PG–387	**MMS–15/ PGDMAT–11**

M.Sc. DEGREE/P.G. DIPLOMA EXAMINATION – JUNE, 2018.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let G be a group in which $(ab)^m = a^m b^m$ for three consecutive integers and for all $a, b \in G$. Prove that G is abelian.

2. Prove that the subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

3. Prove that a finite integral domain is a field.

4. If U, V are ideals of R, let $U+V = \{u+v : u \in U, v \in V\}$. Prove that $U+V$ is also an ideal of R.

5. If V is a finite-dimensional space over F, prove that any two bases of V, have the same number of elements.

6. If V is a vector space and $u, v \in V$, then prove that $|(u,v)| \leq \|u\| \|v\|$.

7. If L is a algebraic extension of K and if K is an algebraic extension of F, then prove that L is an algebraic extension of F.

8. If V is finite-dimensional over F, prove that $T \in A(V)$ is regular if and only if T maps V onto V.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove first part of Sylow's theorem.

10. State and prove Cayley's theorem.

11. If R is a ring with unit element, then for all $a, b \in R$ prove that

 (a) $a.0 = 0.a = 0$

 (b) $a(-b) = (-a)b = -(ab)$

 (c) $(-a)(-b) = ab$

 (d) $(-1)a = -a$

 (e) $(-1)(-1) = 1$.

12. Prove that every integral domain can be imbedded is a field.

13. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ... w_m$ in V are linearly independent over F, prove that $m \leq n$.

14. If V and W are of dimensions m and n respectively over F, then prove that $Hom(V, W)$ is of dimensions mn over F.

15. If F is of characteristic 0 and if a, b are algebraic over F, then prove that there exist an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

16. If $T \in A(V)$ has all its characteristic roots is F, then prove that there is a basis of V is which the matrix of T is triangular.

PG–388	**MMS–16/ PGDMAT–12**

M.Sc. DEGREE/P.G. DIPLOMA
EXAMINATION – JUNE 2018.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the ordered set R has the least upper bound property.

2. Prove that compact subsets of metric spaces are closed.

3. Prove that if f is continuous at a point $p \in E$ and if g is continuous at $f(p)$ then prove that $h = g \circ f$ is continuous at p.

4. Let f be monotonic on (a, b), then prove that the set of pints of (a, b) at which f is discontinuous is almost countable.

5. State and prove mean value theorem.

6. If p^* is a refinement of p then prove that $U(p^*, f, \alpha) \leq U(p, f, \alpha)$.

7. State and prove Weierstrass theorem.

8. Prove that a linear operator A on R^n is invertible if and only if $\det[A] = 0$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that for every real $x > 0$ and every integer $n > 0$ there is one and only one real y such that $y^n = x$.

10. Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

11. Let f be a continuous mapping of a compact metric space X in to a metric space Y, then prove that f is uniformly continuous on X.

12. State and prove L'Hospital rule.

13. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable and $\Lambda(r) = \int_r^b |\gamma'(t)|\, dt$.

14. State and prove the Stone-Weierstrass theorem.

15. State and prove Parseval's theorem.

16. State and prove the contraction principle.

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PG–455	PGDMAT–13

P.G. DIPLOMA IN MATHEMATICS EXAMINATION — JUNE 2018.

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. If Y is a subspace of X, then prove that set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

2. Prove that the image of a connected space under a continuous map is connected.

3. Prove that compactness implies the limit point compartness but not conversely.

4. Define the following :

   (a) First countable space

   (b) Second countable space

   (c) Dense subset.

5. Prove that the space l_p^n of all $n-$tuples $x=(x_1, x_2..........x_n)$ of scalars with the norm defined by $\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}$ is a Banach space.

6. State and prove the uniform Boundedness theorem.

7. State and prove the Schwarz inequality in a Hilbert space H.

8. If M is a closed linear subspace of a Hilbert space H, the prove that $H = M \oplus M^{\perp}$.

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. If f is a continuous function from $X \to Y$, then prove that for every convergent sequence $\{x_n\} \to x$ in X_1, the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y. Show also that the converse holds if X is metrizable.

10. Show that finite Cartesian product of connected spaces is connected.

11. Let X be a non–empty Hausdorff space. If X has no insolated points, then prove that X is uncountable.

12. Suppose that X has a countable basis. Then prove the following :

(a) Every open covering of X contains a countable subcollection covering X.

(b) There exists a countable subset of X that is dense in X.

13. Let M be closed linear subspace of a normed linear space N. If the norm of a coset $x+M$ in the quotient space N/M is defined by

$\|x+M\| = \inf \quad \{\|x+m\| / m \in M\}$, then prove that N/M is a normed linear space. If N is a Banach space, then prove that N/M a Banach space.

14. State and prove the closed graph theorem.

15. Let H be a Hilbert space and let f be an arbitrary functional in H^*. Then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

16. (a) If T is an operator on H then prove that T is normal $\Leftrightarrow$ its real and imaginary parts commute.

(b) Prove that an operator T on H is unitary $\Leftrightarrow$ it is an isometric isomorphism of H onto itself.

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