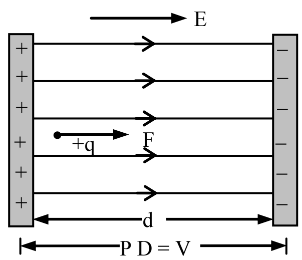
i.e., the motion is accelerated translatory with

$a \propto t^0$; $\upsilon \propto t$ and $s \propto t^2$

Further more in this situation :

$$W = \Delta KE = \frac{1}{2}m\upsilon^2 = \frac{1}{2}m\left[\frac{qE}{m}t\right]^2$$

$$\left[\text{as from Eq.(6)} \ \upsilon = \frac{qEt}{m}\right]$$



Which in the light of Eq. (7) with

s = d, gives

$W = qEd = qV$ [as $E = V/d$] …(8)

(ii) If the particle is projected perpendicular to the field with an initial velocity υ_0

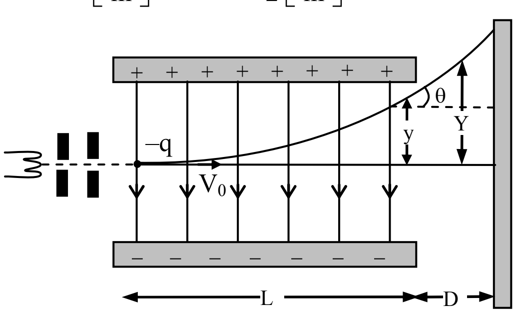
From Eq. $\upsilon = u + at$ and $s = ut + \frac{1}{2}at^2$ respectively

for motion along x-axis as $u = \upsilon_0$ and $a = 0$,

$\upsilon_x = \upsilon_0$ = constt. and $x = \upsilon_0 t$ …(9)

While for motion along y-axis as u = 0 and $a = (qE/m)$,

$$\upsilon_y = \left[\frac{qE}{m}\right]t \ \text{ and } \ y = \frac{1}{2}\left[\frac{qE}{m}\right]t^2 \qquad \ldots(10)$$



So eliminating t between equation for x and y, we have

$$y = \frac{qE}{2m}\left[\frac{x}{\upsilon_0}\right]^2 = \frac{qE}{2m\upsilon_0^2} \qquad \ldots(11)$$

i.e., the path is a parabola.

[However, under same conditions in magnetic field path is a circle.]

Electric Dipole

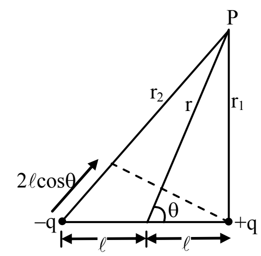
Definition: If two equal and opposite point charges are separated by a distance 2ℓ such that the distance of field point $r >> 2\ell$, the system is called a dipole.

(a) Field of a Dipole

Potential due to dipole at a point (r, θ) as shown in Figure will be

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$



Now as , $r >> 2\ell$

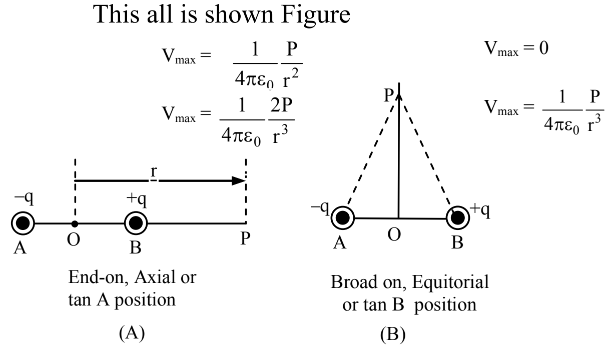
$r_1 \times r_2 = r^2$ and $r_2 - r_1 = 2\ell\cos\theta$

So, $$V = \frac{1}{4\pi\varepsilon_0}\frac{q(2\ell\cos\theta)}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{P\cos\theta}{r^2} \quad [\text{as } p = q \times 2\ell] \quad] \ldots(1)$$

(b) Potential will be minimum when |cos θ| = min = 0, i.e., θ = 90º. So for broad on, equitorial or tan B position, potential is minimum and is zero, i.e., $V_{min} = 0$

This all is shown Figure



(c) Now as electric field,

$$\vec{E} = -\frac{dV}{dx}\vec{n}$$

So component of electric intensity in the direction of r :

$$E_r = -\frac{d}{dr}\left[\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \qquad \ldots(i)$$

And perpendicular to r,

$$E_\theta = -\frac{d}{rd\theta}\left(\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

…(ii)

So that, $\tan\phi = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$

and, $E = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}\sqrt{(1+3\cos^2\theta)}$

…(2)

From this it is clear that :

(1) Intensity due to a dipole varies as $(1/r^3)$ and can never be zero unless $r \to \infty$ or $p \to 0$.

(2) E will be maximum when $\cos^2\theta$ = max = 1, i.e., $\theta = 0^o$, i.e., for end on, axial or tan A position E is maximum and is,

$$E_{max} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

(3) E will be minimum when $\cos^2\theta$ = min = 0, i.e., $\theta = 90^o$, i.e., for broad on, equitorial or tan B position E is minimum and is,

$$E_{min} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

Conductor in Electrostatics

The substances such as metals which allow the charge to flow freely through them are called conductors. In metals conduction envolves the movement of free electrons. In case of conductors in electrostatics, it is worth noting that :

(a) In charging a conductor electrons are removed, conductor becomes positively charged and its potential increases and if added, it becomes negatively charged and its potential decreases.

(b) When a conductor is charges by induction, induced charge (Which is free to move) is equal and opposite to the inducing charge, i.e.,q' = –q

(c) Charge resides on the outer surface of a conductor. However, distribution of charge on the surface is generally not uniform and surface density of charge varies inversly as the radius of curvature of that part of the conductor, i.e.,

$$\sigma \propto (1/R)$$

(d) The dielectric constant of conductors in electrostatics is infinite, i.e., $K = \infty$

(e) Electric intensity inside a conductor is zero while outside (near its surface) is (σ/ε_0), i.e.,

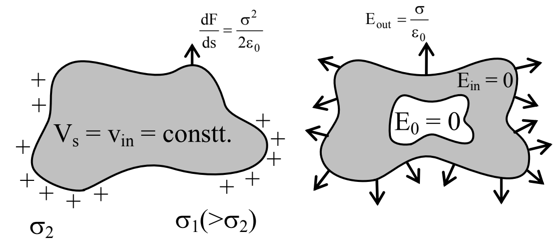
$$E_{in} = 0 \quad \text{and} \quad E_{out} = (\sigma/\varepsilon_0)$$

(f) Conductor is an equipotential surface, i.e., potential at its surface or inside everywhere is same, i.e., for a conductor

V = constt.

(g) Electric field and hence lines of force are normal to the surface of a conductor.

(h) The field in a cavity inside a conductor is zero resulting in **'electrostatic shielding.'**



(A) (B)

Solved Examples

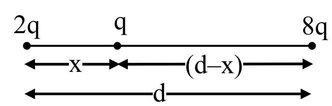
1. Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ?

Sol. As potential energy of two point charges separated by a distance r is given by $U(=q_1q_2/4\pi\varepsilon_0 r)$, so to have minimum potential energy the charges of greater value should be farthest, i.e., q must be between 2q and 8q. Let q be at a distance x from 2q, then potential energy of the system will be

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{2qq}{x} + \frac{8qq}{(d-x)} + \frac{8q\times 2q}{d}\right]$$

For U to be minimum (dU/dx) = 0]

i.e., $-\frac{2q^2}{x^2} + \frac{8q^2}{(d-x)^2} = 0$

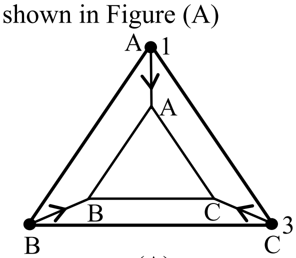


i.e., 2x = (d – x) or x = (d/3) = (9/3) = 3 cm

So to have minimum potential energy, the charge q must be placed at a distance of 3 cm from 2q between 2q and 8q on the line joining the charges. In this situation, Field at q

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{(3)^2} - \frac{8q}{(6)^2}\right] = 0$$ **Ans.**

2. Three point charges 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 1m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m as shown in Figure (A)



(A)

Sol. As potential energy of two charges separated by a distance r is given by $U = [q_1q_2/4\pi\varepsilon_0 r]$, the initial and final potential energy of the system will be

$$(U_S)_I = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times2}{1}+\frac{2\times3}{1}+\frac{3\times1}{1}\right]$$

$$= 9\times10^9\times11 = 9.9\times10^{10}\ \text{J}$$

$$(U_S)_F = \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times2}{0.5}+\frac{2\times3}{0.5}+\frac{3\times1}{0.5}\right]$$

$$= 9\times10^9\times22 = 19.8\times10^{10}\text{J}$$

So work done in changing the configuration of the system:

$$W = (U_S)_F - (U_S)_I = (19.8 - 9.9)\times10^{10}$$

$$= 9.9\times10^{10}\text{J}$$ **Ans.**

3. A particle of mass 40 µg and carrying a charge 5×10^{-9} C is moving directly towards fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has a velocity of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily to rest ? Is the acceleration constant during motion ?

Sol. If the particle comes to rest momentarily at a distance r from the fixed charge, then from 'conservation of energy' we have

$$\frac{1}{2mu^2} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Substituting the given date, we get :

$$\frac{1}{2}\times40\times10^{-6}\times\frac{1}{2}\times\frac{1}{2} = 9\times10^9\times10^{-8}\times5\times10^{-9}\left[\frac{1}{r}-10\right]$$

or $\frac{1}{r} - 10 = \frac{5\times10^{-6}}{9\times5\times10^{-8}} = \frac{190}{9}$

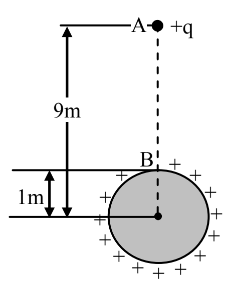
or $\frac{1}{r} = \frac{100}{9} + 10 = \frac{100}{9}$ m

i.e., $r = 4.7\times10^{-2}$ m

As here, $F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$ So acc. $= \frac{F}{m} \propto \frac{1}{r^2}$

i.e., acceleration is not constant during motion.

4. A very small sphere of mass 80 g having a charge q is held at a height 9 m vertically above the centre of a fixed conducting sphere of radius 1m, carrying an equal charge q. When released it falls until it is repelled just before it comes in contact with the sphere. Calculate the charge q. [$g = 9.8$ m/s^2]



Sol. Keeping in mind that here both electric and gravitational potential energy are changing and for external point a charged sphere behaves as whole of its charge were concentrated at its applying conservation of energy between initial and final position, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{qq}{9} + mg\times9 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{1} + mg\times1$$

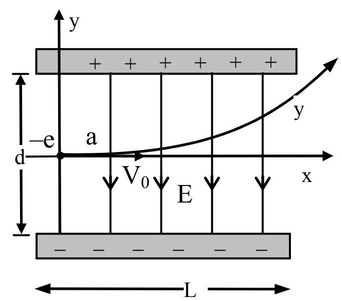
or $$q^2 = \frac{80\times10^{-3}\times9.8}{10^9}$$

$q = 28$ µC **Ans.**

5. The distance between the two plates of a cathode-ray oscillograph is 1 cm and potential difference between them is 1200 volt. If an electron of energy 2000 eV enters at right-angles to the field, what will be its deflection if the plates be 1.5 cm long ?

