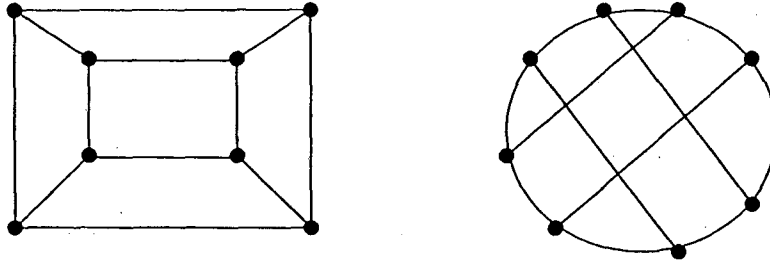


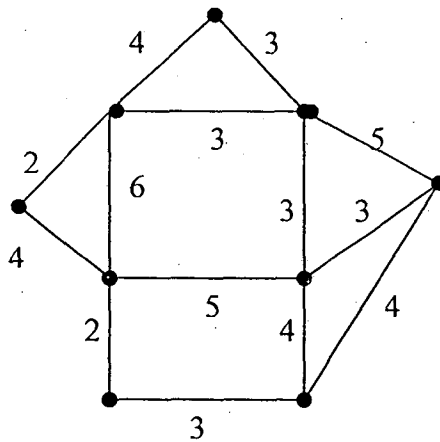
9. What is meant by shortest distance arborescence?
 10. What is meant by an invariant of a graph? Give examples.

Part – B (5 x 16 = 80 marks)

11. (i) Prove that the number of vertices of odd degree in a graph is always even. (5)
 (ii) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. (5)
 (iii) Show whether the following graphs are isomorphic or not. (6)

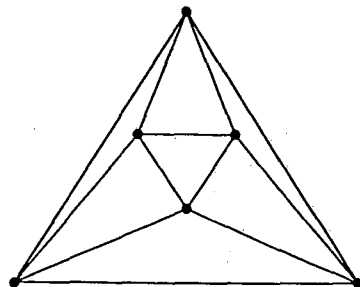


12. a) (i) Prove that the nullity of a graph does not change when you either insert a vertex in the middle of an edge or remove a vertex of degree two by merging two edges incident on it. (8)
 (ii) Find the shortest spanning tree in the following graph using Kruskal's algorithm. (8)



OR

- b) (i) Prove that every circuit has an even number of edges in common with any cut-set. (8)
 (ii) State and prove Euler's formula that gives the number of regions in any planar graph. Verify Euler's formula for the following graph. (8)



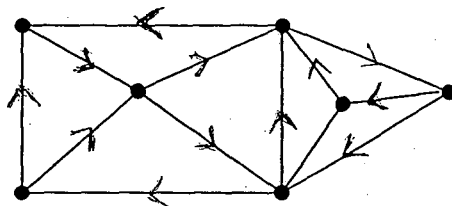
13. a) (i) If A and B are respectively the incidence matrix and the circuit matrix of a graph whose columns are arranged using the same order of edges, prove that

$$A \cdot B^T = B \cdot A^T \quad (8)$$
(ii) Sketch any one orientation of a complete graph of four vertices. Characterize the digraph in terms of binary relations. (8)

OR

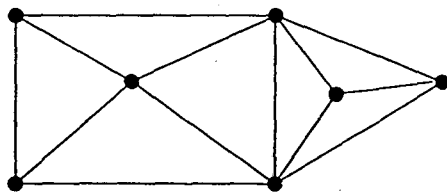
- b) (i) Prove that the covering g of a graph is minimal if and only if g contains no paths of length three or more. (8)
(ii) Prove that in a bipartite graph, a complete matching of a set of vertices V_1 into a set of vertices V_2 exists if there is a positive integer m for which the following condition is satisfied: degree of every vertex in $V_1 \geq m \geq$ degree of every vertex in V_2 . (8)

14. a) Find all the directed circuits in the following graph using Roberts and Flores algorithm, explaining the procedure in detail. (16)

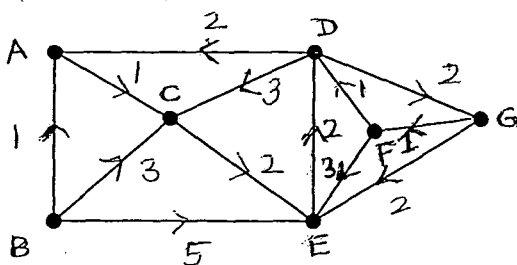


OR

- b) Find all the fundamental circuits in the following graph using Paton's algorithm, explaining the procedure in detail. (16)



15. a) Find the shortest path between all pairs of vertices in the graph given below using Warshall-Floyd algorithm. (16)



OR

- b) Explain the planarity testing algorithm proposed by Hopcroft and Tarjan. How was the ambiguity in finding paths resolved? (16)