CAREER POINT

FOR OFFICE USE ONLY	Form No.	
Enrolment No.		

Please tick the name of the Correspondence & Test Series Course(s) you wish to join -

For Class 12th or 12th pass Student	IIT-JEE Division	AIEEE Division	Pre-Medical Division
Study Material Package	❑	❑	❑
All India Test Series (At Center)	❑	❑	❑
Postal All India Test Series (By Post)	❑	❑	❑
All India Major Test Series (At Center)	❑	❑	❑
Postal Major Test Series (By post)	❑	❑	❑
CP Ranker's Package	❑	❑	❑
For Class 11th Student (2 Years Courses)			
Study Material Package	❑	❑	❑
All India Foundation Test Series(At Center)	❑	❑	❑
Postal All India Foundation Test Series (By Post)	❑	❑	❑

For Class 9th & 10th Students	Class 9th	Class 10th
Synchro School Study Material Package	❑	❑

Currently Studying in	
Class 9th	❑
Class 10th	❑
Class 11th	❑
Class 12th	❑
Class 12th Pass	❑

Please, Affix a Passport Size Photograph here

Other Course (Please Specify):

1. ____________________ 2. ____________________

Medium : ❑ English ❑ Hindi

Test Series Center

Visit our website for list of test centers. If you wish to join Test Series by Post, then write Postal in the Box.

PERSONAL INFORMATION [Use Capital Letters Only]

Name [In Block Letters] ______________________________
[As in the school record]

Father's/Guardian's Name [In Block Letters] ______________________________

Date of Birth [DD/MM/YYYY] __________

Category [Tick appropriate box] ❑ Gen ❑ SC ❑ ST ❑ Ex ❑ PH ❑ DK ❑ OBC

Father's/Guardian's Occupation

❑ Business [Nature of Business] ______________________________

❑ Service [Name of Company/Department with post ______________________________

ACADEMIC DETAILS

Examination	Board CBSE/Raj/MP/UP/Other (specify)	Year of passing	Subject	Max. Marks	Marks Obtained	%age Marks	% age Marks Sc. + Maths/PCM
Class 9th			Science				
			Maths				
Class 10th			Science				
			Maths				
Class 11th			Physics				
			Chemistry				
			Maths				
			Biology				
12th Class			Physics				
			Chemistry				
			Maths				
			Biology				

Tick the appropriate box if eligible for Study Support Scholarship :

Criteria (Please mention in detail)	Study Support Scholarship (%age)
	❑100% ❑ 50% ❑ 35% ❑ 25% ❑ 20%

Note : 1. Student eligible for Study Support Scholarship should submit the fee accordingly and also attach attested copies of supporting documents.
2. A student can avail Study Support Scholarship in only one course.

COURSE FEE DETAIL (Application form received without demand draft of fee will be rejected without intimation)			
Amount	DD Number	Date	Drawn on Bank (Name)

NOTE : YOU CAN ALSO USE THE PHOTOCOPY OF THIS APPLICATION FORM

DETAILS OF ACADEMIC ACHIEVEMENT [IF ANY]	
NTSE Qualified	Yes ☐ No ☐ If yes, upto which level :
IIT-JEE	Year : Qualified Main List ☐ EML List ☐ If Yes then Rank :
AIEEE	Year : All India Rank : State Rank : State :
AIPMT	Year : Qualified Examination Screening ☐ Main ☐ If Yes then Rank :
Other (Specify)	Examination Name : Rank : Year :
Merit/Position in Board	Board : Class: Position : Year :
Other Academic Awards	

Have you joined any other course at Career Point in current or previous session? If yes, give details ______

Name and address of the school where studying presently (or last attended) :

______

______

COMMUNICATION DETAILS

CORRESPONDENCE ADDRESS [IN CAPITAL]

______

______

______ City ______

State ______ Pin ☐☐☐☐☐☐

Residence STD Code ☐ Tel. ☐

(Office) STD Code ☐ Tel. ☐

Mobile ☐

e-mail address ☐

PERMANENT ADDRESS [IN CAPITAL]

______

______

______ City ______

State ______ Pin ☐☐☐☐☐☐

Residence STD Code ☐ Tel. ☐

(Office) STD Code ☐ Tel. ☐

Mobile ☐

e-mail address ☐

DECLARATION

1. I declare that I have read the Rules and Regulations of the institute as mentioned in the Information Booklet and understood the contents fully and agree to abide by them.
2. I am seeking admission in my own interest.
3. I declare that all the information furnished in this Application Form is correct to the best of my knowledge. I understand that in the event of any information found to be incorrect or false, my admission may be cancelled without refund of fee.
4. I shall inform the Director of Career Point about my Roll Nos. of various competitive examination as soon as I receive my Admit Card for those competitive examination. I shall not withhold this information from Career Point, who are entitled to know the results of their efforts to help me get through the exams. I understand that failing above, I shall not be considered for Career Point's reward for success.
5. I agree to abide by the policies, rules & regulations of the Institute as amended from time to time.
6. In all the matters, the decision of Career Point's Director shall be final.

______ Signature (Father/Guardian) Date :

______ Signature (Student) Date :

website : www.careerpointgroup.com | Phone No. 0744-3040000, 2430505 Fax: 0744-3040050 | email : cts@careerpointgroup.com

$$= -(1^2 + 2^2 + ... + 25^2) - \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{25}\right)$$

$$= -\left\{\frac{25 \times 26 \times 51}{6} + K\right\} \text{ where, } K = \sum_{n=1}^{25} \frac{1}{n}$$

$\Rightarrow$ S = –(K + 5525).

Product of roots :

$$1^2 . 2^2 . 3^2 .... 25^2 . 1 . \frac{1}{2} . \frac{1}{3} .... \frac{1}{25} = 1 . 2 . 3 ...25$$

$\therefore$ P = 25 !

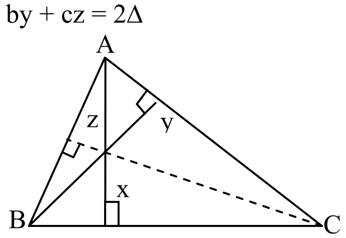
$$\text{Hence } \frac{S}{P} = \frac{-(K+5525)}{25!}, \text{ where } K = \sum_{n=1}^{25} \frac{1}{n}$$

4. Find the point inside a triangle from which the sum of the squares of distance to the three side is minimum. Find also the minimum value of the sum of squares of distance.

Sol. If a, b, c are the lengths of the sides of the Δ and x, y, z are length of perpendicular from the points on the sides BC, CA and AB respectively, we have to minimise : $\Delta = x^2 + y^2 + z^2$

we have, $\frac{1}{2}ax + \frac{1}{2}by + \frac{1}{2}cz = \Delta$

$\Rightarrow ax + by + cz = 2\Delta$



where Δ is the area of ΔABC.

We have the identity :

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2) - (ax + by + cz)^2$
$= (ax - by)^2 + (by - cz)^2 + (cz - ax)^2$

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2) \geq (ax + by + cz)^2$

$\Rightarrow (x^2 + y^2 + z^2)(a^2 + b^2 + c^2 \geq 4\Delta^2$

$$\Rightarrow x^2 + y^2 + z^{-2} \geq \frac{4\Delta^2}{a^2 + b^2 + c^2}$$

Equality holds only when

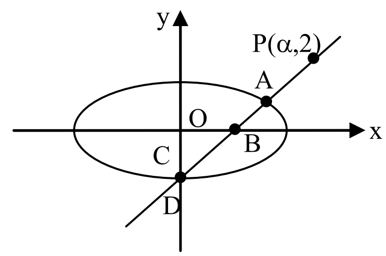
$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \frac{ax + by + cz}{a^2 + b^2 + c^2} = \frac{2\Delta}{a^2 + b^2 + c^2}$$

$\therefore$ The minimum value of Δ is ;

$$\frac{4\Delta^2}{a^2 + b^2 + c^2} = \frac{4(s-a)(s-b)(s-c)s}{a^2 + b^2 + c^2}$$

5. From an external point P(α, 2) a variable line is drawn to meet the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the points A and D. Same line meets the x-axis and y-axis at the points B and C respectively. Find the range of values of 'α' such that PA. PD = PB.PC.

Sol. We have been given,



PA.PD = PB.PC

Equation of any line through point 'P' is :

$$\frac{x - \alpha}{\cos\theta} = \frac{y - 2}{\sin\theta} = r$$

or $x = \alpha + r\cos\theta$, $y = 2 + r\sin\theta$

Putting this point in the equation of given ellipse, we get

$4(r\cos\theta + \alpha)^2 + 9(2 + r\sin\theta)^2 = 36$

$\Rightarrow r^2(4\cos^2\theta + 9\sin^2\theta) + 4r(9\sin\theta + 2\alpha\cos\theta) + 4\alpha^2 = 0$

Since PA and PD are the roots of this quadratic in r, we get

$$PA.PD = \frac{4\alpha^2}{(4\cos^2\theta + 9\sin^2\theta)} \quad ...(i)$$

Similarly, putting $x = r\cos\theta + \alpha$, $y = r\sin\theta + 2$ in the equation of coordinate axis i.e. xy = 0

$(r\cos\theta + \alpha).(r\sin\theta + 2) = 0$

$\Rightarrow r^2\sin\theta\cos\theta + r(2\cos\theta + \alpha\sin\theta) + 2\alpha = 0$

Since PB and PC and the roots of this quadratic in 'r',

$$\text{we get,} \quad PB.PC = \frac{2\alpha}{\sin\theta\cos\theta} = \frac{4\alpha}{\sin 2\alpha} \quad ...(ii)$$

Thus, we get

$$\frac{4\alpha}{\sin 2\theta} = \frac{4\alpha^2}{4\cos^2\theta + 9\sin^2\theta} \quad \text{\{from (i) and (ii)\}}$$

$$\Rightarrow \frac{4\alpha}{\sin 2\theta} = \frac{8\alpha^2}{4(1 + \cos 2\theta) + 9(1 - \cos 2\theta)}$$

$$\Rightarrow \frac{1}{\sin 2\theta} = \frac{2\alpha}{13 - 5\cos 2\theta}$$

$\Rightarrow 13 = 5\cos 2\theta + 2\alpha\sin 2\theta$

where; $5\cos 2\theta + 2\alpha\sin 2\theta \leq \sqrt{25 + 4\alpha^2}$

$$\therefore \sqrt{4\alpha^2 + 25} \geq 13 \Rightarrow \alpha^2 \geq \frac{169 - 25}{4} = 36$$

$\Rightarrow \alpha \in (-\infty, -6] \cup [6, \infty)$

6. Let f(x) be a polynomial with integral coefficients suppose that both f(1) and f(2) are odd. Then, prove that for any integer n, f(n) $\neq$ 0.

Sol. Suppose f(x) = 0 for some integer n.

Then (x – n) divides f(x)

So; f(x) = (x – n) g (x)

Now, f(1) = (1 – n) . g(1) and f(2) = (2 – n) g (2)

Now g (1) and g(2) are both integers, and one of (1 – n) or (2 – n) is even.

So one of f(1) or f(2) is even, which is contradictory, so there is no integer n, for which f(n) = 0

MATHS

INVERSE TRIGONOMETRIC FUNCTION

Mathematics Fundamentals

- Meaning of inverse function :
  1. $\sin\theta = x \Leftrightarrow \sin^{-1} x = \theta$
  2. $\cos\theta = x \Leftrightarrow \cos^{-1}x = \theta$
  3. $\tan\theta = x \Leftrightarrow \tan^{-1}x = \theta$
  4. $\cot\theta = x \Leftrightarrow \cot^{-1}x = \theta$
  5. $\sec\theta = x \Leftrightarrow \sec^{-1}x = \theta$
  6. $\text{cosec}\,\theta = x \Leftrightarrow \text{cosec}^{-1}x = \theta$
- Domains and Range of Functions :

Function	Domain	Range
$\sin^{-1}x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\cos^{-1}x$	$-1 \le x \le 1$	$0 \le \theta \le \pi$
$\tan^{-1}x$	$-\infty < x < \infty$, i.e. $x \in R$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
$\text{cosec}^{-1}x$	$x \le -1, x \ge 1$	$\theta \ne 0, -\frac{\pi}{2} \le \theta < \frac{\pi}{2}$
$\sec^{-1}x$	$x \le -1, x \ge 1$	$\theta \ne \frac{\pi}{2}, 0 \le \theta \le \pi$
$\cot^{-1}x$	$-\infty < x < \infty$ i.e. $x \in R$	$0 < \theta < \pi$

- **Properties of Inverse Functions :**

(a) 1. $\sin^{-1}(\sin\theta) = \theta$, $\sin(\sin^{-1}x) = x$

2. $\cos^{-1}(\cos\theta) = \theta$, $\cos(\cos^{-1}x) = x$
3. $\tan^{-1}(\tan\theta) = \theta$, $\tan(\tan^{-1}x) = x$
4. $\cot^{-1}(\cot\theta) = \theta$, $\cot(\cot^{-1}x) = x$
5. $\sec^{-1}(\sec\theta) = \theta$, $\sec(\sec^{-1}x) = x$
6. $\text{cosec}^{-1}(\text{cosec}\,\theta) = \theta$, $\text{cosec}(\text{cosec}^{-1}x) = x$

(b) 1. $\sin^{-1}x = \text{cosec}^{-1}(1/x)$

2. $\cos^{-1}x = \sec^{-1}(1/x)$
3. $\tan^{-1}x = \cot^{-1}(1/x)$

(c) 1. $\sin^{-1}(-x) = -\sin^{-1}x$

2. $\cos^{-1}(-x) = \pi - \cos^{-1}x$
3. $\tan^{-1}(-x) = -\tan^{-1}x$
4. $\cot^{-1}(-x) = \pi - \cot^{-1}x$
5. $\sec^{-1}(-x) = \pi - \sec^{-1}x$
6. $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$

(d). 1. $\sin^{-1}x + \cos^{-1}x = \pi/2$

2. $\tan^{-1}x + \cot^{-1}x = \pi/2$
3. $\sec^{-1}x + \text{cosec}^{-1}x = \pi/2$

- **Formulae for Sum and Difference of Inverse Function –**

1. $\tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1}\frac{x+y}{1-xy} & \text{where } xy < 1 \\ \pi + \tan^{-1}\frac{x+y}{1-xy} & \text{when } xy > 1 \end{cases}$
2. $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$
3. $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$
4. $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right\}$
5. $\cot^{-1}x \pm \cot^{-1}y = \cot^{-1}\left[\frac{xy \mp 1}{y \pm x}\right]$
6. $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

- **Some Important Results :**

1. $2\sin^{-1}x = \sin^{-1}2x\sqrt{1-x^2}$
2. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$
3. $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$
4. $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
5. $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$
6. $3\tan^{-1}x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$
7. $\tan^{-1}\left[\frac{x}{\sqrt{a^2-x^2}}\right] = \sin^{-1}\left(\frac{x}{a}\right)$
8. $\tan^{-1}\left[\frac{3a^2x-x^3}{a(a^2-3x^2)}\right] = 3\tan^{-1}\left(\frac{x}{a}\right)$
9. $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$

QUADRATIC EQUATION

General quadratic equation :

An equation of the form

$ax^2 + bx + c = 0$ ...(1)

where $a \neq 0$, is called a quadratic equation, in the real or complex coefficients a, b and c.

Roots of a quadratic equation :

The values of x, (say $x = \alpha, \beta$) which satisfy the quadratic equation (1) are called the roots of the equation and they are given by

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} ; \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Discriminant of a quadratic equation :

The quantity $D \equiv b^2 - 4ac$, is known as the discriminant of the equation.

Nature of the Roots :

In the equations $ax^2 + bx + c = 0$, let us suppose that a, b, c are real and $a \neq 0$. The following is true about the nature of its roots-

(i) The equation has real and distinct roots if and only if $D \equiv b^2 - 4ac > 0$.

(ii) The equation has real and coincident (equal) roots if and only if $D \equiv b^2 - 4ac = 0$.

(iii) The equation has complex roots of the form $\alpha \pm i\beta$, $\alpha \neq 0$, $\beta \neq 0 \in R$, if and only if $D \equiv b^2 - 4ac < 0$.

(iv) The equation has rational roots if and only if a, b, $c \in Q$ (the set of rational numbers) and $D \equiv b^2 - 4ac$ is a perfect square (of a rational number).

(v) The equation has (unequal) irrational (surd form) roots if and only if $D \equiv b^2 - 4ac > 0$ and not a perfect square even if a, b and c are rational. In this case if $p + \sqrt{q}$, p, q rational, is an irrational root, then $P - \sqrt{q}$ is also a root (a, b, c being rational).

(vi) $\alpha + i\beta$ ($\beta \neq 0$ and $\alpha, \beta \in R$) is a root if and only if its conjugate $\alpha - i\beta$ is a root, that is complex roots occur in pairs in a quadratic equation. In case the equations is satisfied by more than two complex numbers, then it reduces to an identitiy.

$0.x^2 + 0.x + 0 = 0$, i.e. $a = 0 = b = c$.

Relation between Roots and Coefficients :

If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then the sum and product of the roots is

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Hence the quadratic equation whose roots are α and β is given by

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ or $(x - \alpha)(x - \beta) = 0$

Condition that the two quadratic equations have a common root :

Let α be a common root of two quadratic equations

$a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

where $a_1 \neq 0$, $a_2 \neq 0$ and $a_1b_2 - a_2b_1 \neq 0$.

Then $a_1\alpha^2 + b_1\alpha + c_1 = 0$ and $a_2\alpha^2 + b_2\alpha + c_2 = 0$

which gives (by cross multipication),

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Thus eliminating α, the condition for a common root is given by

$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$ ...(2)

Condition that the two quadratic equations have both the roots common :

The two quadratic equation will have the same roots if and only if their coefficients are proportional, i.e.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Descarte's rule of signs :

The maximum number of positive of a polynomial f(x) is the number of changes of signs in f(x) and the maximum number of negative roots of f(x) is the number of changes of signs in f(–x).

Position of roots :

If f(x) = 0 is an equation and a, b are two real numbers such that f(a) f(b) < 0, then the equation f(x) = 0 has at least one real root or an odd number of real roots between a and b. In case f(a) and f(b) are of the same sign, then either no real root or an even number of real roots of f(x) = 0 lie between a and b.

The quadratic expression :

(A) Let $f(x) \equiv ax^2 + bx + c$, a, b, c $\in$ R, a > d be a quadratic expression. Since,

$$f(x) = a\left\{\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a^2}\right)\right\} \qquad ...(3)$$

The following is true from equation (3)

(i) f(x) > 0 (< 0) for all values of x $\in$ R if and only if a > 0 (< 0) and $D \equiv b^2 - 4ac < 0$.

