# DEPARTMENT OF MATHEMATICS ANNA UNIVERSITY, CHENNAI

### VISION

We, at the Department of Mathematics, Anna University, Chennai, shall strive constantly to

- Achieve excellence in Mathematics education by providing high quality teaching, research and training in Mathematics to all our students to significantly contribute in the fields of Mathematics, Computer Science and all related Engineering fields.
- Contribute to the quality Human Resource Development in Mathematics and Computer Science through our effective Masters and Research Programmes.

### MISSION

- To provide strong Mathematical background to Engineering Students to cope up with the needs of emerging technologies both at National and International levels.
- To popularize and to project the proper perspective of Mathematics and Computer Science towards attracting young talents to take up teaching and research careers in Mathematical Sciences.

#### ANNA UNIVERSITY, CHENNAI

#### UNIVERSITY DEPARTMENTS

#### M. Phil. MATHEMATICS (FT)

#### **REGULATIONS - 2019**

#### CHOICE BASED CREDIT SYSTEM

#### 1. **PROGRAMME EDUCATIONAL OBJECTIVES (PEOs):**

- I. To provide training in advanced topics of Mathematics to professionally carryout Ph. D level research.
- II. To provide deeper knowledge for demonstrating the advanced principles of modern mathematics.
- III. To provide creative and critical thinking in specialized research topics.
- IV. To provide training in undertaking project work, so as to analyze and solve the problem independently.
- V. To provide training for making technical presentation and publishing results in any chosen topic related to the field of specialization.

#### 2. **PROGRAMME OUTCOMES (POs):**

After going through the one years of study, our M.Phil Students will exhibit ability to:

| PO# | Graduate Attribute                         | Programme Outcome   |  |  |  |  |  |
|-----|--|---|--|--|--|--|--|
| 1   | Engineering knowledge                      | Apply knowledge of mathematics, basic science and engineering science.                |  |  |  |  |  |
| 2   | Problem analysis                           | Identify, formulate and solve engineering problems.                                   |  |  |  |  |  |
| 3   | Design/development of solutions            | Design a system or process to improve it performance, satisfying its constraints.     |  |  |  |  |  |
| 4   | Conduct investigations of complex problems | Conduct experiments &collect, analyze and interpret the data.                         |  |  |  |  |  |
| 5   | Modern tool usage                          | Apply various tools and techniques to improve the efficiency of the system.           |  |  |  |  |  |
| 6   | The Engineer and society                   | Conduct themselves to uphold the professional and social obligations.                 |  |  |  |  |  |
| 7   | Environment and sustainability             | Design the system with environment consciousness and sustainable development.         |  |  |  |  |  |
| 8   | Ethics                                     | Interaction with industry, business and society in a professional and ethical manner. |  |  |  |  |  |
| 9   | Individual and team work                   | Function in a multi-disciplinary team.  |  |  |  |  |  |
| 10  | Communication                              | Proficiency in oral and written Communication.  |  |  |  |  |  |
| 11  | Project management and finance             | Implement cost effective and improved system.   |  |  |  |  |  |
| 12  | Life-long learning                         | Continue professional development and learning as a life-long activity.               |  |  |  |  |  |

#### 3. PROGRAMME SPECIFIC OUTCOMES (PSOs):

By the completion of the M.Phil programme in Mathematics the student will have the following Programme specific outcomes.

- 1. To be able to demonstrate advanced principles of Mathematics.
- 2. To be able to identify the research level problems in the area of their research interest.
- 3. To be able to utilize appropriate mathematical tools for solving research level or real world problems.
- 4. To be able to critically analyse the possible solutions of the emerging mathematical research problems.

#### 4. PEO / PO Mapping:

| PROGRAMME                 | PROGRAMME OUTCOMES |              |              |     |              |     |     |     |     |              |      |              |
|---------------------------|--------------------|--------------|--------------|-----|--------------|-----|-----|-----|-----|--------------|------|--------------|
| EDUCATIONAL<br>OBJECTIVES | PO1                | PO2          | PO3          | PO4 | PO5          | P06 | P07 | PO8 | PO9 | PO10         | PO11 | PO12         |
| I                         | V                  | $\checkmark$ | V            | V   |              |     |     |     |     | V            |      | $\checkmark$ |
| II                        | V                  | V            | $\checkmark$ |     | $\checkmark$ | V   |     | V   | V   | V            |      | $\checkmark$ |
| III                       |                    |              | $\checkmark$ |     |              |     |     |     |     | $\checkmark$ |      | $\checkmark$ |
| IV                        |                    | $\checkmark$ | V            |     |              |     |     |     | V   | $\checkmark$ |      | $\checkmark$ |
| V                         |                    |              |              |     |              |     |     | V   |     | V            |      |              |

## Mapping of Course Outcome and Programme Outcome

|      |          | Course Name                     | PO01         | PO0          | 2PO03        | BPO04        | PO05         | PO06 | PO07 | PO08         | PO09         | PO10         | PO11 | PO12         |
|------|----------|---------------------------------|--------------|--------------|--------------|--------------|--------------|------|------|--------------|--------------|--------------|------|--------------|
|      | r 1      | Algebra and Analysis            | $\checkmark$ | $\checkmark$ | V            | $\checkmark$ |              |      |      |              |              | $\checkmark$ |      | $\checkmark$ |
| -    | Semester | Advanced Differential Equations | $\checkmark$ | V            | V            | $\checkmark$ |              |      |      |              |              | $\checkmark$ |      | $\checkmark$ |
| YEAR | Sei      | Elective I                      | V            | $\checkmark$ | $\checkmark$ | $\checkmark$ |              |      |      |              |              | $\checkmark$ |      | $\checkmark$ |
|      | er 2     | Elective II                     | V            | V            | V            | $\checkmark$ |              |      |      |              |              | $\checkmark$ |      | $\checkmark$ |
|      | Semester | Project Work                    | V            | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |      |      | $\checkmark$ | $\checkmark$ | $\checkmark$ |      |              |
|      |          |                                 |              |              |              |              |              |      |      |              |              |              |      |              |

#### ANNA UNIVERSITY, CHENNAI UNIVERSITY DEPARTMENTS

#### M.Phil. MATHEMATICS (FT)

### **REGULATIONS - 2019**

### CHOICE BASED CREDIT SYSTEM

#### CURRICULA AND SYLLABI

#### SEMESTER I

| S.<br>NO. | COURSE               |                                 | CATE  |    | rioe<br>R We |         | TOTAL<br>CONTACT | CREDITS |  |  |
|-----------|----------------------|---------------------------------|-------|----|--------------|---------|------------------|---------|--|--|
| NO.       | CODE GORGE TILL GORY |                                 | L     | Т  | Ρ            | PERIODS |                  |         |  |  |
| THEOF     | THEORY               |                                 |       |    |              |         |                  |         |  |  |
| 1.        | MX5101               | Algebra and Analysis            | PCC   | 4  | 0            | 0       | 4                | 4       |  |  |
| 2.        | MX5102               | Advanced Differential Equations | PCC   | 4  | 0            | 0       | 4                | 4       |  |  |
| 3.        |                      | Program Elective I              | PEC   | 4  | 0            | 0       | 4                | 4       |  |  |
|           |                      |                                 | TOTAL | 12 | 0            | 0       | 12               | 12      |  |  |

#### SEMESTER II

| S.<br>NO. | COURSE<br>CODE | COURSE TITLE        | CATE  | PERIODS<br>PER WEEK |   | -  | TOTAL<br>CONTACT | CREDITS |  |
|-----------|----------------|---------------------|-------|---------------------|---|----|------------------|---------|--|
| NO.       | CODE           |                     | GORY  | L                   | Т | Ρ  | PERIODS          |         |  |
| 1.        |                | Program Elective II | PEC   | 4                   | 0 | 0  | 4                | 4       |  |
| 2.        | MX5211         | Dissertation        | EEC   | 0                   | 0 | 32 | 32               | 16      |  |
|           | L              | ·                   | TOTAL | 4                   | 0 | 32 | 36               | 20      |  |

Total No. of Credits : 32

|       | PROGRAM CORE COURSES (PCC) |                                    |       |          |      |         |                 |  |  |  |
|-------|----------------------------|------------------------------------|-------|----------|------|---------|-----------------|--|--|--|
| SI.No | COURSE                     | COURSE TITLE                       | PERIO | DS PER \ | NEEK | CREDITS | SEMESTER        |  |  |  |
| CODE  | CODE                       |                                    | L     | Т        | Р    | UNEDITO | <u>OLINEOTE</u> |  |  |  |
| 1.    | MX5101                     | Algebra and Analysis               | 4     | 0        | 0    | 4       | 1               |  |  |  |
| 2.    | MX5102                     | Advanced Differential<br>Equations | 4     | 0        | 0    | 4       | 1               |  |  |  |
|       | I                          | Credits                            | 8     |          |      |         |                 |  |  |  |

## PROFESSIONAL ELECTIVE COURSE (PEC)

| S. No | COURSE<br>CODE | COURSE TITLE   | CATEGORY | CONTACT<br>PERIODS | L | т | Р | С |
|-------|----------------|--|----------|--------------------|---|---|---|---|
| 1.    | MX5001         | Abstract Harmonic Analysis   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 2.    | MX5002         | Advanced Analysis  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 3.    | MX5003         | Advanced Number Theory and Cryptography  | PEC 4    |                    | 4 | 0 | 0 | 4 |
| 4.    | MX5004         | Advances in Graph Theory   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 5.    | MX5005         | Algebraic Theory of<br>Semigroups  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 6.    | MX5006         | Applied Combinatorics  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 7.    | MX5007         | Approximation Theory   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 8.    | MX5008         | Basic Hypergeometric Series  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 9.    | MX5009         | Boundary Layer Flows   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 10.   | MX5010         | Characteristic Classes   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 11.   | MX5011         | Differential Topology  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 12.   | MX5012         | Finite Element Method  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 13.   | MX5013         | Finite Volume Method   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 14.   | MX5014         | Fixed Point Theory and its<br>Applications                                       | PEC      | 4                  | 0 | 0 | 4 |   |
| 15.   | MX5015         | Fluid Mechanics  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 16.   | MX5016         | Fractional Differential<br>Equations   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 17.   | MX5017         | Functional Analysis and its<br>Applications to Partial<br>Differential Equations | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 18.   | MX5018         | Fundamentals of Chemical<br>Graph Theory   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 19.   | MX5019         | Fuzzy Analysis, Uncertainty<br>Modeling and Applications                         | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 20.   | MX5020         | Fuzzy Sets and Applications  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 21.   | MX5021         | Fuzzy Sets and Systems   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 22.   | MX5022         | Generalized Inverses   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 23.   | MX5023         | Harmonic Analysis  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 24.   | MX5024         | Heat and Mass Transfer   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 25.   | MX5025         | Homology and Cohomology  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 26.   | MX5026         | Introduction to Algebraic<br>Topology  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 27.   | MX5027         | Introduction to Fibre Bundles  | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 28.   | MX5028         | Introduction to Lie Algebras   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 29.   | MX5029         | Mathematical Aspects of Finite<br>Element Method                                 | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 30.   | MX5030         | Mathematical Finance   | PEC      | 4                  | 4 | 0 | 0 | 4 |
| 31.   | MX5031         | Mathematical Statistics  | PEC      | 4                  | 4 | 0 | 0 | 4 |

| 32. | MX5032 | Measure Theory  | PEC | 4 | 4 | 0 | 0 | 4 |
|-----|--------|---|-----|---|---|---|---|---|
| 33. | MX5033 | Modeling and Simulation                                 | PEC | 4 | 4 | 0 | 0 | 4 |
| 34. | MX5034 | Molecular Computing                                     | PEC | 4 | 4 | 0 | 0 | 4 |
| 35. | MX5035 | Networks, Games and Decisions                           | PEC | 4 | 4 | 0 | 0 | 4 |
| 36. | MX5036 | Nonlinear Dynamics                                      | PEC | 4 | 4 | 0 | 0 | 4 |
| 37. | MX5037 | Number Theory   | PEC | 4 | 4 | 0 | 0 | 4 |
| 38. | MX5038 | Number Theory and<br>Cryptography                       | PEC | 4 | 4 | 0 | 0 | 4 |
| 39. | MX5039 | Numerical Solution of Partial<br>Differential Equations | PEC | 4 | 4 | 0 | 0 | 4 |
| 40. | MX5040 | Operator Theory   | PEC | 4 | 4 | 0 | 0 | 4 |
| 41. | MX5041 | Optimization Techniques                                 | PEC | 4 | 4 | 0 | 0 | 4 |
| 42. | MX5042 | Queueing and Reliability<br>Modeling                    | PEC | 4 | 4 | 0 | 0 | 4 |
| 43. | MX5043 | Representations of Lie<br>Algebras                      | PEC | 4 | 4 | 0 | 0 | 4 |
| 44. | MX5044 | Representation Theory of Finite Groups                  | PEC | 4 | 4 | 0 | 0 | 4 |
| 45. | MX5045 | Special Functions                                       | PEC | 4 | 4 | 0 | 0 | 4 |
| 46. | MX5046 | Stochastic Processes                                    | PEC | 4 | 4 | 0 | 0 | 4 |
| 47. | MX5047 | Univalent Functions                                     | PEC | 4 | 4 | 0 | 0 | 4 |
| 48. | MX5048 | Domination in Graphs                                    | PEC | 4 | 4 | 0 | 0 | 4 |
| 49. | MX5049 | Genetic Algorithms                                      | PEC | 4 | 4 | 0 | 0 | 4 |

## EMPLOYABILITY ENHANCEMENT COURSES (EEC)

| SI.No  | COURSE         | COURSE TITLE | PERIC | DS PER | WEEK | CREDITS | SEMESTER   |
|--------|----------------|--------------|-------|--------|------|---------|------------|
| 01.140 | CODE           |              | L     | Т      | Р    |         | SLIVILSTER |
| 1      | MX5211         | Dissertation | 0     | 0      | 32   | 16      | 2          |
|        | Total Credits: |              |       |        |      | 16      |            |

#### SUMMARY

|    | M. Phil. Mathematics |             |          |               |  |  |  |  |  |  |
|----|----------------------|-------------|----------|---------------|--|--|--|--|--|--|
|    | Subject Area         | Credits per | Semester | Credits Total |  |  |  |  |  |  |
|    |                      |             |          |               |  |  |  |  |  |  |
|    |                      | I           | II       |               |  |  |  |  |  |  |
| 1. | PCC                  | 08          | 00       | 08            |  |  |  |  |  |  |
| 2. | PEC                  | 04          | 04       | 08            |  |  |  |  |  |  |
| 3. | EEC                  | 00          | 16       | 16            |  |  |  |  |  |  |
|    | Total Credit         | 12          | 20       | 32            |  |  |  |  |  |  |

#### ALGEBRA AND ANALYSIS

#### **OBJECTIVES**:

MX5101

- To introduce the advanced topics in algebra such as module.
- To study the classification and structure of module algebra.
- To learn about complex functions and complex integration on measurable space.
- To study about holomorphic functions and its properties over complex space.
- To know more about harmonic functions and Arzela-Ascoli theorem.

#### UNIT I MODULES

Basic Definitions – Quotient Modules – Module Homomorphisms – Generation of Modules- Direct Sums – Free Modules.

#### UNIT II STRUCTURE OF MODULES

Change of scalars – Simple Modules - Semi-simple Modules - Structure of Semi-simple Modules– Chain conditions – The Radical.

#### UNIT III ABSTRACT INTEGRATION

The concept of measurability – Simple functions – Elementary properties of measures – Integration of positive functions – Integration of complex functions – The role played by the sets of measure zero.

#### UNIT IV ELEMENTARY PROPERTIES OF HOLOMORPHIC FUNCTIONS

Complex differentiation – Integration over paths – The local Cauchy theorem – The power series representation – The open mapping theorem – The global Cauchy theorem – The calculus of residues.

#### UNIT V HARMONIC FUNCTIONS

The Cauchy-Riemann Equations - The Poisson Integral - The mean value property - Arzela - Ascoli Theorem.

#### OUTCOMES:

- The students are capable of handling the advanced topics in algebra.
- Ability to understand and apply algebraic structures in other area of Mathematics.
- Students will be knowledgeable on abstract integration.
- Students will understand the properties of analytic functions and use it to evaluate the integrals.
- Students will get more idea about harmonic function and its applications.

#### REFERENCES

- 1. Elras, M. Stein and Ramishakarchi, "Complex Analysis", Princeton University Press, New Jersey, 2003.
- 2. Halmos P.R., "Measure Theory", Springer, New York, 1974.
- 3. Lang S., "Algebra", Reading Mass., Addisson Wesley, Third Edition, Massachusetts, 2005.
- 4. Pierce R.S., "Associative Algebras, Graduate Texts in Mathematics", Springer Verlag, NewYork,1982.
- 5. Royden, H.L., "Real Analysis", Macmillan Company, Fourth Edition, New York, 2010.
- 6. Rudin W., "Real and Complex Analysis", Tata Mc-Graw Hill, Third Edition, New York, 2006.

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**TOTAL: 60 PERIODS** 

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### ADVANCED DIFFERENTIAL EQUATIONS

#### **OBJECTIVES:**

MX5102

- To familiarize in various types of linear equations.
- To understand the method of solving equations with periodic coefficients.
- To understand the concept of stability analysis.
- To understand the concept of partial differential equations.
- To solve nonlinear partial differential equations.

### UNIT I LINEAR EQUATIONS

Uniqueness and existence theorem for a linear system - Homogeneous linear systems - Inhomogeneous linear systems - Second-order linear equations - Linear equations with constant coefficients problems.

#### UNIT II LINEAR EQUATIONS WITH PERIODIC COEFFICIENTS

Floquet theory - Parametric resonance - Peturbation methods for the Mathieu equation - The Mathieu equations with damping problems.

#### UNIT III STABILITY

Preliminary definitions - Stability for linear systems - Principle of linearlized stability - Stability for autonomous systems - Liapunov functions problems.

