Semester-I

Core Course-I

MAT MC1: Field Theory & Commutative Algebra-I

[7 hrs/per week (including Tutorials)]
[Max. Marks: 100]
(Final-80+Internal Assessment-20)
Time : 3hrs

Objective

The objective of this course is to introduce the basic ideas of Field Theory and Galois Theory and to see its application to the solvability of polynomial equations by radicals. Answers to some classical problems of ancient Greeks regarding the ruler and compass constructions shall be obtained as a consequence of the development of the subject. This course also provides the foundation required for more advanced studies in Algebra. The aim is also to develop necessary prerequisites for Math 723S.

Note :
1.   **The question paper will have eight questions. Candidates will attempt five questions.**
2.   **There will be two questions from each UNIT and the students will be required to answer at least one question from each UNIT.**
3.   **All questions carry equal marks.**

UNIT-I

Field Theory: Fields, Examples, Algebraic and Transcedental elements. The degree of a field extension. Adjunction of roots. Splitting fields. Finite fields. Algebraically closed fields. Separable and purely inseparable extensions.

UNIT-II

Perfect fields, primitive elements, Lagrange's theorem on primitive elements. Normal extensions, Galois extensions, The fundamental theorem of Galois Theory.

UNIT-III

Symmetric functions. Cyclotomic extensions. Cyclic extensions, Norms and traces. Quintic Equations and solvability by radicals.

UNIT-IV

Review of Rings and ring homomorphism, ideals, quotient rings, zero divisors, nilpotent elements, units, prime ideals and maximal ideals, Nilradical and Jacobson radical, operation on ideals, extension and contraction of ideals, Modules and module homomorphisms, submodule and quotient module, operation on submodules, direct sum and product, finitely generated modules, exact sequences, tensor product of modules,