M.Sc. (Previous) Degree Examinations

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.01: DPA 510: ALGEBRA

Time: 3 hrs] *[Max. Marks: 70/80*

Instructions to candidates:

1. ***Students who have attended 30 marks IA scheme will have to answer for total 70 marks.***
2. ***Students who have attended 20 marks IA scheme will have to answer for total of 80 marks.***
3. ***Answer any FIVE questions from Section – A. Each question carries 14 marks for both 70 – 80 marks scheme and question No. 9 in Section – B is compulsory for 80 marks scheme.***

PART – A

1. a) State and prove Cayley's theorem.

b) Prove that the set I(G) of all inner automorphisms of a group G is a normal subgroup of A(G), the group of all automorphisms of G and that I(G) is isomorphic to G/Z, where Z is the centre of G.

c) State and prove Cauchy's theorem for finite groups. (4 + 5 + 5)

2. a) State and prove Sylow's theorem.

b) If G is a group of order 231, show that 11 – Sylow subgroup of G is contained in the centre of G.

c) Define a solvable group. If G is a group and H is a subgroup of G; then show that H is solvable. (4 + 6 + 4)

3. a) If in a ring R, $x^2 = x$ for all x then show that $2x = 0$ and $x + y = 0$ imply $x = y$.

b) Prove that the ideal S of the ring I of all integers is maximal if and only if S is generated by some prime integer.

c) Find the units in the ring of Gaussian integers and show that they form a multiplicative abelian group. (3 + 4 + 7)

4. a) If R is a Euclidean ring and 'a' and 'b' be any two elements in R, not both of which are zero. Then prove that 'a' and 'b' have a g.c.d 'd' which can be expressed in the form $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.

b) State and prove unique factorization theorem for Euclidean rings.

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c) If $f(x)$ and $g(x) \neq 0$ are in $F[x]$, then show that there exists two polynomials $q(x)$ and $r(x)$ in $F(x)$ such that $f(x) = q(x).\, g(x) + r(x)$, where either $r(x) = 0$ or $\deg r(x) < \deg g(x)$. (4 + 6 + 4)

5. a) Show that the set of all real valued continuous functions satisfying the differential equation $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0$ is a vector space over F. Find a basis of this.

b) If V and W are finite dimensional vector spaces with dimensions m and n respectively, then show that $\text{Hom}(V, W)$ is also a vector space with dimension $'mn'$.

c) If W is a subspace of a vector space, define the annihilator $A(W)$. If V is finite dimensional, show that $\hat{W}$ and $\frac{\hat{V}}{A(W)}$ are isomorphic and that $\dim A(W) = \dim V - \dim W$, where $\wedge$ denotes dual. (4 + 5 + 5)

6. a) Define rank and nullity of a linear transformation. If V is a finite dimensional vector space and if $T \in A(V)$, then prove that $\dim V = rank\, T + nullity\ T$.

b) If $T \in A(V)$ satisfies a polynomial $f(x) \in F[x]$, then Prove that the minimal polynomial for T over F divides $f(x)$.

c) Let V be a vector space of dimension '*n*' over F and let $T \in A(V)$. If $m_1(T)$ and $m_2(T)$ are the matrices in the basis $\{v_1, v_2, ...v_n\}$ and $\{w_1, w_2, ...w_n\}$ respectively, of V over F, then show that there is an invertible matrix C in F_n such that $m_2(T) = C m_1(T) C^{-1}$. (4 + 4 + 6)

7. a) If $T \in A(V)$ is nilpotent, of index of nilpotence then prove that there exists a basis of V such that the matrix of T in this basis has the form.

$$\begin{pmatrix} M_{ns} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_{nr} \end{pmatrix}$$

where $n_1 \geq n_2 \geq .... \geq n_r$ and $n_1 + n_2 + ... + n_r = \dim V(F)$.

b) Suppose that $V = V_1 \oplus V_2$. where V_1 and V_2 are subspaces over V invariant under T and T_1 and T_2 are the linear transformations induced by T on V_1 and V_2 respectively. If $P_1(x)$ and $P_2(x)$ are the minimal polynomials of T_1 and T_2 respectively, prove that the minimal polynomials of T is the *l.c.m* of $P_1(x)$ and $P_2(x)$.