Sol. As distance between the plates is 1 cm and potential difference 1200 V, the field between the plates

$$E = \frac{V}{d} = \frac{1200}{1\times10^{-2}} = 1.2\times10^5\ \frac{V}{m} \quad \ldots(1)$$



So the electron will experience a force $F_e = eE$ opposite to the field as shown in Figure and hence acceleration of electron along y-axis:

$$a = \frac{F}{m} = \frac{eE}{m}$$

$$= \text{constt.} \quad \ldots(2)$$

So from equation of motion,

$$s = ut + \frac{1}{2}at^2$$

Along x-axis, $L = \upsilon_0 t$ [as a = 0] …(3)

and along y-axis, $y = \frac{1}{2}at^2$ [as u = 0] …(4)

Eliminating t, between Eqs. (3) and (4)

$$y = \frac{1}{2}a\left[\frac{L}{u_0}\right]^2 = \frac{1}{2}\frac{eEL^2}{mu_0^2} \quad \left[\text{as from Eq.(2)}\ a = \frac{eE}{m}\right]$$

or $$y = \frac{1}{4}\frac{eE}{K}L^2 \quad \left[\text{as}\ K = \frac{1}{2}m\upsilon_0^2\right]$$

Substituting the given data and value of E from Eq. (1),

$$y = \frac{1}{4}\frac{(1.6\times10^{-19})(1.2\times10^5)\times(1.5\times10^{-2})^2}{200\times1.6\times10^{-19}}$$

$= 3.375$ mm **Ans.**

Newton's Law of motion

KEY CONCEPTS & PROBLEM SOLVING STRATEGY

Newton's First Law (or Law of Inertia) :

A body continues to maintain its state of equilibrium till disturbed by an unbalanced force i.e. it continues to maintain its state of rest or of uniform motion till an unbalanced external force disturbs it. This law is also called Galileo Law or Law of Inertia.

Newton's second Law :

The rate of change of momentum is equal to the force applied on the body and this change takes place in the direction of force applied i.e. $\vec{F} = \frac{d\vec{p}}{dt}$

Newton's third law (Action-reaction law) :

To every action there is equal and opposite reaction and both act on two different bodies.

Mathematically $\vec{F}_{AB} = \vec{F}_{BA}$

or $m_Aa_A = m_Ba_B$ (in magnitude)

i.e. for the same force acting on two bodies the massive body has less acceleration than a light body.

Impulse :

If two bodies moving along a straight line collide, then the collision is small and the force experienced during collision on any of the two bodies varies with time and has a large value. In such cases the net effect of force can be measured with the help of a physical quantity called Impulse.

since $\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}\,dt = d\vec{I}$

$$\Rightarrow \vec{I} = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \int_{t_1}^{t_2} \vec{F}\,dt$$

$$\Rightarrow \vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt = \vec{p}_2 - \vec{p}_1$$

So, Impulse = total change in momentum

Also called Impulse-Momentum theorem

Important :

- Newton's second law is the real law of motion as the First law and Third law can be derived form it.
- The concept of impulse must be applied at those places where an extremely large force acts on a body for a very small time interval. Then, impulse is just defined as the product of extremely large force with the very small time value.
- Impulse is a vector quantity having direction along the force.

Equilibrium :

A body is said to be in the equilibrium state when

(a) no net force acts on the body

$\sum \vec{F} = \vec{0}$ (Condition for translational equilibrium)

$\Rightarrow \sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$

(b) no net torque acts on the body :

$\sum \vec{p} = \vec{0}$ (condition for rotational equilibrium)

$\Rightarrow \sum \tau_x = 0 \qquad \sum \tau_y = 0 \qquad \sum \tau_z = 0$

This statement is none other than law of conservation of moments according to which the above condition can be restated as

$$\sum \begin{pmatrix} \text{total clockwise} \\ \text{momentus} \end{pmatrix} = \sum \begin{pmatrix} \text{total anticlockwise} \\ \text{momentus} \end{pmatrix}$$

Important : For Rotational Equilibrium

$$\sum \begin{pmatrix} \text{total clockwise} \\ \text{momentus} \end{pmatrix} = \sum \begin{pmatrix} \text{total anticlockwise} \\ \text{momentus} \end{pmatrix}$$

Frames of Reference :

- The system/co-ordinate system/a platform w.r.t. which the position or the motion of a body is determined is called a **frame of reference.** The simplest frame of reference having all the properties of a frame is the **Cartesian co-ordinate frame/system.**
- Frame of reference are of two types :

S.N.	Inertial frame	Non-Inertial frame
1.	Newton's laws are valid in the inertial frames.	Newton's Laws are not valid in the non-inertial frame. They are to be modified by introducing the concept of pseudo force.
2.	All non-accelerated frames (frames at rest or frames moving with uniform velocity) are inertial frames.	All accelerated frames are non-inertial frames.

3.	A particle moves with uniform velocity in the absence of an external force.	In this frame of reference the particle doesn't move with uniform velocity.
4.	A frame of reference moving with constant velocity with respect to an inertial frame is also inertial.	A rotating frame of reference is a non-inertial frames and an example for this is the earth.

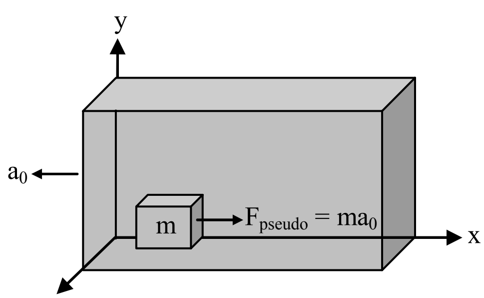
Concept of Pseudo force :

If a body of mass m is placed in a non-inertial frame having acceleration $\vec{a}_0$ then it experiences a pseudo force $m\vec{a}_0$ acting in a direction opposite to the direction of $\vec{a}_0$ (the acceleration of non-inertial frame). So,

$$\vec{F}_{pseudo} = -m\vec{a}_0$$

where, negative sign indicates the pseudo force is always directed in a direction opposite to the direction of the acceleration of the frame.

While drawing free Body Diagrams (FBDs) in which pseudo force is involved, we must first see the acceleration of the non internal frame and then in the FBD, plot the pseudo force with a value ma in a direction opposite to the acceleration of non-inertial frame.



"No negative sign has then to be applied to the value of pseudo force as its direction in the indicated correctly".

Problem Solving strategy

Newton's First Law : Equilibrium of a Particle

Step 1 : Identify the relevant concepts : You must use Newton's first law for any problem that involves forces acting on a body in equilibrium. Remember that "equilibrium" means that the body either remains at rest or moves with constant velocity. For example, a car is in equilibrium when it's parked, but also when it's driving down a straight road at a steady speed.

If the problem involves more than one body, and the bodies interact with each other, you'll also need to use Newton's third law. This law allows you to relate the force that one body exerts on a second body to the force that the second body exerts on the first one.

Be certain that you identify the target variable(s). Common target variables in equilibrium problems include the magnitude of one of the forces, the components of a force, or the direction of a force.

Step 2: Set up the problem using the following steps:

- Draw a very simple sketch of the physical situation, showing dimensions and angles. You don't have to be an artist!
- For each body that is in equilibrium, draw a free-body diagram of this body. For the present, we consider the body as a particle, so a large dot will do to represent it. In your free-body diagram, do not include the other bodies that interact with it, such as surface it may be resting on, or a rope pulling on it.
- Now ask yourself what is interacting with the body by touching it or in any other way. On your free-body diagram, draw a force vector for each interaction. If you know the angle at which a force is directed, draw the angle accurately and label it. A surface in contact with the body exerts a normal force perpendicular to the surface and possibly a friction force parallel to the surface. Remember that a rope or chain can't push on a body, but can only pull in a direction along its length. Be sure to include the body's weight, except in case where the body has negligible mass (and hence negligible weight). If the mass is given, use w = mg to find the weight. Label each force with a symbol representing the magnitude of the force.
- Do not show in the free-body diagram any forces exerted by the body on any other body. The sums in Eq. $\sum \vec{F} = 0$ (particle in equilibrium, vector form) and $\sum F_x = 0$; $\sum F_y = 0$ (particle in equilibrium, component form) include only forces that act on the body. Make sure you can answer the question "What other body causes that force?" for each force. If you can't answer that question, you may be imagining a force that isn't there.
- Choose a set of coordinate axes and include them in your free-body diagram. (If there is more than one body in the problem, you'll need to choose axes for each body separately.) Make sure you label the positive direction for each axis. This will be crucially important when you take components of the force vectors as part of your solution. Often you can simplify the problem by your choice of coordinate axes. For example, when a body rests or sides on a plane surface, it's usually simplest to take the axes in the directions parallel and perpendicular to this surface, even when the plane is tilted.

Step 3 : Execute the solution as follows :

- Find the components of each force along of the body's coordinate axes. Draw a wiggly line through each force vector that has been replaced by its components, so you don't count it twice. Keep in mind that while the magnitude of a force is always positive, the component of a force along a particular direction may be positive or negative.

- Set the algebraic sum of all x-components of force equal to zero. In a separate equation, set the algebraic sum of all y-components equal to zero. (Never add x-and y-components in a single equations.) You can then solve these equations for up to two unknown quantities, which may be force magnitudes, components, or angles.
- If there are two or more bodies, repeat all of the above steps for each body. If the bodies interact with each other, use Newton's third law to relate the forces they exert on each other.
- Make sure that you have as many independent equations as the number of unknown quantities. Then solve these equations to obtain the target variables. This part is algebra, not physics, but it's an essential step.

Step 4 : Evaluate your answer : Look at your results and ask whether they make sense. When the result is a symbolic expression or formula, try to think of special cases (particular values or extreme cases for the various quantities) for which you can guess what the results ought to be. Check to see that your formula works in these particular cases.

Newton's Second Law : Dynamics of Particles

Step 1: Identify the relevant concepts : You have to use Newton's second law for any problem that involves forces acting on an accelerating body.

As with any problem, identify the target variable – usually an acceleration or a force. If the target variable is something else, you'll need to identify another concept to use. For example, suppose the problem asks you to find how fast a sled is moving when it reaches the bottom of a hill. This means your target variable is the sled's final velocity. To find this, you'll first need to use Newton's second law to find the sled's acceleration. Then you'll also have to use the constant –acceleration relationships to find velocity from acceleration.

Step 2: Set up the problem using the following steps:

- Draw a simple sketch of the situation. Identify one or more moving bodies to which you'll apply Newton's second law.
- For each body you identified, draw a free-body diagram that shows all the forces acting on the body. (Don't try to be fancy–just represent the object by a point.) Be careful not to include any forces exerted by the object on some other object. Remember, the acceleration of a body is determined by the forces that act on it, not by the forces that it exerts on anything else. Make sure you can answer the question "What other body is applying this force ?" for each force in your diagram. Furthermore, never include the quantity $m\vec{a}$ in your free-body diagram; it's not a force!
- Label each force with an algebraic symbol for the force's magnitude, as well as a numerical value of the magnitude if it's given in the problem. (Remember that magnitudes are always positive. Minus signs show up later when you take components of the forces.) Usually, one of the forces will be the body's weight; it's usually best to label this as w = mg. If a numerical value of mass is given, you can compute the corresponding weight.
- Choose your x-and y-coordinate axes for each object, and show them explicitly in each free body diagram. Be sure to indicate which is the positive direction for each axis. If you know the direction of the acceleration, it usually simplifies things to take one positive axis along that direction. Note that if your problem involves more than one object and the objects accelerate in different directions, you can use a different set of axes for each object.
- In addition to Newton's second law, $\sum \vec{F} = m\vec{a}$, identify any other equations you might need. (You need one equation for each target variable.) For example, you might need one or more of the equations for motion with constant acceleration. If more than one body is involved, there may be relationships among their motions; for example, they may be connected by a rope. Express any such relationships as equations relating the accelerations of the various bodies.