#### UNIT IV PARTIAL DIFFERENTIAL EQUATIONS

Review of method of characteristics for first order partial differential equations and classification of second order partial differential equations.

#### UNIT V NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

Nonlinear first order partial differential equations - Conservation laws - Lax-Oleinik - Formula - Riemann's problem - Long time behavior separation of variables - Similarity solutions.

### OUTCOMES:

After completing this course, the students will able to

- Solve the linear differential equations with constant coefficients .
- Solve the linear differential equations with the periodic coefficients in Peturbation methods.
- Analysis the stability of the given ordinary differential equations.
- Solve Partial differential equations with given conditions.
- Solve nonlinear partial differential equations in application of real life problems.

#### **REFERENCES:**

- 1. R. Grimshaw, "Nonlinear ordinary differential equations", CRC Press, Boca Raton, 1991
- 2. Lawrence C. Evans, "Partial Differential Equations", American Mathematical Society, Second edition, Vol.19, Rhode Island, 2010.
- 3. David Betounes," Differential equations: Theory and applications", Springer, Second Edition, New York, 2010.
- 4. L. Perko, "Differential equations and dynamical systems", Springer, Vol. 7, New York, 2013.
- 5. Mathew P. Coleman, " An introduction to partial differential equations with Matlab", CRC Press, Second edition, Boca Raton, 2013.
- 6. Sandro Salsa, " Partial differential equations in action: From modelling to theory", Springer, Cham, 2008.

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**TOTAL: 60 PERIODS** 

#### ABSTRACT HARMONIC ANALYSIS

#### Prerequisite: Real Analysis

#### OBJECTIVES

- To give a comprehensive overview of harmonic analysis and its applications to all areas of mathematical sciences.
- To introduce compact and locally compact abelian groups.
- To introduce Haar measure and the invariant means.
- To introduce convolutions of functions and measures.
- To introduce duality theorem.

#### UNIT I TOPOLOGICAL GROUPS

Topological group and its basic properties - Subgroups and quotient groups - Product groups and projective limits - Properties of topological groups involving connectedness -Invariant pseudo-metrics and separation axioms.

#### UNIT II STRUCTURE THEOREMS

Structure theory for compact and locally compact Abelian groups -Some special locally compact Abelian groups.

#### UNIT III HAAR MEASURE

The Haar integral - Haar measure - Invariant means defined for all bounded functions - Invariant means on almost periodic functions.

#### UNIT IV UNITARY REPRESENTATIONS

Convolutions - Convolutions of functions and measures - Elements of representation theory. Unitary representations of locally compact groups.

#### UNIT V DUALS

The character group of a locally compact Abelian group and the duality theorem. TOTAL: 60 PERIODS

#### OUTCOMES

- To familiarize the students with topological groups. Haar, measures and convolutions in the harmonic analysis.
- The student will gain understanding of structure theorems.
- The student will get to know about Haar measure and the invariant means on almost periodic functions.
- The student will be able to understand about the elements of representation theory.
- The student will learn about the duality in locally compact Abelian groups.

#### REFERENCES

- 1. Edwin Hewitt and Kenneth A. Ross, "Abstract Harmonic Analysis-I", Springer-Verlag, Berlin, 1993.
- 2. Lynn H. Loomis, "An introduction to abstract harmonic analysis", Van Nostrand Co. Princeton, Canada, 1953.

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#### ADVANCED ANALYSIS

#### **Prerequisite: Real Analysis**

#### **OBJECTIVES**

- Real Analysis is the fundamental course behind almost all other branches of Mathematics.
- The aim of the course is to make the students understand the basic and advanced concepts of Real analysis.
- To introduce Lebesgue measure and the Fundamental Theorem of Calculus. •
- To introduce the Fourier Transforms.
- To introduce the Holomorphic Fourier Transforms.

#### L<sup>P</sup>SPACES UNIT I

Convex functions – Jenson's inequality - L<sup>p</sup>spaces-Schwartz, Minkowski inequality - Approximation by continuous functions.

#### UNIT II **COMPLEX MEASURES**

Total variation - Positive and negative variation - Absolute Continuity - Radon Nikodym theorem -Bounded linear functional in L<sup>p</sup>- Riesz representation theorem.

#### UNIT III DIFFERENTIATION

Derivatives of measures Lebesque points - Metric density - Fundamental theorem of calculus -Differentiable transformations.

#### UNIT IV FOURIER TRANSFORMS

Formal Properties - Inversion Theorem - Uniqueness theorem - Plancheral Theorem - Translation invariant subspaces - Banach Algebra L<sup>1</sup> - Complex Homomorphisms.

#### HOLOMORPHIC FOURIER TRANSFORM UNIT V

Two theorems of Paley and Wiener - Quasi Analytic classes - Denjoy-Carleman Theorem.

#### OUTCOMES

- The students get introduced to the classical Banach spaces.
- The students will get good understanding of methods of decomposing signed measures which • has applications in probability theory and Functional Analysis.
- The students will be able to use measure theory for differentiation. •
- The students will get good understanding of Fourier Transform and its Holomorphic extensions.
- The students will be able to analyze Holomorphic Fourier Transforms. •

#### REFERENCES

- 1. Avner Friedman, "Foundations of Modern Analysis, Dover Publications, New York, 1982.
- 2. Rudin, W.," Real and Complex Analysis", McGraw-Hill International Edition, Third Edition, New York, 1987.

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#### **TOTAL: 60 PERIODS**

#### ADVANCED NUMBER THEORY AND CRYPTOGRAPHY

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#### Prerequisite: Number Theory

#### **OBJECTIVES:**

- To introduce congruences and solving congruences
- To introduce quadratic residues, Jacobi symbol and different important functions in number theory
- To introduce diophantine equations and Waring's problem
- To introduce traditional symmetric key ciphers
- To introduce asymmetric key cryptography

### UNIT I CONGRUENCES

Congruences, Solutions of congruences, congruences of deg 1, The function 0(n) - Congruences of higher degree, Prime power moduli, Prime modulus, congruences of degree 2, Prime modulus, Power residues.

#### UNIT II QUADRATIC RESIDUES

Quadratic residues, Quadratic reciprocity, The Jacobi symbol, greatest integer function, arithmetic function, The Mobius Inversion formula, The multiplication of arithmetic functions.

#### UNIT III DIOPHANTINE EQUATIONS

Diophantine equations, The equation ax + by = c, Positive solutions, Other linear Equations, Sums of four and five squares, Waring's problem, sum of fourth powers, sum of two Squares.

#### UNIT IV TRADITIONAL SYMMETRIC – KEY CIPHERS

Substitution Ciphers – Transportation Ciphers – Stream and Block Ciphers – Modern Block Ciphers – Modern Stream Ciphers – DES – AES.

#### UNIT V ASYMMETRIC KEY CRYPTOGRAPHY

RSA Cryptosystem – Rabin Cryptosystem – Elgamal Cryptosystem – Elliptic Curve Cryptosystem.

### TOTAL: 60 PERIODS

#### OUTCOMES:

- Students would have learnt to solve congruences and compute power residues
- Students will be able to apply quadratic reciprocity law and the Mobius inversion formula in cryptography
- The student will be able to solve diophantine equations and Waring's problem
- The student would have learnt about traditional and modern stream and Block ciphers
- The student will be equipped to deal with advanced crypto systems like elliptic curve cryptosystem

#### REFERENCES

- 1. Apostol T.M., "Introduction to analytic number theory", Narosa Publishing House, New Delhi, 1980.
- 2. Forouzan B.A., "Cryptography & Network Security", Tata McGraw Hill, Special Indian Edition, New Delhi, 2007.
- 3. Ireland K., Rosen M.,, "A Classical Introduction to Modern Number Theory", Springer International Edition, Second Edition, New York, 2010.
- 4. Koblitz, N., "A course in number theory and Cryptography", Springer Verlag, Second Edition, Washington, 1994.
- 5. Niven I., Zuckermann H.S., Montgomery H.L., "An Introduction to the Theory of Numbers", John Wiley, Fifth Edition, New York, 2006
- 6. Rose H.E., "A Course in Number Theory", Oxford University Press, Second Edition, Oxford, 2007.
- 7. Stinson D.R., "Cryptography: Theory and Practice", CRC Press, Third Edition, Boca Raton, 2002.

#### **ADVANCES IN GRAPH THEORY**

#### **Prerequisite: Graph Theory**

#### OBJECTIVE

- To introduce advanced topics in Graph Theory.
- To study Graph Theory based tools in solving practical problems.
- To understand the concepts of connectivity and colorings in Graphs.
- To understand the connection between Graph Theory and Geometry.
- To relate properties of Graphs with the spectral properties of associated matrices.

#### UNIT I CONNECTIVITY IN GRAPHS

Vertex connectivity – Edge connectivity – Blocks – k-connected and k-edge connected graphs – Network flow problems.

#### UNIT II COLORING OF GRAPHS

Vertex colorings and upper bounds – Brooks' theorem – Graphs with large chromatic number – Turan's theorem – Counting proper colorings – Edge colouring – Characterization of line graphs.

#### UNIT III PLANAR GRAPHS

Embeddings and Euler's formula – Dual graphs – Kuratowski's theorem – 5 colour theorem – Crossing number – Surface of higher genus.

#### UNIT IV MATCHINGS AND COVERS IN GRAPH

Maximum Matchings, Hall's Matching Condition, Min-Max Theorems, Independent Sets and Covers, Dominating Sets.

#### UNIT V EIGENVALUES OF GRAPHS

The characteristic polynomial – Linear algebra of real symmetric matrices – Eigenvalues and graph parameters – Eigenvalues of regular graphs – Strongly regular graphs. TOTAL: 60 PERIODS

#### OUTCOME

After successful completion of the course, students will be able to:

- investigate research problems in Graph Theory.
- write research papers on Graph Theory in a technical manner.
- apply Graph Theory based tools in solving practical problems.
- verify whether a given graph is planar.
- relate properties of Graphs with the spectral properties of associated matrices.

#### REFERENCES

- 1. Balakrishnan R. and Ranganathan K., "A textbook of Graph Theory", Springer, New York, 2012.
- 2. Biggs N., "Algebraic Graph Theory", Cambridge University Press, Second Edition, Cambridge, 2001.
- 3. Douglas B. West, "Introduction to Graph Theory", Prentice Hall of India, Second Edition, Chennai, 2002
- 4. Graham R.L., Rothschild B.L and Spencer J.H., "Ramsey Theory", Wiley Publishers, Second Edition, Hoboken, 1990.
- 5. Murthy U. S. R. and Bondy J. A., "Graph Theory", Pearson, London, 2015.

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#### ALGEBRAIC THEORY OF SEMIGROUPS

L T P C 4 0 0 4

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**TOTAL: 60 PERIODS** 

#### Prerequisite: An introductory course in Algebra

#### **OBJECTIVES:**

- To introduce the branch of Algebraic concepts developed on Semi groups
- To know about more classes of Semigroups,
- To introduce the concept of bands and variety of bands.
- To learn about inverse semigroup and inverse semigroup.
- To study more about auto uniform semi lattices.

#### UNIT I SEMIGROUPS

Monogenic semigroups – Ordered sets, semi lattices and lattices Binary relations, equivalences Congruences – Free semi groups – Ideals and Rees congruences. The equivalence L,R,H,J and D – The structure of D classes – Regular D-classes – Regular semi groups.

#### UNIT II SIMPLE SEMIGROUPS

Certain classes of semigroups – O-Simple semigroups – Principal factors – Primitive Idempotents – Congruences on completely simple O – semigroups.

#### UNIT III BANDS

Union of groups – semi lattice of groups – bands – free bands – varieties of bands.

#### UNIT IV INVERSE SEMIGROUPS AND SIMPLE INVERSE SEMIGROUPS 12

Inverse semigroups – Natural order relation on an inverse semi group – Congruence in Inverse semigroup – Bisimple inverse semigroups – Simple inverse semigroups.

#### UNIT V SEMI LATTICES

Fundamental inverse semigroups – auto uniform semi lattices.

#### OUTCOMES:

- Students would have learnt the basics of semi group theory
- Students would have got the the idea of congruences on semi group.
- Students will understand the relation of groups and lattices,
- Students will be knowledgeable about inverse semigroups.
- Students would have learnt about some important applications of semilattices.

#### REFERENCES

- 1. Gerhard O. Michler, "Theory of Finite Simple Groups", Cambridge University Press, Cambridge, 2006.
- 2. Howie, J.M., "An Introduction to Semigroup Theory", Academic Press, London, 1976.
- 3. John. M. Howie, "Fundamentals of Semigroup Theory", Oxford Science Publications, Oxford, 1996.

#### MX5006

#### **APPLIED COMBINATORICS**

#### LT P C 4 0 0 4

#### OBJECTIVE

- To introduce combinatorial techniques such as generating functions.
- To introduce Polya's Theorem to enumerate discrete combinatorial objects of a given type.
- To arrange elements of a finite set into patterns according to specified rules.
- To introduce codes and its applications in communication.
- To introduce combinatorial techniques to solve discrete optimization problems

#### **TOOLS OF COMBINATORICS** UNIT I

Generating permutations and combinations – Exponential generating function and generating function for permutations – Recurrence relation – Solving recurrence relation using generating function – Principle of inclusion and exclusion and its applications.

#### UNIT II POLYA THEORY OF COUNTING

Burnside's Lemma – Distinct colorings – Cycle index – Polya's theorem.

#### UNIT III **COMBINATORIAL DESIGNS**

Balanced incomplete block designs - Necessary condition for existence of (b, , r, k, ) designs. Resolvable designs – Steiner triple systems – Symmetric balanced incomplete block desians.

#### UNIT IV **CODING THEORY**

Encoding and decoding - Error correcting codes - Linear codes - Use of block designs to find error correcting codes.

#### UNIT V COMBINATORIAL OPTIMIZATION

Matching – Bipartite matching – System of distinct representatives – Algorithm for finding maximum matching - Networks - Maximum flow problem - The max flow algorithm - Complexity of max flow algorithm. **TOTAL : 60 PERIODS** 

### OUTCOME

After successful completion of the course, students will be able to:

- apply combinatorial techniques in design theory, coding theory and optimization problems.
- use generating functions to solve a variety of combinatorial problems. •
- apply Polya's Theorem to enumerate discrete combinatorial objects of a given type.
- use codes in communication problems.
- solve discrete optimization problems. •

### REFERENCES

- 1. Alan Tucker, "Applied Combinatorics", Wiley, Sixth Edition, Hoboken, 2012.
- 2. Daniel I. A. Cohen ,"Basic Techniques of Combinatorial Theory", John Wiley & Sons, New York, 1979.
- 3. Fred S. Roberts and Barry Tesman, "Applied Combinatorics", CRC Press, Second Edition, Boca Raton, 2009.
- 4. Peter J. Cameron," Combinatorics: Topics, Techniques, Algorithms", Cambridge University Press, First Edition, Cambridge, 1995.
- 5. Richard A. Brualdi, "Introductory Combinatorics", Pearson Education, Fifth Edition, New York, 2011.

#### MX5007

# **APPROXIMATION THEORY**

### LTPC 4004

### Prerequisite: A basic course in Analysis

### **OBJECTIVES**

- To introduce the basic concepts of approximation theory and its applications.
- To introduce the concept of Chebyshev polynomials.
- To introduce various interpolation methods.
- To introduce Characterization and Duality.
- To introduce projection and its various properties.

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### UNIT I APPROXIMATION IN NORMED LINEAR SPACES

Existence - Uniqueness - convexity - Characterization of best uniform approximations - Uniqueness results - Haar subspaces - Approximation of real valued functions on an interval.

#### UNIT II CHEBYSHEV POLYNOMIALS

Properties - More on external properties of Chebyshev polynomials - Strong uniqueness and continuity of metric projection - Discretization - Discrete best approximation.

#### UNIT III INTERPOLATION

Introduction - Algebraic formulation of finite interpolation - Lagrange's form - Extended Haar subspaces and Hermite interpolation - Hermite-Fejer interpolation.

#### UNIT IV BEST APPROXIMATION IN NORMED LINEAR SPACES

Introduction - Approximative properties of sets - Characterization and Duality.

#### UNIT V PROJECTION

Continuity of metric projections - Convexity, Solarity and Chebyshevity of sets - Best simultaneous approximation.

#### **TOTAL: 60 PERIODS**

#### OUTCOMES

- The course enables the students to gain better knowledge on topics like interpolation, best approximation and projection.
- The students gain knowledge in Chebyshev polynomials and its properties.
- The students will be able to approximate using different interpolation techniques.
- The students will get to know about approximating in normed linear spaces.
- The students will be able to know about projections and simultaneous approximation.

#### REFERENCES

- 1. Cheney E.W.," Introduction to approximation theory", Tata McGraw Hill Pvt. Ltd., New York, 1966.
- 2. Hrushikesh N.Mhaskar and Devidas V.Pai., "Fundamentals of approximation theory", Narosa Publishing House, New Delhi, 2000.
- 3. Singer I., "Best Approximation in Normed Linear Spaces by element of linear subspaces", Springer Verlag, Berlin, 1970.
- 4. Cheney E.W and Light W.A, "A course in approximation theory", American Mathematical Society, Rhode Island, 2009.

### MX5008

## Prerequisite: Complex Analysis

**BASIC HYPERGEOMETRIC SERIES** 

#### OBJECTIVES

- To introduce an extension of Beta, Gamma functions.
- To introduce Hypergeometric series and its properties.
- To introduce summation formula and also transformation formula.
- To introduce bilateral hypergeometric series.
- To develop the above on q-analogue and their applications on theta and elliptic functions.

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A Summation Formula of terminating very-well-poised Series – Watson's Transformation Formula for Terminating very-well-poised Series – Bailey Transformation Formula for Terminating Series – Twoterm transformation Formula.

#### **UNIT IV BILATERAL BASIC HYPERGEOMETRICSERIES**

Bilateral Basic Hypergeometric Series - Ramanujan's sum - Bailey's sum of a very-well-poised Series - Transformation Formula for an generalized bilateral series - A General Transformation Formula for a very-well-poised Series – Transformation Formulas for very-well-poised Series.