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c) Find all possible Jordon Canonical forms for 10 x 10 matrices whose minimal polynomial is $x^2(x-1)^2\ (x+1)^3$. (6 + 4 + 5)

8. a) If L is a finite extension of K and if K is a finite extension of a field F, show that L is a finite extension of F and $[L:F]=[L:K][K:F]$.

b) If a and b in K are algebraic over F of degrees m and n respectively, then prove that $\pm b$, $a.b$ and $a/b\ (b \neq 0)$ are algebraic over F of degrees atmost *'mn'*.

c) Let R be the field of real numbers and Q, the field of rational numbers. In R, $\sqrt{2}$ and $\sqrt{3}$ are both algebraic over Q.

i) Exhibit a polynomial of degree 4 over Q satisfied by $\sqrt{2}+\sqrt{3}$.

ii) Show that $Q\left(\sqrt{2}+\sqrt{3}\right)=Q\left(\sqrt{2},\sqrt{3}\right)$

What is the degree of $\sqrt{2}+\sqrt{3}$ over Q? (5 + 4 + 5)

PART – B

9. a) Let V be the vector space of all polynomials P from R into R which have degree 2 or less. Define three linear functionals on V by

$$f_1(p)=\int_0^1 p(x)\,dx;\ f_2(p)=\int_0^2 p(x)\,dx;\ f_3(p)=\int_0^{-1} p(x)\,dx$$

Show that $\{f_1, f_2, f_3\}$ is a basis of V^{-1}. Determine a basis of V such that $\{f_1, f_2, f_3\}$ is its dual basis.

b) Find the rank and signature of $x_1^2+2x_1x_2+x_2^2$. (6 + 4)

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M.Sc. (Previous) Degree Examinations

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.02: DPA 520: ANALYSIS – I

Time: 3 hrs] *[Max. Marks: 70/80*

Instructions to candidates:

1. ***Students who have attended 30 marks IA scheme will have to answer for total 70 marks.***
2. ***Students who have attended 20 marks IA scheme will have to answer for total of 80 marks.***
3. ***Answer any FIVE questions from Section – A. Each question carries 14 marks for both 70 – 80 marks scheme and question No. 9 in Section – B is compulsory for 80 marks scheme.***

PART – A

1. a) If $x > 0$, and 'n' be a positive integer then show that $\exists$ unique real y such that $y^n = x$.

b) If α is an irrational number and x is rational then prove that $\alpha + x$ is irrational.

c) Prove that between any two reals there exists infinitely many rationals. (6 + 4 + 4)

2. a) Define open set and closed set. Show that the union of any collection of open sets is open.

b) Define countable set. Prove that a countable union of countable sets is countable.

c) Show that set of irrational numbers is uncountable. (5 + 5 + 4)

3. a) Prove that every K – cell is compact.

b) Prove that every open interval in $\mathbb{R}^1$ is connected.

c) Give an example of an open cover of $(0,1)$ which has no finite sub cover.

(5 + 5 + 4)

4. a) Define convergent sequence. Prove that if sequence $\{x_n\}$, converges to $x \in p$ iff every neighborhood of 'x' contains all but finitely many of the terms of $\{x_n\}$.

b) Let $\{x_n\}$ be a monotonic sequence. Prove that $\{x_n\}$ is convergence if and only if it is bounded.

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c) Let $\{x_n\}$ be the sequence for which there is a constant k, such that $\forall n \in N$, $|x_{n+1} - x_n| < \frac{k}{2^n}$. Then, show that $\{x_n\}$ is Cauchy sequence. (5 + 5 + 4)

5. a) Let X and Y be metric space. Then, show that $f : X \to Y$ is continuous iff for every open set $V \subset Y, \ni f^{-1}(V)$ is open in X.

b) Prove that continuous image of a compact set is compact.

c) Give an example of a function which is continuous but not uniformly continuous on $\mathbb{R}$. (5 + 5 + 4)

6. a) State and prove Cuachy's mean value theorem.

b) Show that if '*f*' is a real differentiable function on $[a, b]$ with $f^1(a) < \lambda < f^1(b)$. Then, there is a point $x \in (a, b)$ such that $f^1(x) = \lambda$.

c) Let $f : \mathbb{R}^1 \to \mathbb{R}^1$ be differentiable function such that f'' exists and $f'' \geq 0$. show that, $f\left(\frac{x+y}{2}\right) \leq \frac{1}{2}\{f(x) + f(y)\}, x, y \in \mathbb{R}^1$. (6 + 4 + 4)

7. a) Define R – S integral. State and prove the Necessary and sufficient condition for R – S integral.

b) Suppose $f \in R(\alpha)$ on $[a, b]$, $m \leq f \leq M$ and ϕ is continuous function on $[m, M]$ then, show that $\phi \circ f\ R(\alpha)$ on $[a, b]$.

c) Define $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$. Compute $\int_0^{-1} f\, dx$ and $\int_{-0}^{1} f.dx$. (6 + 4 + 4)

8. a) Prove the fundamental theorem of integral calculus.

b) If *f* is continuous on $[a,b]$, show that *f* need not be of bounded variation.

c) Using fundamental theorem of integral calculus to compute $\int_0^3 (x^2 - x)dx$.