Step 3 : Execute the solution as follows :

- For each object, determine the components of the forces along each of the object's coordinate axes. When you represent a force in terms of its components, draw a wiggly line through the original force vector to remind you not to include it twice.
- For each object, write a separate equation for each component of Newton's second law, as in Eq. $\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$ (Newton's second law, component form)
- Make a list of all the known and unknown quantities. In your list, identify the target variable or variables.
- Check that you have as many equations as there are unknowns. If you have too few equations, go back to step 5 of "Set up the problem." If you have too many equations, perhaps there is an unknown quantity that you haven't identified as such.
- Do the easy part – the math! Solve the equations to find the target variable(s).

Step 4 : Evaluate your answer : Does your answer have the correct units ? (When appropriate, use the conversion $1N = 1kg . m/s^2$) Does it have the correct algebraic sign ? (If the problem is about a sled sliding downhill, you probably took the positive x-axis to point down the hill. If you then find that the sled has a negative acceleration – that is, the acceleration is uphill – then something went wrong in your calculations.) When possible, consider particular values or extreme cases of quantities and compare the results with your intuitive expectations. Ask, "Does this result make sense ?"

Solved Examples

1. A balloon is descending with a constant acceleration a. The mass of the balloon and its contents is M. What mass m of its contents should be released so that the balloon starts ascending with the same acceleration a ? Assume that the volume of the balloon remains the same when the mass m is released.

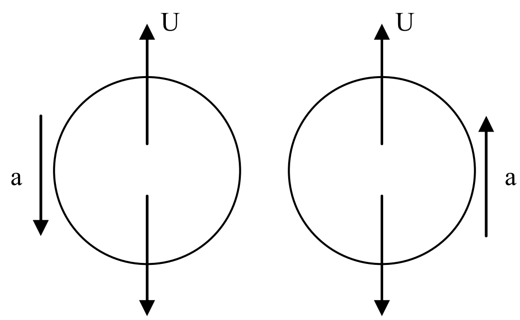
Sol. The forces acting on the balloon are its weight and the upthrust U due to air. Since the volume of the balloon remains the same the upthrust is the same in both the cases. We have, According to Newton's second law,

$$Mg - U = Ma$$

and $U - (M - m)g = (M - m)a$

Solving these, we get

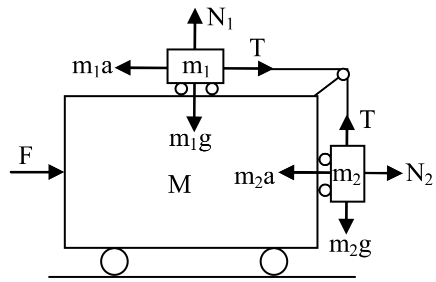
$$m = \frac{2a}{a+g} M$$



2. A frictionless cart of mass M carries two other frictionless carts having masses m_1 and m_2 connected by a string passing over a pulley, as shown in the figure. What horizontal force F must be applied on M so that m_1 and m_2 do not move relative to it ?

Sol. Since m_1 and m_2 are in accelerating frame, we can assume that inertial force m_1a and m_2a act on them, respectively, a being the acceleration of the system. Clearly,

$$a = \frac{F}{M + m_1 + m_2} \qquad ...(1)$$



The forces acting on m_1 and m_2 are shown in fig. We have

For m_1 : $T = m_1a$

For m_2 : $T = m_2g$

$\Rightarrow$ $m_1a = m_2g$

or $a = \frac{m_2}{m_1} g$ ...(2)

Eqs. (1) & (2) give

$$F = (M + m_1 + m_2) \frac{m_2}{m_1} g$$

3. The total mass of an elevator with a 80 kg man in it is 1000 kg. This elevator moving upward with a speed of 8 m/sec, is brought to rest over a distance of 16m. Calculate (a) the tension T in the cables supporting the elevator and (b) the force exerted on the man by the elevator floor.

Sol. (a) The elevator having an initial upward speed of 8 m/sec is brought to rest within a distance of 16 m. Hence,

$$0 = (8)^2 + 2a(16) \qquad (\because v^2 = u^2 + 2as)$$

$$a = -\frac{8\times 8}{2\times 16} = -2 \text{ m/sec}^2$$

Resultant upward force on elevator = T – mg. According to Newton's law,

$$T - mg = ma$$

or $T = mg + ma = m(g + a)$

$= 1000 (9.8 - 2) = 7800$ N.

(b) Let P be the upward force exerted on the man by the elevator floor. If m´ be the mass of the man, then

Weight of the man acting downward = m´g

Upward force on the man = P – m´g.

According to Newton's law,

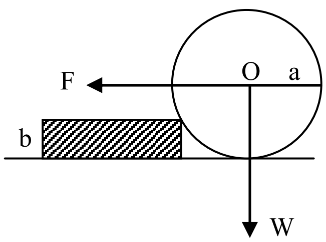
$P - m'g = m'a$

or $P = m'(a + g)$

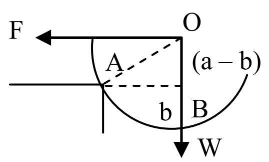
or $= 80\,(-2 + 9.8) = 624$ N

4. What is the least horizontal force needed to pull a cylinder of radius a and weight W over an obstacle of height b ?

Sol. The situation is shown in fig. The different forces acting on the cylinder are shown in fig. The weight W of the cylinder acts downwards. The applied force F is horizontal towards the left.



(a)



(b)

As the cylinder is pulled up and not rolled up, the algebraic sum of the moments about a point A, which is in contact with the obstacle must be zero, hence

$F \times OB = W \times AB$

From fig. (b) , OB = (a – b) and

$AB = \sqrt{[(OA)^2 - (OB)^2]} = \sqrt{[a^2 - (a-b)^2]}$

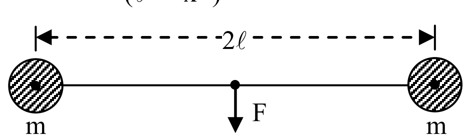
$= \sqrt{(a^2 - a^2 + 2ab - b^2)}$

$= \sqrt{[b(2a - b)]}$

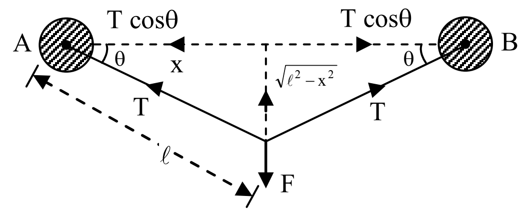
$\therefore\ F \times (a - b) = W\sqrt{[b(2a-b)]}$

or $F = \dfrac{W\sqrt{[b(2a-b)]}}{(a-b)}$

5. Two particles of equals masses m and m are connected by a light string of length 2ℓ as shown in fig. A constant force F is applied continuously at the mid-point of the string, always along the perpendicular bisector of the straight line joining the two particles. Show that when the distance between the two particles is x, the acceleration of the particle is $a = \dfrac{F}{m} \cdot \dfrac{x}{(\ell^2 - x^2)^{1/2}}$



(a)



(b)

Sol. The force acting on A towards B is given by T cos θ.

From figure,

$F = 2T\sin\theta$

or $T = \dfrac{F}{2\sin\theta}$ ...(1)

Now, acceleration of A towards B,

$a_1 = T\cos\theta/m$ ...(2)

and acceleration of B towards A,

$a_2 = T\cos\theta/m$ ...(3)

Substituting the value of T, from eq. (1) in eqs. (2) and (3), we get

$a_1 = a_2 = \dfrac{F\cos\theta}{2m\sin\theta} = \dfrac{F\cot\theta}{2m}$

AUTOMOBILE BATTERY

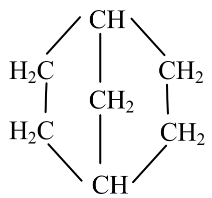
XTRA FUNDA

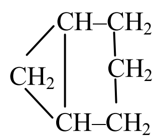
Lets pause briefly to discuss a situation that illustrates the concept of electric potential difference. Consider the common 12-V automobile battery. Such a battery maintains a potential differences across its terminals, where the positive terminal is 12 V higher in potential than the negative terminal. In practice, the negative terminal is usually connected to the metal body of the car, which can be considered at a potential of zero volts. The battery becomes a useful device when it is connected by conducting wires to such things as headlight, a radio, power windows, motors, and so forth. Now consider a charge of +1C, to be moved around a circuit that contains the battery connected to some of these external devices. As the charge is moved inside the battery from the negative terminal (at 0V) to the positive terminal (at 12 V), the work done by the battery on the charge is 12 J. Thus, every coulomb of positive charge that leaves the positive terminal of the battery carries energy of 12 J. As this charge moves through the external circuit towards the negative terminal, it gives up its 12 J of electrical energy to the external devices. When the charge reaches the negative terminal, its electrical energy is zero.

NOMENCLATURE & ISOMERISM

Alicyclic compounds :

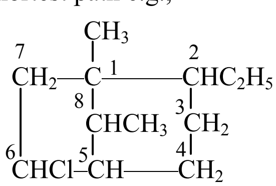
In addition to the simple monocyclic compounds, there are more complicated compounds with bridges linked across the ring e.g.,





bicyclo[2,2,1] heptatne   bicyclo[3,1,0] hexane

According to the I.U.P.A.C system, cycloalkanes consisting of two rings only and having two or more atoms in common, take the prefix bicyclo followed by the name of the alkane containing the same total number of carbon atoms. The number of carbon atoms in each of the three bridges connecting the two tertiary carbon atoms is indicated in brackets in descending order. Numbering begins with one of the bridgeheads and proceeds by the longest possible path to the second bridgehead; numbering is then continued from this atom by the longer unnumbered path back to the first bridgehead and is completed by the shortest path e.g.,

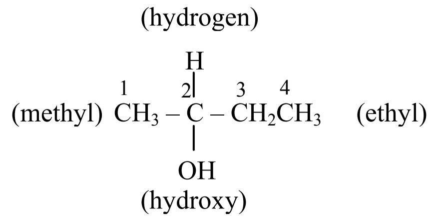


6-chloro-2-ethyl-1, 8-dimethylbicyclo[3,2,1]octane

N.B.A bridged system is considered to have a number of rings equal to the number of scissions required to convert the system into an acyclic compound.

Enantiomers and chiral molecules :

Enantiomers occur only with compounds whose molecules are chiral. A **chiral molecule is defined as one that is not superposable on its mirror image.** Alkene stereoisomers are not chiral, whereas the trans-1, 2-dimethylcyclopentane isomers are chiral. A chiral molecule and its mirror image are called a **pair of enantiomers.** The relationship between them is defined as **enantiomeric.** Molecules (and objects) that are superposable on their image are **achiral** (meaning not chiral). Most socks, for example, are achiral, whereas shoes are chiral. Many familiar objects are chiral, while other objects can be shown to be chiral only by applying the universal test for chirality–the nonsuperposability of the object and its mirror image.



An important property of enantiomers such as these is that interchanging any two groups at the tetrahedral atom that bears four different groups converts one enantiomer into the other. In the figure, it is easy to see that interchanging the OH group and H-atom convert one enantiomer into the other.

Because interchanging two groups at C2 converts one stereoisomer into another, C2 is an example of what is called a **stereogenic carbon.** A stereogenic carbon is defined as a carbon atom bearing groups of such nature that an interchange of any two groups will produce a stereoisomer. Carbon-2 of butanol is an example of a tetrahedral stereogenic carbon. Not all stereogenic cabons are tetrahedral, however. The carbon atoms of cis- and trans-1, 2-dichloroethene are examples of trigonal planar stereogenic carbons because an interchange of groups at either atom also produces a stereoisomer (a diastereomer). In general, any location where an interchange of groups leads to a stereoisomer is called a stereogenic centre.