(ii) f(x) $\geq$ 0 ($\leq$ 0) if and only if a > 0 (< 0) and $D \equiv b^2 - 4ac = 0$.

In this case (D = 0), f(x) = 0 if and only if $x = -\frac{b}{2a}$

(iii) If $D \equiv b^2 - 4ac > 0$ and a > 0 (< 0), then

$$f(x) = \begin{cases} < 0(> 0), & \text{for x lying between the roots of } f(x) = 0 \\ > 0(< 0), & \text{for x not lying between the roots of } f(x) = 0 \\ = 0, & \text{for x = each of the roots of } f(x) = 0 \end{cases}$$

(iv) If a > 0, (< 0), then f(x) has a minimum (maximum) value at $x = -\frac{b}{2a}$ and this value is given by

$$[f(x)]_{\min\,(\max)} = \frac{4ac - b^2}{4a}$$

(B) **The sign of the expression :**

(i) The value of expression (x – a) (x – b); (a < b) is positive if x < a or x > b, in other words x does not lie between a and b.

(ii) The expression (x – a) (x – b); (a < b) is negative if a < x < b i.e. if x lies between a and b.

Some important results :

- If f(α) = 0 and f′(α) = 0, then α is a repeated root of the quadratic equation f(x) = 0 and $f(x) = a(x - \alpha)^2$. In fact $\alpha = -\frac{b}{2a}$.
- Imaginary and irrational roots occur in conjugate pairs (when a, b, c $\in$ R or a, b, c being rational) i.e., if –3 + 2i or $5 - 2\sqrt{7}$ is a root then –3 – 2i or $5 + 2\sqrt{7}$ will also be a root.
- For the quadratic equation $ax^2 + bx + c = 0$

  (i) One root will be reciprocal of the other if a = c.

  (ii) One root is zero if c = 0

  (iii) Roots are equal in magnitude but opposite in sign if b = 0.

  (iv) Both roots are zero if b = c = 0.

  (v) Roots are positive if a and c are of the same sign and b is of the opposite sign.

  (vi) Roots are of opposite sign if a and c are of opposite sign.

  (vii) Roots are negative if a, b, c are of the same sign.
- If the ratio of roots of the quadratic equation $ax^2 + bx + c = 0$ be p : q, then $pqb^2 = (p + q)^2ac$.
- If one root of the quadratic equation $ax^2 + bx + c = 0$ be p : q, then $pqb^2 = (p + q)^2ac$.
- If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then

  $$(ac^n)^{\frac{1}{n+1}} + (a^nc)^{\frac{1}{n+1}} + b = 0$$
- If one roots of the equation $ax^2 + bx + c = 0$ be n times the other root, then $nb^2 = ac(n + 1)^2$.
- If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{k+1}{k}$ and $\frac{k+2}{k+1}$, then $(a + b + c)^2 = b^2 - 4ac$.
- If the roots of $ax^2 + bx + c = 0$ are α, β, then the roots of $cx^2 + bx + a = 0$ will be $\frac{1}{\alpha}, \frac{1}{\beta}$.
- The roots of the equation $ax^2 + bx + c = 0$ are reciprocal to $a'x^2 + b'x + c' = 0$ if $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$.
- Let $f(x) = ax^2 + bx + c$, where a > 0 .Then

  (i) Conditions for both the roots of f(x) = 0 to be greater than a given number K are $b^2 - 4ac \geq 0$; f(K) > 0; $\frac{-b}{2a} > K$.

  (ii) The number K lies between the roots of f(x) = 0 if f(K) < 0.'

  (iii) Condition for exactly one root of f(x) = 0 to lie between d and e is f(d) f(e) < 0.

Puzzle : Bags of Marbles

- You have three bags, each containing two marbles. Bag A contains two white marbles, Bag B contains two black marbles, and Bag C contains one white marble and one black marble.
- You pick a random bag and take out one marble.
- It is a white marble.

Based on New Pattern

IIT-JEE 2011

XtraEdge Test Series # 2

Time : 3 Hours

Syllabus : Physics : Laws of motion, Friction, Work Power Energy, Gravitation, S.H.M., Laws of Conservations of Momentum, Rotational Motion (Rigid Body), Elasticity, Fluid Mechanics, Surface Tension, Viscosity, Refl. At Plane surface, Ref. at Curved surface, Refraction at Plane surface, Prism (Deviation & Dispersion), Refraction at Curved surface, Wave Nature of Light: Interference. **Chemistry :** Gaseous state, Chemical Energetics, Oxidation-Reduction, Equivalent Concept, Volumetric Analysis, Reaction Mechanism, Alkane, Alkene, Alkyne, Alcohol, Ether & Phenol, Practical Organic Chemistry, Aromatic Hydrocarbons, Halogen Derivatives, Carboxylic Acid & Its Derivatives, Nitrogen Compounds, Amines, Carbohydrates, Amino Acid, Protein & Polymers. **Mathematics:** Logarithm & Modulus Function, Quadratic Equation, Progressions, Binomial Theorem, Permutation & Combination, Complex Number, Indefinite Integration, Definite Integration, Area Under the Curve, Defferential Equations.

Instructions :

Section - I
- Question 1 to 8 are multiple choice questions with only one correct answer. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer.

Section - II
- Question 9 to 13 are multiple choice questions with multiple correct answer. ***+3 marks*** will be awarded for correct answer and ***0 mark*** for wrong answer.

Section - III
- Question 14 to 18 are passage based single correct type questions. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer.

Section - IV
- Question 19 to 27 are numerical response questions (with single digit Answer). ***+3 marks*** will be awarded for correct answer and ***0 mark*** for wrong answer.

PHYSICS

Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. If the potential energy between electron and proton at a distance r is given by

$$U = -\frac{Ke^2}{3r^3}$$

The force between electron and proton will be :

(A) $-\frac{Ke^2}{r^4}$ (B) $\frac{Ke^2}{3r^3}$

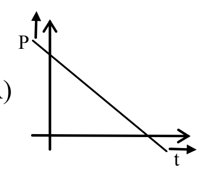
(C) $\frac{Ke^2}{3r^4}$ (D) None of these

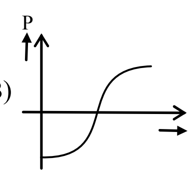
2. A mass m rests on a rough horizontal surface with coefficient of friction μ. If the mass is pulled by a force F = mg at a n angle 30º to the horizontal, the limiting friction between mass and the surface will be-

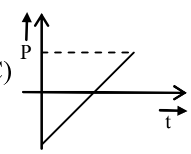
(A) μmg (B) $\mu(mg - \sqrt{3/2}mg)$

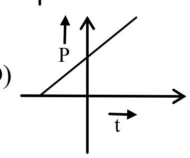
(C) $\mu\frac{mg}{2}$ (D) $\mu\left(mg + \frac{mg}{2}\right)$

3. A particle is projected with speed u in air at an angle θ with the horizontal. The graph showing the variation of instantaneous power due to gravity P with time t will be -

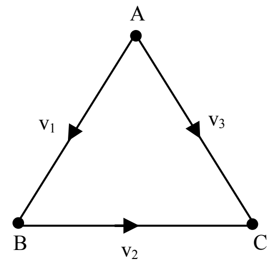
(A) 

(B) 

(C) 

(D) 

4. Three particles of equal masses are placed at the corners of an equilateral triangle, as shown figure. Now particle A starts with a velocity v_1 towards line AB, Particle B starts with a velocity v_2 towards line BC and particle C starts with Velocity v_3 towards line CA. The displacement of C.M. of three particle A, B and C after time "t" will be (given if $\vec{v}_1 = \vec{v}_2 = \vec{v}_3$) :



(A) Zero (B) $\frac{v_1 + v_2 + v_3}{3}$

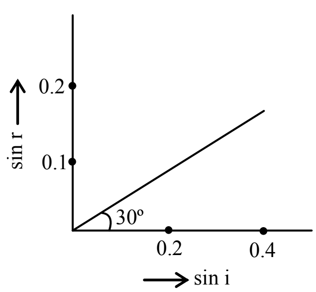
(C) $\frac{v_1 + \frac{\sqrt{3}}{2}v_2 + \frac{v_3}{2}}{3}$ (D) $\frac{v_1 + v_2 + v_3}{4}$

5. For a particular angle of incidence the angle of refraction in three media A, B and C are 15º, 25º and 45º respectively. The speed of light in these media v_a, v_b and v_c can be related as -

(A) $v_a > v_b > V_c$ (B) $v_a = v_b = v_c$

(C) $v_a < v_b = v_c$ (D) $v_a < v_b < v_c$

6. Light is incident from a medium X at angle of incidence i and refracted into a medium Y at angle of refraction r. The graph is sin i versus sin r is shown in the figure. Which of he following conclusions would fit the situation ?



(a) Speed of light in medium Y is $\sqrt{3}$ times that in medium X

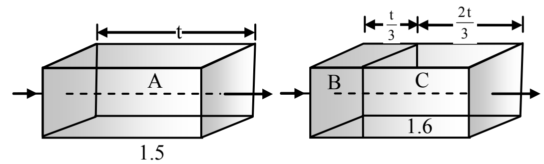
(b) Speed of light in medium Y is $1/\sqrt{3}$ times that in medium X

(c) Total internal reflection will occur above a certain i value

(A) b and c (B) a and c

(C) b only (D) c only

7. Two transparent slabs have the same thickness as shown. One is made of material A of refractive index 1.5. The other is made of two materials B and C with thickness in the ratio 1 : 2. The refractive index of C is 1.6 If a monochromatic parallel beam passing in the slabs has the same number of wavelengths inside both, the refractive index of B is



(A) 1.1 (B) 1.2 (C) 1.3 (D) 1.4

8. The potential energy for a force field $\vec{F}$ is given by U (x, y) = sin (x + y). The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is -

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. A particle of mass 5 kg moving in the x - y plane has its potential energy given by U = (–7x + 24y) J, Where x and y are in metre. The particle is initially at origin and has a velocity $\vec{u} = (14.4\hat{i} + 4.2\hat{j})\ ms^{-1}$.

(A) The particle has a speed of 25 ms^{-1} at t = 4 s

(B) The particle has an acceleration of 5 ms^{-2}

(C) The acceleration of the particle is perpendicular to its initial velocity

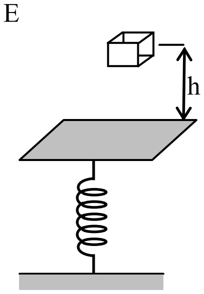
(D) None of the above is correct

10. A wire of length L, cross-sectional area A is made of a material of Young's modulus Y. The wire is stretched by an amount. x, which lies well within the elastic limit. If k be the equivalent force constant of the wire and W be the work done then -

(A) $k = \frac{YA}{2L}$ (B) $k = \frac{YA}{L}$

(C) $W = \frac{1}{2}\frac{YAx^2}{L}$ (D) $W = \frac{YAx^2}{L}$

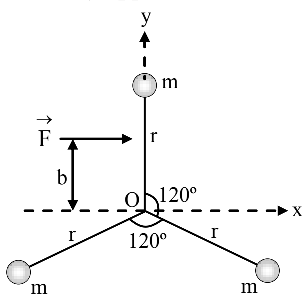
11. A body of mass m falls from a height h onto a pan (of negligible mass) of a spring balance as shown. The spring also possesses negligible mass and has spring constant k. Just after striking the pan, the body starts oscillatory motion on vertical direction of amplitude A and energy E



(A) $A = \frac{mg}{k}$ (B) $A = \frac{mg}{k}\sqrt{1+\frac{2kh}{mg}}$

(C) $E = mgh + \frac{1}{2}kA^2$ (D) $E = mgh + \left(\frac{mg}{2k}\right)^2$

12. Each of the three balls has a mass m and is welded to the rigid equiangular from of negligible mass. The assembly rests on a smooth horizontal surface. A force F is suddenly applied to one bar shown.



(A) Acceleration of point O is $\vec{a} = \frac{1}{3}\frac{\vec{F}}{m}\hat{i}$

(B) Acceleration of point O is $\vec{a} = \frac{\vec{F}}{m}\hat{i}$

(C) The magnitude of angular acceleration of the frame is $\alpha = \frac{1}{3}\frac{Fb}{mr^2}$

(D) The magnitude of angular acceleration of the frame is $\alpha = \frac{Fb}{mr^2}$

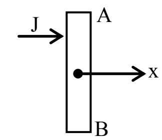
13. The x – y plane is the boundary between tow transparent media. Medium 1 with z ≥ O has a refractive index $\sqrt{2}$ and medium 2 with z < 0 has a refractive index $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. The refracted ray makes angle r with +z axis and incident ray makes an angle i with –z axis. Then,

(A) i = 120º (B) i = 60º
(C) r = 45º (D) r = 135º

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage- II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage : I (Ques. 14 to 15)

A rod AB of length 2 m and mass 2 kg is lying on smooth horizontal x – y plane with its centre at origin O as shown in figure. An impulse J of magnitude 10 Ns applied perpendicular to AB at A.



14. The distance of point P from centre of the rod which is at rest just after the impact is

(A) $\frac{2}{3}$m (B) $\frac{1}{3}$m

(C) $\frac{1}{2}$m (D) $\frac{1}{4}$m

15. If the rod is hinged at B and instead of impulse J, a point mass of 1 kg moving with a speed 10 m/s collide at A and sticks to it. Find the angular velocity of rod just after the collision -

(A) 1 rad/s (B) 2 rad/s
(C) 3 rad/s (D) 4 rad/s

Passage: II (Ques. 16 to 18)

Collision is the transfer of momentum due to only the internal forces between the particles taking part in collision. When exchange of a momentum takes place between two physics bodies due to their mutual interactive force, it is defined as collision between two bodies.

Two bodies move in different directions interact each other at the point of intersection of their line of motion and the reaction due to their physical contact is the interaction force which is the cause of transfer of momentum from one body to another.

Collision may be either elastic or inelastic. In case of elastic collision, momentum and K.E. are both conserved but in case of inelastic collision only momentum is conserved and K.E. is not conserved.

16. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position and released. The ball hits the wall, the coefficient of restitution being $\left(e = \frac{2}{\sqrt{5}}\right)$. The number of collision after which the amplitude of oscillation becomes less than 60º is :

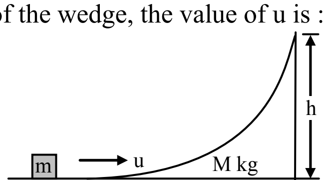
(A) n = 3 (B) n = 2
(C) n = 5 (D) n = 4

17. Two particles having the position $\vec{r}_1 = \left(3\hat{i} + 5\hat{j}\right)$ m and $\vec{r}_2 = \left(-5\hat{i} - 3\hat{j}\right)$ m move with velocities $\vec{v}_1 = \left(4\hat{i} + 3\hat{j}\right)$ m/s and $\vec{v}_2 = \left(a\hat{i} + 7\hat{j}\right)$ m/s. If the particles collide, then value of 'a' must be :

(A) 8 (B) 6

(C) 4 (D) 2

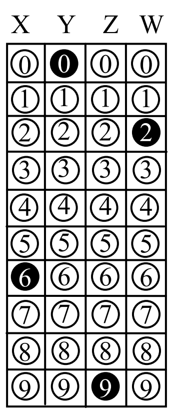
18. A block of mass m is moved towards a movable wedge of mass M kg and height h with velocity u (All surfaces are smooth). If the block just reaches the top of the wedge, the value of u is :



(A) $\sqrt{2gh}$ (B) $\sqrt{\dfrac{2ghK}{1+K}}$

(C) $\sqrt{\dfrac{2gh(1+K)}{K}}$ (D) $\sqrt{2gh\left(1-\dfrac{1}{K}\right)}$

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



19. Find the mass of water flowing in 10 minutes through a tube 0.1 cm in diameter 40 cm long if there is a constant pressure head of 20 cm of water. Given η for water = 0.008 C.G.S. units and $g = 1000$ cm/s^2. [in gram]

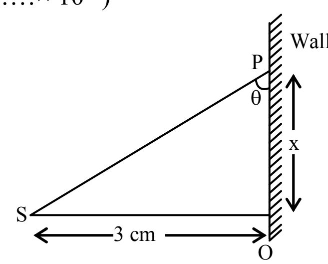
20. A plate of area 100 cm^2 is placed on the upper surface of castor oil, 2mm thick. Taking the co-efficient of viscosity to be 15.5 poise, calculate the horizontal force necessary to move the plate with a velocity 3 cm/sec. [in 10^{-5}N].

21. A ball of 1 kg attached to a string of 0.5 m length is whirled round in a horizontal circle. If the breaking stress is given by 1.5 N find the maximum permissible angular velocity. [in rad-s^{-1}]

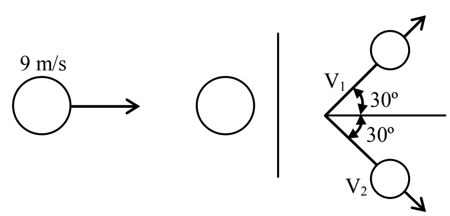
22. Distance between two consecutive dark bands is 0.4 mm when yellow light of 6000 Å wavelength is used. Find the distance between two consecutive bright bands with a wavelength of 4500 Å [in 10^{-4} m]

23. A 4 cm object is placed perpendicular to the principal axis of a convex mirror of focal length 7.5 cm. Find distance of the image from the mirror if it is 0.6 cm is size.

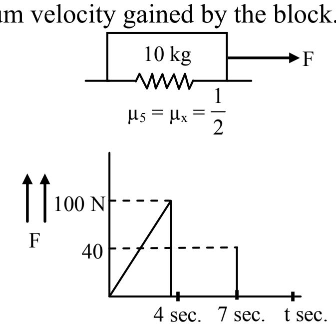
24. A spot light rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when θ = 45º ?
(Ans. ……× 10^{-1})



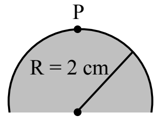
25. A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after the collision the direction of each ball makes an angle of 30º with the original line of motion. Find the speed of each ball after the collision.



26. The 10 kg block is resting on the horizontal surface when the force F is applied to it for 7 sec. What is the maximum velocity gained by the block.