#### UNIT V THETA AND ELLIPTIC FUNCTIONS

Theta Functions – Elementary Properties – Zeros – Relation among Squares of Theta Functions -Pseudo Addition Theorem - Infinite Products - Elliptic Functions - Differential Equations - The Function sn(u), cn(u), dn(u) – Addition Theorem.

#### **OUTCOMES**

- The students have learnt the q-analogue along with an extension of Concepts of Beta, Gamma function and its application on elliptic and theta functions.
- The students will gain an understanding about hypergeometric and Q-hypergeometric series.
- The students will get to know summation and transformation formulae. •
- The students will get introduced to bilateral hypergeometric series.
- The students will be able to apply the above for theta and elliptic functions.

#### REFERENCES

- 1. Exton H., "Multiple Hypergeometric Functions and Applications", John Wiley and Sons, New York, 1976.
- 2. Gasper.G. and Rahman M., "Basic Hypergeometric Series, Encyclopedia of Mathematics and its Applications", Cambridge University Press, New York, 1990.
- 3. Rainville E.D., "Special Functions", Macmillan, New York, 1971.
- 4. Whittaker E. T., Watson G. N., "A Course of Modern Analysis", Cambridge University Press, Cambridge, 1996.

#### MX5009

#### BOUNDARY LAYER FLOWS

LTPC 4 0 0 4

#### **OBJECTIVES**

- To give a comprehensive overview of the boundary layer theory.
- To demonstrate the application of the theory to all areas of fluid mechanics with emphasis on the laminar flow past bodies.
- To enable the students derive the boundary layer equations and study their properties.
- To show how to obtain exact and approximate solutions for specific boundary layer flows.
- To enable the students understand the turbulent boundary layer flows.

A q-Analogue of Differentiation and Integration – Simple q-Differentiation and q-Integration Formulae - The q-Binomial Theorem - q-Exponential Functions - q-Analogue of Circular Functions - q-Gamma and q-Beta Functions.

#### **BASIC HYPERGEOMETRICSERIES** UNIT II

Basic Hypergeometric Series – Heine's Transformation Formula – Heine's q-Analogue of Gauss' Summation Formula - q-Analogue of Saalschiitz's Summation Formula - The Bailey- Daum Summation Formula - Generalized q-Hypergeometric Functions - well-poised, nearly-poised and very-well-poised Basic Hypergeometric Series.

#### SUMMATION AND TRANSFORMATION FORMULAS UNIT III

### **TOTAL: 60 PERIODS**

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### UNIT I DERIVATION AND PROPERTIES OF NAVIER-STOKES EQUATIONS

Equations of motion and continuity – Stress system – relation between stress and strain - Stokes hypothesis – Navier-Stokes equations – Derivation – Interpretation – Limiting case.

#### UNIT II EXACT SOLUTIONS

Hagen – Poiseuille theory – Flow between two concentric rotating cylinders – Couette Motion – Parallel flow – Other exact solutions.

#### UNIT III BOUNDARY LAYER EQUATIONS AND THEIR PROPERTIES

Derivation of boundary layer equations – Separation – Skin friction – Boundary layer along a flat plate – Characteristics of a boundary layer - Similar solutions – Transformation of the boundary layer equations – Momentum and integral equations.

#### UNIT IV EXACT AND APPROXIMATE METHODS

Exact solutions of boundary layer equations – Flow past a wedge - Flow past a cylinder – Approximate methods – Application of the momentum equation – Von Karman and Pohlhausen method – Comparison – Methods of boundary layer control.

#### UNIT V TURBULENT BOUNDARY LAYERS

Introduction – Turbulent flow – Mean motion and fluctuations – Apparent stresses – Derivation of the stress tensor – Fundamental equations of turbulent flows – Prandtl's mixing theory – Turbulent shearing stress.

#### OUTCOMES

At the end of the course, the students will be able to

- derive the governing equations of any flow problem.
- determine the exact solutions for flows in specific geometries.
- formulate the boundary layer flows and analyze their properties.
- solve the boundary layer flows using exact and approximate methods.
- formulate the turbulent boundary layer flows and study their properties.

#### REFERENCES

- 1. Batchelor G.K., "An Introduction to fluid dynamics", Cambridge University Press, Second Edition, Cambridge, 2000.
- 2. Schlichting H., "Boundary layer theory", Springer, Ninth Edition, Berlin, 2017.
- 3. Yuan S.W., "Foundations of fluid mechanics", Prentice-Hall, New Delhi, 1988.

### MX5010

#### CHARACTERISTIC CLASSES

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LTPC

# Pre-requisites: A basic course in Algebraic topology including homology and cohomology and basic knowledge of smooth manifolds

#### **OBJECTIVES:**

- To introduce the notion of a vector bundle
- To introduce the Stiefel-Whitney cohomology classes of a vector bundle through axiomatic definition and study their properties
- To prove the existence and uniqueness of Stiefel-Whitney classes
- To study about the Euler classes and the Chern classes
- To introduce Pontrjagin classes and the relations between Chern classes and Pontrjagin classes and Pontrjagin class

#### **TOTAL: 60 PERIODS**

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#### UNIT I **VECTOR BUNDLES**

Vector bundles, the tangent bundle and normal bundle of a smooth manifold, Euclidean vector spaces. Construction of new vector bundles from the old.

#### STIEFEL WHITNEY CLASSES UNIT II

Axiomatic definition of Stiefel Whitney classes, Whitney product theorem and Whitney duality theorem, parallelizability and immersion of projective spaces, Stiefel Whitney numbers and applications to cobordism.

#### EXISTENCE AND UNIQUENESS OF STIEFEL WHITNEY CLASSES UNIT III

Thom isomorphism and Stiefel Whitney classes, Grassmann manifolds and universal bundles, cell structure for Grassmann manifolds, mod 2 cohomology of infinite real Grassmann manifolds, uniqueness of Stiefel Whitney classes.

#### UNIT IV **CHERN CLASSES**

Euler classes, construction of Chern classes, the integral cohomology ring of an infinite complex Grassmannians, product theorem for Chern classes, dual bundles, total Chern class of the tangent bundle of complex projective space.

#### UNIT V **PONTRJAGIN CLASSES**

Complexification of a real vector bundle, Pontrjagin classes, Chern classes and Pontrjagin classes, Pontrjagin class and the Euler class, the cohomology of the oriented Grassmann manifold, Chern numbers, Pontrjagin numbers.

#### OUTCOMES

- Students would have learnt about vector bundles in detail
- Students would have gained knowledge about Steifel-Whitney classes, Stiefel-Whitney numbers and their applications
- Students would have learnt how to prove the existence of Stiefel-Whitney classes using Thom isomorphism and their uniqueness using the mod 2 cohomology ring of an infinite real Grassmannian
- Students would have learnt about the construction of Chern classes and their computations for some important bundles
- Students would have learnt about Pontrjagin classes, computation of the cohomology of oriented Grassmann manifolds and about Chern numbers and Pontrjagin numbers

#### REFERENCES

- 1. Husemoller D., Fibre Bundles, Springer, Third edition, New York, 1993.
- 2. Milnor J., Stasheff J.D., Characteristic classes, Princeton University Press, Vol. 16, Princeton, 2016.

#### MX5011

#### DIFFERENTIAL TOPOLOGY

LTPC 4 0 0 4

#### **Prerequisite:** Topology

#### **OBJECTIVES**

- To introduce smooth manifolds and to do calculus on manifolds.
- To introduce manifolds with boundary and intersection theory.
- To introduce the concept of orientation and oriented intersection theory.
- To introduce the Hopf degree theorem.
- To introduce the notion of smooth manifolds and classify compact one manifolds and smooth compact surfaces.

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**TOTAL: 60 PERIODS** 

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#### UNIT I MANIFOLDS AND MAPS

Derivatives and tangents-inverse function theorem and immersions-submersions -homotopy and stability-Sard's theorem and Morse functions-embedding manifolds in Euclidean space.

#### UNIT II TRANSVERSALITY AND INTERSECTION

Manifolds with boundary- one manifolds and some consequences - transversality -intersection theory modulo 2-winding numbers and the Jordan – Brouwer separation theorem.

#### **ORIENTED INTERSECTION THEORY** UNIT III

Orientation on manifolds - oriented intersection number-degrees of maps- fundamental theorem of algebra -Euler characteristic as an intersection number.

#### UNIT IV APPLICATIONS OF INTERSECTION THEORY

Lefschetz Fixed point theory -Borsuk Ulam theorem - vector fields- isotopy -Hopf degree theorem.

#### UNIT V COMPACT SMOOTH SURFACES

Morse functions, Morse Lemma, Connected sum, attaching handles, Handle decomposition theorem, Application to smooth classification of compact smooth surfaces.

### OUTCOMES

- Differential manifolds occur in different fields like mathematics, physics, mechanics and economics.
- A course in differential topology will equip the students with techniques and results required to solve problems involving manifolds.
- The students will gain an understanding of manifolds with boundary.
- The students will get more knowledge on orientation intersection theory.
- The students will gain a thorough understanding of Hopf Degree theorem.
- The students will be able to gain knowledge about smooth surfaces and application of their classification.

### REFERENCES

- 1. Guillemin V. and Pollack A., "Differential Topology", Prentice-Hall, Upper Saddle River, 1974.
- 2. Milnor J., "Topology from the differentiable view point, Princeton Landmarks in Mathematics", Princeton University Press, Rhode Island, 1997.
- 3. Morris W. Hirsch, "Differential topology", Springer-Verlag, New York, 1976.
- 4. Shastri A.R., "Elements of Differential Topology", CRC Press, Boca Raton, 2011.

### MX5012

### Prerequisite: Numerical Analysis

FINITE ELEMENT METHOD

### OBJECTIVES

- To introduce the integral formulations and variational methods of solving boundary value problems.
- To enable the students understand various steps in the finite element method of solution.
- To demonstrate finite element method to solve time-dependent problems in one-dimension.
- To discuss the finite element method to solve time-dependent problems in two-dimensions.
- To make the students analyze various measures of errors, convergence and accuracy of solution.

### **TOTAL: 60 PERIODS**

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### UNIT I INTEGRAL FORMULATIONS AND VARIATIONAL METHODS

Weighted integral and weak formulations of boundary value problems - Rayleigh-Ritz method - Method of weighted residuals.

#### UNIT II FINITE ELEMENT ANALYSIS OF ONE- DIMENSIONAL PROBLEMS 12

Discretization of the domain - Derivation of element equations - Connectivity of elements - Imposition of boundary conditions - Solution of equations.

### UNIT III EIGENVALUE AND TIME DEPENDENT PROBLEMS IN ONE DIMENSION 12

Formulation of eigenvalue problem - Finite element models - Applications of semi discrete finite element models for time-dependent problems - Applications to parabolic and hyperbolic equations.

## UNIT IV FINITE ELEMENT ANALYSIS OF TWO- DIMENSIONAL PROBLEMS

Interpolation functions - Evaluation of element matrices - Assembly of element equations - Imposition of boundary conditions - Solution of equations - Applications to parabolic and hyperbolic equations.

### UNIT V FINITE ELEMENT ERROR ANALYSIS

Interpolation Functions - Numerical Integration and Modeling Considerations - Various measures of errors - Convergence of solution - Accuracy of solution.

### TOTAL: 60 PERIODS

#### OUTCOMES

At the end of the course, the students will be able to

- construct integral formulations of boundary value problems.
- implement Finite Element Method for one-dimensional problems.
- formulate and solve eigenvalue problems and time-dependent problems in one-dimension.
- apply finite element method and solve time dependent problems in two-dimensions.
- perform the finite element error analysis.

#### REFERENCES

- 1. Buchanen G.R. and Rudhramoorthy R., "Finite Element Analysis", Schaum's Outline Series, Tata McGraw Hill, New Delhi, 2006.
- 2. Huttan D.V., "Fundamentals of Finite Element Analysis", Tata McGraw Hill, New Delhi, 2005.
- 3. Reddy J.N., "An Introduction to the Finite Element Method", Tata Mc-Graw Hill, Third Edition, New Delhi, 2005

#### MX5013

#### Prerequisite: Numerical Analysis

FINITE VOLUME METHOD

#### OBJECTIVES

- To introduce the ideas of conservation laws and governing equations of fluid flows.
- To demonstrate the finite volume method for diffusion and convection-diffusion problems.
- To present the solution algorithms for momentum equations.
- To exhibit the finite volume methods of solving unsteady flows.
- To extend the above ideas to problems in complex geometries.

#### UNIT I CONSERVATION LAWS AND BOUNDARY CONDITIONS

Governing equation of fluid flow: Mass - Momentum and Energy equations - Equation of state; Navier-Stokes equations for a Newtonian fluid - Conservative form of equations of fluid flow - Differential and integral forms of the transport equation - Classification of PDE's and fluid flow equations - Viscous fluid flow equations - Transonic and supersonic compressible flows.

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#### UNIT II FINITE VOLUME METHOD FOR DIFFUSION & CONVECTION-DIFFUSION PROBLEMS

FVM for Diffusion Problems: one-dimensional steady sate diffusion - Two-dimensional diffusion and three-dimensional diffusion problems; FVM for Convection-Diffusion problems: one-dimensional steady state convection- diffusion - central differencing schemes for one -dimensional convection-diffusion - Upwind differencing scheme - Hybrid differencing scheme -Higher-order differencing scheme for convection - Diffusion problems - TVD schemes.

#### UNIT III SOLUTION ALGORITHMS FOR PRESSURE-VELOCITY LINKED EQUATIONS

Staggered grid - momentum equations - SIMPLE, SIMPLER, SIMPLEC algorithms - PISO algorithm - Solution of discretised equation: Multigrid techniques.

#### UNIT IV FINITE VOLUME METHOD FOR UNSTEADY FLOWS

One-dimensional unsteady heat conduction: Explicit - Crank-Nicolson - fully implicit schemes - Implicit method for two and three dimensional problems - transient convection - Diffusion equation and QUICK differencing scheme - Solution procedures for unsteady flow calculations and implementation of boundary conditions.

### UNIT V METHOD WITH COMPLEX GEOMETRIES

Body-fitted co-ordinate grids for complex geometries - Cartesian Vs. Curvilinear grids -difficulties in Curvilinear grids - Block-structured grids - Unstructured grids and discretisation in unstructured grids - Discretisation of the diffusion term - Discretisation of convective term -Treatment of source terms - Assembly of discretised equations - Pressure-velocity coupling in unstructured meshes - Staggered Vs. co-located grid arrangements - Face velocity interpolation method to unstructured meshes.

#### TOTAL: 60 PERIODS

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#### OUTCOMES

At the end of the course, the students will be able to

- derive the conservation laws and governing equations of fluid flows.
- apply finite volume method for diffusion and convection-diffusion problems.
- solve momentum equations after discretizing.
- solve unsteady flow problems using the finite volume methods.
- apply finite volume methods to solve problems in complex geometries.

#### REFERENCES

- 1. Chung T.J., "Computational Fluid Dynamics", Cambridge University Press, Cambridge, 2002.
- 2. Ferziger J.H and Peric. M, "Computational methods for Fluid Dynamics", Springer, Third Edition, New Delhi, 2005.
- 3. Suhas V. Patankar, "Numerical Heat Transfer and Fluid Flow", Taylor & Francis, Boca Raton, 2009.
- 4. Versteeg H.K. and Malalasekera W. "An Introduction to Computational Fluid Dynamics: The Finite Volume Method", Pearson Education, Second Edition, New Delhi, 2008.

#### MX5014

## FIXED POINT THEORY AND ITS APPLICATIONS

L T P C 4 0 0 4

### **Prerequisite: Functional Analysis**

#### OBJECTIVES

- To identify all self-maps in which at least one element is left invariant.
- To introduce Brouwer fixed point theorem.
- To introduce fixed points for multivalued functions.
- To introduce more fixed point theorems and few operators.
- To introduce fixed point theorem for perturbed operators and their applications to differential equations.

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#### UNIT I THE BANACH FIXED POINT THEOREM AND ITERATIVE METHODS

The Banach fixed point theorem – The significance of Banach fixed point theorem – Applications to nonlinear equations - The Picard - Lindelof theorem - The Main theorem for iterative methods for linear operator equation – Applications to systems of linear equations and to linear integral equations.

#### UNIT II THE SCHAUDER FIXED POINT THEOREM AND COMPACTNESS

Extension theorem – Retracts – The Brouwer fixed point theorem – Existence principle for systems of equations - Compact operators - Schauder fixed - point theorem - Peano's theorem - Systems of Integral equations and semi linear differential equations.

#### FIXED POINTS OF MULTIVALUED MAPS UNIT III

Generalized Banach fixed point theorem - Upper and lower semi continuity of multi-valued maps -Generalized Schauder fixed point theorem – Variational inequalities and Browder fixed point theorem.

#### UNIT IV NON EXPANSIVE OPERATORS AND ITERATIVE METHODS

Uniformly convex Banach spaces - Demi closed operators - The fixed point theorem of Browder, Gohde and Kirk - Demi compact operators - Convergence principles in Banach spaces - Modified successive approximations – Applications to periodic solutions.

#### UNIT V **CONDENSING MAPS**

A non-compactness measure - Condensing maps - Operators with closed range and an approximation technique for constructing fixed points - Sadovskii's fixed point theorem for condensing maps – Fixed point theorem for perturbed operators – Application to differential equations in Banach spaces.

### OUTCOMES

- The student will be able to apply fixed point theory in various branches of applied mathematics.
- The student will get to know about fixed point theorem and its application to systems of integral equations.
- The student will be able to gain an in-depth understanding of various fixed point theorems.
- The student will get to know about approximation methods and applications to periodic solutions. ٠
- The student will get to know more about approximation techniques for constructing fixed points and their applications to differential equations in Banach spaces.