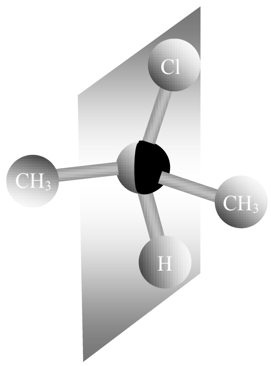
When we discuss interchanging groups like this, we must take care to notice that what we are describing is something we do to a molecular model or something we do on paper. An interchange of groups in a real molecule, if it can be done, requires breaking covalent bonds, and this is something that requires a large input of energy. This means that enantiomers such as the 2-butanol enantiomers do not interconvert spontaneously.

Tests for chirality : Plane of symmetry

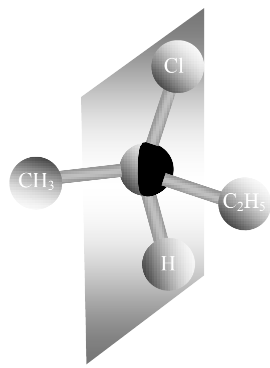
The ultimate way to test for molecular chirality is to construct models of the molecule and its mirror image and then determine whether they are superposable. If the two models are superposable, the molecule that they represent is achiral. If the models are not superposable, then the molecules that they represent are chiral. We can apply this test with actual models, as we have just described, or we can apply it by drawing three-dimensional structures and attempting to superpose them in our minds.

There are other aids, however, that will assist us in a recognizing chiral molecules. We have mentioned one already : the presence of a single tetrahedral stereogenic carbon. The other aids are based on the absence in the molecule of certain symmetry elements. A molecule will not be chiral, for example, if it possesses a plane of symmetry.

A plane of symmetry (also called a mirror plane) is defined as an imaginary plane that bisects a molecule in such a way that the two halves of the molecule are mirror images of each other. The plane may pass through atoms, between atoms, or both. For example, 2-chloropropane has a plane of symmetry fig.(a), whereas 2-chlorobutane does not fig(b). All molecules with a plane of symmetry are achiral.



(a)



(b)

(a) 2-Chloropropane has plane of symmetry & achiral.

(b) 2-Chlorobutane does not possess a plane of symmetry and is chiral.

Specific Rotation :

The number of degrees that the plane of polarization is rotated as the light passes through a solution of an enantiomer depends on the number of chiral molecules that it encounters. This, of course, depends on the length of the tube and the concentration of the enantiomer. In order to place measured rotations on a standard basis, chemists calculate a quantity called the specific rotation, [α], by the following equation:

$$[\alpha] = \frac{\alpha}{c.\ell}$$

where [α] = the specific rotation

α = the observed rotation

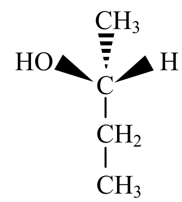
c = the concentration of the solution in grams per milliliter of solution (or density in g mL^{-1} for neat liquids)

ℓ = the length of the tube in decimeters (1 dm = 10 cm)

The specific rotation also depends on the temperature and the wavelength of light that is employed. Specific rotations are reported so as to incorporate these quantities as well. A specific rotation might be given as follows :

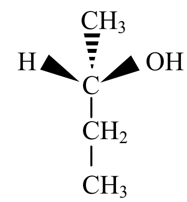
$$[\alpha]_D^{25} = +3.12^o$$

This means that the D line of a sodium lamp (λ = 589.6nm) was used for the light, that a temperature of 25ºC was maintained, and that a sample containing 1.00 g mL^{-1} of the optically active substance, in a 1-dm tube, produced a rotation of 3.12º in a clockwise direction.The specific rotations of (R)-2-butanol and (S)-2-butanol are given here :



(R)-2-Butanol

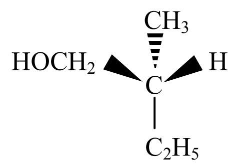
$[\alpha]_{10}^{25} = -13.52^o$



(S)-2-Butanol

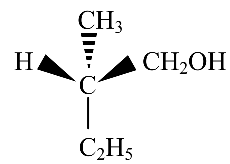
$[\alpha]_{10}^{25} = +13.52^o$

The direction of rotation of plane-polarized light is often incorporated into the names of optically active compounds. The following two sets of enantiomers show how this is done :



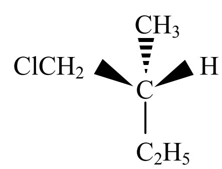
(R)-(+)-2-Methyl-1-Butanol

$[\alpha]_{10}^{25} = +5.756^o$



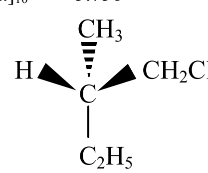
(S)-(–)-2-Methyl-1-Butanol

$[\alpha]_{10}^{25} = -5.756^o$



(R)-(–)-1-Chloro-2-methylbutane

$[\alpha]_{10}^{25} = -1.64^o$

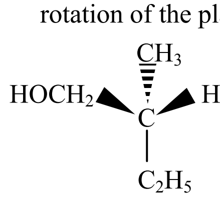


(S)-(+)-1-Chloro-2-methylbutane

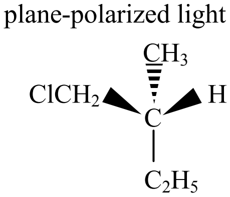
$[\alpha]_{10}^{25} = +1.64^o$

The previous compounds also illustrate an important principle : No obvious correlation exists between the configurations of enantiomers and the direction [(+) or (–)] in which they rotate plane-polarized light.

(R)-(+)-2-Methyl-1-butanol and (R)-(–)-2-Methyl-1-butanol chloro-2-methyl butane have the same configuration; that is, they have the same general arrangement of their atoms in space. They have, however, an opposite effect on the direction of rotation of the plane of plane-polarized light :



(R)-(+)-2-Methyl-1-butanol



(R)-(–)-1-Chloro-2-methylbutane

These same compounds also illustrate a second important principle: No necessary correlation exist between the (R) and (S) designation and the direction of rotation of plane-polarized light. (R)-2-Methyl-1-butanol is dextrorotatory (+), and (R)-1-chloro-2-methyl butane is levorotatory (–)

Racemic Forms

A sample that consists exclusively or predominantly of one enantiomer causes a net rotation of plane – polarized light. A plane of polarized light as it encounters a molecule of (R)-2-butanol, causing the plane of polarization to rotate slightly in one direction. (for the remaining purposes of our discussion we shall limit our description of polarized light to the resultant plane, neglecting consideration of the circularly polarized components from which plane-polarized light arises.) Each additional molecule of (R)-2-butanol that the beam encounters would cause further rotation in the same direction. If, on the other hand, the mixture contained molecules of (S)-2-butanol, each molecule of this enantiomer would cause the plane of polarization to rotate in the opposite direction. If the (R) and (S) enantiomers were present in equal amounts, there would be no net rotation of the plane of polarized light.

An equimolar mixtrure of two enantiomers such as the example above is called a racemic mixture (or racemate or racemic form). A racemic mixture causes no net rotation of plane-polarized light. In a racemic mixture the effect of each molecule of one enantiomer on the in circularly polarized beam cancels the effect of molecules of the other enantiomers, resulting in no net optical activity.

The racemic form of a sample is often designated as being (±).A racemic mixture of (R)-(–)-2-butanol and (S)-(+)-2-butanol might be indicated as

(±)-2-butanol or as (±)- $CH_3CH_2CHOHCH_3$

Racemic forms and enantiomeric excess :

A sample of an optically active substance that consists of a single enantiomer is said to be enantiomerically pure or to have an **enantiomeric excess** of 100%. An enantiomerically pure sample of (S)-(+)-2-butanol shows a specific rotation of +13.52º ($[\alpha]_D^{25}$ = + 13.52º). On the other hand, a sample of (S)-(+)-2-butanol that contains less than an equimolar amount of (R)-(–)-2-butanol that contains less than an equimolar amount of (R)-(–)-2-butanol will show a specific rotation that is less than + 13.52º but greater than 0º. Such a sample is said to have an enantiomeric excess less than 100%. The enantiomeric excess (ee) is defined as follows :

% Enantiomeric excess

$$= \frac{\text{moles of one enantiomer - moles of other enantiomer}}{\text{total moles of both enantiomers}} \times 100$$

The enantiomeric excess can be calculated from optical rotations :

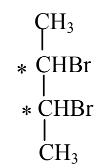
% Enantiomeric excess

$$= \frac{\text{observed specific rotation}}{\text{specific rotation of the pure enantiomer}} \times 100$$

Meso Compounds :

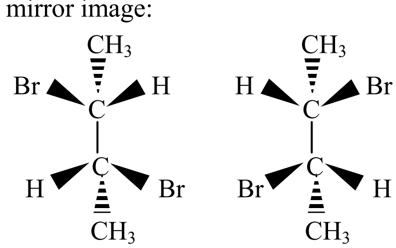
A structure with two stereogenic carbons does not always have four possible stereoisomers. Sometimes there are only three. This happens because some molecules are achiral even though they contain stereogenic carbons.

To understand this, let us write stereochemical formulas for 2,3-dibromobutane shown here:



2,3-Dibromobutane

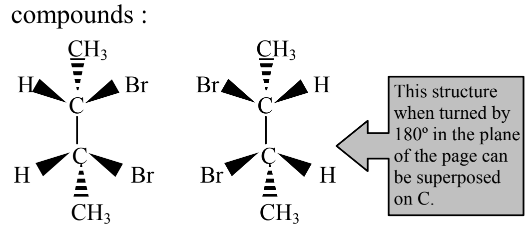
We begin in the same way as we did before. We write the formula for one stereoisomer and for its mirror image:



(A) (B)

Structures A and B are nonsuperposable and represent a pair of enantiomers.

When we write structure C and its mirror image D, however, the situation is different. The two structures are superposable. This means that C and D do not represent a pair of enantiomers. Formulas C and D represent two different orientations of the same compounds :



(C) (D)

The molecule represented by structure C (or D) is not chiral even though it contains tetrahedral atoms with four different attached groups. Such molecules are called meso compounds. Meso compounds, because they are achiral, are optically inactive.

KEY CONCEPT

Physical Chemistry Fundamentals

ELECTRO CHEMISTRY

Equivalent and Molar conductivities :

Since the charges of solute ions are critical in determining the conductance of a solution, the comparison of conductance data is made between values for solutions corresponding to a total of unit charge on each ion of the solute. It is because of this that the equivalent conductivity of the solution is employed for comparison purpose.

Suppose 1 equivalent mass of an electrolyte is dissolved in volume V of the solution. Let this whole solution be placed in a conductivity cell. Multiplying and dividing the right side of Eq. (i) by the distance ℓ between the two electrodes of the cell, we get eq. (ii)

$$\kappa = \left(\frac{\ell}{A}\right)G = \left(\frac{\ell}{A}\right)\frac{1}{R}\ ;\ \ ...(i)\ G = \kappa\frac{A\times\ell}{\ell^2} = \frac{\kappa V}{\ell^2}$$

or $G\ell^2 = \kappa V$ ...(ii)

Note : Both V and ℓ^2 in Eq. (ii) carry the units of $m^3\ eq^{-1}$ and $m^2\ eq^{-1}$, respectively.