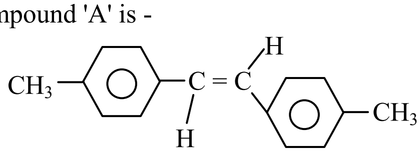
27. Moment of inertia of a semicircular ring of mass π kg and radius 2 cm about the axis passing through point P and perpendicular to its plane is : [in 10^{-5} kg m^2]

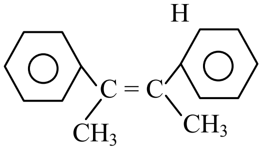


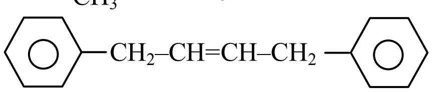
CHEMISTRY

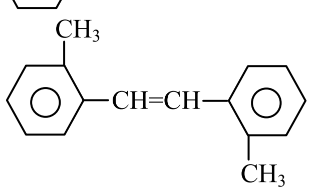
Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. An alkene (A) $C_{16}H_{16}$ on ozonolysis gives only one product (B) (C_8H_8O). Compound (B) on reaction with $NH_2OH/H_2SO_4,\Delta$ gives N-methyl benzamide the compound 'A' is -

(A) 

(B) 

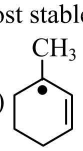
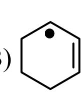
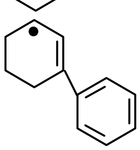
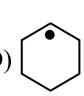
(C) 

(D) 

2. 10^{-2} moles of Fe_3O_4 is treated with excess of KI solution in presence of dilute H_2SO_4, the products are Fe^{2+} and I_2 (g). What volume of 0.1 (M) $Na_2S_2O_3$ will be needed to reduce the liberated I_2 (g)?

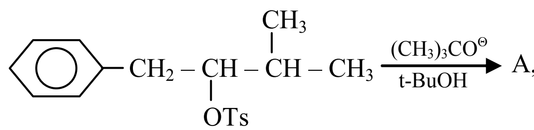
(A) 50 ml (B) 100 ml
(C) 200 ml (D) 400 ml

3. Most stable free radical is

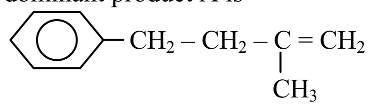
(A) 
(B) 
(C) 
(D) 

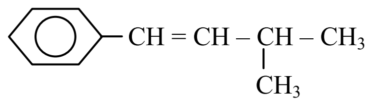
4. A gas is present in a cylinder fitted with movable piston. Above and below of the piston there is equal number of moles of gas. The volume above is two times the volume below at a temperature of 300K. At what temperature will the volume above be four times the volume below-

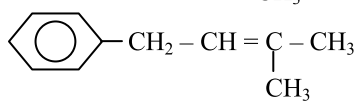
(A) 600 K (B) 400 K
(C) 200 K (D) 120 K

5. 

pre dominant product A is

(A) 

(B) 

(C) 

(D) None of these

6. Given :
$HCO_3^-(aq) + OH^-(aq) \rightarrow H_2O(1) + CO_3^{2-}(aq);$
$\Delta H_1 = -41.84$ kJ mol^{-1}
$H^+(aq) + OH^-(aq) \rightarrow H_2O(1)$; $\Delta H_2 = -57.32$ kJ mol^{-1}
The enthalpy change for the reaction
$HCO_3^-(aq) \rightarrow H^+(aq) + CO_3^{2-}(aq)$ would be
(A) (–41.84 – 57.32) kJ mol^{-1}
(B) (– 41.84 + 57.32) kJ mol^{-1}
(C) (41.84 + 57.32) kJ mol^{-1}
(D) (41.84 – 57.32) kJ mol^{-1}

7. Which of the following reactions is a redox reaction?
(A) $Cr_2O_3 + 6HCl \rightarrow 2CrCl_3 + 3H_2O$
(B) $CrO_3 + 2NaOH \rightarrow Na_2CrO_4 + H_2O$
(C) $2CrO_4^{2-} + H^+ \rightleftharpoons Cr_2O_7^{2-} + OH^-$
(D) $Cr_2O_7^{2-} + 6I^- + 14H^+ \rightleftharpoons 2Cr^{3+} + 3I_2 + 7H_2O$

8. The propagation steps involved in the free radical addition of HX across a double bond are

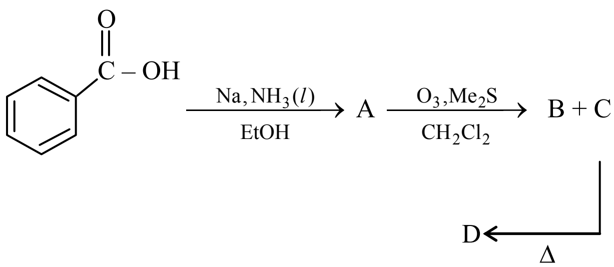
Step 1 $X^\bullet + >C=C< \longrightarrow X-\overset{|}{\underset{|}{C}}-\overset{|}{\underset{|}{C}}^\bullet$

Step 2 $X-\overset{|}{\underset{|}{C}}-\overset{|}{\underset{|}{C}}^\bullet + HX \longrightarrow X-\overset{|}{\underset{|}{C}}-\overset{|}{\underset{|}{C}}-H + X^\bullet$

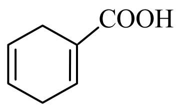
HCl does not follow free-radical addition because
(A) Step 1 is exothermic and step 2 is endothermic
(B) Step 1 is endothermic and step 2 is exothermic
(C) Steps 1 and 2 are exothermic
(D) Steps 1 and 2 are endothermic

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

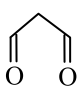
9. Consider the reaction



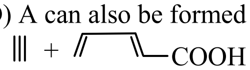
Identify the correct representation of structure of the products -

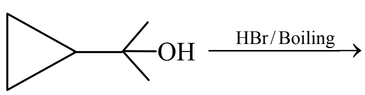
(A) A is 

(B) The intermediate formed in the conversion of B to D is enol

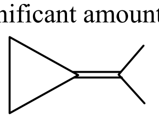
(C) The structure of C is 

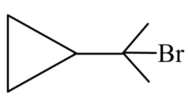
(D) A can also be formed from the reaction

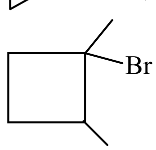


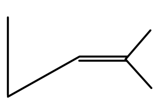
10.  product.

Which of the following are possible products (in significant amount) -

(A) 

(B) 

(C) 

(D) 

11. Which of the following statements is/are correct ?

(A) At 300 ºC, with reaction in the vapour phase, the relative rates of substitution of primary, secondary and tertiary hydrogen atom by chlorine are 1.00 : 3.25 : 4.43

(B) As the temperature increases, the relative rates of substitution of primary, secondary and tertiary hydrogen atom by chlorine become equal, i.e. 1 : 1 : 1

(C) Increasing pressure causes a decrease in the relative rates of primary hydrogen substitution by chlorine

(D) Bromination of alkene is more selective than chlorination

12. Which of the following statements is incorrect

(A) Alkynes are more reactive than alkenes towards halogen additions

(B) Alkynes are less reactive than alkenes towards halogen additions

(C) Both alkynes and alkenes are equally reactive towards halogen additions

(D) Primary vinylic cation ($RCH = \overset{+}{C}H$) is more reactive than secondary vinylic cation ($R\overset{+}{C} = CH_2$)

13. When nitrobenzene is treated with Br_2 in presence of $FeBr_3$, the major product formed is m-bromonitrobenzene. Statements which are related to obtain the m-isomer are :

(A) The electron density on meta carbon is more than that on ortho and para positions

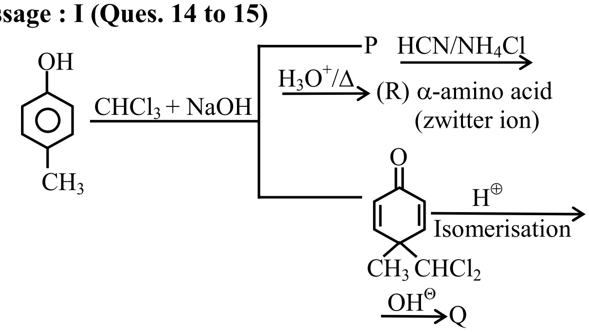
(B) The intermediate carbonium ion formed after initial attack of Br^+ at the meta position is least destabilised

(C) Loss of aromaticity when Br^+ attacks at the ortho and para positions and not at meta position

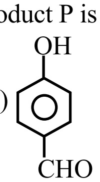
(D) Easier loss of H^+ to regain aromaticity from the meta position than from ortho and para positions

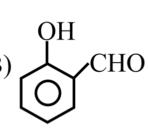
This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage- II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

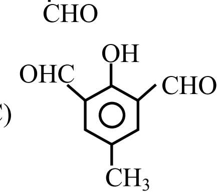
Passage : I (Ques. 14 to 15)

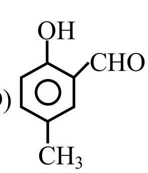


14. Product P is -

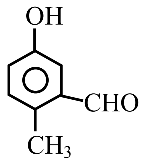
(A) 

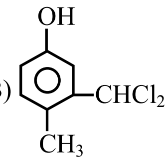
(B) 

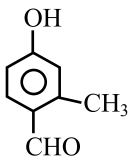
(C) 

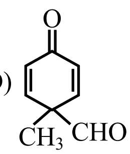
(D) 

15. Product Q is -

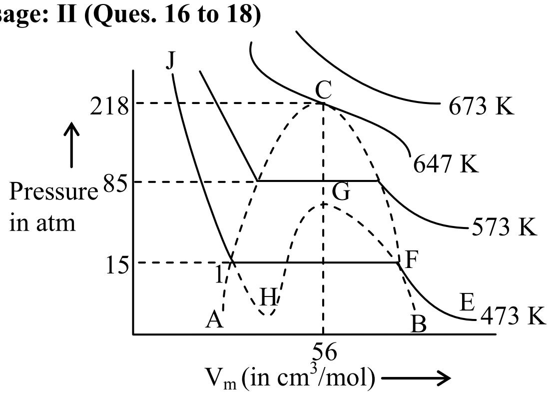
(A) 

(B) 

(C) 

(D) 

Passage: II (Ques. 16 to 18)



In the given figure P-V_m isotherm of H_2O is shown. The line (……) represent, the vanderwaals plot for H_2O at 473 K. The vanderwaals constant of H_2O is represented by a and b.

16 What is the equation of the dotted line (- - -) A I C F B ?

(A) $\left(\dfrac{dP}{dV_m}\right)_T = 0$ and $\left(\dfrac{d^2P}{dV_m{}^2}\right)_T = 0$

(B) $P = \dfrac{a}{V_m{}^2}\left(1+\dfrac{2b}{V_m}\right)$

(C) $P = \dfrac{a}{V_m{}^2}\left(1-\dfrac{2b}{V_m}\right)$

(D) $\left(\dfrac{d^2P}{dV_m^2}\right)_T = 0$

17 As per the vanderwaals line I H G F (- - - -) which of the following section against the behaviour of gas-

(A) I H (B) H G

(C) G F (D) All of the given

18. For H_2O which of the following is / are correct-

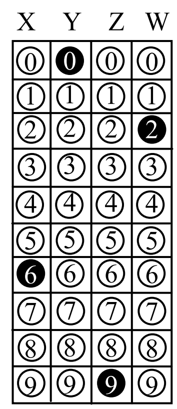
(A)For H_2O, compressibility factor (Z_c) is equal to 0.23.

(B) For H_2O, compressibility factor (Z_c) is lesser than 0.375 because of stronger intermolecular attraction among H_2O molecules.

(C) For H_2O if reduced pressure, reduced volume and reduced temperature are 20, 0.6 and 2 respectively then intermolecular force of repulsion predominate over intermolecular H-bonding among H_2O molecules.

(D) All of the above are correct.

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



19. How many following compounds give positive test with tollen's reagent?

[Glucose, Acetaldehyde, Sucrose, α-Hydroxy Ketone, fructose, Acetone, Maltose, Formic acid, Starch, Acetic acid, Lactose, Cellulose, Glycogen, Benzaldehyde.]

20. 1.245 g of $CuSO_4 . x\ H_2O$ was dissolved in water and H_2S was passed into it till CuS was completely precipitated. The H_2SO_4 produced in the filtrate required 10 mL of 1 N NaOH solution. Calculate x.

21. pH of the half equivalence point of the titration of valine $(CH_3)_2$ CH CH NH_2 COOH, is 2.284 with HCl and 9.716 with NaOH. Calculate the isoelectric point of valine that is, the pH at which the dipole ion does not migrate in an electric field.

22. 80 Volumes of ammonia gas effuse out from a vessel in 8 minutes at –5ºC and 500 mm Hg pressure. How long (in minutes) will 40 volumes of a certain gas (X) with molecular mass 68 take to effuse out from the same vessel at the same temperature & pressure ?

23. Kinetic energy (in atm × litre) of 0.30 moles of He gas in a container of maximum capacity of 4 litres at 5 atm. must be : (R = 0.0821 atm litre mol^{-1} K^{-1})

24. Maltose molecule reacts with 'X' number of phenyl hydrazine molecules to yield monophenylosa zone. The value of 'X' is.

25. For the reaction

$M^{x+} + MnO_4^- \rightarrow MO_3^- + Mn^{2+} + \frac{1}{2} O_2$ if one mole of MnO_4^- oxidises 1.67 moles of M^{x+} to MO_3^-, then the value of x in the reaction is :

26. 2 moles of an ideal gas (C_v = 5/2 R) was compressed adiabatically against constant pressure of 2 atm. Which was initially at 350 K and 1 atm pressure. The work involve (in term R) in the process is equal to : Give your answer after divide actual answer by 100 R.

27. 1.8 g of a carbohydrate (molar mass 180 g mol^{-1}) requires 3.925 g acetyl chloride (molar mass 78.5 g mol^{-1}) in acetylation of hydroxyl group. Calculate number of hydroxyl groups per unit of carbohydrate.

MATHEMATICS

Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. The value of $\sum_{\substack{r=0 \\ r \le s}}^{s} \sum_{s=1}^{n} {}^nC_s \, {}^sC_r$ is

(A) $3^n - 1$ (B) $3^n + 1$
(C) 3^n (D) $3(3^n - 1)$

2. The sum to n terms of the series

$\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots\ldots$ is

(A) $\frac{3n}{n+1}$ (B) $\frac{6n}{n+1}$

(C) $\frac{9n}{n+1}$ (D) $\frac{12n}{n+1}$

3. Let $x^2 \ne n\pi - 1$, $n \in N$ then

$\int x\sqrt{\frac{2\sin(x^2+1) - \sin 2(x^2+1)}{2\sin(x^2+1)+\sin 2(x^2+1)}}$ dx equals

(A) $\ell n \left|\frac{1}{2}\sec(x^2+1)\right| + C$

(B) $\ell n \left|\sec\left(\frac{x^2+1}{2}\right)\right| + C$

(C) $\frac{1}{2}\ \ell n\ |\sec(x^2+1)| + C$

(D) $\frac{1}{2}\ \ell n \left|\frac{2}{\sec(x^2+1)}\right| + C$

4. $\int \frac{(x-1)dx}{(x+1)\sqrt{x(x^2+x+1)}}$ is

(A) $\tan^{-1}\sqrt{x + \frac{1}{x} + 1} + c$

(B) $\sqrt{2}\tan^{-1}\sqrt{x + \frac{1}{x} + 1} + \lambda$

(C) $2\tan^{-1}\sqrt{x + \frac{1}{x} + 1} + c$

(D) $\frac{1}{2}\tan^{-1}\sqrt{x + \frac{1}{x} + 1} + c$

5. $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$

then $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} =$

(A) $\frac{\pi}{60}$ (B) $\frac{\pi}{20}$ (C) $\frac{\pi}{40}$ (D) $\frac{\pi}{80}$

6. The positive value of parameter 'a' for which the area of figure bounded by y = sin ax, y = 0, $x = \frac{\pi}{a}$ and $x = \frac{\pi}{3a}$ is 3 unit, is

(A) 1 (B) $\frac{1}{3}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

7. Solution of $(1 + e^{x/y})\,dx + e^{x/y}(1 - x/y)\,dy = 0$ is

(A) $xe^{x/y} + x = c$ (B) $ye^{x/y} - x = c$
(C) $ye^{x/y} + y = c$ (D) $ye^{x/y} + x = c$

8. If $I_n = \int_0^{\pi/2} x^n \sin x\, dx$ then $I_7 + 42\, I_5 =$

(A) $\left(\frac{\pi}{2}\right)^7$ (B) $\left(\frac{\pi}{2}\right)^6$

(C) $7\left(\frac{\pi}{2}\right)^6$ (D) $7\left(\frac{\pi}{2}\right)^7$

Questions 9 to 13 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which MULTIPLE (ONE OR MORE) is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and no negative marks.

9. Value of $\int_0^{1/2} \frac{dx}{\sqrt{1-x^{2n}}}$; ($n\in N$) is

(A) less than or equal to $\pi/6$
(B) greater than or equal to 1/2
(C) non-negative
(D) greater than 1

10. If $\int x\, \ell n\,(1 + x^2)\, dx = \phi(x)\, \ell n(1 + x^2) + \psi(x) + c$ then

(A) $\phi(x) = \frac{1+x^2}{2}$ (B) $\psi(x) = \frac{1+x^2}{2}$

(C) $\psi(x) = \frac{-1+x^2}{2}$ (D) None of these

11. Solution of equation

$$x \cos x\left(\frac{dy}{dx}\right) + y(x\sin x + \cos x) = 1 \text{ is}$$

(A) $xy = \sin x + c \cos x$
(B) $xy \sec x = \tan x + c$
(C) $xy + \sin x + c \cos x = 0$
(D) $xy = \tan x + c$

12. If $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$ for all x and f(x) is a function for which $\int_0^2 f(x)\,dx = 5$, then $\int_0^{50} f(x)\,dx$ is equal to

(A) 125 (B) $\int_{-4}^{46} f(x)\,dx$

(C) $\int_1^{51} f(x)\,dx$ (D) $\int_2^{52} f(x)\,dx$

13 If A_i is the area bounded by $|x - a_i| + |y| = b_i$, $i \in N$, where $a_{i+1} = a_i + \frac{3}{2} b_i$ and $b_{i+1} = \frac{b_i}{2}$, $a_1 = 0$, $b_1 = 32$, then:

(A) $A_3 = 128$ (B) $A_3 = 256$

(C) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{8}{3}(32)^2$ (D) $\lim_{n\to\infty} \sum_{i=1}^{n} A_i = \frac{4}{3}(16)^2$

This section contains 2 paragraphs; passage- I has 2 multiple choice questions (No. 14 & 15) and passage- II has 3 multiple (No. 16 to 18). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

Passage : I (Ques. 14 to 15)

Consider the differential equation

$$e^x\,(ydx - dy) = e^{-x}\,(ydx + dy).$$

Let $y = f(x)$ be a particular solution to this differential equation which passes through the point (0, 2). Let $C \equiv y = \log_{1/4}\left(x - \frac{1}{4}\right) + \frac{1}{2}\log_4(16x^2 - 8x + 1)$, be another curve

14. The area bounded by the curve C, parabola $x = y^2 + 1/4$ and the line $x = 1/4$ is

(A) 1 (B) 3

(C) $\frac{2}{3}$ (D) $\frac{1}{3}$

15. If the area bounded by the curve $y = f(x)$, curve C, ordinate $x = 1/4$ & the ordinate $x = a$ is $4 - \ell n\ 4 + \frac{1}{e^{1/4}} - e^{1/4}$, then value of a is

(A) ℓn 6 (B) ℓn 4

(C) 4 (D) ℓn 12

Passage: II (Ques. 16 to 18)

By using by parts it is possible to reduce an integral dependent on the integer n (n > 0), to an integral of the same type with smaller value of n.

