

CET – MATHEMATICS – 2011

VERSION CODE: A – 4

1. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, then the value of $x^{b+c} \cdot y^{c+a} \cdot z^{a+b}$ is

- 1) -1 2) 0 3) 2 4) 1

Ans: (4)

$$\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b} = k \Rightarrow x = e^{k(b-c)}, y = e^{k(c-a)}, z = e^{k(a-b)}$$

$$x^{b+c} \cdot y^{c+a} \cdot z^{a+b} = e^{k(b^2-c^2)} \cdot e^{k(c^2-a^2)} \cdot e^{k(a^2-b^2)} = e^0 = 1$$

2. The sum of the first n terms of $\frac{1^2}{1} + \frac{1^2+2^2}{1+2} + \frac{1^2+2^2+3^2}{1+2+3} + \dots$ is

- 1) $\frac{2n^2-n}{3}$ 2) $\frac{n(n+2)}{3}$ 3) $\frac{2n^2+n}{3}$ 4) $\frac{n^2-2n}{3}$

Ans: (2)

$$\text{Put } n = 2 \quad \frac{1^2}{1} + \frac{1^2+2^2}{1+2} = 1 + \frac{5}{3} = \frac{8}{3}$$

Check with options, option (2) only satisfies

3. If n is an odd positive integer and $(1+x+x^2+x^3)^n = \sum_{r=0}^{3n} a_r x^r$, then

$a_0 - a_1 + a_2 - a_3 + \dots - a_{3n}$ is **(print mistake)**

- 1) 0 2) -1 3) 1 4) 4^n

Ans: (1)

Put $x = -1$

$$(1-1+1-1)^n = \sum_{r=0}^{3n} a_r x^r \Rightarrow 0 = a_0 - a_1 + a_2 - a_3 + \dots$$

4. If r^{th} and $(r+1)^{\text{th}}$ terms in the expansion of $(p+q)^n$ are equal, then $\frac{(n+1)q}{r(p+q)}$ is

- 1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) 1 4) 0

Ans: (3)

$$T_r = {}^nC_{r-1} p^{n-r+1} \cdot q^{r-1} = (T_{r+1}) = {}^nC_r p^{n-r} \cdot q^r \text{ (given)}$$

$$\Rightarrow \frac{n!}{(n-r+1)!(r-1)!} p^{n-r+1} \cdot q^{r-1} = \frac{n!}{(n-1)!r!} p^{n-r} \cdot q^r$$

$$\frac{p}{(n-r+1)} = \frac{q}{r} \Rightarrow pr = nq - rq + q$$

$$(p+q)r = q(n+1)$$

$$\frac{(n+1)q}{(p+q)r} = 1$$

5. If α, β and γ are roots of $x^3 - 2x + 1 = 0$, then the value of $\sum \left(\frac{1}{\alpha + \beta - \gamma} \right)$ is

1) $\frac{1}{2}$

2) 0

3) -1

4) $-\frac{1}{2}$

Ans: (3)

$$x^3 - x - x + 1 = 0 \quad x^2(x^2 - 1) - (x - 1) = 0$$

$$x = 1, x^2 + x - 1 = 0 \quad \alpha = 1, \beta = \frac{-1 + \sqrt{5}}{2}, \gamma = \frac{-1 - \sqrt{5}}{2}$$

$$\frac{1}{\alpha + \beta - \gamma} = \frac{1}{1 + \sqrt{5}} \quad \frac{1}{\beta + \gamma - \alpha} = \frac{1}{-2} \quad \frac{1}{\gamma + \alpha - \beta} = \frac{1}{1 - \sqrt{5}}$$

$$\sum \frac{1}{\alpha + \beta + \gamma} = -\frac{1}{2} - \frac{1}{2} = -1$$

6. Define a relation R on $A = \{1, 2, 3, 4\}$ as xRy if x divides y. R is

1) equivalence

2) symmetric and transitive

3) reflexive and symmetric

4) reflexive and transitive

Ans: (4)

Reflexive and transitive

7. The negation of $p \rightarrow (\sim p \vee q)$ is

1) $p \wedge \sim q$

2) $p \rightarrow q$

3) $p \rightarrow \sim(p \vee q)$

4) $p \vee (p \vee \sim q)$

Ans: (1)

$$\sim(p \rightarrow (\sim p \vee q)) \text{ is } p \wedge \sim(\sim p \vee q)$$

$$\equiv p \wedge (p \wedge \sim q) \equiv (p \wedge \sim q)$$

8. In any triangle ABC, the simplified form of $\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2}$ is

- 1) $a^2 + b^2$ 2) $\frac{1}{a^2} - \frac{1}{b^2}$ 3) $\frac{1}{a^2 - b^2}$ d) $a^2 - b^2$