The term $G\ell^2$ is known as equivalent conductivity, abbreviated as Λ_{eq}. Thus, we have , $\Lambda_{eq} = G\ell^2 = \kappa V$

The equivalent conductivity of an electrolyte may be defined as the conductance of a volume of solution containing one equivalent mass of a dissolved substance when placed between two parallel electrodes which are at a unit distance apart, and large enough to contain between them the whole solution. The equivalent conductivity thus gives the conducting power of the ions produced by 1 equivalent mass (i.e., mass corresponding to a total of unit charge on each ion) of an electrolyte at any particular concentration.

The unit of Λ_{eq} in CGS units are :

$\Lambda_{eq} = (\Omega^{-1}\ cm^{-1})\ (cm^3\ eq^{-1}) = \Omega^{-1}\ cm^2\ eq^{-1} \equiv S\ cm^2\ eq^{-1}$

If c is the concentration of the solution (in equivalent per unit volume), then V (which carry a unit of $m^3\ eq^{-1}$) will be equal to 1/c. Hence, Eq. (ii) may be written as

$\Lambda_{eq} = \kappa(1/c)$ i.e. $\Lambda_{eq} = \kappa/c$ ...(iii)

In SI units, c is expressed as the amount per unit volume instead of equivalent per unit volume, and thus one uses the term molar conductivity as defined below. The molar conductivity, Λ_m, of an electrolyte may be defined as the conductance of a volume of solution containing one mole of a dissolved substance when placed between two parallel electrodes which are at a unit distance apart, and large enough to contain between them the whole solution. Thus, it gives the conducting power of the ions produced by 1 mole of an electrolyte at any particular concentration. This can be calculated using the expressions analogous to eqs (ii) and (iii).

$\Lambda_m = \kappa V = \kappa(1/c)$ i.e. $\Lambda_m = \kappa/c$

where V is the volume of the solution containing one mole of the substance and c is the resultant molar concentration. Note V carries the unit of $m^3\ mol^{-1}$

The unit of Λ_m will be

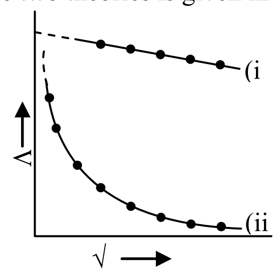
$\Lambda_m = (\Omega^{-1}m^{-1})(m^3\ mol^{-1}) = \Omega^{-1}\ m^2\ mol^{-1} \equiv Sm^2\ mol^{-1}$

Variation of conductivity and molar conductivity with concentration :

Both the conductivity and molar conductivity of a solution vary with concentration. The conductivity increases with increase in concentration whereas the molar conductivity increases on dilution (i.e. decrease in concentration). For strong electrolytes, conductivity increases sharply with increase in concentration while for weak electrolytes it starts at lower value in dilute solutions and increases much more gradually. In both the cases, this increase is due to the increase in the number of ions per unit volume of the solution. For strong electrolytes, the number of ions per unit volume increases in proportion to the concentration and that is why the increase in conductivity is very rapid. In weak electrolytes, however, the increase in the number of ions is basically due to the change in the partial ionization of the solute, and consequently, the conductivity increases very gradually.

As stated above, molar conductivity Λ_m of both strong and weak electrolytes increases on dilution. The basic reason for this is that the decreases in conductivity is more than compensated by increase in the value of 1/c on dilution.

The variation of molar conductivity on dilution for strong and weak electrolytes shows altogether different behavior as can be seen from figure where Λ_m has been plotted against $\sqrt{c}$. For strong electrolytes, the variation is almost linear in dilute solutions while that for weak electrolytes, the variation is very rapid. As the molar conductivity is a measure of the conducting power of all the ions that are available in 1 mole of the substance, it is, therefore, obvious that the number of ions that are available for conductance increases on dilution. For weak electrolytes, the increase in the number of ions has been explained on the basis of Arrhenius theory of electrolytic dissociation whereas that for strong electrolytes has been explained on the basis of Debye-Huckel-Onsager theory. In brief, the increase in the number of ions in case of weak electrolytes is due to the increase in the degree of ionization of the electrolyte on dilution, whereas in the case of strong electrolytes, this increase is due to the weakening of the ion-ion interactions on dilution. A brief account of the above two theories is given in the following.



Variation of molar conductivity on dilution (i) for strong electrolyte and (ii) weak electrolyte

Kohlrausch's law of independent migration of ions:

For a strong electrolyte, the value of Λ_m in a very dilute solution, is very close to the limiting value of the conductivity Λ_m^∞ at infinite dilution (or at zero concentration obtained by extrapolation). On the other hand, the corresponding value for a weak electrolyte is very far away from the limiting value at zero concentration. For example, at 25 ºC for 0.001 M sodium chloride solution Λ_m is 123.7 $ohm^{-1}\ cm^2\ mol^{-1}$ as against Λ_m^∞ of 126.5 $ohm^{-1}\ cm^2\ mol^{-1}$. At the same concentration and temperature, the value for acetic acid is 49.2 $ohm^{-1}\ cm^2\ mol^{-1}$ as compared to 390.7 $ohm^{-1}\ cm^2\ mol^{-1}$ for the value of Λ_m^∞.

Kohlrausch was the first to point out that when Λ_m for a uni-univalent strong electrolyte is plotted against $\sqrt{c}$, the curve approaches linearity in dilute solutions, i.e.

$$\Lambda_m = \Lambda_m^\infty - b\sqrt{c} \qquad \text{...(i)}$$

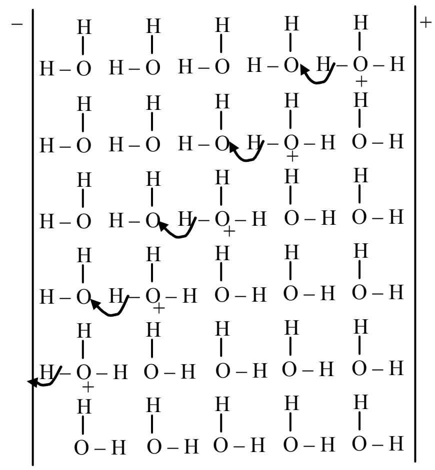
where b is constant. The value of Λ_m^∞ can, thus, be obtained by extrapolating the above curve to a value of $\sqrt{c} = 0$.

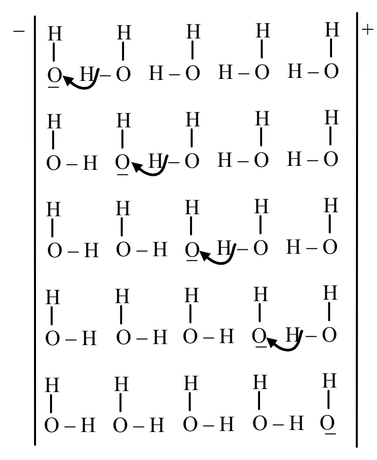
Statement of Kohlrausch's Law : The extrapolation method cannot be employed for a weak electrolyte as Λ_m versus $\sqrt{c}$ curve does not approach linearity in solutions as dilute as 0.0001 M. In fact, the variation of Λ_m with dilution is very rapid. However, the value of Λ_m^∞ for a weak electrolyte can be determined by the application of Kohlrausch´s law of independent migration of ions. This law states that at infinite dilution, where dissociation for all electrolytes is complete (including weak electrolytes since $\alpha \to 1$ as $c \to 0$; Ostwald dilution law) and where all interionic effects disappear (because of larger distance between ions), each ion migrates independently of its co-ion and contributes to the total molar conductivity of an electrolyte a definite share which depends only on its own nature and not at all on the ion with which it is associated. Thus, Λ_m^∞ of the electrolyte must be equal to the sum of the molar conductivities of the ions composing it. Thus

$$\Lambda_m^\infty(AB) = \lambda^\infty(A^+) + \lambda^\infty(B^-) \qquad \text{...(ii)}$$

Abnormally high conductivities of H^+ and OH^-

The molar conductivities of the hydrogen ion and the hydroxyl ion are much larger than those of other ions. This was first explained by von Grotthus and hence is known as **Grotthus conductance**. It is explained on the basis of a proton jump from one water molecule to another. The process of proton transfer results in a more rapid transfer of positive charge from one region of the solution to another, than would be possible if the ion H_3O^+ has to push its way through the solution as other ions do. The mechanisms of conduction of H^+ ion and OH^- ion are shown below.





This type of mechanism also prevails in any other solvent. Thus, in a given solvent (for example, liquor ammonia) the molar conductivities of its characteristic cation and anion (namely, NH_4^+ and NH_2^-) will have unusually high values than any other cations and anions.

UNDERSTANDING

Inorganic Chemistry

1. A metal (A) gives the following observations :

(i) It gives golden yellow flame.

(ii) It is highly reactive and used in photoelectric cells as well as used in the preparation of Lassaigane solution.

(iii) (A) on fusion with NaN_3 and $NaNO_3$ separately, yields an alkaline oxide (B) and an inert gas (C). The gas (C) when mixed with H_2 in Haber's process gives another gas (D). (D) turns red litmus blue and gives white dense fumes with HCl.

(iv) Compound (B) react with water forming on alkaline solution (E). (E) is used for the saponification of oils and fats to give glycerol and a hard soap.

(v) (B) on heating at 670 K give (F) and (A). The compound (F) liberates H_2O_2 on reaction with dil. mineral acids. It is an oxidising agent and oxidises $Cr(OH)_3$ to chromate, manganous salt to manganate, sulphides to sulphates.

(vi) (B) reacts with liquid ammonia to give (G) and (E). (G) is used for the conversion of 1, 2 dihaloalkanes into alkynes.

What are (A) to (G)? Explain the reactions involved.

Sol. (i) (A) appears to be Na as it gives the golden yellow flame. It is also used in the preparation of Lassaigane solution which is sodium extract of organic compounds.

$$Na + C + N \rightarrow NaCN$$
$$Na + Cl \rightarrow NaCl$$
$$2Na + S \rightarrow Na_2S$$

(ii) Compound (B) is Na_2O and (C) is N_2 while (D) is NH_3, as (D) is alkaline and turns red litmus blue and gives white fumes with HCl

$$(C) + H_2 \rightarrow NH_3$$
$$N_2 + 3H_2 \rightleftharpoons \underset{(D)}{2NH_3}$$
$$NH_3 + HCl \rightarrow \underset{\text{White fumes}}{NH_4Cl}$$

(iii) is prepared from Na as follows.

$$2NaNO_3 + 10\,Na \rightarrow \underset{(B)}{6Na_2O} + \underset{(C)}{N_2}$$
$$3NaN_3 + NaNO_2 \rightarrow \underset{(B)}{2Na_2O} + \underset{(C)}{N_2}$$

(iv) Compound (E) is NaOH as it is used in the preparation of soaps.

$$\underset{(B)}{Na_2O} + H_2O \rightarrow \underset{(E)}{2NaOH}$$

$$\begin{array}{l} CH_2OOCC_{17}H_{35} \\ | \\ CHOOCC_{17}H_{35} \\ | \\ CH_2OOCC_{17}H_{35} \end{array} + 3NaOH \xrightarrow{\Delta} \begin{array}{l} CH_2OH \\ | \\ CH_2OH \\ | \\ CH_2OH \end{array} + \underset{\text{(soap)}}{3C_{17}H_{35}COONa}$$

(v) (F) is sodium peroxide as only peroxides gives H_2O_2 on reaction with dil. acids.

$$\underset{(B)}{2Na_2O} \xrightarrow[\Delta]{670\text{ K}} \underset{(F)}{Na_2O_2} + \underset{(A)}{2Na}$$
$$\underset{(F)}{Na_2O_2} + \underset{\text{dil.}}{H_2SO_4} \rightarrow H_2O_2 + Na_2SO_4$$

(F) gives the following oxidations :

$$Cr(OH)_3 + 5OH^- \rightarrow CrO_4^{2-} + 4H_2O + 3e^-$$
$$Mn^{2+} + 8OH^- \rightarrow MnO_4^- + 4H_2O + 5e^-$$
$$S^{2-} + 8OH^- \rightarrow SO_4^{2-} + 4H_2O + 8e^-$$

The reduction equation of (F) is

$$O_2^{2-} + 2H_2O + 2e^- \rightarrow 4OH^-$$

(vi) (G) is sodamide because it is used in the dehydrohalogenation reactions.

$$\underset{(B)}{Na_2O} + NH_3(l) \rightarrow \underset{(G)}{NaNH_2} + \underset{(E)}{NaOH}$$

$$CH_3 - \underset{Br}{\underset{|}{CH}} - \underset{Br}{\underset{|}{CH_2}} + 2NaNH_2 \xrightarrow{\Delta} \underset{\text{Propyne}}{CH_3 - C \equiv CH} + 2NaBr + 2NH_3$$

2. A green coloured compound (A) gave the following reactions :

(i) (A) dissolves in water to give a green solution. The solution on reaction with $AgNO_3$ gives a white ppt. (B) which dissolves in NH_4OH solution and reappears on addition of dil. HNO_3. It on heating with $K_2Cr_2O_7$ and conc. H_2SO_4 produced a red gas which dissolves in NaOH to give yellow solution (C). Addition of lead acetate solution to (C) gives a yellow ppt. which is used as a paint.