16. If $I_n = \int \frac{dx}{(x^2+a^2)^n}$ then $I_5 - \frac{7}{8a^2}\, I_4$ is equal to

(A) $\frac{x}{(x^2+a^2)^4}$

(B) $\frac{1}{8a^2(x^2+a^2)^3}$

(C) $\frac{x}{8a^2(x^2+a^2)^4}$

(D) None of these

17. If $I_{n,-m} = \int_0^{\pi/4} \frac{\sin^n x}{\cos^m x} dx$ then $I_{n,-m} + \frac{n-1}{m-1} I_{n-2,\, 2-m}$ is equal to

(A) $\left(\frac{1}{\sqrt{2}}\right)^{n-m}$ (B) $\frac{1}{m-1}\left(\frac{1}{\sqrt{2}}\right)^{n-m}$

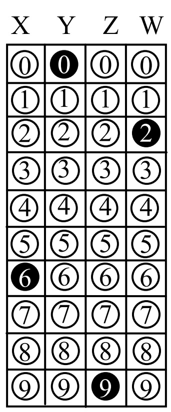
(C) $\frac{1}{n-1}\left(\frac{1}{\sqrt{2}}\right)^{n-m}$ (D) $\frac{n-1}{m-1}\left(\frac{1}{\sqrt{2}}\right)^{n-m}$

18. If $I_{m,n} = \int_0^1 x^m (1-x)^n dx$ then value of $I_{3,4}$ is equal to

(A) $\frac{1}{280}$ (B) $\frac{1}{140}$

(C) $\frac{2}{173}$ (D) $\frac{3}{155}$

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :



19 If $\int xe^x \cos x\, dx = ae^x (b(1-x) \sin x + cx \cos x) + d$, then find the value of $2a + b + c$.

20 Determine a positive integer n such that

$$\int_0^{\pi/2} x^n \sin x\, dx = \frac{3}{4}(\pi^2 - 8)$$

21 Find the area enclosed between the curves

$y = \log_e (x + e)$, $x = \log_e \left(\frac{1}{y}\right)$ and the x-axis.

22 If the solution of differential equation $x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 12y$ is $y = Ax^m + Bx^{-n}$ then find the value of m + n, if $m \,\&\, n \in N$.

23 Let $f(x) = \min.\{x - [x], -x - [-x]\}$.

Then find value of $\int_{-2}^{2} f(x)\, dx$.

(Here [·] stands for G. I. F)

24 $\lim\limits_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \sin^{2k}\left(\frac{r\pi}{2n}\right) = \frac{\lfloor 2k}{(x)^k (k!)^2}$. Find the value of x.

25 y = f(x) passes through (1, 2) and satisfies the relation $y(1 + xy)\, dx - x dy = 0$. Find value of $7f\left(\frac{1}{2}\right)$.

26 $\int \frac{(x+\sqrt{1+x^2})^{15}}{\sqrt{1+x^2}} dx = \frac{\left\{x+\sqrt{1+x^2}\right\}^a}{b} + \lambda$

then find the value of (a + b)/ 6

27 Area bounded between maxima and minima of function $y = x^3 - 3x + 4$ with curve and X-axis is A. Find number of even divisors of 3A.

THE OLDEST LIVING THING ON EARTH

Trees live much longer than any other type of plant or animal. In fact it's possible to know the age of a tree by counting the rings in its trunk (One ring generally equates to one year).

The oldest living tree and hence one of the oldest known living things on Earth is a bristlecone pine tree located in California, North America, it's actually over 4600 years old, now thats old.

Based on New Pattern

IIT-JEE 2012

XtraEdge Test Series # 2

Time : 3 Hours

Syllabus : Physics : Laws of motion, Friction, Work Power Energy, Gravitation, S.H.M., Laws of Conservations of Momentum, Rotational Motion (Rigid Body), Elasticity, Fluid Mechanics, Surface Tension, Viscosity. **Chemistry :** Gaseous state, Chemical Energetics, Oxidation-Reduction, Equivalent Concept, Volumetric Analysis. **Mathematics:** Logarithm & Modulus Function, Quadratic Equation, Progressions, Binomial Theorem, Permutation & Combination, Complex Number

Instructions :

Section - I

- Question 1 to 8 are multiple choice questions with only one correct answer. ***+3 marks*** will be awarded for correct answer and
  -1 mark for wrong answer.

Section - II

- Question 9 to 13 are multiple choice questions with multiple correct answer. ***+3 marks*** will be awarded for correct answer and ***0 mark*** for wrong answer.

Section - III

- Question 14 to 18 are passage based single correct type questions. ***+3 marks*** will be awarded for correct answer and ***-1 mark*** for wrong answer.

Section - IV

- Question 19 to 27 are numerical response questions (with single digit Answer). ***+3 marks*** will be awarded for correct answer and ***0 mark*** for wrong answer.

PHYSICS

Questions 1 to 8 are multiple choice questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Mark your response in OMR sheet against the question number of that question. + 3 marks will be given for each correct answer and – 1 mark for each wrong answer.

1. A body at rest on a rough horizontal surface starts with an acceleration when a force 'F' is applied. When the force is doubled, the acceleration increases to 4 times. The force necessary just to start the body will be –

(A) F (B) 2F

(C) $\frac{F}{2}$ (D) $\frac{2F}{3}$

2. A block of mass M rests on a rough horizontal surface. The coefficient of friction between the block and the surface is μ. A force F = mg acting at an angle ϕ = 60º with the vertical side of block pulls it. In which of the following case, the block can be pulled along the surface.

(A) $\mu \geq \frac{\sqrt{3}}{2}$ (B) $\mu \leq \frac{\sqrt{3}}{2}$

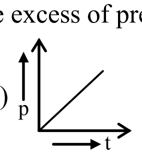
(C) $\mu \geq \sqrt{3}$ (D) $\mu \leq \sqrt{3}$

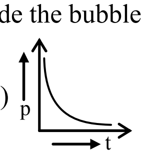
3. Power supplied to a particle of mass 2 kg varies with time at $P = \frac{3t^2}{2}$ watt. Here t is in seconds. If velocity of particle at t = 0 is v = 0, the velocity of particle at time t = 2 sec will be -

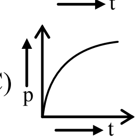
(A) 1 m/s (B) 4 m/s

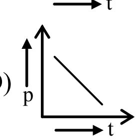
(C) 3 m/s (D) $2\sqrt{2}$ m/s

4. A soap bubble is blown slowly at the end of a tube by a pump supplying air at a constant rate. Which one of the following graphs represent the correct variation of the excess of pressure inside the bubble with time

(A) 

(B) 

(C) 

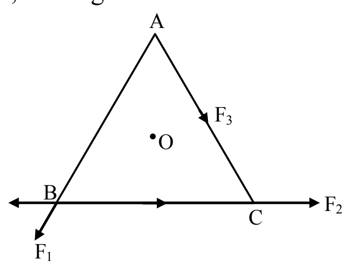
(D) 

5. A bob of mass M is hung using a string of length ℓ. A mss m moving with a velocity u pierces through the bob and emerges out with a velocity $\frac{u}{3}$. The frequency of oscillation of the bob considering an amplitude A is -

(A) $2\pi\sqrt{\frac{2mu}{MA}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2m}{3MA}}$

(C) $\frac{1}{2\pi}\frac{2}{3}\frac{mu}{MA}$ (D) cannot be found

6. What should be the value of F_3. So that torque about O is zero, in the given situation :



(A) 2 ($F_1 + F_2$) (B) $\frac{F_1 + F_2}{2}$

(C) $F_1 - F_2$ (D) $F_1 + F_2$

7. A bar of mass m resting on a smooth horizontal plane starts moving due to a force F = $\frac{mg}{3}$ of constant magnitude. In the process of its rectilinear motion, the angle θ between the direction of this force and the horizontal varies as θ = ks, where k is a constant. The velocity of bar as a function of θ is -

(A) $\upsilon = \sqrt{\frac{2g}{3}\sin\theta}$ (B) $\upsilon = \sqrt{\frac{2g}{3k}\cos\theta}$

(C) $\upsilon = \sqrt{\frac{2g}{3k}\sin\theta}$ (D) $\upsilon = \sqrt{\frac{2g}{3}\cos\theta}$

8. A planet of mass m is revolving round the sun (of mass M_s) in an elliptical orbit. If $\vec{v}$ is the velocity of the planet when its position vector from the sun is $\vec{r}$, then areal velocity of the planet is -

(A) $\vec{v}\times\vec{r}$ (B) $\vec{r}\times\vec{v}$

(C) $\frac{1}{2}\left(\vec{v}\times\vec{r}\right)$ (D) $\frac{1}{2}\left(\vec{r}\times\vec{v}\right)$

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9. A planet is revolving round the sun. Its distance from the sun at Apogee is r_A and that at perigee is r_p. The mass of planet and sun is m and M respectively v_A and v_p is the velocity of planet at Apogee and Perigee respectively and T is the time period of revolution of planet round the sun -

(A) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_P)^3$

(B) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_P)^3$

(C) $v_Ar_A = v_Pr_P$

(D) $v_A < v_P$; $r_A > r_P$

10. A student holds the axle of a spinning bicycle wheel while seated on a pivoted stool. The student and the stool are initially at rest while the wheel is spinning in a horizontal plane with an angular momentum $\vec{L}_0$ pointing upward. The wheel is inverted about its centre by 180º -

(A) The angular momentum of the system is conserved

(B) The angular momentum of the system is not conserved

(C) The final angular momentum of "student plus stool" will be $2\vec{L}_0$

(D) The final angular momentum of "student plus stool" will be zero

11. The potential energy U for a force field $\vec{F}$ is such that U = – kxy, where k is a constant -

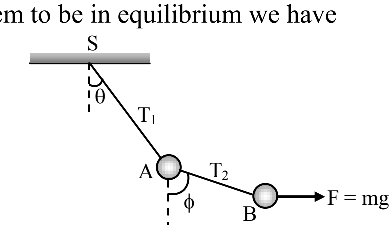
(A) $\vec{F} = ky\hat{i} + kx\hat{j}$

(B) $\vec{F} = kx\hat{i} + ky\hat{j}$

(C) The force $\vec{F}$ is a conservative force

(D) The force $\vec{F}$ is a non-conservative force

12. The spheres A and B as shown have mass M each. The strings SA and AB are light and inextensible with tensions T_1 and T_2 respectively. A constant horizontal force F = Mg is acting on B. For the system to be in equilibrium we have



(A) tan ϕ = 1 (B) tan θ = 0.5

(C) $T_2 = \sqrt{2}$ mg (D) $T_1 = \sqrt{5}$ mg

13. Which of the following statement(s) is (are) correct about friction ?

(A) The coefficient of friction between two bodies is largely independent of area of contact

(B) The frictional force can never exceed the reaction force on a body from the supporting surface

(C) Rolling friction is generally smaller than sliding friction

(D) Friction is due to irregularities of the surfaces in contact.

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Passage : I (Ques. 14 to 15)

Two pendulum bobs of mass m and 2m collide elastically at the lowest point in their motion. If both the balls are released from height H above the lowest point .

14. Velocity of the bob of mass m is -

(A) $\frac{\sqrt{2gH}}{3}$ (B) $\frac{5}{3}\sqrt{2gH}$

(C) $\sqrt{2gH}$ (D) none of these

15. The height of the bob of mass rise after the collision-

(A) $\frac{25}{9}H$ (B) $\frac{H}{9}$

(C) $\frac{16H}{9}$ (D) none of these

Passage: II (No. 16 to 18)

If a hole is made at a height x from the ground for a cylinder holding water to height 'h', the velocity of efflux is given by toricellian theorem as $v = \sqrt{2g(h-x)}$. For streamlined liquid flow the time depends on the height at which the hole is made. Range-the horizontal distance at which the water touches the ground is given by R = vt where $t = \sqrt{\frac{2x}{g}}$. Rate of flow will obviously change as the position of the hole is altered and also when the liquid level drops.

16. The ration of the time taken to reduce the height to $\frac{3H}{4}$ and then $\frac{H}{2}$ from H is (liquid flowing through a hole at its base) :

(A) $\sqrt{H_1H_2}:1$ (B) $\sqrt{H_1}:\sqrt{H_2}$

(C) $\sqrt{\frac{3H}{4}}:\sqrt{\frac{H}{2}}$ (D) $(2-\sqrt{3}):(\sqrt{3}-\sqrt{2})$

17. For two holes made at distance h from the top and bottom, the range are R_1 and R_2. Then :

(A) $R_1 = R_2$ (B) $R_1 = \sqrt{(H-h)}\ R_2$

(C) $R_1 = 4R_2$ (D) $hR_1 = (H-h)R_2$

18. The principle used to arrive at the velocity of low can be derived from :

(A) Pascal's law (B) Bernoulli's theorem

(C) Stoke's law (D) All of these

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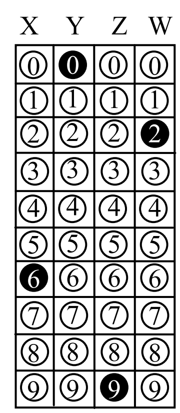
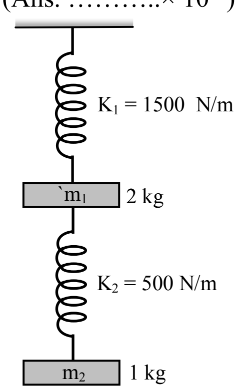


19. Find the potential energy stored in the springs in equilibrium. (Ans. ………..× 10^{-1})

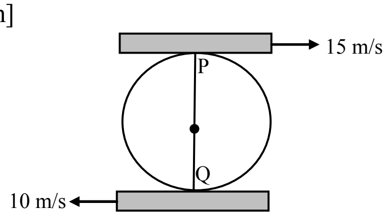


20. A wire of length 1 m and area of cross section 2×10^{-6} m^2 is suspended from the top of a roof at one end and a load of 20 N is applied at the other end. If the length of the wire is increased by 0.5×10^{-4} m. Calculate its Young's modulus ? [in 10^{11} N/m^2]

21. A body of mass 2 kg is attached to a spring. Now a mass of 300 g is also attached to it and its length is increased by 2 cm. What will be the time period of 2 kg mass, if the second body is now removed ? (Take g = 10 ms^{-2})

22. The gravitational potential in a region is given by V = 20 (x + y) J/kg. Find the magnitude of gravitational force on the particle of mass 0.5 kg placed at the origin.

23. A cylinder of radius 10 cm rides between two horizontal bars moving in opposite direction as shown in the figure. Where is the instantaneous axis of rotation of roller ? [Neglect slipping at P and Q] [in cm]



24. An object of mass 1 kg moves under the action of a force F in a straight line with its velocity V, changing with displacement x as $v = 10\sqrt{x}$.
Find the work done in a displacement from x = 0 to x = 2. [in Joule] (Ans. ………..× 10^2)

25. An object falling from height of 80 m explodes into two parts of mass ratio 1 : 2 after 2 second of free fall. Velocity of the bigger part in the frame of reference of C.O.M. is 20 m/s. What is velocity of each part w.r.t. ground ? (Ans. ………..× 10^{-1})

26. A block of mass 2 kg is pulled by a constant power 100 W is placed on a rough horizontal plane. The frictional coefficient between block and surface is $\frac{1}{4}$.
Find the maximum velocity of block.
(Ans. …….× 10^1)

27. A solid sphere (a) rolls (b) slides, from rest down an inclined plane. Find the ratio of their accelerations.

CHEMISTRY

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1. Adiabatic reversible expansion of a monoatomic gas (M) and a diatomic gas (D) at an initial temperatrue T_i has been carried out independently from initial volume V_1 to final volume V_2. The final temperature (T_M for monoatomic gas and T_D for diatomic gas) attained will be :
(A) $T_M = T_D > T_i$ (B) $T_M < T_D < T_i$
(C) $T_M > T_D > T_i$ (D) $T_M = T_D = T_i$

2. For a given mixture of $NaHCO_3$ and Na_2CO_3, volume of a given HCl required is x mL with phenolphthalein indicator and further y mL required with methyl orange indicator. Hence, volume of HCl for complete reaction of $NaHCO_3$ is :
(A) 2x (B) x/2
(C) y (D) (y – x)

3. Helium atom is two times heavier than a hydrogen molecule. At 298 K, the average kinetic energy of a helium atom is
(A) two times that of a hydrogen molecule
(B) same as that of a hydrogen molecule
(C) four times that of a hydrogen molecule
(D) half that of hydrogen molecule

4. In van der Waals equation of state for a nonideal gas the term that accounts for intermolecular forces is
(A) (V – b) (B) $\left(p + \frac{a}{V^2}\right)$
(C) RT (D) $(RT)^{-1}$

5. White phosphorus reacts with caustic soda. The products are PH_3 and NaH_2PO_2. This reaction is an example of
(A) oxidation
(B) reduction
(C) oxidation and reduction
(D) neutralization

6. Which of the following reactions is a redox reaction?
(A) $Cr_2O_3 + 6HCl \rightarrow 2CrCl_3 + 3H_2O$
(B) $CrO_3 + 2NaOH \rightarrow Na_2CrO_4 + H_2O$
(C) $2CrO_4^{2-} + H^+ \rightleftharpoons Cr_2O_7^{2-} + OH^-$
(D) $Cr_2O_7^{2-} + 6I^- + 14H^+ \rightleftharpoons 2Cr^{3+} + 3I_2 + 7H_2O$

7. The reaction of cyanamide, $NH_2CN(s)$, with oxygen was run in a bomb calorimeter and ΔU at 300 K was found to be –743 kJ mol^{-1}. The value of ΔH at 300K for the combustion reaction
$NH_2CN(s) + (3/2)\, O_2(g) \rightarrow N_2(g) + CO_2(g) + H_2O\ (1)$
would be
(A) – 741.75 kJ mol^{-1} (B) –743 kJ mol^{-1}
(C) – 744.25 kJ mol^{-1} (D) – 740.5 kJ mol^{-1}

8. The combustion reaction occurring in an automobile is $2C_8H_{18}(s) + 5O_2(g) \rightarrow 16CO_2(g) + 18H_2O(1)$. This reaction is accompanied with

(A) $\Delta H = -ve, \Delta S = +ve, \Delta G = +ve$

(B) $\Delta H = +ve, \Delta S = -ve, \Delta G = +ve$

(C) $\Delta H = -ve, \Delta S = +ve, \Delta G = -ve$

(D) $\Delta H = +ve, \Delta S = +ve, \Delta G = -ve$

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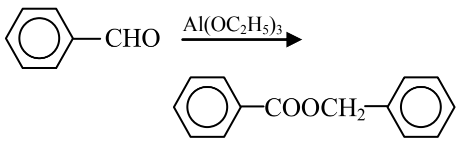
9. Which of the following are disproportionation reactions ?