### REFERENCES

- 1. Deimling K., "Nonlinear Functional Analysis", Springer-Verlag, New York, 1985.
- 2. Istratescu V.L., "Fixed Point Theory: An Introduction", D. Reidel Publishing Company, Boston.1981.
- 3. Mohamed A. Khamsi & William A. Kirk, "An Introduction to Metric Spaces and Fixed Point Theory" John Wiley & Sons, New York, 2001.
- 4. Mohan C. Joshi, Ramendra K. Bose," Some Topics in Nonlinear Functional Analysis", John Wiley& Sons Australia, 1985.
- 5. Smart D.R., "Fixed Point Theory", Cambridge University Press, Cambridge, 1980.
- 6. ZeidlerE.,"NonlinearFunctionalAnalysisanditsapplications", Vol.1, Springer-Verlag, New York,1989.

### MX5015

#### FLUID MECHANICS

LTPC 4004

#### **OBJECTIVES:**

- To give a comprehensive overview of basic concepts of fluid mechanics.
- To introduce the concepts of kinematics and kinetics in fluid flows.
- To enable the students understand the two-dimensional flows in various geometries.
- To introduce the hydrodynamical aspects of conformal transformation. •
- To demonstrate various viscous fluid flows.

## **TOTAL: 60 PERIODS**

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#### UNIT I KINEMATICS OF FLUIDS IN MOTION

Real and Ideal fluids - Velocity - Acceleration - Streamlines - Path lines - Steady & unsteady flows -Velocity potential - Vorticity vector - Local and particle rates of change - Equation of continuity -Conditions at a rigid boundary.

#### UNIT II **EQUATIONS OF MOTION OF A FLUID**

Pressure at a point in a fluid – Boundary conditions of two inviscid immiscible fluids – Euler's equations of motion - Bernoulli's equation - Some potential theorems - Flows involving axial symmetry.

#### UNIT III TWO DIMENSIONAL FLOWS

Two-Dimensional flows – Use of cylindrical polar co-ordinates – Stream function, complex potential for two-dimensional flows, irrotational, incompressible flow - Complex potential for standard twodimensional flows -Two dimensional image systems - Milne-Thomson circle theorem - Theorem of Blasius.

#### UNIT IV CONFORMAL TRANSFORMATION AND ITS APPLICATIONS

Use of conformal transformations – Hydrodynamical aspects of conformal mapping – Schwarz Christoffel transformation - Vortex rows.

#### UNIT V **VISCOUS FLOWS**

Stress - Rate of strain - Stress analysis - Relation between stress and rate of strain - Coefficient of viscosity – Laminar flow – Navier-Stokes equations of motion – Some problems in viscous flow.

#### **TOTAL: 60 PERIODS**

#### **OUTCOMES**

At the end of the course, the students will be able to

- understand the concepts of kinematics and kinetics of fluid flows.
- derive the governing equations of fluid flows.
- solve the fluid flows in two-dimensional and axisymmetric geometries. •
- apply conformal transformation to fluid flows. •
- solve the viscous fluid flow problems in different geometries.

#### REFERENCES

- 1. Batchelor G.K., "An Introduction to Fluid Dynamics", Cambridge University Press, Cambridge, 2000
- 2. Frank Chorlton, "Textbook of Fluid Dynamics", CBS Publishers, New Delhi, 1985
- 3. Milne Thomson L.M., "Theoretical Hydrodynamics", Macmillan, New York, 1967.
- 4. White F.M., "Fluid Mechanics", McGraw-Hill, Seventh Edition, New York, 2011.
- 5. White F.M., "Viscous Fluid Flow", McGraw-Hill, New York, 1991.

#### MX5016

### FRACTIONAL DIFFERENTIAL EQUATIONS

LTPC 4 0 0 4

### **Prerequisite: Differential Calculus**

#### **OBJECTIVES**

- To introduce functions of fractional calculus.
- To introduce fractional derivatives and fractional integrals.
- To propose new methods to approximate Fractional differential equations solution. •
- To use the new method to approximate the solution of partial Fractional differential equations.
- To discuss the perturbation of the solution of Fractional differential equations.

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### UNIT I SPECIAL FUNCTIONS OF FRACTIONAL CALCULUS

Gamma Function - Mittag-Leffler Function – Wright Function.

### UNIT II FRACTIONAL DERIVATIVES AND INTEGRALS

Grunwald-Letnikov Derivatives – Riemann-Liouville Fractional Derivatives – Caputo's Fractional Derivatives – Properties of Fractional Derivatives – Laplace Transform of Grunwald-Letnikov, Riemann-Liouville and Caputo's Derivatives.

#### UNIT III LINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

Fractional Derivatives of a General Form – Existence and Uniqueness Theorems as Method of Solutions. Dependence of a solution on initial data.

#### UNIT IV FRACTIONAL GREEN'S FUNCTIONS

Definition and some properties. One-Term Equation – Two –Term Equation – Three Term Equation Four Term Equation – General n-term Equation.

#### UNIT V OTHER METHODS OF SOLUTIONS OF FRACTIONAL- ORDER EQUATIONS

The Mellin Transform Method – Power Series Method – Babenko's Symbolic Calculus Method – Method of Orthogonal Polynomials. Numerical Evaluation of Fractional Derivatives.

#### **TOTAL : 60 PERIODS**

#### OUTCOMES

After the completion of this course students can able to

- Explain the basic concepts of Fractional order derivatives.
- Obtain and Explain the Fundamental Definitions, Concepts, Theorems, Stability and Applications of Fractional Dynamical Systems.
- Gain Experience on Fractional order Differential Equations.
- Generalize, Emphasize and Apply the concept of Theory of Ordinary Differential Equations to the Fractional order Differential Equations.
- Interpret the Stability results and Applications of Fractional Dynamical Systems.

#### REFERENCES

- 1. Kelin Oldham and J. Spanier, "The fractional Calculus", Academic Press, New York, 1974.
- 2. Kilbass, A.A., H.M. Srivastava and J.J. Trujillo, "Theory and applications of Fractional differential equations", North-Holland mathematics Studies, 204, Elsevier, Amsterdam, 2006.
- 3. Miller, K.S., and B. Ross, "An Introduction to the Fractional Calculus and Fractional differential equations", John Wiley and Sons, New York, 1993.
- 4. Podlubny, I., "Fractional Differential Equations", Academic Press, New York, 1998

#### MX5017

# FUNCTIONAL ANALYSIS AND ITS APPLICATIONS TO L PARTIAL DIFFERENTIAL EQUATIONS 4

L T P C 4 0 0 4

#### Prerequisite: Functional Analysis and Partial Differential Equations

#### OBJECTIVES

- The aim of the course is to make the students understand the functional analytic concepts and techniques used in Partial Differential Equations.
- To introduce Sobolev Spaces and their properties.
- To find weak solutions to elliptic boundary value problems.
- To introduce finite element method and the analysis of the method.
- To introduce semigroups in Hilbert spaces..

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#### UNIT I **DISTRIBUTION THEORY**

Distributions - operations with distributions - support and singular support - convolutions fundamental solutions - Fourier transform - tempered distributions.

#### SOBOLEV SPACES **UNIT II**

Basic properties - approximation by smooth functions and consequences - imbedding theorems -Rellich- Kondrasov compactness theorem - fractional order spaces - trace spaces - dual spaces trace theory.

#### UNIT III WEAK SOLUTIONS OF ELLIPTIC EQUATIONS

Abstract variational results (Lax-Milgram lemma, Babuska- Brezzi theorem) - existence and uniqueness of weak solutions for elliptic boundary value problems (Dirichlet Neumann and mixed problems) - regularity results.

#### UNIT IV **GALERKIN METHODS**

Galerkin method - maximum principles - eigenvalue problems - introduction to the mathematical theory of the finite element method.

#### UNIT V **EVOLUTION EQUATIONS**

Unbounded operators - exponential map - C0-semigroups - Hille-Yosida theorem - contraction semigroups in Hilbert spaces - applications to the heat - wave and Schrodinger equations in homogeneous problems.

#### OUTCOMES

- The course, apart from providing a thorough understanding of the functional analytic concepts. and techniques used in partial differential equations, will enable them to solve the partial differential equations of various problems arising in Science and Engineering.
- The student will gain more understanding of Sobolev spaces, trace spaces etc.
- The student will be able to solve various elliptic boundary value problems.
- The student will be able to find solutions to partial differential equations through Galerkin's finite • element method.
- The student will be in a position to apply the technique to the heat wave problem.

#### REFERENCES

- 1. Evans L. C., "Partial Differential Equations, Graduate Studies in Mathematics" 19, AMS, University Press, Hyderabad, 2009.
- 2. Kesavan, S., "Topics in Functional Analysis and Applications", New Age International Ltd., New Delhi, 2008.
- 3. McOwen R.C., "Partial differential Equations", Pearson Education, New Delhi, 2003.

### MX5018

#### Prerequisite: Graph Theory

FUNDAMENTALS OF CHEMICAL GRAPH THEORY

#### **OBJECTIVES:**

- To study the connection between Chemical structures and Topological indices.
- To represent chemical compounds as molecular graphs.
- To study the various type of polynomials of molecular graphs. •
- To study a constructive algorithm for finding a structures and enumerating valence isomers.
- To study the elements of Graph Spectral Theory and Topological Resonance Theory.

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### **TOTAL : 60 PERIODS**

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#### UNIT I THE ORIGINS OF CHEMICAL GRAPH THEORY

The first use of Chemical Graphs – The emergence of Structure Theory – The concept of valence – The growth of Chemical Graph Theory – The introduction to Topological Indices – Elementary Bonding Theory.

#### UNIT II ELEMENTS OF GRAPH THEORY FOR CHEMIST

Some Graph Theoretical Terms – Connectedness of Graphs – Planarity of Graphs – Operations on Graphs – Matrix Representation of graphs – Distances in Graphs and Digraphs – Metric and Topological Spaces for simple graphs – Graphs in Quantum Chemistry.

#### UNIT III POLYNOMIALS IN GRAPH THEORY

On Chemical Applications of Graphic Polynomials - Polynomials - The Characteristic Polynomial -Matching Polynomial – More graphic polynomials.

#### UNIT IV ENUMERSATIONS OF ISOMERS

Introduction – Definitions and Mathematical background – Polya's theorem – Generalized polya theorem - Valence isomers - Polyhexes - Isomers and computer programme for their generations -Isomerism and Reaction Graphs.

#### UNIT V **GRAPH THEORY AND MOLECULAR ORBITALS**

Introduction - Elements of Graph Spectral Theory - Huckel Theory - Isomorphism of Huckel Theory and Graph Spectral Theory – Topological Resonance Theory.

#### OUTCOMES

- Students will be able to connect chemical structures and Topological indices.
- A knowledge for interpreting molecular structure as a graph can be achieved.
- Students should be able to derive polynomials for respective chemical graphs.
- Students could successfully construct algorithms for generating isomerism and reaction graphs.
- One can understand the application of spectral graph theory.

#### REFERENCES

- 1. Bonchev D. and Rouvray D.H, "Chemical Graph Theory: Introduction and Fundamentals", Abacus Press / Gordon & Breach Science Publishers, New York, 1991.
- 2. Douglas B. West, "Introduction to Graph Theory", Prentice Hall of India, New Delhi, 2002.
- 3. Trinajstic N., "Chemical Graph Theory", CRC Press, Volume I and II, Florida, 2000.

### MX5019

## Prerequisite: Fuzzy Set Theory

FUZZY ANALYSIS, UNCERTAINTY MODELING

AND APPLICATIONS

#### OBJECTIVES

- To impart knowledge in understanding the Applications of fuzzy relations.
- To give a clear picture about the Uncertainty Modeling.
- To get a clear understanding of the various applications of fuzzy sets both in Engineering and Management.
- To acquire knowledge in solving Scheduling, Inventory and Marketing.
- To obtain the most optimal solution for a problem with given constraints.

#### TOTAL : 60 PERIODS

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### UNIT I FUZZY RELATIONS, FUZZY GRAPHS AND FUZZY ANALYSIS

Fuzzy Relations on sets and Fuzzy Sets-compositions of Fuzzy Relations – Properties of the Min-Max composition- Fuzzy Graphs- Special Fuzzy Relations- Fuzzy functions on Fuzzy Sets- Extrema of fuzzy functions- Integration of fuzzy functions- Integration of a Fuzzy function over a crisp interval-Integration of a (crisp) Real-valued function over a Fuzzy interval- Fuzzy differentiation.

### UNIT II UNCERTAINTY MODELING

Application-Oriented Modeling of Uncertainty-Causes of Uncertainty- types of available information-Uncertainty theorems as transformers of information- Matching Uncertainty theory and Uncertain phenomena-Possibility theory-Fuzzy sets and Possibility Distribution-Possibility and necessity measures- Possibility of Fuzzy events-Probability of a Fuzzy event as a scalar-Probability of a Fuzzy event as a Fuzzy set -Possibility vs. Probability

UNIT IIIAPPLICATIONS OF FUZZY SETS IN ENGINEERING AND MANAGEMENT12Introduction-Engineering Applications- Linguistic Evaluation and Ranking of Machine Tools- FaultDetection in Gearboxes- Applications in Management- A Discrete Location Model- Fuzzy set Modelsin Logistics- Fuzzy Approach to the Transportation Problem- Fuzzy linear Programming in Logistics.

#### UNIT IV SCHEDULING, INVENTORY AND MARKETING

Fuzzy Sets in Scheduling- Job- Shop Scheduling with Expert System- A method to control Flexible Manufacturing Systems- Aggregate Production and Inventory Planning- Fuzzy Mathematical Programming for Maintenance Scheduling-Scheduling Courses, Instructors and Classrooms- Fuzzy set Models in Inventory Control- Fuzzy sets in Marketing- Customer Segmentation in Banking and Finance- Bank Customer Segmentation based on Customer Behavior.

### UNIT V EMPIRICAL RESEARCH IN FUZZY SET THEORY

Formal Theories vs. Factual Theories vs. Decision Technologies- Models in Operations Research and Management Science- Testing Factual Models- Empirical Research on Membership Functions- Type A- Membership Model- Type B- Membership Model- Empirical Research on Aggregators.

#### **TOTAL : 60 PERIODS**

#### OUTCOMES

- It helps the students to understand the applications of fuzzy relations.
- It gives the ability to perform the uncertainty modeling.
- It enables them to solve the real time engineering problems with uncertainty modeling.
- It sets up a base for techniques of solving scheduling, inventory and Marketing.
- It paves way to obtain the most optimal solution for a constrained problem.

#### REFERENCES

- 1. George J. Klir and Bo Yuan, "Fuzzy Sets and Fuzzy Logic: Theory and Applications", Prentice Hall of India Private Limited, New Delhi, 1995.
- 2. Zimmermann, H.J, "Fuzzy Set Theory and Its Applications", Kluwer Academic Publishers, Fourth Edition, Boston, 2001.

#### MX5020

#### FUZZY SETS AND APPLICATIONS

LT PC 4 0 0 4

#### OBJECTIVE

- To introduce the basic methods that are applicable to the construction of membership Functions of Fuzzy Sets and to the selection of appropriate operations on Fuzzy Sets.
- To cover fundamentals of reasoning based on approximate reasoning.
- To overview the applications of Fuzzy sets on varied topics such as fuzzy
- Neural Networks, fuzzy Automata and fuzzy dynamic systems.

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- To get exposed to the ideas of fuzzy clustering, pattern recognition and image processing.
- It paves way to adopt the methods which gives most optimal solution for decision making, Ranking and Linear Programming.

#### UNIT I CONSTRUCTING FUZZY SETS AND OPERATIONS ON APPROXIMATE REASONING

General Discussion – Methods of construction: An overview – Direct methods with one expert – Direct methods with multiple experts – Indirect methods with one expert - direct methods with multiple experts – Constructions from Sample Data

### UNIT II APPROXIMATE REASONING

Fuzzy Expert Systems : An overview – Fuzzy Implications – Selection of Fuzzy Implications –Multi conditional Approximate Reasoning – the Role of Fuzzy Relation Equations – interval – Valued Approximate Reasoning.

#### UNIT III FUZZY SYSTEMS

General Discussion – Fuzzy Controllers : An overview and an example-Fuzzy Systems and Neural Networks – Fuzzy Neural Networks – Fuzzy Automata – Fuzzy Dynamic Systems.

#### UNIT IV PATTERN RECOGNITION AND FUZZY DATABASES

Introduction – Fuzzy Clustering - Fuzzy Pattern Recognition – Fuzzy Image Processing – Fuzzy Databases – Fuzzy Information Retrieval.

#### UNIT V FUZZY DECISION MAKING

General Discussion – Individual Decision Making – Multiperson Decision Making – Multicriteria Decision Making – Fuzzy Ranking Methods – Fuzzy Linear Programming.

#### OUTCOMES

- It helps to acquire a strong foundations in the construction of Membership Functions of Fuzzy Sets.
- It helps the students to study the fundamentals of reasoning based on approximate reasoning.
- It gives a overview of the applications of Fuzzy sets on fuzzy Networks, fuzzy automata and fuzzy dynamic systems.
- It motivates the students to explore the concepts of fuzzy clustering, pattern recognition and image processing.

• It gives a better understanding of fuzzy decision making, Ranking and Linear Programming.

#### REFERENCES

- 1. George J. Klir and Bo Yuan, "Fuzzy Sets and Fuzzy Logic: Theory and Applications", Prentice Hall of India Private Limited, New Delhi,1995.
- 2. Zimmermann, H.J, "Fuzzy Set Theory and Its Applications", Kluwer Academic Publishers, Fourth Edition, Boston, 2001.

#### MX5021

#### OBJECTIVES

- To introduce the basic concepts of fuzzy sets and operations on fuzzy sets.
- To discuss the most successful application area of fuzzy systems such as the area of fuzzy control.

FUZZY SETS AND SYSTEMS

- To define Intuitionistic Fuzzy Sets and its properties.
- To extend the concept of Intuitionistic Fuzzy Sets to interval valued fuzzy sets.
- To explore some other extensions of Intuitionistic Fuzzy Sets.