(ii) The hydroxide of cation of (A) in borax bead test gives brown colour in oxidising flame and grey colour in reducing flame.

(iii) Aqueous solution of (A) gives a black ppt. on passing H_2S gas. The black ppt. dissolves in aquaregia and gives back (A).

(iv) (A) on boiling with $NaHCO_3$ and Br_2 water gives a black ppt. (D)

(v) (A) on treatment with KCN gives a light green ppt. (E) which dissolves in excess of KCN to give

(F). (F) on heating with alkaline bromine water gives the same black ppt. as (D).

Identify compounds (A) to (F) and give balanced equations of the reactions.

Sol. Reaction (i) indicates that (A) contains Cl^- ions because, it gives white ppt. soluble in NH_4OH. It is again confirmed because it gives chromyl chloride test. The colour of oxidising and reducing flames indicate that (A) also contains Ni^{2+} ions. Hence, (A) is $NiCl_2$. The different reactions are :

(i) $NiCl_2 + 2AgNO_3 \rightarrow 2AgCl + Ni(NO_3)_2$

$AgCl + 2NH_3 \rightarrow \underset{\text{Soluble}}{[Ag(NH_3)_2]Cl}$

$Ag(NH_3)_2Cl + 2HNO_3 \rightarrow \underset{\text{white ppt.(B)}}{AgCl\downarrow} + 2NH_4NO_3$

The equations of chromyl chloride tests are :

$NiCl_2 + Na_2CO_3 \rightarrow 2NaCl + NiCO_3$

$4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \rightarrow 4NaHSO_4 + 2KHSO_4 + 3H_2O + \underset{\text{Red gas}}{2CrO_2Cl_2}$

$CrO_2Cl_2 + 4NaOH \rightarrow \underset{\text{Yellow solution (C)}}{Na_2CrO_4} + 2NaCl + 2H_2O$

$Na_2CrO_4 + (CH_3COO)_2Pb \rightarrow \underset{\text{Yellow ppt.}}{PbCrO_4} + 2CH_3COONa$

(ii) $Na_2B_4O_7 . 10H_2O \xrightarrow{\Delta} Na_2B_4O_7 + 10H_2O$

$Na_2B_4O_7 \xrightarrow{\Delta} \underbrace{2NaBO_2 + B_2O_3}_{\text{Transparent bead}}$

$NiO + B_2O_3 \xrightarrow{\Delta} \underset{\text{Nickel meta borate (Brown)}}{Ni(BO_2)_2}$ [Oxidising flame]

$Ni(BO_2)_2 + C \xrightarrow{\Delta} \underset{\text{Grey}}{Ni} + B_2O_3 + CO$

[Reducing flame]

(iii) $NiCl_2 + H_2S \rightarrow 2HCl + \underset{\text{Black ppt.}}{NiS\downarrow}$

$NiS + 2HCl + [O] \rightarrow \underset{(A)}{NiCl_2} + H_2S\uparrow$

(iv) $\underset{(A)}{NiCl_2} + 2NaHCO_3 \rightarrow NiCO_3 + 2NaCl + CO_2 + H_2O$

$2NiCO_3 + 4NaOH + [O] \xrightarrow{\Delta} \underset{\text{Black ppt. (D)}}{Ni_2O_3\downarrow} + 2Na_2CO_3 + H_2O$

(v) $\underset{(A)}{NiCl_2} + 2KCN \rightarrow \underset{\text{Green ppt. (E)}}{Ni(CN)_2} + 2KCl$

$Ni(CN)_2 + 2KCN \rightarrow \underset{(F)}{K_2[Ni(CN)_4]}$

$NaOH + Br_2 \rightarrow NaOBr + HBr$

$2K_2[Ni(CN)_4] + 4NaOH + 9NaOBr \xrightarrow{\Delta} \underset{(D)}{Ni_2O_3}\downarrow + 4KCNO + 9NaBr + 4NaCNO$

3. Calculate mol of $Ca(OH)_2$ required to carry out following conversion taking one mol in each case :

(a) $\begin{matrix} COOH \\ | \\ COOH \end{matrix}$ into $\begin{matrix} COO \\ | \\ COO \end{matrix} \rangle Ca$

(b) H_3PO_4 into $CaHPO_4$

(c) NH_4Cl into NH_3

(d) $NaHCO_3$ into $CaCO_3$

Sol. (a) $\begin{matrix} COOH \\ | \\ COOH \end{matrix}$ is a dibasic acid

$\begin{matrix} COOH \\ | \\ COOH \end{matrix} + Ca(OH)_2 \longrightarrow \begin{matrix} COO \\ | \\ COO \end{matrix} \rangle Ca$

1 mol 1 mol

$Ca(OH)_2$ required = 1 mol

(b) $H_3PO_4 + Ca(OH)_2 \longrightarrow CaHPO_4 + 2H_2O$

1 mol of $H_3PO_4 \equiv 2H^+$ neutralised by 1 mol of $Ca(OH)_2$

$Ca(OH)_2$ required = 1 mol

(c) $2NH_4Cl + Ca(OH)_2 \rightarrow CaCl_2 + 2NH_3 + 2H_2O$

2 mol $NH_4Cl \equiv$ 1 mol $Ca(OH)_2$

1 mol $NH_4Cl \equiv$ 0.5 mol $Ca(OH)_2$

(d) $2NaHCO_3 + Ca(OH)_2 \rightarrow Na_2CO_3 + CaCO_3 + 2H_2O$

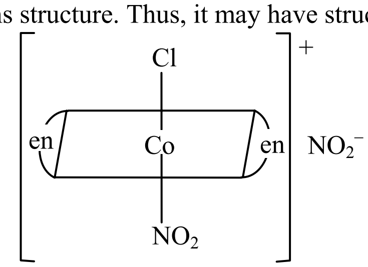
2 mol $NaHCO_3 \equiv$ 1 mol $Ca(OH)_2$

1 mol $NaHCO_3 \equiv$ 0.5 mol $Ca(OH)_2$

4. A compound $Co(en)_2 (NO_2)_2Cl$ has been prepared in a number of isomeric forms. One form undergoes no reaction with $AgNO_3$ or (en) and is optically inactive. A second form reacts with $AgNO_3$ but not with (en) and is optically inactive. A third form is optically active and reacts with both $AgNO_3$ and (en). Identify each of these isomeric forms. Name and sketch each of the structures.

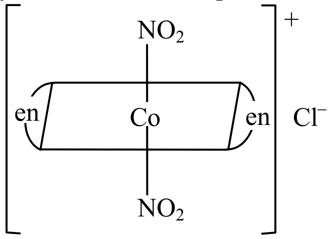
Sol. Based on reactions with $AgNO_3$ and (en), and optical activity, isomers can be identified.

First form : There is no reaction with $AgNO_3$, hence no Cl^- ions outside coordination sphere. Also there is no reaction with bidentate (en), hence these ligands are trans to each other. Optical inactivity is also due to trans structure. Thus, it may have structure :



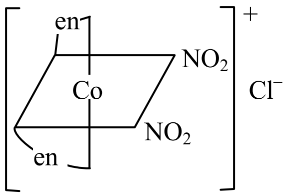
trans-chloronitrobis (ethylenediamine) cobalt (III) nitrite.

Secon form : In this Cl^- is outside coordination sphere since it reacts with $AgNO_3$. As in the first form NO_2^- ligands are trans to each other being optically inactive. This is represented as,



trans-bis (ethylenediamine) dinitrocobalt (III) chloride

Third form : In this case also, Cl^- is outside coordination sphere. Also it shows reaction with (en) hence monodentate ligands are cis to each other. Being optically active, mirror image should not superimpose. Thus, it can have structure :



cis-bis (ethylenediamine) dinitrocobalt (III) chloride

5. A colourless salt (A), soluble in water, gives a mixture of three gases (B), (C) and (D) along with water vapours. (B) is blue, (C) is red and (D) is neutral towards litmus paper. Gas (B) is also obtained when (A) is heated with NaOH and gives brown ppt with K_2HgI_4. Solution thus obtained gives white ppt (E) with $CaCl_2$ solution in presence of CH_3COOH. (E) decolorises MnO_4^-/H^+. Gas (C) turns lime water milky while gas (D) burns with blue flame and is fatal when inhaled. Identify (A) to (D) and explain reactions.

Sol. Gas (B) gives brown ppt with K_2HgI_4

$\Rightarrow$ gas (B) is NH_3

$\Rightarrow$ (A) has NH_4^+ ion

Gas (C) turns lime water milky

$\Rightarrow$ gas (C) can be SO_2 or CO_2

Gas (D) is also obtained along with (C). Gas (D) burns with blue flame and is fatal when inhaled

$\Rightarrow$ gas (D) is CO

$\Rightarrow$ gas (C) is CO_2

$\Rightarrow$ (A) has $C_2O_4^{2-}$ ion

It is confirmed by the fact that $CaCl_2$ gives white ppt CaC_2O_4 (E) which decolorises MnO_4^-/H^+

$\Rightarrow$ (A) is $(NH_4)_2C_2O_4$

Explanation :

$$\underset{(A)}{(NH_4)_2C_2O_4} \xrightarrow{\Delta} \underset{(B)}{2NH_3} + \underset{(C)}{CO_2} + \underset{(D)}{CO} + H_2O$$

(B) is blue towards litmus (basic)

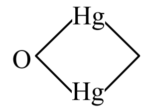
(C) is red towards litmus (acidic)

(D) is neutral

$$(NH_4)_2C_2O_4 + 2NaOH \xrightarrow{\Delta} Na_2C_2O_4 + \underset{(B)}{2NH_3} + 2H_2O$$

$$Na_2C_2O_4 + CaCl_2 \rightarrow \underset{\text{white ppt (E)}}{CaC_2O_4} \downarrow + 2NaCl$$

$$3NaOH + NH_3 + 2K_2HgI_4 \longrightarrow$$



$NH_2I \downarrow + 4KI + 3NaI + 2H_2O$

brown ppt

(Iodide of Millon's base)

$$\underset{\text{violet}}{2MnO_4^-} + 16H^+ + 5C_2O_4^{2-}$$

$$\rightarrow 10\ CO_2 + \underset{\text{colourless}}{2Mn^{2+}} + 8H_2O$$

Science Facts

1. There is clear geological evidence that there has been water on Mars.

2. George Ellery Hale was the 20th century's most important builder of telescopes. In 1897, Hale built a 40 inch wide telescope, the largest ever built at that time. Later, during the building of his 100 inch lens Hale spent time in a sanatorium and would only discuss his plans for the telescope with 'a sympathetic green elf'.