(5 + 5 + 4)

PART – B

9. a) State and prove Taylor's theorem.

b) If $f \in BV[a, b]$ does $\sqrt{f} \in BV[a, b]$? (5 + 5)

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M.Sc. (Previous) Degree Examinations

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.03: DPA 530: ANALYSIS – II

Time: 3 hrs] *[Max. Marks: 70/80*

Instructions to candidates:

1. ***Students who have attended 30 marks IA scheme will have to answer for total 70 marks.***
2. ***Students who have attended 20 marks IA scheme will have to answer for total of 80 marks.***
3. ***Answer any FIVE questions from Section – A. Each question carries 14 marks for both 70 – 80 marks scheme and question No. 9 in Section – B is compulsory for 80 marks scheme.***

PART – A

1. a) Suppose (i) The partial sums A_n of $\sum a_n$ form a bounded sequence

ii) $b_0 \geq b_1 \geq b_2 \geq ......,$

iii) $\lim_{n\to\infty} b_n = 0$

Then prove that $\sum a_n b_n$ converges.

b) State and prove Marten's theorem.

c) Give an example to show that the product of two convergent series need not be convergent. (4 + 5 + 5)

2. a) State and prove the Riemannian rearrangement theorem.

b) Prove the Cauchy's criterion for uniform convergence of sequence of function.

c) Give an example to show every where discontinues limit function, which is not Riemann integrable. (5 + 5 + 4)

3. a) State and prove Weirstrass M – Test.

b) Prove that if $\{f_n\}$ is sequence of continuous function on E and if $f_n \to f$ uniformly on E then prove that f is continuous on E.

c) Give an example to show that convergent sequence of continuous functions may converge to discontinuous limit function. (5 + 5 + 4)

4. a) State and prove the relation between uniform convergence and differentiation.

b) Show that if α be monotonically increasing on $[a,b]$. Suppose $f_n \in R(\alpha)$ on $[a,b]$ for $n = 1,2,3......$ and $f_n \to f$ uniformly on $[a,b]$, then prove that $f_n \in R(\alpha)$ on $[a,b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty}\int_a^b f_n d\alpha$.

c) Show that the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n+x^2}$ is uniformly convergent but not absolutely for all real values of x. (5 + 5 + 4)

5. a) Define Exponential function and Trigonometric functions $c(x)$ and $s(x)$. Show that the functions $c(x)$ and $s(x)$ are periodic with period 2π.

b) With usual notations prove the following:

i) Prove that $E(z)$ is periodic with period $2\pi i$.

ii) If $0 < t < 2\pi$, prove that $E(it) \neq 1$.

c) Derive Exponential function $E(x) = e^x$. Further show that $\lim_{n\to\infty} x^{n-x}e = 0$ every n. (5 + 4 + 5)

6. a) If $f(x)$ and $g(x)$ be two positive functions in $[a, b]$ such that $\lim_{x\to a^+}\frac{f(x)}{g(x)} = l$, where l is a non zero finite number, then show that the two integrals $\int_a^b f\,dx$ and $\int_a^b g\,dx$ converge and diverge together at 'a'.

b) If ϕ is bounded and monotonic in $[a, \infty]$ and $\int_a^{\infty} f\,dx$ is convergent at ∞ then show that $\int_a^{\infty} f_0\phi\,dx$ is convergent at ∞.

c) Test the convergence of the following integrals

$i)\ \int_0^2 \frac{dx}{(2x-x^2)}$ $\qquad ii)\ \int_0^{\pi}\frac{dx}{\sin x}$ (5 + 5 + 4)

7. a) If $u = f(x, y)$ is a homogeneous function of degree 'n', then show that $X.\frac{\partial u}{\partial x} + Y.\frac{\partial u}{\partial y} = nu$.

b) Prove that to every $A \in L\ (\mathbb{R}'', \mathbb{R}')$ corresponds a unique $y \in \mathbb{R}^n$ such that $Ax = x.y$ and also $\|A\| = y$.

