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ANNA UNIVERSITY

B.E./B.TECH. DEGREE END SEMESTER EXAMS, APR/MAY 2011

COMPUTER SCEINCE AND ENGINEERING BRANCH

FOURTH SEMESTER – (REGULATIONS 2005)

MA 505 – PROABILITY AND QUEUEING THEORY

Time : 3 Hours **Answer All Questions** **Max. Marks : 100**

Part A **(10 × 2 = 20)**

1. A box contains 3 good and 7 faulty items. Two items are drawn at random without replacement. If the second selection is given to be good, what is the probability that the first is also good.
2. Find the moment generating function of continuous probability distribution whose density is $4e^{-2x}, x \geq 0$.
3. If on average 8 ships out of 10 arrive safely to ports, find the mean and standard deviation of ships returning safely out of a total of 400 ships.
4. Find the mean of the gamma distribution.
5. If X and Y are independent Poisson random variables such that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$, find the $Var(X + 2Y)$
6. The joint probability density function of X and Y is given by $f(x, y) = kxy, 0 < x < 1, 0 < y < 1$. Find the value of k and $E(XY)$.
7. Define Markov process.
8. Suppose that customers arrive at a bank according to Poisson process with a mean rate of 3 per minute. Find the probability that during a time inter arrival of 2 minutes more than 4 customers arrive.
9. Find the traffic intensity for an $(M/M/C):(\infty/FIFO)$ queue with $\lambda = 10$ per hour, $\mu = 15$ per hour and two servers.
10. Define Kendall's notation.

Part-B **(5×16=80)**

11. (i) Let X be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} \frac{1}{4}, & 2 \leq x \leq 6 \\ 0, & otherwise \end{cases}$$

. Find the Mean and variance of X. (8)

(ii) Suppose that there is a chance for a newly constructed flyover to collapse whether the design is faulty or not. The chance that the design is faulty is 5 percent. The chance that the flyover collapses if the design is 95 percent faulty, otherwise it is 30 percent.

The flyover collapsed. What is the probability that it collapsed because of faulty design? (8)

12. (a) (i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability 1) that exactly two messages arrive within one hour 2)no messages arrives within one hour 3) atleast three messages arrive within one hour. (8)

(ii) State and prove memoryless property of geometric distribution. (8)

(OR)

(b) (i) If X is a uniformly distributed over (3,6), find the p.d.f. of $Y = -2logX$. (8)

(ii) Find moment generating function and hence find the mean and variance of a exponential distribution. (8)

13. (a) The joint probability density function of a two-dimensional random variable is

$$f_{XY}(x,y) = \begin{cases} xy^2 + \frac{x^2}{8}, & 0 \le x \le 2, 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$. Compute $P(X > 1)$, $P\left(Y < \frac{1}{2}\right)$, $P\left(X > 1/Y < \frac{1}{2}\right)$, $P(Y < \frac{1}{2}/X > 1)$, $P(X < Y)$ and $P(X + Y \le 1)$. Also verify whether X and Y are independent. (16)

(OR)

(b) (i) Find the conditional density function of X and Y for the bivariate distribution

$$f(x,y) = \begin{cases} \frac{9(1+x+y)}{2(1+x)^4(1+y)^4}, 0 \le x \le \infty,\ 0 \le y \le \infty \\ 0, otherwise \end{cases}.$$ (8)

(ii) The following table gives the joint probability distribution of two random variables X and Y. Find $E(X), E(Y), E(XY)$. Verify whether X and Y are uncorrelated. (8)

Y/X	0	1	2	3
2	1/8	1/8	1/8	1/8
3	1/16	1/8	0	1/16
4	1/16	0	1/8	1/16

14. (a)(i) Show that the random process $X(t) = A\cos(w_0 t + \theta)$ is WSS, if A and w_0 are constants and θ is uniformly distributed random variables in θ to 2π . (8)

(ii) Prove the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution with mean $\frac{1}{\lambda}$. (8)

(OR)

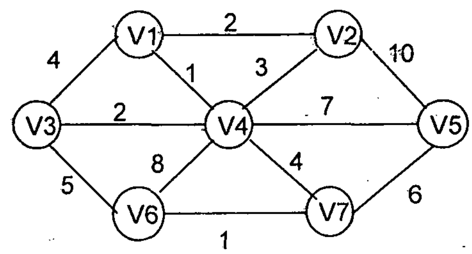
(b) (i) Verify whether the Process $X(t)$ defined by $X(t) = \sin(t + \theta)$, where θ is uniformly distributed in $(0,2\pi)$ is wide sense stationary or not. (8)

(ii) Show the result of running Shellsort on the input 9, 8, 7, 6, 5, 4, 3, 2, 1 using the increments {1, 3, 7}. **(6)**

15 (a)

(i) A graph is implemented by Adjacency matrix of N nodes. Write non recursive algorithm for breadth first search. **(8)**

(ii) Consider the graph: **(8)**



Construct a minimum cost spanning tree using Prim's algorithm and calculate the cost of this tree.

(OR)

15 (b)

(i) Discuss the Dijkstra's algorithm for finding the shortest paths from a source to all other vertices in a directed graph. What is its time complexity? **(6)**

(ii) What is topological sort? Write an algorithm (Topological sort) and to find the result of the following graph. **(10)**