3. Have you ever seen a ring around the moon? Folklore has it that this means bad weather is coming.

4. Every autumn, monarch butterflies fly 3500 kilometres from the Northern US to Mexico. It has been hypothesised that they have some kind of sun compass linked to their body clock which tells them which way to go.

5. 'The Adventures of Tom Sawyer', written by Mark Twain, was the first novel ever to be written on a typewriter.

6. A mature yew (if stripped) will yield between five and 20 pounds of bark.

7. Since about 1990, GHB (gamma-hydroxybutyrate) has been abused in the U.S. for euphoric, sedative, and anabolic (body-building) effects. GHB use associated with sexual assault has surpassed Rohypnol use associated with sexual assault.

Mathematical Challenges

Set 2

This section is designed to give IIT JEE aspirants a thorough grinding & exposure to variety of possible twists and turns of problems in mathematics that would be very helpful in facing IIT JEE. Each and every problem is well thought of in order to strengthen the concepts and we hope that this section would prove a rich resource for practicing challenging problems and enhancing the preparation level of IIT JEE aspirants.

By : Shailendra Maheshwari
Joint Director Academics, Career Point, Kota

Solutions will be published in next issue

1. Six different boxes are placed in a row. A ball is to be put in each box. If unlimited balls of red, blue, green and white colour are available, then find the number of ways in which all the boxes can be filled so that no two adjacent boxes have balls of the same colour in them.

2. If the non singular matrix A is symmetric, then justify that A^{-1} is also symmetric.

3. If θ_1, θ_2, θ_3 and θ_4 are eccentric angles of four conormal points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the normals at which are concurrent, then prove that $\Sigma\cos(\theta_1 + \theta_2) = 0$.

4. Show that the origin lies in the acute angle between the planes $x + 2y + 2z = 9$ and $4x - 3y + 12z + 13 = 0$.

5. Let f(x) be a differentiable function on the interval $a \le x \le b$, with $a > 0$ and suppose that f(x) has a differentiable inverse, $f^{-1}(x)$. Evaluate :

$$\frac{\int_a^b x(f(b) - f(x))\,dx}{\int_{f(a)}^{f(b)} ((f^{-1}(y))^2 - a^2)\,dy}$$

6. If $x + \frac{1}{x} = \sqrt{\sqrt{5}+2}$ and $y + \frac{1}{y} = 2$, then $x^{16} + \frac{1}{x^{16}} + y^{20} + \frac{1}{y^{20}}$ is

(A) 49 (B) 51

(C) 2209 (D) 102

7. A circle S_1 of area 36π cm^2 touches the coordinate axes. Another circle S_2 smaller than S_1 also touches the coordinate axes as well as S_1 also, then the radius of S_2 is –

(A) 2cm (B) $\sqrt{2}$ cm

(C) $12 - 6\sqrt{2}$ cm (D) $18 - 12\sqrt{2}$ cm

8. Find the sum of all the natural number 1 to 120 which are divisible neither by 2 nor 7.

(A) 3042 (B) 3084

(C) 2529 (D) 3033

Passage :

Given below are the definitions of functions F(x), f(x), g(x) and h(x)

$F(x) = x^3$; $0 \le x \le 1$

$= -x^3$; $-1 \le x < 0$

$= 1$; otherwise

$f(x) = F(-x)$; $x \in R$

$g(x) = -F(x)$; $x \in R$

$h(x) = -F(-x)$; $x \in R$

Answer the following questions based on these information's.

9. How many of the following expressions are necessarily zero for every real value of x :

$f(x) + h(x)$; $g(x) - h(x)$; $F(x) + f(x)$; $f(x) - g(x)$?

(A) 1 (B) 2 (C) 3 (D) 4

10. Which of the following relations is necessarily true ?

(A) $h(x) = f(-x)$; $x \in R$

(B) $F(x) = -f(-x)$; $x \in R$

(C) $g(x) + f(-x) = 0$, $x \in R$

(D) $h(x) - F(-x) = 0$; $x \in R$

MATHEMATICAL CHALLENGES

SOLUTION FOR MAY ISSUE (SET # 1)

1. **[A]** Let Y draws any card, the probability that X draws the same card = 1/n

2. **[B]** Let 'X' draws a card marked with r, then y can draw any card marked 1, 2, 3, ... r –1. Hence the required probability = $\sum_{r=1}^{n}\frac{1}{n}\frac{(r-1)}{n} = \frac{n-1}{2n}$

3. **[B]** Let 'Y' draws r^{th} card, then 'X' draws any card marked 1, 2, 3 ...... (r – 1),

Hence required probability = $\sum_{r=1}^{n}\frac{1}{n}\left(\frac{r-1}{n}\right) = \frac{n-1}{2n}$

4. $I_{m,\,n} = \int_0^1 x^m(1-x)^n\,dx, \qquad \text{let } x = \sin^2\theta$

$= 2\int_0^1 \sin^{2m+1}\theta \,.\, \cos^{2n}+1\,\theta\, d\theta$

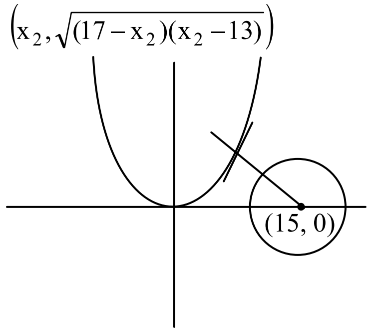
$= \frac{\Gamma(m+1)\,\Gamma(n+1)}{\Gamma(m+n+2)}$

$$\frac{I_{25,50}}{I_{24,49}} = \frac{\dfrac{\Gamma(26).\Gamma(51)}{\Gamma(77)}}{\dfrac{\Gamma(25)\,\Gamma(50)}{\Gamma(75)}} = \frac{\lfloor 25\,\lfloor 50.\lfloor 74}{\lfloor 76\,\lfloor 24\,\lfloor 49} = \frac{25\times 50}{76\times 75}$$

$228\,\frac{I_{26,51}}{I_{25,50}} = 50$

5. $d^2 = (x_1 - x_2)^2 + \left(\frac{x_1^2}{20} - \sqrt{(17-x_2)(x_2-13)}\right)^2$

it is the sq. of distance b/w two points $\left(x_1, \frac{x_1^2}{20}\right)$ & $\left(x_2, \sqrt{(17-x_2)(x_2-13)}\right)$



First point lies on y = $\frac{x_1^2}{20}$

2^{nd} point lies on $y^2 + x^2 - 30x + 221 = 0$

centre (15, 0), $\quad r = \sqrt{225-221} = 2$

$y = \frac{x^2}{20} \Rightarrow x^2 = 20y$

normal is

$x = my - 10m - 5m^3$

it passes through (15, 0)

$15 + 10m + 5m^3 = 0$

$m^3 + 2m + 3 = 0$

$(m + 1)(m^2 - m + 3) = 0$

$\Rightarrow m = -1$

so point on parabola $(-2am, am^2) = (+10, 5)$

Required minimum distance is

$d = \sqrt{(15-10)^2 + (5)^2}$

$= \sqrt{25+25} = 5\sqrt{2} - 2$

so $d_{min}^{\,2} = (5\sqrt{2} - 2)^2$

$= 50 + 4 - 20\sqrt{2} = 54 - 20\sqrt{2}$

6. as $|f(x)| \le |\tan x|$ for $\forall\, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

so f(0) = 0

so $|f(x) - f(0)| \le |\tan x|$

divide both sides by | x |

$\left|\frac{f(x)-f(0)}{x}\right| \le \left|\frac{\tan x}{x}\right|$

$\lim_{x\to 0}\left|\frac{f(x)-f(0)}{x}\right| \le \lim_{x\to 0}\left|\frac{\tan x}{x}\right|$

$|f'(0)| \le 1.$

$\left|a_1 + \frac{1}{2}a_2 + \frac{1}{3}a_3 + ... + \frac{1}{n}a_n\right| \le 1$

$\left|\sum_{i=1}^{n}\frac{a_i}{i}\right| \le 1$

7. $\left|\sqrt{b^2 - 4ac}\right| \le |b|\sqrt{1 - \frac{4ac}{b^2}} \le |b|\sqrt{1 + \left|\frac{4ac}{b^2}\right|}$

$\le |b|\left(1 + \left|\frac{2ac}{b^2}\right|\right)$

so $\left|\sqrt{b^2 - 4ac}\right| \le |b| + \left|\frac{2ac}{b}\right|$

so that $\left|-\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}\right| \le \left|\frac{b}{2a}\right| + \left|\frac{b}{2a}\right| + \left|\frac{c}{b}\right|$

$= \left|\frac{b}{a}\right| + \left|\frac{c}{b}\right|$

Hence the solutions of $az^2 + bz + c = 0$ satisfy the condition

$|z| \le + \left|\frac{b}{a}\right| + \left|\frac{c}{b}\right|$

8. $f(x) = (x - 1)^3 g(x) + 1$

$\Rightarrow f'(x)$ is divisible by $(x - 1)^2$

& $f(x) = (x + 1)^3 g(x) - 1$

$\Rightarrow f'(x)$ is divisible by $(x + 1)^2$

so $f'(x) = a(x^2 - 1)^2 = a(x^4 - 2x^2 + 1)$

Now, $f(x) = \frac{ax^5}{5} - \frac{2x^3}{3}a + ax + c$

$f(+1) = \frac{a}{5} - \frac{2a}{3} + a + c = 1$

$\Rightarrow 8a + 15c = 15$ ...(1)

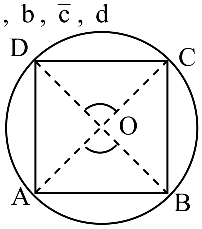
& $f(-1) = -\frac{a}{5} + \frac{2a}{3} - a + c = -1$

$\Rightarrow -8a + 15c = -15$ ...(2)

so $c = 0$, so $a = \frac{15}{8}$

so $f(x) = \frac{1}{8}(3x^5 - 10x^3 + 15x)$

9. Let the centre be O and P.V. of the pt. are respectively $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$



so $|\bar{a}| = |\bar{b}| = |\bar{c}| = |\bar{d}| = R$.

given $AB^2 + CD^2 = 4R^2$

$|\bar{b} - \bar{a}|^2 + |\bar{d} - \bar{c}|^2 = 4R^2$

$|\bar{b}|^2 + |\bar{a}|^2 - 2\bar{a}.\bar{b} + |\bar{d}|^2 + |\bar{c}|^2 - 2\bar{c}.\bar{d} = 4R^2$

$4R^2 - 2\bar{a}.\bar{b} - 2\bar{c}.\bar{d} = 4R^2$

so $\bar{a}.\bar{b} + \bar{c}.\bar{d} = 0$ ...(1)

$\cos(\angle AOB) + \cos(\angle COD) = 0$

$\cos\angle AOB = \cos(\pi - \angle COD)$

so $(\angle AOB) + \angle COD = \pi$

so $\angle AOD + \angle BOC = \pi$

$\Rightarrow \cos(\angle AOD) + \cos(\angle BOC) = 0$

so $\bar{a}.\bar{d} + \bar{c}.\bar{b} = 0$ ...(2)

Now, $\overline{AC}.\overline{BD} = (\bar{c} - \bar{a}).(\bar{d} - \bar{b})$

$= \bar{c}.\bar{d} - \bar{c}.\bar{b} - \bar{a}.\bar{d} + \bar{a}.\bar{b}$

using (1) & (2) in it

$\overline{AC}.\overline{BD} = 0$

so $AC \perp BD$

10. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the ellipse and

$y = m_1(x - ae)$ and $y = m_2(x - ae)$ are two chords through its focus (ae, 0). Any conic through the extrimities of these chords can be defined as

$\{y - m_1(x - ae)\}\ \{y - m_2(x - ae)\}$

$+ \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right) = 0$ ...(1)

If it passes through origin, then

$m_1m_2a^2e^2 - \lambda = 0$ ...(2)

solving (1) with x axis

$m_1m_2(x - ae)^2 + \lambda\left(\frac{x^2}{a^2} - 1\right) = 0$

using (2) in it

$(x - ae)^2 + e^2(x^2 - a^2) = 0$

$(1 + e^2)x^2 - 2aex = 0$

$x = 0$ & $x = \frac{2ae}{1+e^2}$

so other point on x-axis through which this conic passes is $\left(\frac{2ae}{1+e^2}, 0\right)$ which is a fixed point.