(A) 2 

(B) $4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$

(C) $NH_4NO_3 \xrightarrow{\Delta} N_2O + 2H_2O$

(D) $PCl_5 \xrightarrow{\Delta} PCl_3 + Cl_2$

10. 10 volume H_2O_2 solution is present, then it means

(A) 10 ml of H_2O_2 solution liberates 1 ml of oxygen at STP

(B) 1 ml of H_2O_2 solution liberates 10 ml of oxygen at STP

(C) 0.0303 g of H_2O_2 in 10 ml of solution liberates 10 ml O_2 at STP

(D) 0.0303 g of H_2O_2 in 1 ml of the solution liberates 10 ml O_2 at STP

11. Silver metal in ore is dissolved by potassium cyanide solution in the presence of air by the reaction

$4Ag + 8KCN + O_2 + 2H_2O \rightarrow 4KAg(CN)_2 + 4KOH$

(A) The amount of KCN required to dissolve 100 g of pure Ag is 120 g.

(B) The amount of oxygen used in this process is 0.742 g

(C) The amount of oxygen used in this process is 7.40g

(D) The volume of oxygen used at STP is 5.20 litres.

12. Identify the intensive properties among the following:

(A) Enthalpy (B) Temperature

(C) Volume (D) Refractive index

13. During the Joule Thomson effect

(A) A gas is allowed to expand adiabatically from a high pressure region to a low pressure region

(B) A gas is allowed to expand adiabatically at constant pressure

(C) $\Delta H = 0$ for ideal gas

(D) $\Delta U = 0$ for ideal gas

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Passage : I (Ques. 14 to 15)

The kinetic-molecular theory accounts for the properties of gases by assuming that gas particles act independently of one another. Because the attractive forces between them are so weak, the particles in gases are free to move about at random and occupy whatever space is available. The same is not true in liquids and solids, however liquids and solids are distinguished from gases by the presence of strong attractive forces between particles. In liquids, these attractive forces are strong enough to hold the particles in close contact while still letting them slip and slide over one another. In solids, the forces are strong that they hold the particles rigidly in place and prevent their movement.

14. Which pair of molecules have the strongest dipole-dipole interactions ?

(A) NH_3 and CH_4 (B) NH_3 and NH_3

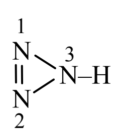
(C) CH_4 and CH_4 (D) CO_2 and CO_2

15. For one mole of an ideal gas $\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial V}{\partial P}\right)_T =$

(A) –1 (B) $-\frac{R^2}{P^2}$ (C) $\frac{R^2}{P^2}$ (D) +1

Passage: II (Ques. 16 to 18)

Redox reactions are those in which oxidation and reduction take place simultaneously. Oxidising agent can gain electron whereas reducing agent can lose electron easily. The oxidation state of any element can never be in fraction. If oxidation number of any element comes out be in fraction, it is average oxidation number of that element which is present in different oxidation states.

16. 

In this compound HN_3 (hydrazoic acid), oxidation state of N^1 N^2 and N^3 are

(A) 0, 0, 3 (B) 0, 0, –1

(C) 1, 1, –3 (D) –3, –3, –3

17. Equivalant weight of chlorine molecule in the equation

$3Cl_2 + 6NaOH \rightarrow 5NaCl + NaClO_3 + 3H_2O$

(A) 42.6 (B) 35.5 (C) 59.1 (D) 71

18. The oxidation number of sulphur in $K_2S_2O_8$ is
(A) + 2 (B) + 4
(C) + 7 (D) +

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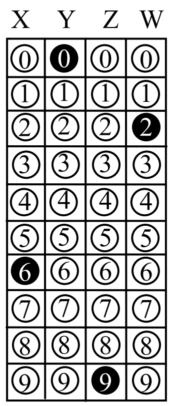


19. A weather balloon is inflated with helium. The balloon has a volume of 100 m^3 and it must be inflated to a pressure of 0.10 atm. If 50 L gas cylinders of helium at a pressure of 100 atm are used, how many cylinders are needed ?
Assume that the temperature is constant.

20. The compressibility factor for nitrogen at 330 K and 800 atm is 1.90 and at 570 K and 200 atm is 1.10. A certain mass of N_2 occupies a volume of $1 dm^3$ at 300 K and 800 atm. Calculate volume occupied (in litre) by same quantity of N_2 gas at 750 K and 200 atm.

21. The ion A^{n+} oxidised to AO_3^- by MnO_4^-, changing to Mn^{2+} in acidic solution. Given that 2.68×10^{-3} mol of A^{n+} requires 1.61×10^{-3} mole of MnO_4^-. What is the value of n ?

22. 100 mL of 1.44% solution of pure FeC_2O_4 in dil. HCl is oxidised by 0.01 M $KMnO_4$. Then volume (in mL) of $KMnO_4$ required is :
Given your answer after divide actual answer by 100.

23. For an isomerisation reaction A $\rightleftharpoons$ B, the temperature dependence of equilibrium constant is given by

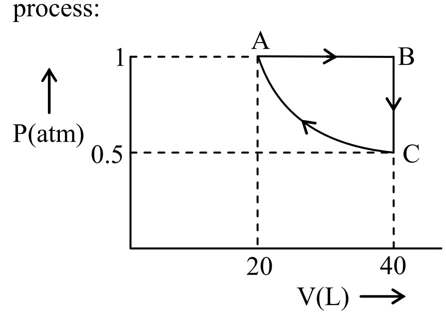
$$\log_e K = 4.0 - \frac{2000}{T}$$

The value of ΔS^o at 300 K
(in term ……….× R) is :

24. A → B, ΔH = +ve. Graph between $\log_{10}P$ and $\frac{1}{T}$ is a straight line of slope $\frac{1}{4.606}$. Hence, ΔH (in cal) is :

25. 9.8 g of $FeSO_4(NH_4)_2SO_4.xH_2O$ was dissolved in 250 mL of its solution. 20 mL of this solution required 20 ml of $KMnO_4$ solution containing 3.53 g of 90% pure $KMnO_4$ dissolved per litre. Calculate x.

26. Two moles of a perfect gas undergo the following process:



What is the value of ΔS for the overall process ?

27. Heat of formation of ethylene from the following data at 20ºC is given by 2^n kcal. What is the value of n ?
$H_2(g) + 1/2O_2(g) \rightarrow H_2O(\ell)$; ΔH = – 65 kcal
$C(s) + O_2(g) \rightarrow CO_2(g)$; ΔH = – 97 kcal
$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(\ell)$;
ΔH = – 340 kcal

MATHEMATICS

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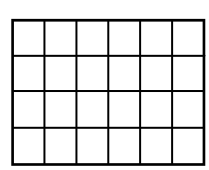
1. Let a,b,c be the three sides of a triangle, then the quadratic equation $b^2x^2 + (b^2+c^2-a^2)x + c^2 = 0$ has
(A) both roots positive
(B) both roots negative
(C) both roots imaginary
(D) None of these

2. Coefficient of abc^3de^2 in expansion of $(a+b+c+d+e)^8$ is
(A) 3630 (B) 3600
(C) 3360 (D) None

3 Number of rectangles in fig. shown which are not squares is



(A) 159 (B) 160 (C) 161 (D) None

4 Let $f(x) = x^3 - 3x^2 + 2x$. If the equation $f(x) = k$ has exactly one positive & one negative solution then k equals

(A) $\frac{-2}{9}$ (B) $\frac{2\sqrt{3}}{9}$

(C) $\frac{-2\sqrt{3}}{9}$ (D) $\frac{2}{9}$

5 The number of solutions of equation
$\log_6(x + 3) = 7 - x$ is

(A) 0 (B) 2

(C) 1 (D) None of these

6 If $a_n = \sum_{k=0}^{n} \frac{(\log_e 10)^n}{k!(n-k)!}$ for $n \ge 0$, then

$a_0 + a_1 + a_2 + a_3$ …………infinite equals to

(A) 10 (B) 100

(C) 1000 (D) None of these

7 If $a_1, a_2, a_3 \ldots\ldots a_n$ are in H.P. &

$f(k)=\left(\sum_{r=1}^{n} a_r\right) - a_k$ then

$\frac{a_1}{f(1)}, \frac{a_2}{f(2)}, \frac{a_3}{f(3)}, \ldots\ldots\ldots, \frac{a_n}{f(n)}$ are in

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

8 If 'z' is complex number then the locus of 'z' satisfying the condition

$|2z - 1| = |z - 1|$ is

(A) perpendicular bisector of line segment joining $\frac{1}{2}$ and 1

(B) circle

(C) parabola

(D) none of the above curves

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9. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then

(A) $3 \le |z_1 - 2z_2| \le 5$ (B) $1 \le |z_1 + z_2| \le 3$

(C) $|z_1 - 3z_2| \ge 5$ (D) $|z_1 - z_2| \ge 1$

10 If $\sec^2\theta$ & $\text{cosec}^2\theta$ are roots of $ax^2 + bx + c = 0$ $(a > 0)$ then

(A) $b + c = 0$ (B) $b^2 - 4ac \ge 0$

(C) $c \ge 4a$ (D) $b + 4a \ge 0$

11 If $y = \log_{7-a}(2x^2 + 2x + a + 3)$ is defined $\forall x \in R$ then possible integer value of a is/are

(A) 4 (B) – 3 (C) – 2 (D) 5

12 If $12! = 2^a 3^b 5^c 7^d 11^e$ then

(A) $a = 5c$ (B) $a = 10d$

(C) $b = 5e$ (D) None of these

13 If $\sum_{r=0}^{n} \frac{r}{^nC_r} = \sum_{r=0}^{n} \frac{n^2 - 3n + 3}{2.\,^nC_r}$, then

(A) $n = 1$ (B) $n = 2$

(C) $n = 3$ (D) None of these

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Passage : I (Ques. 14 to 15)

Given three equations in the complex plane :

$iz + \bar{z} + 1 + i = 0$ …...(1)

$(2 - i)z = (2 + i)\bar{z}$ ......(2)

$(2 + i)z + (i - 2)\bar{z} - 4i = 0$ ......(3)

14. If the intersection point of the lines (2) and (3) is denoted by z_1, then z_1 equals

(A) $\left(\frac{1}{2} + i\right)$ (B) $(1 + i)$

(C) $\left(1 + \frac{i}{2}\right)$ (D) none of these

15 The distance between the point z_1 and the line (1) is

(A) $\frac{3}{\sqrt{2}}$ (B) $3\sqrt{2}$

(C) $\frac{3}{2\sqrt{2}}$ (D) none of these

Passage: II (Ques. 16 to 18)

The integer a,b,c are selected from 3n consecutive integer (1,2,3,......3n), then in how many ways can these integers be selected such that

16 Their sum is divisible by 3 is

(A) $\frac{n}{2}(3n^2 - 3n + 2)$ (B) $3n^2 - 3n + 2$

(C) $\frac{n}{2}$ (D) None

17 $(a^2 - b^2)$ is divisible by 3 is

(A) $n^2 + \frac{n(n-1)}{2}$ (B) $n^2 + \frac{n(n+1)}{2}$

(C) $n^2 + \frac{3n(n-1)}{2}$ (D) None

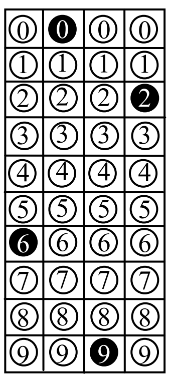
18 $(a^3 + b^3)$ is divisible by 3 is

(A) $\frac{3n^2 - n}{2}$ (B) $\frac{3n^2 + n}{2}$

(C) $\frac{n(n+1)}{2}$ (D) None

This section contains 9 questions (Q.19 to 27). +3 marks will be given for each correct answer and no negative marking. The answer to each of the questions is a SINGLE-DIGIT INTEGER, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the OMR have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following :

X Y Z W



19 The value of expression $x^4 - 8x^3 + 18x^2 - 8x + 2$ when $x = 2 + \sqrt{3}$ is ………

20 Suppose x, y, z > 0 & different from one and $\ell n\, x + \ell n\, y + \ell n\, z = 0$, then value of $x^{\frac{1}{\ell ny}+\frac{1}{\ell nz}} \cdot y^{\frac{1}{\ell nz}+\frac{1}{\ell nx}} \cdot z^{\frac{1}{\ell nx}+\frac{1}{\ell ny}}$ is e^{-k} then k equals ………

21 There are 2n white & 2n red counters. Counters are all alike except for the colour. If the number of ways in which they can be arranged in a line so that they are symmetric w.r.t. a central mark is 70 then n equals ……..

22 If the fraction $\frac{x^3 + (a-10)x^2 - x + a - 6}{x^3 + (a-6)x^2 - x + a - 10}$ reduces to a quotient of two linear functions then a equals………

23 If $\log\left(\frac{x^2}{y^3}\right) = 1$ & $\log(x^2y^3) = 7$ then log |xy| is equal to……..

24 If sum of all solutions of the equation $(x^{\log_{10} 3})^2 - (3^{\log_{10} x}) - 2 = 0$ is $a^{\log_b c}$ where $a,b,c \in N$ & a, b are prime numbers then a + b equals

25 Number of irrational terms in expansion of $(\sqrt{3} + \sqrt{7})^{17}$ is equal to $3k + k^2$ then k equals

26 Number of zeroes at the end in product of $5^6 \times 6^6 \times 7^6 \ldots\ldots 31^6$ is $8k^2 + 2k + 1$ then k equals

27 The value of ''a'' for which all roots of quadratic equation, $f(x) = (a-2)x^2 + 2ax + a + 3 = 0$ lies in (– 2, 1) blongs to $\left(-\infty, -\frac{1}{4}\right) \cup (m, n]$ then value of n – m is

Abundances of the Elements in the Earth's Crust

XTRA FUNDA

Element	Approximate % by weight
Oxygen	46.6
Silicon	27.7
Aluminum	8.1
Iron	5.0
Calcium	3.6
Sodium	2.8
Potassium	2.6
Magnesium	2.1
All others	1.5

Given the abundance of oxygen and silicon in the crust, it should not be surprising that the most abundant minerals in the earth's crust are the silicates. Although the Earth's material must have had the same composition as the Sun originally, the present composition of the Sun is quite different. The elemental composition of the human body and life in general is quite different.

These general element abundances are reflected in the composition of igneous rocks

AIEEE PAPER 2010
(PAPER & SOLUTION)

Time : 3 Hours **Total Marks : 432**

Instructions :

- **Part A – Physics (144 Marks) –**Questions No. 1 to 20 and 23 to 26 consist of **FOUR (4)** marks each and Questions No. 21 to 22 and 27 to 30 consist of **EIGHT (8)** marks each for each correct response.
  Part B – Chemistry (144 Marks) – Questions No. 31 to 39 and 43 to 57 consist of **FOUR (4)** marks each and Questions No.40 to 42 and 58 to 60 consist of **EIGHT (8)** marks each for each correct response.
  Part C – Mathematics (144 Marks) – Questions No.61 to 66, 70 to 83 and 87 to 90 consist of **FOUR (4)** marks each and Questions No. 67 to 69 and 84 to 86 consist of **EIGHT (8)** marks each for each correct response
- Candidates will be awarded marks as stated above in instructions No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

PHYSICS

Directions: Questions number 1 – 3 are based on the following paragraph.
An initially parallel cylindrical beam travels in a medium of refractive index μ (I) = $\mu_0 + \mu_2$I, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is -
(1) planar
(2) convex
(3) concave
(4) convex near the axis and concave near the periphery

Ans.[3]

2. The speed of light in the medium is –
(1) maximum on the axis of the beam
(2) minimum on the axis of the beam
(3) the same everywhere in the beam
(4) directly proportional to the intensity I

Ans. [2]

3. As the beam enters the medium, it will -
(1) travel as a cylindrical beam
(2) diverge
(3) converge
(4) diverge near the axis and converge near the periphery

Ans. [3]

Directions: Questions number 4 – 5 are based on the following paragraph.
A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

4 The speed of daughter nuclei is –
(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$
(3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$

Sol. $\Delta mc^2 = 2\times\frac{1}{2}\times\left(\frac{M}{2}\right)v^2$

$v^2 = \frac{2\Delta mc^2}{M}$, $v = c\sqrt{\frac{2\Delta m}{M}}$

Ans. (3)

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then –
(1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$
(3) $E_1 > E_2$ (4) $E_2 > E_1$

Ans. [4]

Directions: Questions number 6 – 7 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement- 1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase –
Statement – 2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because the range of frequencies present in the incident light.
(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

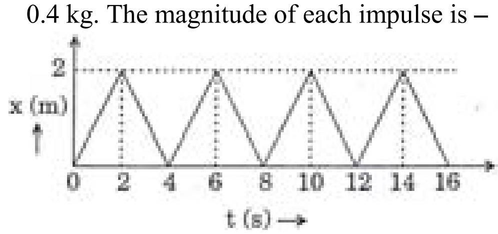
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true.

Ans. [1]

7. **Statement- 1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement- 2: Principle of conservation of momentum holds true for all kinds of collisions.
(1) Statement-1 is true, Statement-2 is false.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** the correct explanation of Statement-1.
(4) Statement-1 is false, Statement-2 is true.