#### TOTAL : 60 PERIODS

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### UNIT I CRISP SETS AND FUZZY SETS

Introduction - Crisp Sets: An Overview - The Notion of Fuzzy Sets - Classical Logic: An Overview-Fuzzy Logic. Operations of Fuzzy Sets: General Discussion - Fuzzy Complement - Fuzzy Union -Fuzzy Intersection - Combinations of Operations – General Aggregation Operations. Fuzzy Numbers: Belief and Plausibility measures – Probability measures – Possibility and Necessity measures.

#### UNIT II FUZZY SYSTEMS

General Discussion - Fuzzy Controllers: An Overview - Fuzzy Controllers: An Example - Fuzzy Systems and Neural Networks - Fuzzy Automata - Fuzzy Dynamic Systems. Pattern Recognition: Introduction - Fuzzy Clustering- Fuzzy Pattern Recognition - Fuzzy Image Processing. Applications: General Discussion - Natural, Life, and Social Sciences – Engineering – Medicine - Management and Decision Making - Computer Science - Systems Science - Other Applications.

#### UNIT III INTUITIONISTIC FUZZY SETS

Definition – Operations and Relations - Properties – Intuitionistic Fuzzy sets of a certain level - Cartesian product and Intuitionistic Fuzzy Relations - Necessity and Possibility Operators - Topological Operators.

#### UNIT IV INTERVAL VALUED INTUITIONISTIC FUZZY SETS

Intuitionistic Fuzzy Sets and Interval Valued Fuzzy Sets - Definition, Operations, and Relations on Interval Valued Intuitionistic Fuzzy Sets - Norms and Metrics on Interval Valued Intuitionistic Fuzzy Sets.

### UNIT V OTHER EXTENSIONS OF INTUITIONISTIC FUZZY SETS

Intuitionistic L-Fuzzy Sets - Intuitionistic Fuzzy Sets over Different Universes - Temporal Intuitionistic Fuzzy Sets - Intuitionistic Fuzzy Sets of Second Type - Some Future Extensions of Intuitionistic Fuzzy Sets.

#### OUTCOMES

- It gives the ability to analyse the various operations on fuzzy sets.
- It helps to visualize the applications of fuzzy sets in various fields.
- It motivates to have a concrete idea about fuzzy relations and Intuitionistic Fuzzy Sets.
- It sets a base to study the extension of Intuitionistic Fuzzy Sets in terms of interval valued fuzzy sets.
- It paves way for further extensions of Intuitionistic Fuzzy Sets.

#### REFERENCES

- 1. George J. Klir and Bo Yuan, "Fuzzy sets and fuzzy logic: Theory and Applications", Prentice Hall of India Private Limited, New Delhi, 2008.
- 2. Kaufmann,A.," Introduction to the Theory of Fuzzy Subsets", Vol. 1: Fundamental Theoretical Elements, Academic Press, New York,1975.
- 3. Krassimir T Atanassov, "Intuitionistic Fuzzy Sets: Theory and Applications" , Physica Verlag, Heidelberg, 1999.
- 4. Zimmermann, H.J., "Fuzzy Set Theory—and Its Applications", Kluwer Academic Publishers, Boston, 2001.

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### TOTAL : 60 PERIODS

#### GENERALIZED INVERSES

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TOTAL: 60 PERIODS

#### Prerequisite: Linear Algebra

#### OBJECTIVES

- To acquaint the students with various techniques of generalized inverses related with optimal and spectral theory.
- To introduce the concept of generalized inverses and their applications.
- To introduce extremal property of inverses.
- To introduce various spectral inverses.
- To develop generalized inverses of partitioned matrices.

### UNIT I EXISTENCE AND CONSTRUCTION OF GENERALIZED INVERSES 12

The penrose equations – Existence and construction of generalized inverses – Properties - Full rank factorizations – Explicit formula for Moore – Penrose inverse of a matrix.

#### UNIT II LINEAR SYSTEMS AND CHARACTERIZATION OF GENERALIZED INVERSES 12

Solution of linear systems – Characterization of classes of generalized inverses - Generalized inverses and orthogonal projectors – Application of Generalized inverses in Interval Linear Programming.

#### UNIT III MINIMAL PROPERTIES OF GENERALIZED INVERSES

Least - squares solutions of inconsistent linear systems – Solutions of minimum norm – Extremal property of the Bott-Duffin inverse with application to electrical Network.

#### UNIT IV SPECTRAL GENERALIZED INVERSES

Introduction – The group inverse – Spectral properties of the group inverse – The Drazin inverse Spectral properties of the Drazin - Inverse – Other spectral generalized inverses.

#### UNIT V GENERALIZED INVERSES OF PARTITIONED MATRICES

Introduction – Partitioned matrices and linear equations – Generalized inverses of partitioned matrices and bordered matrices.

#### OUTCOMES

- The students are expected to have good knowledge of generalized inverses which will be helpful for research in this field.
- The students will have a thorough understanding of generalized inverses and their applications in interval linear programming.
- The students will be able to apply extremal properties to electrical networks.
- The students will gain the knowledge about spectral inverses and their properties.
- The students will get to know about generalized inverses of partitioned matrices.

#### REFERENCES

- 1. Ben-Israel A., and Greville T.N.E., "Generalized Inverses: Theory and Applications", Springer Verlag, Second Edition, New York, 2003.
- 2. Nashed M.Z., "Generalized Inverses and Applications", Academic Press, New York, 1976.
- 3. Rao C.R. and Mitra S. K., "Generalized inverses of Matrices and its Applications", John Wiley, New York, 1971.

#### HARMONIC ANALYSIS

### OBJECTIVES

MX5023

- The aim of the course is to make the students to understand the basic concepts of Harmonic Analysis.
- To introduce Fourier series and Fourier integrals.
- To give an introduction to Hardy spaces.
- To introduce conjugated functions and the related theorems.
- To introduce convolution theorem.

### UNIT I FOURIER SERIES

Basic properties of topological groups, subgroups, quotient groups and connected groups. Discussion of Haar Measure without proof on R, T, Z, and some simple matrix groups.  $L^1$  (G) and convolution with special emphasis on  $L^1(R)$ ,  $L^1(T)$ ,  $L^1(Z)$ . Approximate identities. Fourier series. Fejer's theorem.

### UNIT II FOURIER INTEGRALS

The classical kernels. Fejer's Poisson's and Dirichlet's summability in norm and point wise summability. Fatou's theorem. The inequalities of Hausdorff and Young. Examples of conjugate function series. The Fourier transform. Kernels on R. The Plancheral theorem on R. Plancheral

measure on R, T, Z. Maximal ideal space of  $L^1(R)$ ,  $L^1(T)$ ,  $L^1(Z)$ .

### UNIT III HARDY SPACES

Hardy spaces on the unit circle, Invariant subspaces. Factoring. Proof of the F. and M. Riesz theorem. Theorems of Beurings and Szego in multiplication operator form. Structure of inner and outer functions. The inequalities of Hardy and Hilbert.

#### UNIT IV MAXIMAL FUNCTIONS

Conjugate functions. Theorems of Kolmogorov & Zygmund and M. Rieze & Zygmund on conjugate functions. The conjugate function as a singular integral. Statement of the Burkholder- Gundy Silverstein Theorem on T. Maximal functions of Hardy and Littlewood translation.

#### UNIT V WIENER TAUBERIAN THEOREM

The Theorems of Wiener and Beurling. The Titchmarsh Convolution Theorem. Wiener's Tauberian theorem. Spectral sets of bounded functions.

#### OUTCOMES

- The students will have good understanding of Fourier series and intricacies of convergence.
- The students will be able to understand Fourier integrals and their properties.
- The students will gain knowledge of Hardy spaces and the inequalities of Hardy and Hilbert.
- The students will get to know about conjugate function as a singular integral.
- The student will be able to understand the intricacies of Wiener Tauberian Theorem and invariant subspace problem.

#### REFERENCES

- 1. Henry Helson, "Harmonic Analysis", Hindustan Book Agency, Second Edition, Gurgoan, 2010.
- 2. Hewitt E. and Ross K.A., "Abstract Harmonic Analysis", Springer-Verlag, Vol. 1, Fourth Edition, Berlin, 1993.
- 3. Paul Koosis, "Introduction of H<sub>p</sub> spaces", Cambridge University Press, Second Edition, Cambridge, 1999.
- 4. Yitzhak Katznelson., "An introduction to Harmonic Analysis", Cambridge University Press, Third Edition, Cambridge, 2004.

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**TOTAL : 60 PERIODS** 

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### HEAT AND MASS TRANSFER

#### **OBJECTIVES**

MX5024

- To enable the students understand the concepts of heat and mass transfer and its applications.
- To demonstrate the properties of heat conduction in solving heat equations.
- To introduce the methods of solving flow along surfaces and in channels. •
- To familiarize the students with the properties of free and forced convection in laminar flows
- To present the basic ideas of mass transfer in real life problems.

#### UNIT I **HEAT CONDUCTION**

Concept of Heat conduction - Fundamental law of heat conduction - Steady state heat conduction -Unsteady heat conduction – Numerical solutions of heat conduction equations.

#### UNIT II FLOW ALONG SURFACES AND IN CHANNELS

Boundary layers and turbulence - momentum equation- laminar flow boundary layer equation plane plate in longitudinal flow – pressure gradients along a surface – exact solutions for a flat plate.

#### **FREE CONVECTION** UNIT III

Laminar heat transfer on a vertical plate and horizontal tube - turbulent heat transfer on a vertical plate - free convection in a fluid enclosed between two plane walls - mixed free and forced convection.

#### **UNIT IV** FORCED CONVECTION IN LAMINAR FLOW

Heat flow equation - energy equation - plane plate in longitudinal flow - arbitrarily varying wall temperature – exact solutions of energy equation.

#### UNIT V MASS TRANSFER

Diffusion – flat plate with heat and mass transfer – integrated boundary layer equations of mass transfer - similarity relations for mass transfer - evaporation of water into air.

### **OUTCOMES**

At the end of the course, the students will be able to

- solve heat conduction problems and obtain numerical solutions.
- solve the problems of flow along surfaces and in channels.
- apply finite element method to solve complex problems in free and forced convection flows. •
- solve the heat flow equations in various situations with different boundary conditions.
- analyze the mass transfer properties in various fluid flow problems.

### REFERENCES

- 1. Ebhart B., "Heat transfer ", McGraw Hill Publishing Co., Second Edition, New York, 1971.
- 2. Eckert E.R.G., and Drake R.M., "Heat and mass transfer", Tata McGraw Hill Publishing Co., Second Edition, New York, 1979.
- 3. Cengel Y.A., "Heat Transfer", Mc Graw Hill, Second Edition, New York, 2003.
- 4. Schlichting H., "Boundary Layer Theory", McGraw Hill Publishing Co., Ninth Edition, 2017.



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**TOTAL: 60 PERIODS** 

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#### HOMOLOGY AND COHOMOLOGY

#### Pre-requisites: A basic course in Algebraic Topology

#### **OBJECTIVES:**

MX5025

- To learn about different types of complexes, homology and its properties
- To exhibit computation of homology for certain spaces
- To introduce the notion of cohomology, a dual concept of homology
- To introduce a product operation on cohomology and make it a ring
- To explain the Poincare duality theorem

#### UNIT I SIMPLICIAL AND SINGULAR HOMOLOGY

-complexes, simplicial complexes, simplicial homology, singular homology, Homotopy invariance, exact sequences and excision, Equivalence of simplicial and singular homology

#### UNIT II COMPUTATIONS, APPLICATIONS, FORMAL VIEW POINT

Degree, cell complexes, cellular homology, Mayer-Vietoris sequences, homology with coefficients, Axioms for homology, homology and fundamental group, simplicial approximation, Lefschetz fixed point theorem

#### UNIT III COHOMOLOGY GROUPS

The Universal coefficient theorem, Cohomology of spaces, Axioms for cohomology, simplicial cohomology, cellular cohomology, Mayer-Vietoris sequences

#### UNIT IV CUP PRODUCT

Cup product, the cohomology ring, Kunneth formula, spaces with polynomial cohomology.

### UNIT V POINCARÉ DUALITY

Orientations and homology, Poincaré duality theorem, connection with cup product, Lefschetz duality, Alexander duality.

#### OUTCOMES:

- The students would have learnt about simplicial homology and singular homology theories
- Students would have obtained the skill to compute the homology for certain spaces and the connection between homology and the fundamental group of a topological space
- Students would have learnt how to compute the cohomology of spaces and the Universal coefficient theorem for cohomology
- Students would have learnt the Kunneth theorem and seen examples of important spaces whose cohomology rings are polynomial rings
- Students would have learnt about different duality theorems

#### **REFERENCES:**

- 1. Hatcher A., Algebraic Topology, Cambridge University Press, Cambridge, 2002.
- 2. Mac Lane S., Homology, Springer-Verlag, New York, 1995.
- 3. Spanier E.H, Algebraic topology, Springer-Verlag, paper-back, New York, 1994.
- 4. Vick J. W., Homology Theory: An Introduction to Algebraic Topology, Graduate Texts in Mathematics, Springer-Verlag, 2<sup>nd</sup> Edition, New York, 1994.

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**TOTAL: 60 PERIODS** 

#### INTRODUCTION TO ALGEBRAIC TOPOLOGY

#### **OBJECTIVES:**

- To introduce the notion of homotopy and some geometric constructions
- To introduce the notion of the fundamental group of a space and to see some applications
- To introduce Van Kampen's theorem and see some of its applications
- To introduce covering spaces and to understand the connection between covering spaces and the fundamental group of the base space of a covering space
- To learn about the classification of covering spaces

#### UNIT I HOMOTOPY AND SOME GEOMETRIC CONCEPTS

Homotopy and Homotopy type, contractible spaces, retraction and deformation, cell complexesoperations on cell complexes-criteria for homotopy equivalence, Homotopy extension property.

#### UNIT II THE FUNDAMENTAL GROUP

Fundamental groups, the Fundamental group of the circle, applications: fundamental theorem of algebra, Brouwer fixed point theorem in dimension 2, Borsuk-Ulam theorem in dimension 2, Fundamental group of a product, induced homomorphisms-applications.

#### UNIT III VAN KAMPEN'S THEOREM

Free product of groups, Van Kampen theorem, simple applications, applications to cell complexesconstruction of a K(G,1) space

#### UNIT IV COVERING SPACES

Covering projections, relations with fundamental group, the homotopy lifting property, the lifting problem.

#### UNIT V MORE ON COVERING SPACES

universal covering space, the classification of covering spaces, group actions, deck transformation groups

- OUTCOMES:
  - The students would have learnt about homotopy between maps and homotopically equivalent spaces
  - Students would have learnt some important applications of fundamental groups including the Brouwer fixed point theorem and Borsuk-Ulam theorem in dimension 2
  - Students will be able to compute fundamental groups of spaces using Van Kampen's theorem
  - Students would have understood about the lifting problems for covering spaces
  - The students will have an understanding of universal covering spaces, group actions and deck transformation groups

#### REFERENCES

- 1. Hatcher A., Algebraic topology, Cambridge University Press, New York, 2002.
- 2. Rotman J.J., An introduction to algebraic topology, Graduate text in Mathematics 119, Springer-Verlag, New York, 1988.
- 3. Spanier E.H., "Algebraic topology", Springer-Verlag, paper-back, New York, 1994.

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TOTAL: 60 PERIODS

#### **INTRODUCTION TO FIBRE BUNDLES**

#### Pre-requisites: A course in algebraic topology, knowledge of differentiable manifolds

#### **OBJECTIVES:**

- To introduce the notion of a bundle
- To introduce and study about vector bundles
- To learn what is a fibre bundle and fibre bundles with structure group
- To learn about the restriction and prolongation of structure group for fibre bundles
- Using fibres bundles to study about the homotopy groups and classifying spaces for the classical groups.

#### UNIT I THE GENERALITIES ON BUNDLES

Bundles and cross sections, morphisms of bundles, Products and fibre products, restrictions of bundles and induced bundles.

#### UNIT II VECTOR BUNDLES

Vector bundles, morphism of vector bundles, induced vector bundles, Homotopy properties of vector bundles, Gauss maps, Functorial description of the Homotopy classification of vector bundles.

#### UNIT III GENERAL FIBRE BUNDLES

Bundles defined by transformation groups, Principal bundles, induced bundles of principal bundles, Fibre bundles, Numerable principal bundles, Milnor construction, Homotopy classification of Principal G-bundles over CW-complexes.

#### UNIT IV CHANGE OF STRUCTURE GROUP IN FIBRE BUNDLES

Fibre bundles with homogeneous spaces as fibres, Prolongation and restriction of Principal Bundles, restriction and prolongation of structure group for fibre bundles, Classifying spaces and reduction of structure group.

#### UNIT V CALCULATION INVOLVING CLASSICAL GROUPS

Stiefel varieties, Grassmann manifolds, Stability of the Homotopy groups of the classical groups, Vanishing of lower Homotopy groups of Stiefel varieties, Universal bundles and classifying spaces for the classical groups.

#### OUTCOMES:

- Students would have gained knowledge about the generalities of bundles
- Students will be able to apply homotopy classification of vector bundles
- Students would have learnt about the Milnor construction of a Universal principal bundle
- Students would have learnt about classifying spaces and reduction of structure group for a fibre bundle
- Students would have learnt, using fibre bundles, how to get certain results about the classical groups

#### **REFERENCES:**

- 1. Husemoller D., Fibre Bundles, Springer, Third Edition, New York, 1994.
- 2. Steenrod N., The topology of Fibre bundles, Princeton University Press, Princeton, 1999.

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**TOTAL: 60 PERIODS** 

MX5028

#### INTRODUCTION TO LIE ALGEBRAS

### Pre-requisites: A basic course in abstract algebra and linear algebra

#### **OBJECTIVES:**

- To introduce the notion of a Lie algebra with suitable examples
- To introduce the notion of a subalgebra, ideal of a Lie algebra and homomorphism between Lie algebras
- To introduce some special types of Lie algebras like solvable and nilpotent Lie algebras
- To learn about semi-simple Lie algebras
- To understand the maximal toral subalgebras and root system for a semisimple Lie algebra

#### UNIT I LIE ALGEBRAS

The notion of a Lie algebra, Linear Lie algebras, Lie algebras of derivations, abstract Lie algebras

#### UNIT II **IDEALS AND HOMOMORPHISMS**

Ideals, homomorphisms and representations, adjoint representation, automorphisms, inner automorphisms

#### UNIT III SOLVABLE AND NILPOTENT LIE ALGEBRAS

Solvability, nilpotency, Engel's theorem, Lie's theorem, Jordan Chevalley decomposition, Cartan's criterion.