Hence proved.

MATHS HUMOUR

A constant function and e^x are walking on Broadway. Then suddenly the constant function sees a differential operator approaching and runs away. so e to-the x follows him and asks why the hurry.

"Well, you see, there's this diff.operator coming this way, and when we meet, he'll differentiate me and nothing will be left of me...!"

"Ah," says e^x, "he won't bother ME, I'm e to-the x!" and he walks on. Of course he meets the differential operator after a short distance.

e^x : "Hi, I'm e^x"

diff.op. : "Hi, I'm d/dy"

Students' Forum

Expert's Solution for Question asked by IIT-JEE Aspirants

MATHS

1. Find all possible negative real values of 'a' such that :

$$\int_a^0 (9^{-2t} - 2.9^{-t})\,dt \ge 0$$

Sol. Here $\int_a^0 (9^{-2t} - 2.9^{-t})\,dt \ge 0$

$$\Rightarrow \left(\frac{9^{-2t}}{-2\log 9} - \frac{2.9^{-t}}{-\log 9}\right)_a^0 \ge 0 \Rightarrow \left(-9^{-2t} + (49)^{-t}\right)_a^0 \ge 0$$

$\Rightarrow$ $9^{-2a} - 4.9^{-a} + 3 \ge 0$ $\Rightarrow$ $t^2 - 4t + 3 \ge 0$;

where $t = 9^{-a}$ and $t \in (1, \infty)$

$\Rightarrow$ $(t-1)(t-3) \ge 0$

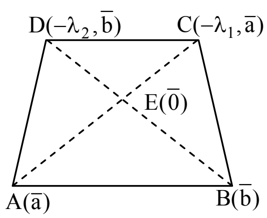
$\Rightarrow t \le 1$ or $t \ge 3$ is possible as $t > 1$.

$\therefore$ $9^{-a} \ge 3 \Rightarrow a \le -\frac{1}{2}$; $|3^{-a} \ge 3 \Rightarrow a \le -1$

2. Let ABCD be any arbitrary plane quadrilateral in the space having E as the point of intersection of its diagonals. If Δ_1 and Δ_2 be the areas of triangles DEC and AEB, using vector method prove that $\sqrt{\Delta} \ge \sqrt{\Delta_1} + \sqrt{\Delta_2}$, where Δ is the area of the quadrilateral ABCD. Also discuss the case when the equality holds.

Sol. Let the position vector of the points A, B, C and and D with respect to E be $\vec{a}$, $\vec{b}$, $-\lambda_1.\ \vec{a}$ and $-\lambda_2 \vec{b}$; where $\lambda, \lambda_2 \in R^+$

Now, $\Delta_1 = \frac{1}{2}|\vec{E}C \times \vec{E}D| = \frac{\lambda_1\lambda_2}{2}|\vec{a} \times \vec{b}|$



$$\Rightarrow \sqrt{\Delta_1} = \sqrt{\frac{1}{2}|\vec{a} \times \vec{b}||\lambda_1\lambda 2|} = \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}}\sqrt{\lambda_1\lambda_2} \quad ...(i)$$

and $\Delta_2 = \frac{1}{2}|\vec{E}B \times \vec{E}A| = \frac{1}{2}|\vec{a} \times \vec{b}|$

$$\Rightarrow \sqrt{\Delta_2} = \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}} \quad ...(ii)$$

also $\Delta = \frac{1}{2}|\vec{A}C \times \vec{B}D| = \frac{(1+\lambda_1)(1+\lambda_2)}{2}|\vec{a} \times \vec{b}|$

$$\therefore \sqrt{\Delta} = \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}}\sqrt{(1+\lambda_1)(1+\lambda_2)}$$

$$\sqrt{\Delta} = \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}}\sqrt{1+\lambda_1+\lambda_2+\lambda_1\lambda_2}$$

where, $\frac{\lambda_1+\lambda_2}{2} \ge \sqrt{\lambda_1\lambda_2}$

$$\therefore \sqrt{\Delta} = \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}}\sqrt{1+2\sqrt{\lambda_1}\lambda_2+\lambda_1\lambda_2}$$

$$\ge \sqrt{\frac{1}{2}|\vec{a} \times \vec{b}|} + \sqrt{(1+\sqrt{\lambda_1\lambda_2})^2}$$

$$= \sqrt{\frac{|\vec{a} \times \vec{b}|}{2}} + \sqrt{\frac{|\vec{a} \times \vec{b}||\lambda_1\lambda_2|}{2}}$$

$\therefore$ $\sqrt{\Delta} \ge \sqrt{\Delta_2} + \sqrt{\Delta_1}$ {using (i) and (ii)

It is clear that equality holds if $\lambda_1 = \lambda_2$ and in this case side AB and DC will become parallel.

3. Let S be the coefficients of x^{49} in given expression f(x) and if P be product of roots of the equation f(x) = 0, then find the value of $\frac{S}{P}$, given that :

$$f(x) = (x-1)^2\left(\frac{x}{2}-2\right)\left(x-\frac{1}{2}\right)\left(\frac{x}{3}-3\right)\left(x-\frac{1}{3}\right), \ ....\left(\frac{x}{25}-25\right)\left(x-\frac{1}{25}\right)$$

Sol. Here we can write f(x) as :

$$f(x) = \left\{(x-1)\left(\frac{x}{2}-2\right)\left(\frac{x}{3}-3\right)...\left(\frac{x}{25}-25\right)\right\} \times \left\{(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{3}\right)...\left(x-\frac{1}{25}\right)\right\}$$

Now roots of f(x) = 0 are;

$1^2, 2^2, 3^2, ..., 25^2$ and $1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{25}$

Now f(x) is the polynomial of degree 50,

So coefficient of x^{49} will be :

S = – (sum of roots)


ESTD: 1975
Glorious 35 Years
(REG. No. 225/75-76)
GOVERNED BY CHILDREN SCHOOL SOCIETY (REGD.)
CHILDREN SR. SEC. SCHOOL
&
CHILDREN GIRLS SR. SEC. SCHOOL
(AFFILIATED TO CBSE/RBSE/NIOS)
Raj. Board
R.B.S.E. Ajmer
Nursery to XII
Central Board
C.B.S.E. New Delhi
Nursery to XII
National Open Board
NIOS, New Delhi
VIII, X & XII
MEDIUM: English & Hindi
GROUP: Science, Agri, Comm., Art's & Computer
★ Special Facilities For Coaching Students ★
Website : childrenschoolkota.com
ADMISSION
Open
Dadabari : 2502160, 2502760
Fax : 0744-2502060
Chhawani : 2362100, 2362262
Fax : 0744-2362277

CAREER POINT

Correspondence & Test Series Courses Information

Courses of IIT-JEE

COURSE → INFORMATION ↓	For Class 12th/ 12th Pass Students						For Class 11th Students		
	Study Material Package IIT-JEE 2011	All India Test Series IIT-JEE 2011 (At Center)	Postal All India Test Series IIT-JEE 2011	Major Test Series IIT-JEE 2011 (At Center)	Postal Major Test Series IIT-JEE 2011 (By Post)	CP Ranker's Package IIT-JEE 2011	Study Material Package IIT-JEE 2012	All India Foundation Test Series IIT-JEE 2012 (At Center)	Postal All India Test Series IIT-JEE 2012
Eligibility for Admission	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 11th Students	Class 11th Students	Class 11th Students
Medium	English or Hindi	English	English	English	English	English	English or Hindi	English	English
Test Center	Postal	visit our website or contact us	Postal	visit our website or contact us	Postal	Postal	Postal	visit our website or contact us	Postal
Issue of Application Kit	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch
Date of Dispatch/ First Test	Immediate Dispatch	15 Aug 10 onwards	15 Aug 10 onwards	16 Jan 2011 onwards	16 Jan 2011 onwards	Last week of Sept.2010	Immidiate Dispatch	15 Aug 10 onwards	15 Aug 10 onwards
Course Fee	9000	3600	1500	1000	700	900	9800	4500	2500

Courses of AIEEE

COURSE → INFORMATION ↓	For Class 12th/ 12th Pass Students						For Class 11th Students
	Study Material Package AIEEE 2011	All India Test Series AIEEE 2011 (At Center)	Postal All India Test Series AIEEE 2011	Major Test Series AIEEE 2011 (At Center)	Postal Major Test Series AIEEE 2011 (By Post)	CP Ranker's Package AIEEE 2011	Study Material Package AIEEE 2012
Eligibility for Admission	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 11th Students
Medium	English or Hindi	English or Hindi	English or Hindi	English or Hindi	English or Hindi	English or Hindi	English or Hindi
Test Center	Postal	visit our website or contact us	Postal	visit our website or contact us	Postal	Postal	Postal
Issue of Application Kit	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch
Date of Dispatch/ First Test	Immediate Dispatch	15 Aug 10 onwards	15 Aug 10 onwards	16 Jan 2011 onwards	16 Jan 2011 onwards	Last week of Sept.2010	Immidiate Dispatch
Course Fee	8000	3600	1500	1000	700	900	9000

Courses of Pre-Medical

COURSE → INFORMATION ↓	For Class 12th/ 12th Pass Students						For Class 11th Students		
	Study Material Package AIPMT 2011	All India Test Series AIPMT 2011 (At Center)	Postal All India Test Series AIPMT 2011	Major Test Series AIPMT 2011 (At Center)	Postal Major Test Series AIPMT 2011 (By Post)	CP Ranker's Package AIPMT 2011	Study Material Package AIPMT 2012	All India Foundation Test Series AIPMT 2012 (At Center)	Postal All India Test Series AIPMT 2012
Eligibility for Admission	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 12th or 12th Pass Students	Class 11th Students	Class 11th Students	Class 11th Students
Medium	English or Hindi	English or Hindi	English or Hindi	English or Hindi	English	English	English or Hindi	English	English
Test Center	Postal	visit our website or contact us	Postal	visit our website or contact us	Postal	Postal	Postal	visit our website or contact us	Postal
Issue of Application Kit	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch	Immediate Dispatch
Date of Dispatch/ First Test	Immediate Dispatch	15 Aug 10 onwards	15 Aug 10 onwards	16 Jan 2011 onwards	16 Jan 2011 onwards	Last week of Sept.2010	Immidiate Dispatch	15 Aug 10 onwards	15 Aug 10 onwards
Course Fee	8000	3600	1500	1000	700	900	8500	4500	2500

• For details about all correspondence & lest Series Course Information, Please visit our website.: www.careerpointgroup.com