Sol. Both are true but not explain the Ist.
Ans. (3)

8. The figure shows the position – time (x – t) graph of one-dimensional motion of the body of mass 0.4 kg. The magnitude of each impulse is –



(1) 0.2 Ns (2) 0.4 Ns
(3) 0.8 Ns (4) 1.6 Ns

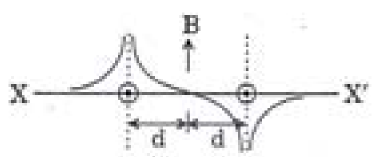
Sol. From graph,
$v_1 = 1\ ms^{-1}$, $v_2 = -1\ ms^{-1}$

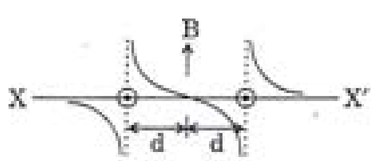
$\therefore\ J = \int F\,dt = \int dP = m\,\Delta V$

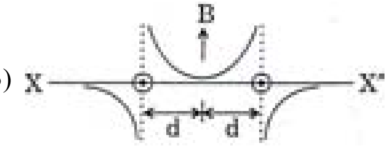
$= 0.4 \times 2 = 0.8$ N.s.

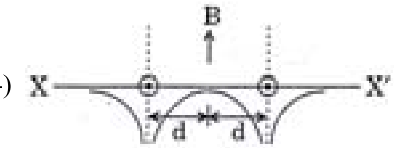
Ans. (3)

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by -

(1) 

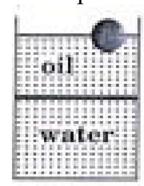
(2) 

(3) 

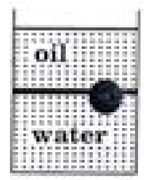
(4) 

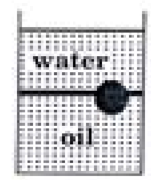
Ans. [2]

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(1) 

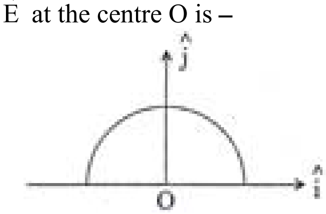
(2) 

(3) 

(4) 

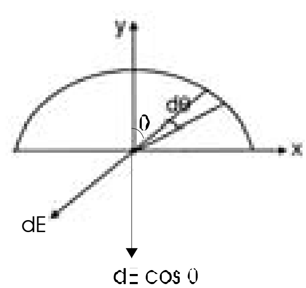
Sol. $\because\ \rho_{oil} < \rho < \rho_{water}$,
so ball will not sink in water but sink in oil.
Ans. (3)

11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is –



(1) $\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(3) $-\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Sol. 

$$E = \int_{-\pi/2}^{\pi/2} dE\cos\theta = 2\int_{0}^{\pi/2} \frac{k\lambda R d\theta}{R^2}\cos\theta$$

$$\vec{E} = \frac{2}{4\pi\varepsilon_0}\frac{qR}{\pi R\, R^2}[\sin\theta]_0^{\pi/2} = \frac{q}{2\pi^2\varepsilon_0 R^2}[Sin90 - \sin 0](-\hat{j})$$

$$\vec{E} = \frac{q}{2\pi^2\varepsilon_0 R^2}(-\hat{j})$$

Ans. (4)

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is-

(1) 0.25 (2) 0.5
(3) 0.75 (4) 0.99

Sol. $\eta = \left(1 - \frac{T_2}{T_1}\right)$

$\because\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\therefore\ \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{32}\right)^{\gamma-1}$

Putting $\gamma = 7/5$

$\eta = \left(1 - \frac{1}{4}\right) = \frac{3}{4} = 0.75$

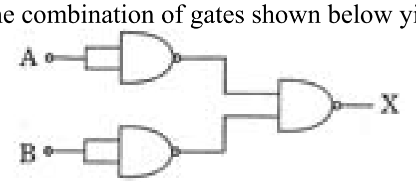
Ans. (3)

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are **-**

(1) 4, 4, 2 (2) 5, 1, 2
(3) 5, 1, 5 (4) 5, 5, 2

Sol. 23.023 significant fig. 5
0.0003 significant fig. 1
2.1×10^{-3} significant fig. 2
Ans. (2)

14. The combination of gates shown below yields **–**



(1) NAND gate (2) OR gate
(3) NOT gate (4) XOR gate

Sol. $X = \overline{\overline{A}\cdot\overline{B}} = A + B$
i.e. OR gate
Ans. (2)

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called –

(1) γ-rays (2) X-rays
(3) ultraviolet rays (4) microwaves

Sol. $P = n\frac{hc}{\lambda}$

Ans. (2)

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be –

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$
(3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

Sol. ${}_{Z}^{A}X \xrightarrow{(3\alpha+2\,\text{positron})} {}_{Z-3\times2-2\times1}^{A-3\times4}X = {}_{Z-8}^{A-12}X$

$$\therefore\ \frac{\text{No. of Neutrons}}{\text{No. of Protons}} = \frac{(A-12)-(Z-8)}{Z-8}$$

$$= \frac{A-Z-4}{Z-8}$$

Ans. (3)

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto r = R, and ρ(r) = 0 for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by –

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Sol. r < R

$$\oint E\cdot dS = \frac{\int \rho_v dv}{\varepsilon_0}$$

$$E\cdot 4\pi r^2 = \int_0^r \frac{\rho_0}{\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)4\pi r^2 dr$$

$$E\cdot 4\pi r^2 = \frac{\rho_0 4\pi}{\varepsilon_0}\left[\int_0^r \frac{5}{4}r^2 dr - \int_0^r \frac{r^3}{R}dr\right]$$

$$E\cdot 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{5}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$E = \frac{\rho_0}{\varepsilon_0}\left[\frac{5}{4}\frac{r}{3} - \frac{r^2}{4R}\right]$$

$$\boxed{E = \frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)}$$

Ans. (3)

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is –

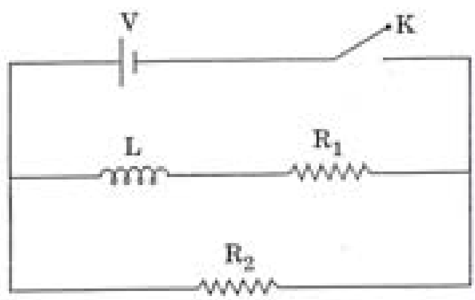
(1) 242 W (2) 305 W
(3) 210 W (4) Zero W

Sol. $X_L = X_C$

$$P = \frac{V^2}{R} = 242 \text{ W}$$

Ans. (1)

19. In the circuit shown below, the key K is closed at t = 0. the current through the battery is –



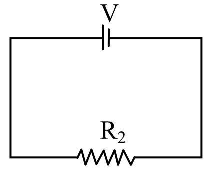
(1) $\frac{V(R_1 + R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(2) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1 + R_2)}{R_1R_2}$ at t = ∞

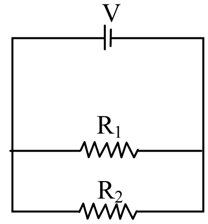
(4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at t = ∞

Sol. at t = 0



$$I = \frac{V}{R_2}$$

at t = ∞



$$I = \frac{V(R_1 +}{R_1R}$$

Ans. (3)

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is –

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant
(3) $y^2 = x$ + constant (4) xy = constant

Sol. $\vec{v} = ky\hat{i} + kx\hat{j}$

$$\Rightarrow \frac{dx}{dt} = ky, \ \frac{dy}{dt} = kx$$

$$\therefore \ \frac{dy}{dx} = \frac{x}{y} \ \Rightarrow \ \int y\,dy = \int x\,dx$$

$$\boxed{y^2 = x^2 + \text{constant}}$$

Ans. (1)

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1 / t_2 will be –

(1) 2 (2) 1
(3) $\frac{1}{2}$ (4) $\frac{1}{4}$

Sol. $U = \frac{Q^2}{2C} = \frac{Q_0^2 e^{-\frac{2t}{RC}}}{2C}$ $Q = Q_0 e^{-t/RC}$

$$U = \frac{U_0}{2}$$

$$\frac{Q_0^2}{2 \times 2C} = \frac{Q_0^2 e^{-\frac{2t_1}{RC}}}{2C} \qquad \frac{Q_0}{4} = Q_0 e^{-\frac{t_2}{RC}}$$

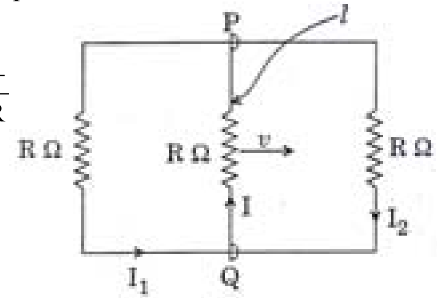
$$\frac{1}{2} = e^{-\frac{2t_1}{RC}} \qquad \log_e 4 = \frac{t_2}{RC}$$

$$t_1 = \frac{RC \log_e 2}{2} \qquad t_2 = RC \log_e 4$$

$$\frac{t_1}{t_2} = \frac{1}{4}$$

Ans. (4)

22. A rectangular loop has a sliding connector PQ of length *l* and resistance R Ω and it is moving with a speed *v* as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are –



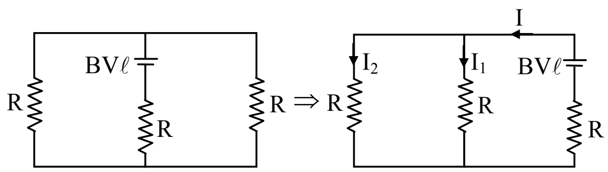
(1) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

(2) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(3) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(4) $I_1 = I_2 = I = \frac{Blv}{R}$

Sol.



$I = \frac{2BVl}{3R}$ $\qquad I_1 = I_2 = \frac{BVl}{3R}$

Ans. (3)

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by y = 0.02 (m) $\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is –

(1) 6.25 N (2) 4.0 N

(3) 12.5 N (4) 0.5 N

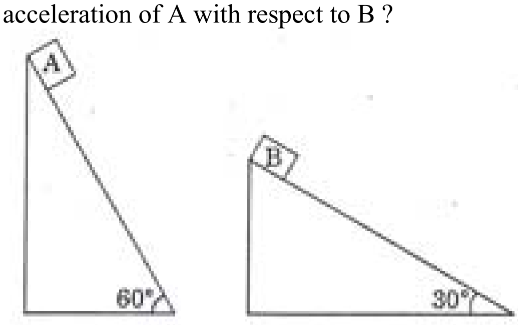
Sol. Putting $\omega = \frac{2\pi}{.04}, k = \frac{2\pi}{0.5}$

in equation $T = \mu v^2 = \mu\left(\frac{\omega}{k}\right)^2$

= 6.25 N

Ans. (1)

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



(1) 4.9 ms^{-2} in vertical direction

(2) 4.9 ms^{-2} in horizontal direction

(3) 9.8 ms^{-2} in vertical direction

(4) Zero

Sol. $a_{A_{vertical}} = g\sin^2 60° = \frac{3g}{4}$

$a_{B_{vertical}} = g\sin^2 30° = \frac{g}{4}$

So, $a_{AB} = \frac{g}{2} = 4.9\, ms^{-2}$ vertical

Ans. (1)

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

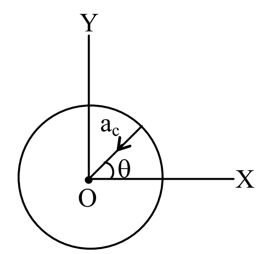
(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(2) $-\frac{v^2}{R}\cos\theta\ \hat{i} + \frac{v^2}{R}\sin\theta\ \hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\ \hat{i} + \frac{v^2}{R}\cos\theta\ \hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\ \hat{j}$

Sol.

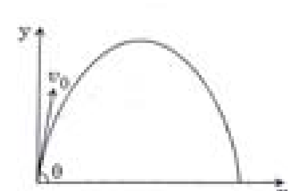


$\vec{a} = a_c\cos\theta(-\hat{i}) + a_c\sin\theta(-\hat{j})$

$\vec{a} = -\frac{V^2}{R}\cos\theta\ \hat{i} - \frac{V^2}{R}\sin\theta\ \hat{j}$

Ans. (4)

26. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0\sin\theta}{g}$, the angular momentum of the particle is –



(1) $\frac{1}{2}mg\,v_0t^2\cos\theta\ \hat{i}$ (2) $-mg\,v_0t^2\cos\theta\ \hat{j}$

(3) $mg\,v_0t\cos\theta\ \hat{k}$ (4) $-\frac{1}{2}mg\,v_0t^2\cos\theta\ \hat{k}$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Sol. at any time t

$$\vec{r} = (v_0 \cos\theta)\, t\, \hat{i} + \left((v_0 \sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

$$\vec{v} = v_0 \cos\theta\, \hat{i} + (v_0 \sin\theta - gt)\,\hat{j}$$

so, $\vec{L} = m(\vec{r} \times \vec{v}) = -\frac{1}{2} m g v_0 t^2 \cos\theta\, \hat{k}$

Ans. (4)

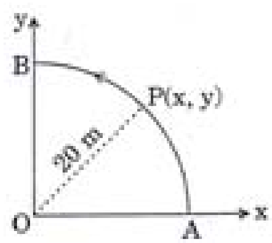
27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is –

(1) 1 (2) 4
(3) 3 (4) 2

Sol. $\dfrac{1}{1-\dfrac{\rho}{\sigma}} = \dfrac{1}{1-\dfrac{.8}{1.6}} = 2$

Ans. (4)

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length s = t^3 + 5, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly.



(1) 14 m/s^2 (2) 13 m/s^2
(3) 12 m/s^2 (4) 7.2 m/s^2

Sol. $s = t^3 + 5$

$v = \dfrac{ds}{dt} = 3t^2$ $\qquad \dfrac{dv}{dt} = 6t$

$a = \sqrt{(a_r^2 + a_t^2)}$

$= \sqrt{\left(\dfrac{v^2}{R}\right)^2 + \left(\dfrac{dv}{dt}\right)^2} = 14\, m/s^2$

at t = 2s

Ans. (1)

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is D = [U(x = ∞) – $U_{\text{at equilibrium}}$], D is –

(1) $\dfrac{b^2}{6a}$ (2) $\dfrac{b^2}{2a}$

(3) $\dfrac{b^2}{12a}$ (4) $\dfrac{b^2}{4a}$

Sol. $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

so $U_{x=\infty} = 0$

At equilibrium:

$$F = 0 = -\frac{dU}{dx} = -12\,ax^{-13} + 6bx^{-7} = 0$$

$\Rightarrow \dfrac{1}{x^6} = \dfrac{b}{2a}$; So $x = \left(\dfrac{2a}{b}\right)^{\frac{1}{6}}$

$U_{eq} = -\dfrac{b^2}{4a}$; $D = \left[0 - \left(-\dfrac{b^2}{4a}\right)\right] = \dfrac{b^2}{4a}$

Ans. (4)

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. the respective temperature coefficients of their series and parallel combinations are nearly –

(1) $\dfrac{\alpha_1+\alpha_2}{2}, \dfrac{\alpha_1+\alpha_2}{2}$

(2) $\dfrac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(3) $\alpha_1+\alpha_2 \quad \dfrac{\alpha_1+\alpha_2}{2},$

(4) $\alpha_1+\alpha_2 \quad \dfrac{\alpha_1\alpha_2}{\alpha_1+\alpha_2},$

Sol. In series, $R_1 + R_2 = R_s$

$R(1 + \alpha_1 T) + R(1 + \alpha_2 T) = 2R(1 + \alpha_s T)$

$2R + RT(\alpha_1 + \alpha_2) = 2R + 2R\,\alpha_s T$

$\alpha_s = \dfrac{\alpha_1+\alpha_2}{2}$

In parallel $\dfrac{1}{Rp} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$$\frac{1}{\frac{R}{2}(1+\alpha_p T)} = \frac{1}{R(1+\alpha_1 T)} + \frac{1}{R(1+\alpha_2 T)}$$

$2(1 - \alpha_p T) = 1 - \alpha_1 T + 1 - \alpha_2 T$

$\alpha_p = \dfrac{\alpha_1+\alpha_2}{2}$

Ans. (1)

CHEMISTRY

31. In aqueous solution the ionization constants for carbonic acid are $K_1 = 4.2 \text{ x } 10^{-7}$ and $K_2 = 4.8 \text{ x } 10^{-11}$ Selection the correct statement for a saturated 0.034 M solution of the carbonic acid.

(1) The concentration of H^+ is double that of CO_3^{2-}
(2) The concentration of CO_3^{2-} is 0.034 M.
(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-
(4) The concentration of H^+ and HCO_3^- are approximately equal.

Sol. $H_2CO_3 \rightleftharpoons H^+ + HCO_3^- \quad k_1 = 4.2 \times 10^{-7}$

0.034 M x x

$HCO_3^- \rightleftharpoons H^+ + CO_3^{-2} \quad k_1 = 4.8 \times 10^{-11}$

x – y x + 5

As $k_2 << k_1$, disso in second is negligible

$\therefore \quad x + y \cong x$

and hence $(H^+) \cong [HCO_3^-]$

Ans. (4)

32. Solution product of silver bromide is 5.0 x 10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 5.0 x 10^{-8} g (2) 1.2 x 10^{-10} g
(3) 1.2 x 10^{-9} g (4) 6.2 x 10^{-5} g

Sol. $k_{sp} = [Ag^+][Br^-] = 5 \times 10^{-13}$

$[Ag^+] = 0.05M$

$[Br^-] = \dfrac{5 \times 10^{-13}}{0.05} = 10^{-11} M$

$[Br^-] = [kBr]$

$\therefore$ Mass added in gms $= 10^{-11} \times 120$ g

$= 1.2 \times 10^{-9}$ g

Ans. (3)

33. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $O^{2-} > F^- > Na^+ > Al^{3+}$
(2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$
(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$
(4) $Na^+ > F^- > Mg^{2+} O^{2-} Al^{3+}$

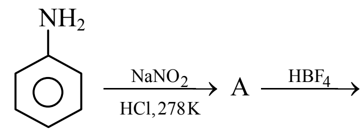
Sol. Isoelectronic series

$r \downarrow \longrightarrow \dfrac{z}{e} \uparrow$

$\therefore \; O^{-2} > F^- > Na^+ > Mg^{+2} > Al^{+3}$

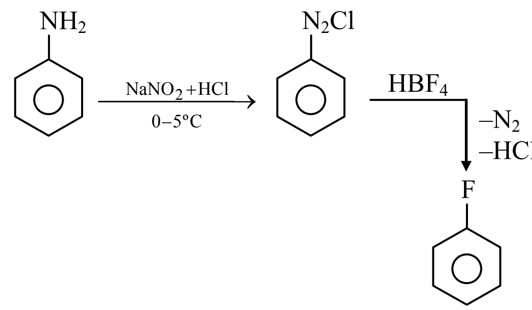
Ans. (1)

34. In the chemical reactions.



The compounds 'A' and 'B' respectively are
(1) nitrobenzene and chlorobenzene
(2) nitrobenzene and flurobenzene
(3) phenol and benzene
(4) benzene diazonium chloride and flurobenzene

Sol.