#### **UNIT IV** SEMI-SIMPLE LIE ALGEBRAS

semi-simple Lie algebras, Killing form, complete reducibility of representations, representations of sl(2,F).

#### UNIT V ROOT SPACE DECOMPOSITION OF A SEMI-SIMPLE LIE ALGEBRA

Maximal toral subalgebras and roots, root space decomposition, orthogonality properties, integrality properties and rationality properties.

#### OUTCOMES:

- Students would have learnt the basic axioms defining a Lie algebra •
- Students will be knowledgeable about the representations of a Lie algebra
- Students would have learnt important results like Lie's theorem and Engel's theorem
- They would have a thorough understanding of basic properties of semi-simple Lie algebras
- Students will have a good understanding of the root space decomposition of a semi-simple Lie • algebra

#### REFERENCES

- 1. Carter R., Segal G., Macdonald I., Lectures on Lie Groups and Lie Algebras, Mathematical Society Students Texts 32, Cambridge University Press, South Asian Edition, London, 2010.
- 2. Erdmann K., Wildon M. J., Introduction to Lie Algebras, Springer-Verlag London, 2006.
- 3. Hall B., Lie Groups, Lie Algebras, and Representations. An Elementary Introduction, Graduate Texts in Mathematics, Springer-Verlag, Berlin, 2015.
- 4. Humphreys J. E., Introduction to Lie Algebras and Representation Theory, Graduate Texts in Mathematics 9 , Springer-Verlag, New York, 1997.

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**TOTAL: 60 PERIODS** 

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MX5029

#### MATHEMATICAL ASPECTS OF FINITE ELEMENT METHOD

#### Pre-requisites: A basic course in Functional Analysis

#### OBJECTIVES

- The aim of the course is to make the students understand the mathematical aspects of finite element method required for solving partial differential equations.
- To introduce Sobolev Spaces and their properties.
- To introduce the concept of variational formulation of elliptic and parabolic boundary value problems.
- To introduce various element and approximation property.
- To introduce higher dimensional variational problems.

#### UNIT I BASIC CONCEPTS

Weak formulation of Boundary Value Problems - Ritz-Galerkin approximation - Error Estimates - Piecewise polynomial spaces - Finite Element Method - Relationship to Difference Methods - Local Estimates.

#### UNIT II SOBOLEV SPACES

Review of Lebesgue integration theory - Weak derivatives - Sobolev norms and associated spaces - Inclusion relations and Sobolev's inequality - Trace Theorems - Negative norms and duality.

#### UNIT III VARIATIONAL FORMULATIONS

Review of Hilbert spaces - Projections onto subspaces and Riesz representation theorem - Symmetric and non-symmetric variational formulation of elliptic and parabolic boundary value problems - Lax-Milgram Theorem - Error estimates for General Finite Approximation.

# UNIT IV CONSTRUCTION OF FINITE ELEMENT SPACE AND APPROXIMATION THEORY IN SOBOLEV SPACES

The Finite Element - Triangular finite elements - Lagrange element - Hermite element, Rectangular elements - Interpolant - Averaged Taylor polynomials - Error representation - Bounds for the Interpolation error - Inverse estimates.

#### UNIT V HIGHER DIMENSIONAL VARIATIONAL PROBLEMS

Higher-dimensional examples - Variational formulation and approximation of Poisson's and Neumann equations - Coercivity of the variational problem - Elliptic regularity estimates - Variational approximations of general Elliptic and Parabolic problems.

#### OUTCOMES

- The students will be in position to tackle complex problems involving partial differential equations arising in the mathematical models of various problems in Science and Engineering by finite element techniques.
- The student will gain more understanding of Sobolev spaces.
- The student will have more knowledge about variational formulations of elliptic and parabolic boundary value problems.
- The student will get to know about different finite elements and be able to find error estimates for different methods.

• The student will be able to extend the knowledge to higher dimensional variational problems.

#### REFERENCES

- 1. Brenner S. and Scott R., "The Mathematical Theory of Finite Element Methods", Springer-Verlag, New York, 2007.
- 2. Ciarlet P.G., "The Finite Element Methods for Elliptic Problems", North Holland, Amsterdam, 1978.
- 3. Claes Johnson, "Numerical Solutions of Partial Differential Equations by the Finite Element Method", Cambridge University Press, Cambridge,1987.

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4. Thomee V., "Galerkin Finite Element Methods for Parabolic Problems", Lecture Notes in Mathematics, Vol.1054, Springer-Verlag, Berlin, 1997.

### MATHEMATICAL FINANCE

#### **OBJECTIVES:**

MX5030

- To understand the basic probability concepts in association with random variables and significance of the Central Limit theorem with respect to the Brownian motion.
- To understand the basic concepts of present value and accumulated value and apply these concepts toward solving more complicated financial problems and complex annuity problems.
- To appreciate the Arbitrage theorem in the context of the Black Scholes formula.
- To obtain a practical knowledge on the Portfolio selection problem
- To understand option pricing with respect to various options via multi-period binomial models.

#### UNIT I PROBABILITY AND RANDOM VARIABLES

Probability and Events - Conditional probability - Random Variables and Expected values - Covariance and Correlation - Normal Random Variables - Properties of Normal Random Variables - Central Limit theorem - Geometric Brownian Motion as a limit of simpler models - Brownian motion.

#### UNIT II PRESENT VALUE ANALYSIS AND ARBITRAGE

Interest rates - Present value analysis - Rate of return - Continuously varying interest rates - Pricing contracts via Arbitrage - An example in options pricing.

#### UNIT III ARBITRAGE THEOREM AND BLACK-SCHOLES FORMULA

The Arbitrage theorem – Multi-period binomial model - Black-Scholes formula - Properties of Black - Scholes option cost - Delta Hedging Arbitrage Strategy - Pricing American put options.

#### UNIT IV EXPECTED UTILITY

Limitations of arbitrage pricing - Valuing investments by expected utility - The portfolio section problem - Capital assets pricing model - Rates of return - Single period and geometric Brownian motion.

#### UNIT V EXOTIC OPTIONS

Barrier options - Asian and look back options - Monte Carlo Simulation - Pricing exotic option by simulation - More efficient simulation estimators - Options with non-linear pay offs - pricing approximations via multi-period binomial models.

#### OUTCOME

- To demonstrate a comprehensive understanding of the probability concepts
- To locate and use information to solve problems in interest theory and financial engineering
- To know the main features of models commonly drawn from industry and financial firms in order to explore arbitrage strategy
- To understand and appraise utility and effectiveness in option pricing
- To simulate appropriate models treating Exotic options

#### REFERENCES

- 1. Sheldon M. Ross, "An Elementary Introduction to Mathematical Finance", Cambridge University Press, 3<sup>rd</sup> Edition, Cambridge, 2011.
- 2. Steven Roman, "Introduction to the Mathematics of finance", Springer-Verlag New York, 2<sup>nd</sup> Edition, 2012.
- 3. Williams, R.J., "Introduction to the Mathematics of finance", AMS, Universities Press Pvt. Ltd, India, 2006.

#### TOTAL: 60 PERIODS

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### MATHEMATICAL STATISTICS

### **OBJECTIVES:**

MX5031

- To understand the basic concepts of sampling distributions and statistical properties of point and interval estimators.
- To apply the small/ large sample tests through Tests of hypothesis.
- To understand the correlation and regression concepts in empirical statistics.
- To understand the concept of analysis of variance and use them to investigate factorial dependence
- To appreciate the classical multivariate methods and computational techniques.

### UNIT I SAMPLING DISTRIBUTIONS AND ESTIMATION THEORY

Sampling distributions - Characteristics of good estimators - Method of Moments - Maximum Likelihood Estimation - Interval estimates for mean, variance and proportions.

#### UNIT II TESTING OF HYPOTHESIS

Type I and Type II errors - Tests based on Normal, t, <sup>2</sup> and F distributions for testing of mean, variance and proportions - Tests for Independence of attributes and Goodness of fit.

#### UNIT III CORRELATION AND REGRESSION

Method of Least Squares - Linear Regression - Normal Regression Analysis - Normal Correlation Analysis - Partial and Multiple Correlation - Multiple Linear Regression.

#### UNIT IV DESIGN OF EXPERIMENTS

Analysis of Variance - One-way and two-way Classifications - Completely Randomized Design - Randomized Block Design - Latin Square Design.

### UNIT V MULTIVARIATE ANALYSIS

Mean Vector and Covariance Matrices - Partitioning of Covariance Matrices - Combination of Random Variables for Mean Vector and Covariance Matrix - Multivariate, Normal Density and its Properties - Principal Components: Population principal components - Principal components from standardized variables.

#### OUTCOMES:

On successful completion of this course students will be able to:

- Demonstrate knowledge of, and properties of, statistical models in common use.
- Apply the basic principles underlying statistical inference (estimation and hypothesis testing)
- Be able to construct tests and estimators, and derive their properties
- Demonstrate knowledge of applicable large sample theory of estimators and tests.
- Recognize the importance of Multivariate analysis in various practical application.

#### REFERENCES

- 1. Devore J.L. "Probability and Statistics for Engineering and the Sciences", Cengage Learning, 8<sup>th</sup> Edition, New Delhi, 2012.
- 2. Gupta S.C. and Kapoor V.K.," Fundamentals of Mathematical Statistics", Sultan Chand & Sons, 11<sup>th</sup> Edition, New Delhi, 2019.
- 3. Johnson R.A. and Wichern D.W., "Applied Multivariate Statistical Analysis", Pearson, 6<sup>th</sup> Edition, Chennai, 2008.
- 4. Miller I. and Miller M., "John E. Freund's Mathematical Statistics with Applications", Pearson, 8<sup>th</sup> Edition, Chennai, 2014.

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**TOTAL: 60 PERIODS** 

#### MX5032

#### **MEASURE THEORY**

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TOTAL : 60 PERIODS

#### Pre-requisite: Real Analysis

#### **OBJECTIVES**

- To gain understanding of the abstract measure theory and definition and main properties of the integral. To construct Lebesgue's measure on the real line and in *n*-dimensional Euclidean space.
- To explain the basic advanced directions of the theory.
- To introduce the completeness and convergence in measures.
- To introduce the concept of signed measures.
- To introduce the product measures.

#### UNIT I MEASURES ON THE REAL LINE

Lebesgue Outer Measure- Measurable sets – Regularity – Measurable functions-Borel and Lebesgue measurability-Hausdorff measures

#### UNIT II ABSTRACT MEASURES SPACES

Measures and outer measures-Extension of a measure-Uniqueness of the extension- Completion of a measure- Integration with respect to a measure.

#### UNIT III CONVERGENCE

L<sup>p</sup>spaces-completeness-Convergence in measure-Almost Uniform convergence

#### UNIT IV SIGNED MEASURES

Hahn-Jordan Decompositions-Radon-Nikodym theorem- Applications.

#### UNIT V MEASURES IN PRODUCT SPACES

Measurability in a product space- Product measures-Fubini's Theorem-Lebesgue measure in Euclidean space-Laplace and Fourier Transforms.

#### OUTCOMES

- The student learns the concepts of measure and integral with respect to a measure, to show their basic properties, and to provide a basis for further studies in Analysis, Probability, and Dynamical Systems.
- The student will be able to perform integration with respect to measure.
- The student will gain knowledge on convergence in measures.
- The student will get to know about signed measures and their properties.
- The student will be able to learn about measurability in product spaces.

#### REFERENCES

- 1. Avner Friedman, "Foundations of Modern Analysis", Hold Rinehart Winston, Canada, 1970.
- 2. De Barra G, "Measure Theory and Integration", New Age International Publishers, Second Edition, Chennai, 2013.

MODELLING AND SIMULATION

#### MX5033

#### **OBJECTIVES:**

- To understand the basics of simulation and its types.
- To understand how to generate random numbers using various methods and test them for different standard probability distributions.
- To analyze models and simulate experiments to meet real world system specifications and evaluate the performance using logical flowchart.
- To appreciate the comparison of available simulation languages and study any one such language in detail.
- To build a simulation model for any of the industrial systems using the simulation models that are studied.

#### UNIT I INTRODUCTION

Systems – modeling – general – systems theory – concept of simulation – simulation as a decision making tool – types of simulation.

#### UNIT II **RANDOM NUMBERS**

Pseudo random numbers - methods of generating random variables - discrete and continuous distributions – testing of random numbers.

#### UNIT III **DESIGN OF SIMULATION EXPERIMENTS**

Problem formulation – data collection and reduction – time flow mechanism – key variables – logic flow chart - starting condition - run size - experimental design consideration - output analysis and interpretation validation.

#### UNIT IV SIMULATION LANGUAGES

Comparison and selection of simulation languages – study of any one simulation language.

#### UNIT V **CASE STUDY**

Development of simulation models using the simulation language studied for systems like, queuing systems - production systems - inventory systems - maintenance and replacement systems investment analysis and network.

#### **OUTCOMES:**

On successful completion of this course students will be able to:

- Develop simulation in software •
- Apply the experimental process to acquire desired simulation results
- Apply visualization techniques to support the simulation process
- Use appropriate techniques to verify and validate models and simulation
- Analyze simulation results to reach an appropriate conclusion

#### **REFERENCES:**

- 1. Altiok, T., Melamed, B. "Simulation Modeling and Analysis with ARENA", Elsevier, 1<sup>st</sup> Edition, Amsterdam, 2007.
- Banks, J., Carson, J.S., Nelson, B.L., Nicol, D.M. "Discrete Event System Simulation", Pearson 2. Education, 5<sup>th</sup> Edition, Chennai, 2009.
- Deo, N. "Systems Simulation with Digital Computer", Prentice Hall of India, New Jersey, 2006. 3.
- 4. Gordon, G. "Systems Simulation", Prentice Hall of India, 2<sup>nd</sup> Edition, New Jersey, 1978.
- Law, A.M. "Simulation Modeling & Analysis", McGraw Hill Education, 5<sup>th</sup> Edition, New York, 5. 2015.
- 6. Schriber, T.J. "Simulation using GPSS", John Wiley, 2<sup>nd</sup> Edition, 1991.
- Shannon, R.E., "Systems Simulation: the Art and Science", Prentice Hall, New Jersey, 1975. 7.

**TOTAL: 60 PERIODS** 

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| DNA based co   | o the data Encryption standard", In proceedings of the 2 <sup>nd</sup> DIMA<br>omputers, 1996.<br>DNA Computing", Springer Verlag, Berlin, 1997.   | CS Workshop on                 |
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|  | I., Rothemund PWK, Roweis, S. and Winfree E., "On ap   |                                |
| <ul> <li>Students will b</li> <li>Students get fa</li> <li>Students succa</li> <li>Students able</li> <li>Students gain<br/>Theoretical Control</li> </ul> | be able to understand DNA structures and their operations.<br>amiliarity in Molecular Computing.<br>ressfully understand the characterization of Languages.<br>to iterate splicing is an operation with Languages.<br>the knowledge of applications of Molecular Computation in Math<br>omputer Science. | ematics and                    |
| OUTCOMES:  | τοτ  | AL : 60 PERIODS                |
|  | LICATIONS OF MOLECULAR COMPUTING<br>s of Molecular Computing to various problems of Mathematics  | <b>12</b><br>s and Theoretical |
| Dperations of Stick  | <b>KER SYSTEM AND SPLICING SYSTEM</b><br>ing – Sticker systems classifications – Generative capacity of<br>ng – Non-Iterated Splicing as an operation with Languages – Itera<br>ages.  |                                |
|  | <b>RESENTATION OF LANGUAGES</b><br>Regular and Linear Languages – Characterizations of Recurs  | <b>12</b><br>ively Enumerable  |
| -  | INNINGS OF MOLECULAR COMPUTING<br>ent – SAT problem – Breaking DES code.   | 12                             |
|  | OGICAL INTRODUCTION (DNA STRUCTURE AND PROCESS<br>Operations on DNA molecules – Reading out the sequence.  | SING) 12                       |
| • To highlight re  | cent application of Molecular Computing.   |                                |

#### To study the structure of DNA.

- To study the methods of Molecular computing.
- To represent Languages

MX5034

**OBJECTIVES:** 

- To introduce Sticker system and Splicing system

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#### **OBJECTIVES:**

- To introduce the certain algorithms for solving the network models •
- To expose them to different project management techniques like PERT and CPM •
- To familiarize with the various aspects of game theory which involves decision situation in which two intelligent opponents with conflicting objectives are vying to outdo one another
- To introduce the students to the idea of making decision for problems involving various alternatives
- To get an idea of certain topics on random processes such as Weiner process and OU process. ٠

### UNIT I NETWORK MODELS

# Scope and definition of network models - Minimal spanning tree algorithm - Shortest - route problem - Maximal-flow Model.

## UNIT II CPM AND PERT

Network representation - Critical path (CPM) computations - Construction of the time schedule - Linear programming formulation of CPM - PERT calculations.

### UNIT III GAME THEORY

Optimal solution of two-person zero-sum games - Mixed strategies - Graphical solution of  $(2 \times n)$  and  $(m \times 2)$  games - Solution of  $m \times n$  games by linear programming.

### UNIT IV DECISION ANALYSIS

Decision making under certainty: analytic hierarchy process (AHP) - Decision making under risk - Decision under uncertainty.

### UNIT V MARKOVIAN DECISION PROCESS

Scope of the Markovian decision problem - Finite stage dynamic programming model - Infinite stage model - Linear programming solution.

