POST-GRADUATE COURSE

Assignment — June, 2019

MATHEMATICS

Paper - 7A : Integral Transformations

Full Marks : 50 **Weightage of Marks : 20%**

Special credit will be given for accuracy and relevance in the answer. Marks will be deducted for incorrect spelling, untidy work and illegible handwriting. The weightage for each question has been indicated in the margin.

(Notations have their usual meanings.)

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2 \times 5 = 10$

   a) State the properties of a class of functions $f(t)$ for which their Laplace transform exists.

   b) State the conditions for the existence of Laplace transform of the n^{th} derivative of a function $f(t)$ and write the formula for the evaluation of $L[f^{(n)}(t)]$, where $f^{(n)}(t)$ is the n^{th} derivative of $f(t)$.

   c) Write the formula for the Laplace transform of the convolution of the two functions $f_1(t)$ and $f_2(t)$.

   d) Show that : $L[\sin \omega t] = \dfrac{\omega}{p^2 + \omega^2}$.

   e) State and prove the final value theorem of Laplace transform.

   f) State Riemann-Lebesgue's theorem.

   g) State Dirichlet's conditions.

2. Considering the function $f(x)$ defined by

   $f(x) = 1 - |x|$, when $x \le 1$

   $\quad\quad 0$, when $x > 1$

   and using Fourier inversion theorem, show that $\displaystyle\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} \mathrm{d}x = \pi$. 10

3. Deduce the Parseval's relation for Fourier transform. 10

4. Considering the two functions,

$f_1(x) = e^{-a|x|}$, $f_2(x) = e^{-bx}$, $(a, b > 0)$

and using the Parseval's relation, show that

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{ab(a+b)}.$$ 10

5. Use convolution theorem to find $L^{-1}\left[\frac{1}{\sqrt{p}}, \frac{1}{p-1}\right]$,

where L^{-1} denotes Laplace inversion operator.

10

6. Solve the following problem for the stationary temperature in a semi-infinite body :

i) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty$, $y \geq 0$

ii) $u(x, 0) = f(x)$, $-\infty < x < \infty$

iii) $u(x, y) \to 0$ as $y \to \infty$

iv) $u(x, y)$, $u_x(x, y) \to 0$ as $|x| \to \infty$. 10

7. Obtain the Laplace inversion of $F(p) = \frac{1}{\sqrt{1 + p^2}}$ in a series of powers *t*. Assuming that the sum of this series is $\Im_0(t)$, which is Bessel function of order 0, show that $\sin t = \int_0^t \Im_0(\tau)\, \Im_0(t - \tau)$. 10

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