Ans. (4)

35. If 10^{-4} dm^{-3} of water is introduced into a 1.0 dm^{-3} flask at 300 K, how many moles of water are in in the vapour phase when equilibrium is established ?
(Given : Vapour pressure of H_2O at 300 is 3170 pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27 x 10^{-3} mol (2) 5.56 x 10^{-3} mol
(3) 1.53 x10^{-2} mol (4) 4346 x 10^{-2} mol

Sol. PV = nRT

$3170 \dfrac{N}{m^2} \times 10^{-3} m^3 = n \times 8.314 \times 300$

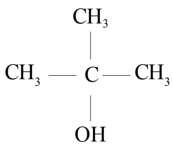
$n = \dfrac{3170 \times 10^{-3}}{24.93 \times 10^{-2}} = \dfrac{31.7 \times 10^{-1}}{24.93 \times 10^{-2}} = 1.287 \times 10^{-1} m$

Ans. (1)

36. From amongst the following alcohols the one that would react fastest with conc, HCl and anhydrous $ZnCl_2$ is

(1) 1- Butanol
(2) 2- Butanol
(3) 2- Methylpropan -2-ol
(4) 2- Methylpropanol

Sol. Reactivity of Lucas reagent for alcohol
$= 3° > 2° > 1° > CH_3$



2-methyl. 2-propanol (3° -alcohol) highest reactivity

Ans. (3)

37. If sodium sulphate is considered to be completely dissociated into cations and anions in equeous solution, the change in freezing point of water (ΔT_f), When 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is ($K_f = 1.86 K\ Kg\ mol^{-1}$)

(1) 0.0186 K (2) 0.0372 K
(3) 0.0558 K (4) 0.0744 K

Sol. $Na_2SO_4 \rightleftharpoons 2Na^2 + SO_4^{-2}$
i = 3
$\Delta T_f = iK_f.m.$
= 3 × 1.86 × 0.0
= 0.0558 K

Ans. (3)

38. Three reactions involving $H_2PO_4^-$ are given below:

(1) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(2) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(3) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid?

(1) (i) Only (2) (ii) Only
(3) (iii) and (ii) (4) (iii) only

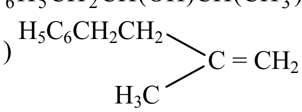
Sol. (ii) only

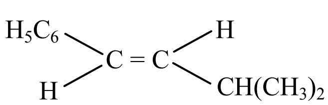
$H_2PO_4^- + H_2O \longrightarrow HPO_4^{-2} + H_3O^+$
acid base

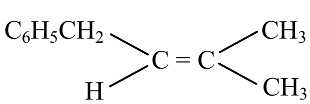
Ans. (2)

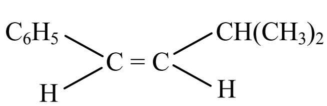
39. The main product of the following reaction is

$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{conc.H_2SO_4} ?$

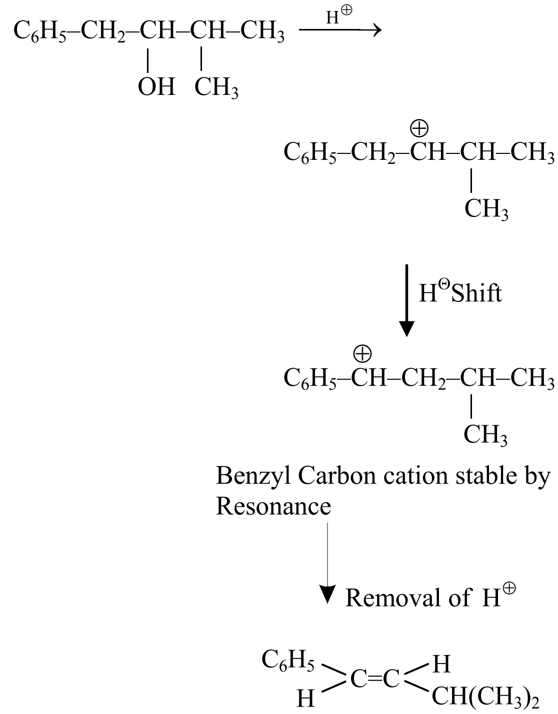
(1) 

(2) 

(3) 

(4) 

Sol.



Trans is more stable than cis Alkene

Ans. (2)

40. The energy required to break one mole of Cl–Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl–Cl bond is

($C = 3\ x\ 10^8\ ms^{-1}$ and $N_A = 6.02\ x\ 10_{23}\ mol^{-1}$)

(1) 494 nm (2) 594
(3) 640 nm (4) 700 nm

Sol. $E = \frac{hc}{\lambda}$

$= \frac{242 \times 10^{+3}}{6.02 \times 10^{23}} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\lambda}$

$\lambda = \frac{6.62 \times 10^{-26} \times 3 \times 6.02 \times 10^{20}}{242}$

$= \frac{6.62 \times 18.06 \times 10^{-6}}{242}$

$= 0.494 \times 10^{-6} = 4.94 \times 10^{-7}$ m

= 494 nm

Ans. (1)

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is

(1) 29.5 (2) 59.0 (3) 47.4 (4) 23.7

Sol. Equivalent of NH_3

$= (0.1 \times 20) - (0.1 \times 15) = 0.5$

wt. of NH_3

$= 0.5 \times 17 = 8.5mg$

wt. of 'N'

$= \frac{14}{17} \times 8.5mg = 7mg$

% of 'N' = $\frac{7}{29.5} \times 100 = 23.7$

Ans. (4)

42. Ionisation energy of He^+ is 19.6 x 10^{-18} J $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82 x 10^{-17} J $atom^{-1}$

(2) 4.41 x 10^{-16} J $atom^{-1}$

(3) -4.41×10^{-17} J $atom^{-1}$

(4) -2.2 x 10^{-15} J $atom^{-1}$

Sol. $\frac{I.E_1}{I.E_2} = \frac{Z_1^2}{Z_2^2}$

$= \frac{19.6 \times 10^{-18}}{x} = \frac{4}{9}$

$x = \frac{9}{4} \times 19.6 \times 10^{-18} = 44.1 \times 10^{-18}$ J/atm.

$= 4.41 \times 10^{-17}$ J/atm

Ans. (3)

43. On mixing, heptane and octane form an ideal solution At 373 K, the vapour pressures of the two liquid components (Heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane =114 g mol^{-1})

(1) 144.5 kPa (2) 72.0 kPa

(3) 36.1 kPa (4) 96.2 kPa

Sol. $P_T = P_0 x_0 + P_{hep} x_{hep}$

$= 45 \times \frac{0.3}{0.55} + 105 \times 25 \frac{0.25}{0.55}$

= 45 × 0.545 + 105 × 0.454

= 72.25 kPa.

Ans. (2)

44. Which one of the following has an optical isomer ?

(1) $[Zn(en)_2]^{2+}$ (2) $[Zn(en)(NH_3)_2]^{2+}$

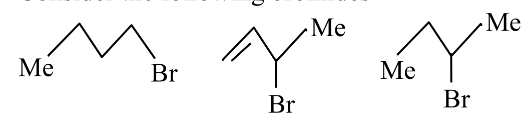
(3) $[Co(en)_3]^{3+}$ (4) $[Co(H_2O)_4(en)]^{3+}$

Sol. $[M(AA)_3]$ type of compound

$\therefore$ optically active

Ans. (3)

45. Consider the following bromides



(A) (B) (C)

The correct sorder of S_N1 reactivity is

(1) A > B > C (2) B > C > A

(3) B > A > C (4) C > B > A

Sol. Reactivity for SN' $\propto$ Stability of carbocation

1° carbocation (A) | stable by resonance (B) | 2° Carbocation (C)

Ans. (2)

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is -

(1) ethene (2) propene

(3) 1-butene (4) 2-butene

Sol. Molecular weight = 44 $\therefore [CH_3 - CHO]$

$$CH_3 - CH = O + O = CH - CH_3 \longrightarrow \underset{\text{2-butene}}{CH_3 - CH = CH - CH_3}$$

Ans. (4)

47. Consider the reaction :

$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+(aq) + 2\,Cl^-(aq)$

The rate equation for this reaction is

Rate = k $[Cl_2][H_2S]$

Which of these mechanisms is/are consistent with this rate equation ?

(A) $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl + +HS^- \rightarrow H^+ + Cl^- + S$ (fast)

(B) $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (slow)

(1) A only (2) B only

(3) Both A and B (4) Neither A nor B

Sol. $r = k[Cl_2][H_2S]$

$\therefore$ According to A $\rightarrow r = k[H_2S][Cl_2]$

$\therefore$ According to B $\rightarrow r = k[Cl_2][HS]$

or $K_{eq} = \frac{[H^+][HS]}{[H_2S]}$

$[HS] = K_{eq} \frac{[H_2S]}{H^+}$

$r = k[Cl_2]\ K_{eq} \frac{[H_2S]}{[H^+]}$

$= K' \frac{[Cl_2][H_2S]}{[H^+]}$

$\therefore$ (A) Only

Ans. (1)

48. The Gibbs energy for the decomposition of Al_2O_3 at 500 °C is as follows:

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2, \Delta_rG = +966\text{kJ mol}^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500 °C is at least

(1) 5.0 V (2) 4.5 V
(3) 3.0 V (4) 2.5 V

Sol. $\Delta G = -nFE$ $n = 4$

$966 \times 10^3 = -4 \times 96500 \times E = 2.5$ V

Ans. (4)

49. The correct order of increasing basicity of the given conjugate bases $(R = CH_3)$ is

(1) $RCO\overline{O} < HC \equiv \overline{C} < \overline{N}H_2 < \overline{R}$

(2) $RCO\overline{O} < HC \equiv \overline{C} < \overline{R} < \overline{N}H_2$

(3) $\overline{R} < HC \equiv \overline{C} < RCO\overline{O} < \overline{N}H_2$

(4) $RCO\overline{O} < \overline{N}H_2 < HC \equiv \overline{C} < \overline{R}$

Sol. Conjugated acid

RCOOH $CH \equiv CH$ NH_3 $R - H$

Order to A.S. $\Rightarrow RCOOH > CH \equiv CH > NH_3 > R - H$

Order to B.S.

$\Rightarrow RCOO^- < CH \equiv C^- < NH_2^- < R^-$

Ans. (1)

50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 144 pm (1) 288 pm
(3) 398 pm (4) 618 pm

Sol. $r_{\oplus} + r_{(-)} = \frac{a}{2}$

$110 + r_{(-)} = \frac{508}{2}$

$r_{(-)} = 254 - 110$

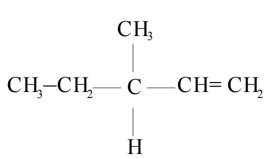
$= 144$ nm

Ans. (1)

51 Out of the following the alkene that exhibits optical isomerism is

(1) 2-methyl-2-pentene
(2) 3-methyl-2-pentene
(3) 4-methyl-pentene
(4) 3-methyl-1-pentene

Sol.



Due to presence of chiral carbon atom 4 is show optical isomerism.

Ans. (4)

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) $T = T_e$ (2) $T_e > T$
(3) $T > T_e$ (4) T_e is 5 times T

Sol. $\Delta G = \Delta H - T\Delta S$

+ve +ve

$T > T_e$ for $\Delta G = -ve$

Ans. (3)

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

(1) 48% and 26% (2) 30% and 26%
(3) 26% and 32% (4) 32% and 48%

Sol. ccp : p.f. = 74%; 100 – 74 = 26%

bcc : p.f. = 68%; 100 – 68 = 32%

Ans. (3)

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) natural rubber (2) teflon
(3) nylon 6,6 (4) polystyrene

Sol. Fact

Ans. (3)

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions ?

(1) 8 (2) 9
(3) 10 (4) 11

Sol. $K_{sp} = [Mg^{+2}][OH^-]^2$

$1 \times 10^{-11} = [0.001][OH^-]^2$

$[OH^-] = 10^{-4}$

pOH = 4; pH = 10

Ans. (3)

56. The correct order of $E^\circ_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co (2) Mn > Cr > Fe > Co
(3) Cr > Fe > Mn > Co (4) Fe > Mn > Cr > Co

Sol.

$E^0_{M+2/M}$	Ti	V	Cr	Mn	Fe	Co
	–1.67	–1.18	– 0.91	–1.18	– 0.44	– 0.28

Ni	Cu	Zn
– 0.24	+ o.34	– 0.76

Ans. (2)

57. Biuret test is *not* given by

(1) proteins (2) carbohydrates
(3) polypeptides (4) urea

Sol. Carbohydrate does not give biuret test. Due to absence of amide group.
Ans. (2)

58. The time for half life period of a certain reaction A $\rightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction ?
(1) 1h (2) 4 h
(3) 0.5 h (4) 0.25 h
Ans.[4]

59. A solution containing 2.675 g of $CoCl_3 \cdot 6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger . The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass =143.5 g mol^{-1}). The formula of the complex is (At. mass of Ag = 108 u)
(1) $[CoCl(NH_3)_5]Cl_2$ (2) $[Co(NH_3)_6]Cl_3$
(3) $[CoCl_2(NH_3)_4]Cl$ (4) $[CoCl_3(NH_3)_3]$

Sol. $CoCl_3.6NH_3 \longrightarrow AgCl$

$$4.78 \text{ g or } \frac{4.78}{143.5} = 0.03 \text{ moles}$$

Ans. (2)

60. The standard enthalpy of formation of NH_3 is -46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1}, the average bond enthalpy of N – H bond in NH_3]
(1) -1102 kJ mol^{-1} (2) -964 kJ mol^{-1}
(3) $+352$ kJ mol^{-1} (4) $+1056$ kJ mol^{-1}

Sol. $\frac{1}{2}N_2(g) + \frac{3}{2}H(g) \longrightarrow NH_3$

$$\Delta H_f = \frac{1}{2}B-E_{N-N} + \frac{3}{2}BE_{H-H} - 3.B.E_{N-H}$$

$$-46 = \frac{1}{2}\times(-712) + \frac{3}{2}\times(-436) - 3\times x$$

$$x = \frac{1056}{3} = 352 \text{ kJ/ml.}$$

Ans. (3)

MATHEMATICS

61. Consider the following relations
R = {(x, y) | x, y are real numbers and x = wy for some rational number w};
$S = \{(\frac{m}{n}, \frac{p}{q})$ | m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn}. Then -
(1) R is an equivalence relation but S is not an equivalence relation
(2) Neither R nor S is an equivalence relation
(3) S is an equivalence relation but R is not an equivalence relation
(4) R and S both are equivalence relations

Sol. Probable part of R is
{(0, 1), (0, 2)}
But (1, 0) $\notin$ R
as 1 = (w) 0
So not symmetric
ie. not equivalence Relation

$$\frac{m}{n} S \frac{p}{q} \rightarrow qm = pn$$

Reflexive $\frac{m}{n} S \frac{m}{n} \rightarrow mm = mn$

hence function reflexive .

Let $\frac{m}{n} S \frac{p}{q} \rightarrow qm = pn$

Then $\frac{p}{q} S \frac{m}{n} \rightarrow pn = mq$

hence function symmetric

$\frac{m}{n} S \frac{p}{q} \rightarrow mq = pn$ (1)

$\frac{p}{q} S \frac{r}{s} \rightarrow ps = qr$ (2)

eqn. (1)/(2)

$$\frac{m}{n} = \frac{r}{s} \rightarrow \frac{m}{n} S \frac{r}{s}$$

hence transitive
So S is equivalence relation
Ans. (3)

62. The number of complex numbers z such that |z –1 | = | z + 1| = | z – i| equals -
(1) 0 (2) 1
(3) 2 (4) ∞

Sol. |z – 1| = |z + 1| = |z – i|
The point z is equidistance from (–1, 0), (1, 0) and (0, 1) is only (0, 0)
hence z is only point (0, 0)
Ans. (2)

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) – 2 (2) – 1 (3) 1 (4) 2

Sol . Here roots of $x^2 - x + 1 = 0$ are $-\omega$ and $-\omega^2$.

$(-\omega)^{2009} + (-\omega^2)^{2009} = (-\omega)^{2007} \times (-\omega)^2 + (-\omega^2)^{2007} \times (-\omega^2)^2 - (\omega^2 + \omega) = 1$.