### OUTCOMES:

- It helps in formulating many practical problems in the frame work of Networks.
- It helps the students understand that CPM is a deterministic method whereas PERT uses a probabilistic model which deals with unpredictable activities.
- It enables the students to identifies competitive situations which can be modeled and solved by game theoretic formulations.
- It molds the students to make decisions for various real time problem subject to uncertainty and risk.
- It offers interesting techniques to quantity and effectively obtain the solution of various decision making situations.

### **REFERENCES:**

- 1. Taha, H.A. "Operations Research: An Introduction", Pearson Education India, Tenth Edition, 2017.
- 2. Hillier F.S., Lieberman G.J., Nag, Basu, "Introduction to Operations Research", Tata Mc- Graw Hill, New Delhi, Ninth Edition, 2011.
- 3. Winston W.L., "Operations Research", Brooks/Cole Cengage Learning, Fourth Edition, New York, 2004.

#### MX5036

#### NONLINEAR DYNAMICS

L T PC 4 0 0 4

### OBJECTIVES

- To introduce the method of solving nonlinear differential equations using Jacobi elliptic functions.
- To demonstrate the linear stability analysis for autonomous and non-autonomous systems.
- To give the Lagrangian and Hamiltonian formulation of Mechanics.
- To introduce the classical perturbation theory.
- To establish the properties of nonlinear evolution equations with emphasis on solving Kdv equation.

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**TOTAL: 60 PERIODS** 

#### UNIT I DYNAMICS OF DIFFERENTIAL EQUATIONS

Integration of linear second order equations - Integration of nonlinear second order equations - Jacobi elliptic functions – Periodic Structure of Elliptic functions - Dynamics in the phase plane.

#### UNIT II LINEAR STABILITY ANALYSIS

Stability Matrix - Classification of Fixed points - Examples of fixed point analysis - Limit Cycles - Non-Autonomous Systems.

#### HAMILTONIAN DYNAMICS UNIT III

Lagrangian formulation of Mechanics – Hamiltonian formulation of Mechanics – Canonical transformations - Hamilton-Jacobi equation and Action-Angle variables - Integrable Hamiltonians -Two-dimensional Harmonic Oscillator.

#### UNIT IV **CLASSICAL PERTURBATION THEORY**

Elementary perturbation theory – Canonical perturbation theory –Many degrees of Freedom and the problem of small divisors - The Kolmogorov-Arnold-Moser theorem.

#### UNIT V NONLINEAR EVOLUTION EQUATIONS AND SOLITONS

Basic properties of the Kdv equation - The inverse Scattering transforms: Basic principles, Kdv equation – Other soliton systems. **TOTAL : 60 PERIODS** 

#### **OUTCOMES**

At the end of the course, the students will be able to

- solve nonlinear differential equations using Jacobi elliptic functions. •
- perform the linear stability analysis for autonomous and non-autonomous systems.
- obtain the Lagrangian and Hamiltonian formulation of Mechanics.
- investigate problems using perturbation theory.
- analyze the Kdv equation and its properties.

#### REFERENCES

- 1. Lakshmanan M. and Rajasekar S., "Nonlinear Dynamics", Springer-Verlag, First Edition, New York. 2003.
- 2. Strogatz S.H., "Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering (Studies in Nonlinearity), CRC Press, Second Edition, New York, 2014.
- 3. Tabor M., "Chaos and Inerrability in Nonlinear Dynamics", John Wiley and Sons, New York, 1989.

#### MX5037

#### NUMBER THEORY

#### LT PC 4004

#### **OBJECTIVES:**

- To introduce the concepts of divisibility and congruences.
- To know about application of congruences.
- To study some functions of number theory like greatest integer function, arithmetic functions and mobius inversion formula
- To introduce diophantine equations.
- To introduce Farey Fractions and simple continued Fractions.

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### UNIT I DIVISIBILITY AND CONGRUENCES

Introduction - Divisibility - Primes - The binomial theorem. Congruences - Solutions of congruences - The chinese - Remainder theorem - Techniques of numerical calculation.

### UNIT II APPLICATION OF CONGRUENCE AND QUADRATIC RECIPROCITY

Public - Key cryptography - Prime power moduli - Prime modulus - Primitive roots and power residues - Quadratic residues - The Gaussian reciprocity law.

### UNIT III FUNCTIONS OF NUMBER THEORY

Greatest integer function - Arithmetic functions - Mobius inversion formula - Recurrence functions - Combinational number theory.

### UNIT IV DIOPHANTINE EQUATIONS

The equations ax + by = c Pythagorean triangle - Shortest examples.

#### UNIT V FAREY FRACTIONS AND SIMPLE COTINUED FRACTIONS

Farey sequences- Rational approximations-The Euclidean Algorithm-Infinite continued fractions.

## OUTCOMES:

- The student would have learnt to solve divisibility problems some techniques of numerical calculations using congruences
- Students would have learnt application of congruences.
- Students will be able to apply the Gaussian reciprocity law in public-key cryptography
- The students will be able to solve some diophantine equations .
- Students will have a good foundation in Farey Fractions and simple continued Fractions

### REFERENCES

- 1. Bressoud D., Wagon S., "A Course in Computational Number Theory", Key College Publishing, New York, 2000.
- 2. Graham R.L., Knuth D.E. and Patachink O., "Concrete Mathematics", Pearson education, Second Edition, London, 2002.
- 3. Niven I., Zuckerman H.S., and Montgomery H.L., "An introduction to the theory of numbers", John Wiley & Sons, Fifth Edition, Singapore, 2004.

# MX5038 NUMBER THEORY AND CRYPTOGRAPHY L T P C 4 0 0 4

#### **OBJECTIVES:**

- To study divisibility.
- To study congruences and solving congruences.
- To introduce quadratic residues, Jacobi symbol and different important functions in number theory
- To introduce diophantine equations and Waring's problem
- To introduce traditional symmetric key ciphers

#### UNIT I DIVISIBILITY

Introduction - Divisibility - Primes - The binomial theorem.

#### UNIT II CONGRUENCES

Congruences, Solutions of congruences, congruences of deg 1, The function 0(n) - Congruences of higher degree, Prime power moduli, Prime modulus, congruences of degree 2, Prime modulus, Power residues.

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**TOTAL: 60 PERIODS** 

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#### UNIT III QUADRATIC RESIDUES

Quadratic residues, Quadratic reciprocity, The Jacobi symbol, greatest integer function, arithmetic function, The Moebius Inversion formula, The multiplication of arithmetic functions.

#### UNIT IV **DIOPHANTINE EQUATIONS**

Diophantine equations, The equation ax + by = c, Positive solutions, Other linear Equations, Sums of four and five squares, warings problem, sum of fourth powers, sum of two Squares.

#### **TRADITION SYMMETRIC – KEY CIPHERS** UNIT V

Substitution Ciphers - Transportation Ciphers - Steam and Block Ciphers - Modern Block Ciphers -Modern Steam Ciphers – DES – AES.

#### **OUTCOMES:**

- The student would have learnt to solve divisibility problems using binomial theorem
- Students would have learnt to solve congruences and compute power residues
- Students will be able to apply quadratic reciprocity law and the Mobius inversion formula in cryptography
- The student will be able to solve diophantine equations and Waring's problem
- The student would have learnt about traditional and modern stream and Block ciphers

#### REFERENCES

- 1. Behrouz A. Forouzan, "Cryptography & Network Security", Tata McGraw Hill, Special Indian Edition, Third Edition, New Delhi, 2015.
- 2. Kenneth Ireland & Michael Rosen, "A Classical Introduction to Modern Number Theory", Springer International Edition, Second Edition, New York, 2010
- Koblitz, N., "A course in number theory and Cryptography", Springer Verlag, New York, 1994
   Niven.I, Herbert S.Zuckermann, Hugh L. Montgomery, "An Introduction to the Theory of Numbers", John Wiley, Fifth Edition, New York, 2013.
- 5. Rose H.E., "A Course in Number Theory", Clarendon Press, Second Edition, Oxford, 1995.
- 6. Stinson D.R., "Cryptography: Theory and Practice", CRC Press, Fourth Edition, New York, 2018
- 7. Tom M. Apostol, "Introduction to analytic number theory", Narosa Publishing House, New Delhi, 1980.

#### NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS MX5039 LTPC 4004

#### **Pre-requisite: Partial Differential Equations**

#### **OBJECTIVES:**

- To make the students understand the numerical methods of solving partial differential equations.
- To introduce the methods of solving one-dimensional parabolic equations.
- To demonstrate the methods of solving two-dimensional parabolic equations.
- To display the methods of solving hyperbolic equations.
- To reveal the ideas of solving elliptic equations.

#### UNIT I LINEAR SYSTEMS OF EQUATIONS

Iterative methods for solving large linear systems of algebraic equations: Jacobi, Gauss-Seidel and S.O.R methods - Conditions for convergence of them - Methods for accelerating convergence: Lyusternite's & Aitken's methods - Optimum acceleration parameter for S.O.R method.

#### UNIT II **ONE DIMENSIONAL PARABOLIC EQUATIONS**

Explicit and Crank-Nicolson Schemes for  $u_t = u_{xx}$  - Weighted average approximation - Derivative boundary conditions - Truncation errors - Consistency, Stability and convergence - Lax Equivalence theorem.

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### TOTAL: 60 PERIODS

### UNIT III MATRIX NORMS & TWO DIMENSIONAL PARABOLIC EQUATION

Vector and matrix norms - Eigenvalues of a common tridiagonal matrix - Gerischgorin's theorems - Stability by matrix and Fourier-series methods - A.D.I. methods.

#### UNIT IV HYPERBOLIC EQUATIONS

First order quasi-linear equations and characteristics - Numerical integration along a characteristic - Lax-Wendroff explicit method - Second order quasi-linear hyperbolic equation - Characteristics - Solution by the method of characteristics.

#### UNIT V ELLIPTIC EQUATIONS

Solution of Laplace and Poisson equations in a rectangular region - Finite difference in Polar coordinate Formulas for derivatives near a curved boundary when using a square mesh -discretisation error - Mixed Boundary value problems.

#### OUTCOMES

At the end of the course, the students will be able to

- learn various numerical methods of solving partial differential equations.
- solve one-dimensional parabolic equations using explicit and implicit schemes.
- solve two-dimensional parabolic equations and analyze the stability of the schemes.
- understand the methods of solving hyperbolic equations.
- solve elliptic equations in Cartesian and Polar coordinates.

#### REFERENCES

- 1. Iserles A., "A first course in the Numerical Analysis of Differential Equations", Cambridge University press, New Delhi, 2010.
- 2. Mitchel A.R. and Griffiths S.D.F., "The Finite Difference Methods in Partial Differential Equations", John Wiley and sons, New York, 1980.
- 3. Morton K.W., Mayers, D.F., "Numerical Solutions of Partial Differential Equations", Cambridge University Press, Cambridge, 2005.
- 4. Smith G.D., "Numerical Solution of P.D.E.", Oxford University Press, New York, 1995.

#### MX5040

#### **OPERATOR THEORY**

### OBJECTIVES

- The interplay between the ideas and methods from operator theory and functional analysis with methods and ideas from function theory, commutative algebra and algebraic, analytic and complex geometry gives the field a strong interdisciplinary character.
- To introduce generalized Kato decomposition.
- To introduce the concept of local spectrum and its related theorem.
- To introduce Algebraic spectral spaces.
- To introduce the spectra of some operators.

#### UNIT I KATO DECOMPOSITION PROPERTY

Hyper-Kernel and Hyper-Range of an operator - Semi-regular operators on Banach spaces - Analytical core of an operator - The Semi-regular spectrum of an operator.

#### UNIT II GENERALIZED KATO DECOMPOSITION PROPERTY

The Generalized Kato decomposition - Semi-Fredholm operators- Quasi-nilpotent of operator - Two-Spectral mapping theorems.

**TOTAL: 60 PERIODS** 

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#### UNIT III SINGLE-VALUED EXTENSION PROPERTY (SVEP)

Local spectrum and SVEP- The SVEP at a point- A Local spectral mapping theorem.

#### UNIT IV SVEP ANDFREDHOLM THEORY

The Single-valued Extension Property (SVEP): Algebraic spectral subspaces. The SVEP and Fredholm Theory: Ascent, descent the SVEP- The SVEP for operators of Katotype.

#### SPECTRA OF SOME SPECIAL OPERATORS UNIT V

The SVEP on the components of The Fredholm, Weyl and Browder spectra-Compressions.

#### **OUTCOMES**

- Operator Theory provides an introduction to functional analysis with an emphasis on the theory of linear operators and its application to differential and integral equations, approximation theory, and numerical analysis.
- The student will gain the knowledge on two-spectral mapping theorems.
- The student will be introduced to Single Valued Extension Property. •
- The student will get to know about Fredholm theory.
- The student will be able to know the spectra of Fredholm. Weyl and Browder.

#### REFERENCES

- 1. Aiena, P., "Fredholm and Local Spectral Theory with Applications to Multipliers", Kluwer Academic Publishers, New York, 2004.
- 2. Conway, J. B., "A Course in Functional Analysis", Springer- Verlag, Second Edition, New York,1990.
- 3. Lawsen, K. B., M. M. Neumann, "An Introduction to Local Spectral Theory", London Mathematical Society, Monographs 20, Clarendon press, Oxford, 2000.

| MX5041 | OPTIMIZATION TECHNIQUES | LTPC |
|--------|-------------------------|------|
|        |                         | 4004 |

#### OBJECTIVE

- To introduce Advanced Linear Programming Algorithms
- To introduce the Non Linear Programming Algorithms
- To introduce the Basic Inventory Models
- To introduce the Basic Queueing Models
- To introduce the Network Models.

#### UNIT I ADVANCED LINEAR PROGRAMMING ALGORITHMS

Revised Simplex Method –Bounded Variables Algorithm – Dual Simplex Method and Parametric Linear Programming.

#### NON LINEAR PROGRAMMING ALGORITHMS UNIT II

Direct Search Method-Gradient Method-Constraint Algorithm –Separable Programming and Quadratic Programming.

#### INVENTORY MODELS UNIT III

Static economic- Order Quantity models: Classical EOQ model- EOQ with price breaks- Dynamic EOQ model: No set up EOQ Model - Setup EOQ model- Continuous review models: Probabilitized EOQ model, Probabilistic EOQ model- Single -period models: No -Setup model, Setup model (s-S policy)

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TOTAL : 60 PERIODS

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### UNIT IV QUEUEING MODELS

General Poisson Queueing model- Single server models – Multiple server models- Self service models- Queueing networks.

#### UNIT V NETWORK MODELS

Scope and definition of network models- Minimal Spanning tree algorithm – Shortest –route problem-Maximal –flow model.

#### OUTCOME

- Helps in formulating many Decision Making Problems as a Linear Programming Model
- Students will be capable of using advanced techniques in solving various decision making Problems
- Students should be able to formulate organization problems into Inventory models for seeking optimal solutions.
- Students should be able to formulate organization problems into Queueing models for optimizing the cost.
- Helps in formulating many practical Problems in the frame work of Networks.

#### REFERENCES

- 1. Hamdy A. Taha, "Operations Research-An Introduction", Pearson Education, Tenth Edition, New Delhi, 2015.
- 2. Harvey M. Wagner,"Principles of Operations Research with Applications to Managerial Decisions", Prentice-Hall of India Pvt.Ltd., Second Edition, New Delhi, 1975.
- 3. Rao S.S., "Engineering Optimization : Theory and Practice", Wiley and New Age International, Fourth Edition, New Delhi, 2009.
- 4. Mokhtar S. Bazara, Hanif D. Sherali and Shetty C.M.,"Non-Linear Programming-Theory and Algorithms", John Wiley & Sons Inc, Second Edition, Singapore, 1993.
- 5. J.K. Sharma,"Operations Research Theory and Applications ", Trinity Press, Sixth Edition, New Delhi, 2016.

#### MX5042

#### QUEUEING AND RELIABILITY MODELLING

#### OBJECTIVES

- To introduce the basic concepts of Markovian queues.
- To study the bulk arrival and service systems along with various priority queueing systems.
- To gain the knowledge of non-Markovian queues and their performance measures.
- To discuss the fundamental of the system reliability and hazard rate functions.
- To get knowledge of maintainability and availability along with functioning of two unit systems.

#### UNIT I MARKOVIAN QUEUES

Steady State Analysis - Single and multiple channel queues - Erlang's formula - Queues with unlimited service - Finite source queues - Transient behavior - Busy period analysis for M/M/1 queue.

#### UNIT II ADVANCED MARKOVIAN QUEUES

Bulk input model - Bulk service model - Erlangian models - Preemptive and Non-preemptive priority queue disciplines.

#### UNIT III NON-MARKOVIAN QUEUES

M/G/1 queueing model - Pollaczek-Khintchine formula - Steady-state system size probabilities - Waiting time distributions - Generalization of Little's formula - Busy period analysis of M/G/1 queue.

#### TOTAL : 60 PERIODS

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## UNIT IV SYSTEM RELIABILITY

Reliability and hazard functions - Exponential, normal, weibull and Gamma failure distributions - Time-dependent hazard models, Reliability of series and parallel systems.

## UNIT V MAINTAINABILITY AND AVAILABILITY

Maintainability and Availability functions - Frequency of failures - Two unit parallel system with repair - Two unit series system with repair - k out of m systems.

### TOTAL: 60 PERIODS

### OUTCOME

- Students can evaluate the various system performance measures of the Markovian queueing systems.
- Acquaint with various mathematical techniques of advanced Markovian queues.
- Students will able to formulate the various kinds of Non-Markovian queueing models.
- Aware of various models of reliability of the systems for different probability distributions.
- Understanding of system availability multi units series and parallel systems with repairs.