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c) Show that $2x^4 - 3x^2 y + y^2$ has neither a maximum nor a minimum at $(0, 0)$.
(6 + 4 + 4)

8. a) State and prove Inverse function theorem.

b) Define f in $\mathbb{R}^3$ by $f(x, y_1, y_2) = x^2 y_1 - e^x + y_2$ show that $f(0,1,-1) = 0, (D, f)(0,1,-1) \neq 0$ and there exists a differentiable function g in some neighborhood of $(1,-1)$ in $\mathbb{R}^2$ such that $g(1,-1) = 0$ and $f(g(y_1, y_2)), y_1, y_2) = 0$. Find $(D_1\ g)(1, -1)$ and $(D_2\ g)(1, -1)$. (9 + 5)

PART – B

9. a) Define contraction. Prove that the contraction principle.

b) Show that $\pi(m)\ \pi(1-m) = \dfrac{\pi}{\sin m\pi},\ 0 < m < 1.$ (6 + 4)

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M.Sc. (Previous) Degree Examination

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – PM 10.04: DPA 540: DIFFERENTIAL EQUATIONS

Time: 3 hrs] *[Max. Marks: 70/80*

Instructions to candidates:

1. ***Students who have attended 30 marks IA scheme will have to answer for total 70 marks.***
2. ***Students who have attended 20 marks IA scheme will have to answer for total of 80 marks.***
3. ***Answer any FIVE questions from Section – A. Each question carries 14 marks for both 70 – 80 marks scheme and question No. 9 in Section – B is compulsory for 80 marks scheme.***

PART – A

1. a) If ψ_1 and ψ_2 are two linearly independent solutions of $y'' + a_1 y' + a_2 y = 0$ on an interval I, then prove that their Wronskien is identically zero or never zero on I.

b) Find the function ϕ which is continuous on $0 \le x \le 2$ and which satisfies $\phi(0) = 0,\ \phi'(0) = 1$ and
$y'' - 4y = 0,\ 0 \le x \le 1$ and
$y'' - 16y = 0,\ 1 \le x \le 2$. (7 + 7)

2. a) Find the solution of the Legendre differential equation $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$ about $x = 0$.

b) State and prove Sturm comparison theorem. (8 + 6)

3. a) Obtain Green's function for the B.V.P $y'' + y = f(x),\ y(0) = y(1) = 0$.

b) Obtain the Rodrigues relation for Legendre solution in the form $P_n(x) = \dfrac{1}{2^n\, n!} \dfrac{d^n}{dx^n}(x^2 - 1)^n$ (6 + 8)

4. a) Show that ∞ is a regular singular point of hyper geometric equation $x(1 - x)y'' + [r(\alpha + \beta + 1)x]y' - \alpha\beta y = 0$ about $x = 0$, and hence find its solution.

b) Prove that *i*) $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$

Contd………... 2

ii) $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$ (8 + 6)

5. State and prove Picard's theorem for existence and uniqueness of an initial value problem; $y' = f(x, y),\ y(x_0) = y_0$. And hence find the solution of $y' = 1 - y$ with $y(0) = 0$. (14)

6. a) Find the inverse Laplace transform of

i) $\frac{1}{s\sqrt{s+1}}$ ii) $\log\left(\frac{s^2+1}{(s-1)^2}\right)$ iii) $\cot^{-1}\left(s/k\right)$

b) Employ Laplace transform method to solve $y'' - 7y' + 6y = \sin x$ with $y(0) = 0$ and $y'(0) = 1$. (8 + 6)

7. a) Using Charpits method, obtain solution for $\left(p^2 + q^2\right)y = qz$, which passes through $y = 0\ \&\ z^2 = 4x$.

b) Find the integral surface of linear P.D.E $2y(z-3)p + (2x - z)q = y(2x-3)$ which passes through the curve $z = 0,\ x^2 + y^2 = 2x$. (7 + 7)

8. a) Find the Canonical form of the equation.

$$e^y \frac{\partial^2 z}{\partial x^2} + e^x \frac{\partial^2 z}{\partial y^2} + \sin x \frac{\partial z}{\partial x} + \cos y \frac{\partial z}{\partial y} = 0.$$

b) Solve $u_t = c^2 u_{xx}$ $0 \le x \le l,\ t \le 0$ satisfying the boundary conditions $u(0, t) = K_1$ and $u(l, t) = K_2$ with initial condition $u(x, 0) = \phi(x)$. (7 + 7)

PART – B

9. a) Solve Bessel's Equation of order zero $x^2 y'' + xy' + x^2 y = 0$ about $x = 0$.

b) Find the solution of $y'' + y = \sin 3x$ using method of variation of parameters.

(5 + 5)

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