Ans. (3)

64. Consider the system of linear equations :

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(1) Infinite number of solutions

(2) Exactly 3 solutions

(3) a unique solution

(4) No solution

Sol. Here $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 1\ (1) - 2(1) + 1\ (1) = 0$

$\Delta_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} = 3\ (1)\ - 2\ (5) + 1\ (12) = 5$

$\Delta_1 \neq 0$

When $\Delta = 0$ and if $\Delta_1, \Delta_2, \Delta_3$, are not zero then no solution

Ans. (4)

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is -

(1) 3 (2) 36 (3) 66 (4) 108

Sol. By $^3C_2$ way we can select 2 balls from A and By $^9C_2$ ways we can select 2 balls from B

Total no. of ways $^3C_2 \times {}^9C_2 = 108$

Ans. (4)

66. Let f : (–1, 1) → R be a differentiable function with f(0) = – 1 and f '(0) = 1. Let $g(x) = [f(2f(x) + 2)]^2$,. Then g ' (0) =

(1) 4 (2) –4 (3) 0 (4) –2

Sol. g'(x) = 2 [f(2f(x) + 2)]. f '(2 f(x) + 2) .2f ' (x)

g'(0) = 2 [f(2.f(0)+2)] . f ' (2. f(0) + 2). 2f '(0)

= 2[f(0)]. f '(0) .2

= 2(–1) .(1).2

= – 4

Ans. (2)

67. Let f : R → R be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 3

Sol. Function (↑)

$f(x) \le f(2x) \le f(3x)$

$1 \le \frac{f(2x)}{f(x)} \le \frac{f(3x)}{f(x)}$

given that $\frac{f(3x)}{f(x)} = 1$

hence $1 \le \frac{f(2x)}{f(x)} \le 1$

hence $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$ (by sandwich theorem)

Ans. (1)

68. Let p(x) be a function defined on R such that p'(x) = p'(1 – x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1 p(x)dx$ equals -

(1) $\sqrt{41}$ (2) 21 (3) 41 (4) 42

Sol. P'(x) = P' (1 – x)

integrate

P(x) = – P (1 – x) + k ------- (1)

put x = 1

P(1) = – P(0) + k

41 = – 1 + k

K = 42

Put in (1)

P(x) = –P(1 – x) + 42 ------(2)

Now $I = \int_0^1 P(x)dx$

also $I = \int_0^1 P(1-x)dx$

$2I = \int_0^1 (P(x) + P(1-x))dx$

using (2) $2I = \int_0^1 42\ dx = 42(x)_0^1$

I = 21

Ans. (2)

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference – 2, then the time taken by him to count all notes is -

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

Sol. $a_1 = a_2 = a_3 \ldots\ldots a_9 = 150$

$a_1 + a_2 + a_3 + \ldots. + a_9 = 1350$

$a_{10} + a_{11} + \ldots\ldots\ldots + a_n = 4500 - 1350 = 3150$

$\frac{n}{2}[2\times150+(n-1)(-2)] = 3150$

$150\,n - n^2 + n = 3150$

$n^2 - 151n + 3150 = 0$

n = 25 min

hence total time = 25 + 9 = 34 min

Ans. (2)

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x – axis, is -

(1) y = 0 (2) y = 1

(3) y = 2 (4) y = 3

Sol. $y = x + \frac{4}{x^2}$

$\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0$

$1 = \frac{8}{x^3}$

$x^3 = 8$

x = 2

at x = 2, $y = x + \frac{4}{x^2}$

$= 2 + \frac{4}{4} = 3$

tangent y – 3 = 0 (x – 2)

y = 3

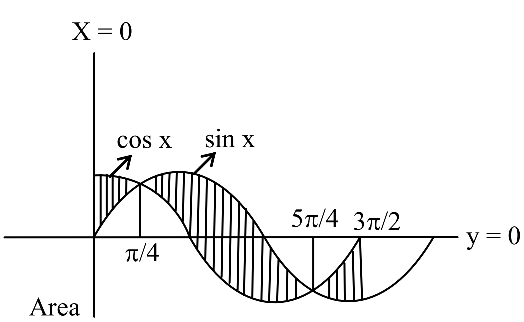
Ans. (4)

71. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is -

(1) $4\sqrt{2}-2$ (2) $4\sqrt{2}+2$

(3) $4\sqrt{2}-1$ (4) $4\sqrt{2}+1$

Sol.



$$\int_0^{\frac{\pi}{4}}(\cos x-\sin x)dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)dx + \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}(\cos x-\sin x)dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + [\sin x + \cos x]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$$

$$= \left[\frac{1}{\sqrt2}+\frac{1}{\sqrt2}-(0+1)\right] - \left[-\frac{1}{\sqrt2}-\frac{1}{\sqrt2}-\frac{1}{\sqrt2}-\frac{1}{\sqrt2}\right] + \left[-1+0-\left(-\frac{1}{\sqrt2}-\frac{1}{\sqrt2}\right)\right]$$

$$= \sqrt2 - 1 + \frac{4}{\sqrt2} - 1 + \sqrt2 \;\; = 4\sqrt2 - 2$$

Ans. (1)

72. Solution of the differential equation

cos x dy = y (sin x – y) dx, $0 < x < \frac{\pi}{2}$ is -

(1) sec x = (tan x + c) y (2) y sec x = tan x + c

(3) y tan x = sec x + c (4) tan x = (sec x + c) y

Sol. $\cos x\frac{dy}{dx} = y\sin x - y^2$

$\cos x\frac{dy}{dx} - \sin x.y = -y^2$

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = \sec x$ $\quad -\frac{1}{y} = z$

$\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$\frac{dz}{dx} + \tan x.z = \sec x$

I.F. = $e^{\int \tan x\,dx}$

Solution of above differential equation is

z. sec x = $\int \sec^2 x\,dx$

$\frac{\sec x}{y} = \tan x + c$

sec x = y (tan x + c)

Ans. (1)

73. Let $\vec a = \hat j - \hat k$ and $\vec c = \hat i - \hat j - \hat k$. Then the vector $\vec b$ satisfying $\vec a \times \vec b + \vec c = \vec 0$ and $\vec a.\vec b = 3$ is -

(1) $-\hat i + \hat j - 2\hat k$ (2) $2\hat i - \hat j + 2\hat k$

(3) $\hat i - \hat j - 2\hat k$ (4) $\hat i + \hat j - 2\hat k$

Sol. Let $b = b_1\hat i + b_2\hat j + b_3\hat k$

given $\vec a\,.\,\vec b = 3$

$b_2 - b_3 = 3$ (1)

and $\vec{a} \times \vec{b} + \vec{c} = 0$

$\vec{a} \times \vec{b} = -\vec{c}$

$$\begin{vmatrix} i & j & k \\ 0 & 1 & -1 \\ b_1 & b_2 & b_3 \end{vmatrix} = -\hat{i} + \hat{j} + \hat{k}$$

$b_3 + b_2 = -1$ (2)

$-b_1 = 1$ ----- (3)

$-b_1 = 1$ ------(4)

$b_1 = -1$

from (1) and (2)

$b_2 = 1$

$b_3 = -2$

$\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

Ans. (1)

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) (–3, 2) (2) (2, –3)

(3) (–2, 3) (4) (3, –2)

Sol. $\vec{a} \perp \vec{b}$ $\therefore \vec{a} \cdot \vec{b} = 0$

$\vec{b} \perp \vec{c} \therefore \vec{b}.\vec{c} = 0$

$2\lambda + 4 + \mu = 0$ .....(1)

$\vec{a} \perp \vec{c} \therefore \vec{a}.\vec{c} = 0$

$\lambda - 1 + 2\mu = 0$ .....(2)

solving (1) and (2), we get

$\lambda = -3$

$\mu = 2$

Ans. (1)

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $x = 1$ (2) $2x + 1 = 0$

(3) $x = -1$ (4) $2x - 1 = 0$

Sol. $y^2 = 4x$ comparing with $y^2 = 4ax$

$a = 1$

Locus of point P will be directrix of given parabola as tangents drawn from P are at right angles, therefore

required locus is $x = -a$

$x = -1$

Ans. (3)

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is -

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$ (3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Sol. $\frac{x}{5} + \frac{y}{b} = 1$ (1) Passess through (13, 32)

$\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow 13b + 160 = 5b \Rightarrow b = -20$

so line is $-20x + 5y = -100$ (1)

second line

$\frac{x}{c} + \frac{y}{3} = 1$

$3x + cy = 3c$ (2)

(1) and (2) are parallel

$\frac{3}{-20} = \frac{c}{5} \Rightarrow c = \frac{-3}{4}$

Line $3x - \frac{3}{4}y = -\frac{9}{4}$

$12x - 3y = -9$

$-20x + 5y = -9 \times \left(-\frac{5}{3}\right)$

$-20x + 5y = 15$ ......(2)

Distance between (1) and (2)

$= \frac{|-100-15|}{\sqrt{400+25}} = \frac{115}{\sqrt{425}} = \frac{115}{5\sqrt{17}} = \frac{23}{\sqrt{17}}$

Ans. (4)

77. A line AB in three dimensional space makes angles 45° and 120° with the positive x – axis and the positive y – axis respectively. If AB makes an acute angle θ with the positive z – axis, then θ equals -

(1) 30° (2) 45° (3) 60° (4) 75°

Sol. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\cos^2 45° + \cos^2 120° + \cos^2 \gamma = 1$

$\frac{1}{2} + \frac{1}{4} + \cos^2 y = 1$

$\text{Cos}^2 y = \frac{1}{4}$

$\cos y = \pm \frac{1}{2}$

$y = 60^o$

Ans. (3)

78. Let S be a non- empty subset of R. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$

Which of the following statements is the negation of the statement P ?

(1) There is a rational number $x \in S$ such that $x \le 0$.

(2) There is no rational number $x \in S$ such that $x \le 0$.

(3) Every rational number $x \in S$ satisfies $x \le 0$.

(4) $x \in S$ and $x \le 0 \Rightarrow$ x is not rational

Ans. [3]

79. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$

(3) $\frac{19}{12}$ (4) $\frac{20}{7}$

Sol.

$$\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)] = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

as $\cos(\alpha + \beta) = 4/5$, $\sin(\alpha - \beta) = 5/13$

$$\tan 2\alpha = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{16-5}{16}} = \frac{56}{33}$$

Ans. (2)

80 The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-85 < m < -35$ (2) $-35 < m < 15$

(3) $15 < m < 65$ (4) $35 < m < 85$

Sol. $x^2 + y^2 - 4x - 8y - 5 = 0$

centre = (2, 4) and radius = 5

$P <$ r for if line is intersecting the circle at two points

$$P = \left|\frac{3(2)-4(4)-m}{\sqrt{3^2+4^2}}\right| < 5$$

$$|-10-m| < 25$$

$$|10+m| < 25$$

$-35 < m < 15$

Ans. (2)

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is -

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$ (3) 6 (4) $\frac{13}{2}$

Sol. $n_1 = 5$ $n_2 = 5$

$\sigma_1^2 = 4$ $\sigma_2^2 = 5$

$\bar{x}_1 = 2$ $\bar{x}_2 = 4$

sum of data = 10 sum of data = 20

$4 = \frac{1}{5}(sum\ of\ squares) - 4$ $5 = \frac{1}{5}(sum\ of\ squares) - 16$

(as variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$)

sum of squares = 40 sum of squares = 105

$\bar{x} = \frac{10+20}{10} = 3$

new variance $= \frac{1}{10}(145) - 9 = \frac{11}{2}$

Ans. (2)

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is -

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{21}$ (4) $\frac{2}{23}$

Sol. Total balls = 3 red balls + 4 blue balls + 2 green balls = 9 balls

required probability $= \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3} = \frac{2}{7}$

Ans. (2)

83. For a regular polygon, let *r* and *R* be the radii of the inscribed and the circumscribed circles. A false statement among the following is -

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Sol. $\tan\left(\frac{\pi}{n}\right) = \frac{\frac{x}{2}}{r} = \frac{x}{2r}$

$r = \frac{x}{2}\cot\left(\frac{\pi}{n}\right)$

and $\sin\frac{\pi}{n} = \frac{x}{2R}$

$R = \frac{x}{2}\operatorname{cosec}\frac{\pi}{n}$

$$\frac{r}{R} = \frac{\cot\left(\frac{\pi}{n}\right)}{\operatorname{cosec}\left(\frac{\pi}{n}\right)} = \cos\left(\frac{\pi}{n}\right)$$

(1) $n = 3, \quad \frac{r}{R} = \frac{1}{2} = .5$

(2) $n = 4, \quad \frac{r}{R} = \frac{1}{\sqrt{2}} = .707$

(3) $n = 5, \quad \frac{r}{R} = \frac{2}{3} = .6$

(4) $n = 6, \quad \frac{r}{R} = \frac{\sqrt{3}}{2}$

(3) is not possible because .6 comes between n = 3 and n = 4 but no integer between n = 3 and n = 4

Ans. (3)

84. The number of 3 × 3 non - singular matrices, with four entries as 1 and all other entries as 0, is -

(1) Less than 4 (2) 5

(3) 6 (4) at least 7

Sol.

$$A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$; \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} ; \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

So at least 7 non singular matrices are there

Ans. (4)

85. Let f : R → R be defined by
$$f(x) = \begin{cases} k - 2x, & if \ x \le -1 \\ 2x + 3, & if \ x > -1 \end{cases}$$

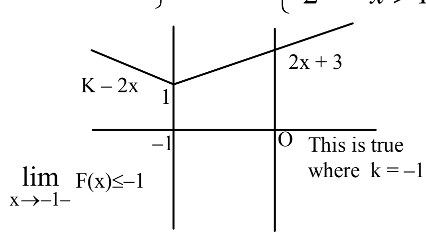
If f has a local minimum at x = –1, then a possible value of k is

(1) 1 (2) 0

(3) $-\frac{1}{2}$ (4) –1

Sol. f:R → R f(x) =

$$\begin{Bmatrix} k - 2x & x \le -1 \\ 2x + 3 & x > -1 \end{Bmatrix} \quad f'(x) = \begin{Bmatrix} -2 & x < -1 \\ 2 & x > 1 \end{Bmatrix}$$



$k - 2x = +1$

$k = -1$

Ans. (4)

Directions : Questions number 86 to 90 are Assertion – Reason type questions. Each of these questions contains two statements:

Statement – 1 (Assertion) and

Statement – 2 (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement – 1 :

The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement – 2 :

If the four chosen numbers form an AP, then the set of all possible values of common difference is {± 1, ± 2, ± 3, ± 4 , ± 5}

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is true.

Sol. S : 1 required no of groups

(1,2,3,4) ………………….. (17,18,19,20) = 17 ways

(1,3,5,7) ………………….. (14,16,18,20) = 14 ways

(1,4,7,10) …………………(11,14,17,20) = 11 ways

(1,5,9,13) ………………….. (8,12,16,20) = 8 ways

(1,6,11,16) ………………….. (5,10,15,20) = 5 ways

(1,7,13,19) ………………….. (2,8,14,20) = 2 ways

required arability = $\frac{(17 + 14 + 11 + 8 + 5 + 2)4!}{{}^{20}C_4 \, 4!}$

$$= \frac{57 \ 4!}{20.19.18.17} = \frac{3.4.3.2.1}{20.18.17.} = \frac{1}{85}$$

S : 1 is true. S : 2

possible cases of common difference are

$[\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6]$

S:2 is false

Ans. (3)

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j, S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$.

Statement – 1 : $S_3 = 55 \times 2^9$.

Statement – 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol.

$$S_1 = \sum_{j=1}^{10} j(j-1)\frac{10!}{j(j-1)(j-2)!(10-j)!}$$

$$= 90\sum_{j=2}^{10}\frac{8!}{(j-2)!(8-(j-2))!} = 90 \times 2^8$$

$$S_2 = \sum_{j=1}^{10} j\frac{10!}{j(j-1)!(9-(j-1))!}$$

$$= 10\sum_{j=1}^{10}\frac{9!}{(j-1)!(9-(j-1))!} = 10 \times 2^9$$

$$S_3 = \sum_{j=1}^{10}[j(j-1)+j]\frac{10!}{j(10-j)!} = \sum_{j=1}^{10}(j-1)\ {}^{10}C_j$$

$$= \sum_{j=1}^{10}(j)\ {}^{10}C_j = 90.2^8 + 10.2^9 = 110.2^8 = 55.2^9$$

Hence statement 1 is true, statement 2 is false

Ans. (3)

88. **Statement – 1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$.

Statement – 2 : The plane $x - y + z = 5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is true.

Sol. Mid point of A(3, 1, 6) and B(1, 3, 4) should lie in the plane

mid point : (2, 2, 5) it satisfies the plane $x - y + z = 5$.

Also AB $\perp$ to plane. Hence Dr's of AB are < 1, – 1, 1 >

statement 1 and 2 are true

Ans. (1)

89. Let f : R $\to$ R be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement – 1 : $f(c) = \frac{1}{3}$, for some $c \in$ R.

Statement – 2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x $\in$ R.

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. AM $\ge$ GM

$$\frac{e^x + \frac{2}{e^x}}{2} \ge \sqrt{(e^x)\left(\frac{2}{e^x}\right)}$$

$$e^x + \frac{2}{e^x} \ge 2\sqrt{2} \qquad (1)$$

$$\because e^x > 0 \Rightarrow e^x + \frac{2}{e^x} > 0 \qquad (2)$$

$$0 < \frac{1}{e^x + \frac{2}{e^x}} \le \frac{1}{2\sqrt{2}}$$

also f(c) = 1/3 for c = 0

so statement 1 : is true

statement 2 : is also true with correct explanation

Ans. (1)

90. Let A be a 2 × 2 matrix with non zero entries and let A^2 = I, where I is 2 × 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.

Statement – 1 : Tr(A) = 0

Statement – 2 : |A| = 1

(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is ture.

Sol. let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2 + bc & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

ab + bd = 0

b (a + d) =0

$b \ne 0$

so, a = – d

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad a + d = 0$$

$T_r(A) = 0$

But $|A| \ne 1$.

So, statement I is true and statement 2 is false.

Ans. (3)

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IIT- JEE 2011 (June issue)

PHYSICS

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	A	C	C	A	D	C	C	B	B,C	B,C
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	B,D	A,D	B,C	B	C	D	A	C		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	2	2	2	3	6	6	5	5	2

CHEMISTRY

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	B	C	C	D	B	B	D	A	B,C,D	B,C
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	A,B,D	A,C,D	A,B	D	C	C	B	D		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	8	5	6	8	9	2	2	5	5

MATHEMATICS

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	A	B	B	C	A	C	D	C	A,B,C	A,C
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	A,B	A,B,D	A,C	D	B	C	B	A		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	1	3	2	7	1	4	4	5	6

IIT- JEE 2012 (June issue)

PHYSICS

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	D	D	C	B	C	D	C	D	B,C,D	A,C
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	A,C	A,B,C,D	A,D	B	B	D	A	B		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	4	2	2	1	8	1	3	2	7

CHEMISTRY

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	B	D	B	B	C	D	A	C	B,C	B,D
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	A,C,D	B,D	A,C,D	B	B	B	A	D		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	2	4	2	6	4	1	6	0	4

MATHEMATICS

Ques	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Ans	C	C	B	C	C	B	C	B	A,B,C,D	A,B,C
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**		
Ans	A,C,D	A,B,C	A,C	C	C	A	C	A		
Numerical Response	**Ques**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
	Ans	1	3	4	8	3	5	3	4	1

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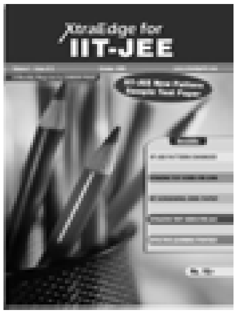

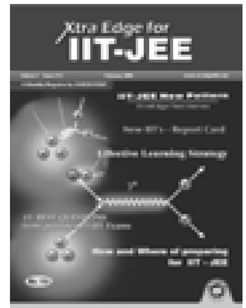

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