### REFERENCES

- 1. Gross D., Shortie J.F, Thompson J.M and Harris C.M., "Fundamentals of Queueing Theory", John Wiley and Sons, Fourth edition, New York, 2013.
- 2. Balagurusamy E., "Reliability Engineering", Tata McGraw Hill Publishing Company Ltd., First edition, New Delhi, 2002.
- 3. Govil A.K., "Reliability Engineering", Tata-McGraw Hill Publishing Company Ltd., New Delhi, 1983.
- 4. Charless E. Ebeling, "An Introduction to Reliability and Maintainability Engineering", Waveland Pr. Inc., Second Edition, Long Grove, 2009.
- 5. Kleinrock. L., "Queueing Systems: Volume 1", John Wiley and Sons, New York, 1975.
- 6. Medhi J, "Stochastic models of Queueing Theory", Academic Press, Amsterdam, 2003.
- 7. K.S. Trivedi," Probability and Statistics with Reliability, Queueing and Computer Science Applications", Wiley, 2<sup>nd</sup> Edition, New Delhi, 2016.
- 8. Thomas, G., Robertzzi," Computer Network and Systems: Queueing Theory and Performance Evaluation", 3<sup>rd</sup> Edition, New Delhi, 2013.

#### MX5043

#### Prerequisite: Lie algebra

**REPRESENTATIONS OF LIE ALGEBRAS** 

#### **OBJECTIVES:**

- To understand what is a semi-simple Lie algebra and some of its properties
- To understand about the root system of a semi-simple Lie algebra
- To gain knowledge about the finite dimensional representations of a semisimple Lie algebra
- To learn the Harish-Chandra's theorem on characters associated with an infinite dimensional module
- To understand how to obtain some remarkable formulas for characters and multiplicities of finite dimensional modules using Harish-Chandra's theorem

### UNIT I SEMI SIMPLE LIE ALGEBRAS

Definitions of radical, semi-simple Lie algebra, Killing form, criterion for semi-simplicity, inner derivations, representation of a Lie algebra, Schur's Lemma, Casimir element, Weyl's theorem, Jordan decomposition, representations of sl(2,F).

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#### UNIT II ROOT SYSTEM

Cartan subalgebra, root system, Weyl group, weights, dominant weights, saturated set of weights.

#### UNIT III FINITE DIMENSIONAL MODULES

universal enveloping algebra, Poincaré–Birkhoff-Witt theorem, Weight spaces, standard cyclic modules, existence and uniqueness theorems, necessary and sufficient conditions for finite dimension, weight strings and weight diagrams, generators and relations for V().

#### UNIT IV MULTIPLICITY FORMULA, CHARACTERS

A universal casimir element, traces on weight spaces, Freudenthal's formula, examples, formal characters, invariant polynomial functions, standard cyclic modules and characters, Harish-Chandra's theorem.

#### UNIT V FORMULAS OF WEYL, KONSTANT, STEINBERG

Some functions on H\*, Konstant's multiplicity formula, Weyl's formulas, Steinberg's formula.

#### TOTAL: 60 PERIODS

#### OUTCOMES:

- Students would have learnt about basic properties of a semi-simple Lie algebra and representations of sl(2,F)
- They would have understood clearly the structure of a semi-simple Lie algebra in terms of its root system
- Students would have a detailed understanding of finite dimensional representations
- Students would have a good understanding of the theorem of Harish-Chandra and some of ite remarkable applications
- The students will have a thorough understanding of the theory of semi-simple Lie algebras over an algebraically closed field of characteristic 0 and their representations.

#### **REFERENCES:**

- 1. Fulton W., Harris J., Representation theory, A First Course, Springer, New York, 2004.
- 2. Humphreys J.E., Introduction to Lie algebras and Representation theory, Springer, New York, 1972.

#### MX5044 REPRESENTATION THEORY OF FINITE GROUPS L T P C 4 0 0 4

**Prerequisite: Algebra** 

#### **OBJECTIVES:**

- Familiarize the concept of modules and its techniques
- To impart knowledge on representation theory on finite groups and relation between modules and representation theory
- To enable the students to analyze the irreducible representations of finite group through character theory.
- To make the students to construct all possible irreducible representation of symmetric group.
- To describe some the ideas of the representation theory in purely combinatorial terms.

#### UNIT I MODULES

Modules – Simple and Semisimple Modules- Tensor product- Restricted and induced modules-Group algebra.

#### UNIT II GROUP REPRESENTATION

Linear and Matrix representations- Reducibility – Complete reducibility and Maschke's theorem – Schur's lemma – Commutant and Endomorphism algebras.

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### UNIT III CHARACTERS AND TENSOR PRODUCTS

Group characters – Inner product of characters – Orthogonality relations- Tensor products – restricted and induced representations.

#### UNIT IV REPRESENTATION OF SYMMETRIC GROUPS

Representation of symmetric groups- Young subgroups, tableaux and tabloids – Specht modules – Standard tableaux- branching rule.

### UNIT V APPLICATIONS IN COMBINATORICS

The Robinson-Schensted algorithm – increasing and decreasing subsequences – the hook formula – the determinant formula.

### OUTCOMES

- Understanding the concept of module theory'
- Fluency in representation theory on finite groups.
- The students will acquire on the sound knowledge on the technical tools to construct a irreducible representations of finite groups
- Able to construct all possible irreducible representations of symmetric groups explicitly.
- Understanding combinatorial ideas which involves in representation of symmetric groups.

#### REFERENCES

- 1. C.W. Curtis and I. Reiner., "Representation theory of finite groups and associative algebras", AMS Chelsea Publishing, Providence, Rhode Island, 2006.
- Bruce E. Sagan., "The symmetric group. Representations, combinatorial algorithms, and symmetric functions", The Wadsworth & Brooks/Cole Mathematics Series. Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, CA, 1991.
- 3. William Fulton, "Young tableaux, with applications to representation theory and geometry", London Mathematical Society Student Texts, 35, Cambridge University Press, Cambridge, 1997.
- G. James and A. Kerber., "The Representation theory of the symmetric group", Encyclopedia of Mathematics and its Applications, 16. Addison-Wesley Publishing Co., Reading, Mass., Boston, 1981.

#### MX5045

## Prerequisite: Complex Analysis

SPECIAL FUNCTIONS

#### OBJECTIVES

- To give an expertise treatment in various special function and orthogonal polynomial.
- To introduce hypergeometric functions and their properties.
- To introduce generalized hypergeometric functions and their properties.
- To introduce orthogonal polynomials with some spectral analysis.
- To introduce a few particular orthogonal polynomials.

#### UNIT I SPECIAL FUNCTIONS

Beta and Gamma Functions – Euler Reflection Formula – The Hurwitz and Riemann zeta functions – Stirling's Asymptotic Formula – Gauss's Multiplication Formula – Ratio of two gamma functions – Integral Representations for Logarithm of Gamma function - The Bohr- Mollerup Theorem

#### UNIT II HYPERGEOMETRIC FUNCTIONS

Hypergeometric Differential Equations – Gauss Hypergeometric Function – Elementary Properties – Contiguous Relations – Integral Representation – Linear and Quadratic Transformation and Summation Formulae.

### TOTAL: 60 PERIODS

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### UNIT III GENERALIZED HYPERGEOMETRIC FUNCTIONS

Generalized Hypergeometric Functions – Elementary Properties – Contiguous Relations – Integral Representation – Transformation and Summation Formulae – Whipple's Transformation.

#### UNIT IV ORTHOGONAL POLYNOMIALS

Zeros – Fundamental Recurrence Formula, Systematic Moment Functions – Representation Theorem – Spectral Points and zeros of Orthogonal Polynomials – Chain Sequence and Orthogonal Polynomials – Some Spectral Analysis – Orthogonal Polynomials whose zeros are dense in intervals – Kreine's Theorem.

#### UNIT V SPECIFIC ORTHOGONAL POLYNOMIALS

Some specific systems of orthogonal polynomials like Hermite – Laguerre – Jacobi, Ultra spherical – q-Polynomials of Al-Salam and Carlitz – Wall Polynomials.

#### OUTCOMES

- Students are exposed to various special functions and orthogonal polynomials.
- The student will gain insight into hypergeometric functions and hypergeometric differential equations.
- The student will get to know about integral representation of generalized hypergeometric functions.
- The student will be able to know the properties of orthogonal polynomials.
- The student will get introduced to some particular orthogonal polynomials.

#### REFERENCES

- 1. Andrews G.E., Askey, R., Ranjan Roy, "Special Functions, Encyclopedia of Mathematics and its Applications", Cambridge University Press, Cambridge, 1999.
- 2. Chihara T.S., "An Introduction to Orthogonal Polynomials", Gordon and Breach, Washington, 1978.
- 3. Copson .E.T., "Theory of Functions of Complex Variables", Oxford University Press, London, 1935.
- 4. Nevai P.G., "Orthogonal Polynomials", Memoirs of AMS, New York, 1981.
- 5. Rainville E.D., "Special Functions", Macmillan, New York, 1971.
- 6. Szego G., "Orthogonal Polynomials", Memoirs of AMS, Fourth Edition, New York, 1975.

#### MX5046

#### OBJECTIVE

 To understand the basic concepts of stochastic processes and be able to develop and analyse the stochastic models that capture the significant features of the probability models in order to predict the short and long term effects in the system.

STOCHASTIC PROCESSES

- To Learn and model the renewal processes and study its theorems and their behavior.
- To study about the combination of renewal processes and Markov process.
- To understand the concept of branching processes and its nature. Also, to learn the variety of models in branching process.
- To find the nature of Wiener process and study its properties.

#### UNIT I MARKOV AND STATIONARY PROCESSES

Specification of Stochastic Processes - Stationary Processes - Poisson Process - Generalizations - Birth and Death Processes - Martingales - Erlang Process.

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TOTAL : 60 PERIODS

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### UNIT II RENEWAL PROCESSES

Renewal processes in discrete and continuous time - Renewal equation - Stopping time - Wald's equation - Renewal theorems - Delayed and Equilibrium renewal processes - Residual and excess life times - Renewal reward process - Alternating renewal process - Regenerative stochastic process.

#### UNIT III MARKOV RENEWAL AND SEMI – MARKOV PROCESSES

Definition and preliminary results - Markov renewal equation - Limiting behaviour – First passage time.

### UNIT IV BRANCHING PROCESSES

Generating functions of branching processes - Probability of extinction - Distribution of the total number of progeny - Generalization of classical Galton - Watson process - Continuous time Markov branching process - Age dependent branching process.

#### UNIT V MARKOV PROCESSES WITH CONTINUOUS STATE SPACE

Brownian motion - Wiener process - Diffusion and Kolmogorov equations - First passage time distribution for Wiener process - Ornstein - Uhlenbeck process.

#### OUTCOME

After the completion of the course, the students will be able to

- Understand and characterize the random phenomena and model a stochastic system.
- Connect the real life situation and renewal processes.
- Obtain the knowledge about the advanced studies of renewal processes.
- Understand stochastic population models through branching processes.
- Obtain the knowledge about Wiener processes.

#### REFERENCES

- 1. Medhi J., "Stochastic Processes", New Age International (P) Ltd., Fourth Edition, New Delhi, 2017.
- 2. Narayan Bhat U. and Gregory K. Miller, "Elements of Applied Stochastic Processes", Wiley Inter science, Third Edition, New York, 2002.
- 3. Karlin S "A First Course in Stochastic Processes", Academic press, New York, 2014.
- 4. Cox D.R. and Miller H.D., "The theory of Stochastic Process", Methuen, London, 1965.
- 5. Ross S. M. ,"Stochastic Processes", Wiley, Second Edition, New York, 1996.

#### MX5047

#### Prerequisite: Complex Analysis

UNIVALENT FUNCTIONS

#### OBJECTIVES

- To introduce theory and advanced techniques in Univalent functions (advanced Complex Analysis)
- To introduce primitive variational method.
- To introduce the concept of subordination.
- To introduce extremal problems and properties.
- To introduce the integral transforms.

#### UNIT I ELEMENTARY THEORY OF UNIVALENT FUNCTIONS

The Area theorem-Growth and Distortion Theorems-Coefficient Estimates-Convex and Star like functions-Close to Convex functions-Spiral like functions-Typically Real functions.

#### TOTAL : 60 PERIODS

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## UNIT II VARIATIONAL METHODS

A Primitive Variational Method - Growth of Integral Means-Odd Univalent functions-Asymptotic Bieberbach Conjecture.

### UNIT III SUB ORDINATION

Basic Principles - Coefficient Inequalities - Sharpened Forms of the Schwartz Lemma– Majorization - Univalent Sub ordinate Functions.

#### UNIT IV GENERAL EXTREMAL PROBLEMS

Functionals of Linear Spaces - Representation of Linear Functionals - Extreme Points and Support Points- Properties of extremal Functions - Extreme Points.

#### UNIT V INTEGRALTRANSFORMS

Linear Operators – Nonlinear operators – Conclusion operators - Alexander Transforms – Libera Transforms – Bernardi Transforms.

#### OUTCOMES

- Students will gain in-depth knowledge in Univalent functions theory to pursue research.
- The students will have a thorough understanding of univalent functions.
- The students will gain knowledge in subordination and univalent subordinate functions.
- The students will get an understanding in solving extremal problems.
- The students will get introduced to the integral transforms.

#### REFERENCES

- 1. Goodman, A.W., "Univalent Functions", Volumes I and II, Polygonal Publishing House, Washington, 1983.
- 2. Peter L. Duren., "Univalent Functions", Springer Verlag, Berlin, 2001.
- 3. Sanford S. Miller, Petru T. Mocanu," Differential Subordinations: Theory and Applications", Marcel Dekker, New York, 2000.

#### MX5048

## DOMINATION IN GRAPHS

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#### Prerequisite: Graph Theory

#### **OBJECTIVES:**

- To introduce Domination concepts in graphs.
- To introduce various type of Dominate concepts.
- To study bonds and conditions of dominating set.
- To study Paired Domination Sets.
- Various types of Dominations are introduced.

#### UNIT I DOMINATING SETS

Dominating sets in graphs – Minimal dominating sets – Hereditary and superhereditary properties – Minimal and Maximal P-sets – Independent sets – Every maximal independent set is a minimal dominating set – Irredundant sets– Domination chain – Bounds involving domination, independence and irredundance numbers.

#### UNIT II CHANGING AND UNCHANGING DOMINATION

Changing and unchanging domination – Basic terminology – Changing – Vertex removal - CVR graphs – Edge removal – CER graphs – Edge addition – CEA graphs – Unchanging – Vertex removal – UVR graphs – Edge removal – UER graphs – Edge addition – UEA graphs.

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**TOTAL: 60 PERIODS** 

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## MX5049

#### **OBJECTIVES:**

- To familiarize with the fundamental concepts of genetic algorithm.
- To study the usage of genetic algorithms in problem encoding.
- To understand and analyze the genetic based techniques for problem solving through cellular automata and neural networks.
- To understand the concepts of evolutionary computing paradigm and to design an appropriate evolutionary algorithm.
- To appreciate construction of mathematical models using genetic algorithm.

#### UNIT I **OVERVIEW OF GENETIC ALGORITHMS**

The appeal of evolution - Search spaces and Fitness landscapes - Elements of genetic algorithms -Genetic algorithms and traditional search methods - A simple genetic algorithm - Applications and working of genetic algorithms.

#### UNIT II IMPLEMENTING A GENETIC ALGORITHM

Usage of a genetic algorithm - Encoding a problem for a genetic algorithm - Adapting the Encoding selection methods - Genetic operators Parameters/ genetic algorithm.

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#### UNIT III **GENETIC ALGORITHMS IN PROBLEM SOLVING**

Evolving cellular automata - Data analysis and prediction - Evolving Neural Networks

#### UNIT III CONDITION ON DOMINATING SET

Condition on the dominating set - Independent dominating sets - Total dominating sets - Connected dominating sets – Bounds for connected domination number – External graphs attaining the bounds.

#### DOMINATING CLIQUES UNIT IV

Dominating cliques – Sufficient condition for existence of a dominating clique – Bounds for the clique domination number - Paired dominating sets - Paired domination number - Bounds for paired domination number - Inequalities connecting paired domination number and other domination parameters

#### UNIT V VARIETIES OF DOMINATION

Varieties of domination - Multiple domination - Bounds for the multiple domination number - k dependence number - Inequality connected k-domination number and k-dependence number -Locating domination – Locating domination number – Bounds – Strong and weak domination – Strong and weak domination number - Bounds.

#### **OUTCOMES:**

- Students understand the Domination Theory in graphs.
- Students get familiar with variations of Domination.
- Students gain the fundamental knowledge on Domination Theory.
- Students understand the bonds for Paired Domination number.
- Students are able to construct various type of dominating sets.

#### **TEXT BOOK:**

1. T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1997.

## **GENETIC ALGORITHMS**

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**TOTAL: 60 PERIODS** 

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### UNIT IV GENETIC ALGORITHMS IN SCIENTIFIC MODELS

Modeling interactions between learning and evolution - Modeling Ecosystems - Measuring Evolutionary activity.

#### UNIT V THEORETICAL FOUNDATIONS OF GENETIC ALGORITHMS

Schemes and the two-armed Bandit problem - Exact mathematical models of simple genetic algorithms - Statistical mechanics approaches

#### OUTCOME

- To appreciate and identify real life problems to be solved using Genetic algorithms
- To apply the encoding selection method to the formulated models.
- To obtain knowledge of the Data analysis and prediction and to evolve Cellular automata and Neural Networks
- To bridge the gap between modeling and application of genetic algorithms
- To demonstrate the application of Genetic algorithms for a few selected problems

#### REFERENCES

- 1. Goldberg, D.E., "Genetic algorithms in Search, Optimization and Machine learning", Addison Wesley, Boston, 1989.
- 2. Koza, J.R. "Genetic Programming: On the Programming of Computers by means of Natural Selection", MIT Press, Cambridge, 1992.
- 3. Man, K.F., Tang, K.S., Kwong, S. "Genetic Algorithms: Concepts and Designs", Springer-Verlag, London, 1999.
- 4. Michalewicz, Z. "Genetic Algorithms + Data Structure = Evolution Programs", Springer-Verlag, 3<sup>rd</sup> Edition, London, 1996.
- 5. Mitchell, M., "An Introduction to Genetic Algorithms", Prentice Hall of India, New Delhi, 1998.
- 6. Rao, S.S., "Engineering Optimization", Wiley Interscience, Hoboken, 1996.

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**TOTAL: 60 PERIODS**