Ans: (2)

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1 - 2\sin^2 A}{a^2} - \frac{1 - 2\sin^2 B}{b^2} = \frac{1}{a^2} - \frac{1}{2R^2} - \frac{1}{b^2} + \frac{1}{2R^2} = \frac{1}{a^2} - \frac{1}{b^2}$$

9. Angles of elevation of the top of a tower from three points (collinear) A, B and C on a road leading to the foot of the tower are 30° , 45° and 60° respectively. The ratio of AB to BC is

- 1) $2 : \sqrt{3}$ 2) $1 : 2$ 3) $\sqrt{3} : 2$ 4) $\sqrt{3} : 1$

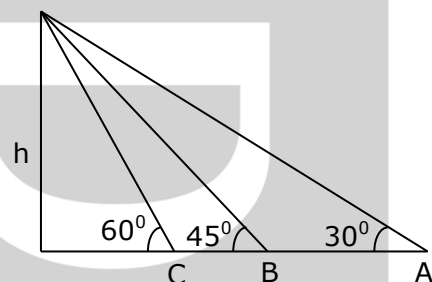
Ans: (4)

$$AB = h (\cot 30^\circ - \cot 45^\circ) = h (\sqrt{3} - 1)$$

$$BC = h (\cot 45^\circ - \cot 60^\circ) = h (1 - \frac{1}{\sqrt{3}})$$

$$\frac{AB}{BC} = \frac{\sqrt{3} - 1}{1 - \frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sqrt{3} : 1$$



10. The value of $\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 70^\circ$ is **(unit is not given)**

- 1) $\frac{1}{16}$ 2) $\frac{\sqrt{3}}{16}$ 3) $\frac{3}{16}$ 4) $\frac{1}{8}$

Ans: (1)

$$\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$$

$$\sin 10^\circ \left(\frac{1}{2}\right) \frac{1}{2} \{\cos 20^\circ - \cos 120^\circ\} = \frac{1}{4} \sin 10^\circ (\cos 20^\circ + \frac{1}{2})$$

$$\frac{1}{4} \sin 10^\circ \cos 20^\circ - \frac{1}{8} \sin 10^\circ = \frac{1}{4} \times \frac{1}{2} (\sin 30^\circ - \sin 10^\circ) + \frac{1}{8} \sin 10^\circ = \frac{1}{16}$$

11. Locus of a point which moves such that its distance from the X-axis is twice its distance from the line $x - y = 0$ is

- 1) $x^2 - 4xy - y^2 = 0$ 2) $x^2 - 4xy + y^2 = 0$
3) $2x^2 - 4xy + y^2 = 0$ 4) $x^2 + 4xy - y^2 = 0$

Ans : (3)

Let P (x, y) be a point on locus cond $|y| = 2 \left| \frac{x-y}{\sqrt{2}} \right| \Rightarrow |y| = \sqrt{2} |x-y|$

squaring $y^2 = 2(x^2 + y^2 - 2xy)$

$$2x^2 + y^2 - 4xy = 0$$

12. The points A (1, 2), B (2, 4) and C (4, 8) form a/an ...

1) right angled triangle

2) straight line

3) equilateral triangle

4) isosceles triangle

Ans: (2)

slope of AB = 2
slope of BC = 2 $\Rightarrow$ A, B, C represent straight line

13. If lines represented by $x + 3y - 6 = 0$, $2x + y - 4 = 0$ and $kx - 3y + 1 = 0$ are concurrent, then the value of k is

1) $-\frac{6}{19}$

2) $-\frac{19}{6}$

c) $\frac{19}{6}$

4) $\frac{6}{19}$

Ans: (3)

Solving $x + 3y = 6$

$$2x + y = 4$$

$$\Rightarrow (x, y) = \left(\frac{6}{5}, \frac{8}{5}\right)$$

Sub in $kx - 3y + 1 = 0$

$$\frac{6k}{5} - \frac{24}{5} + 1 = 0 \Rightarrow k = \frac{19}{6}$$

14. $\lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - \sqrt{x}} \right] = \dots\dots\dots$ (Question incorrect)

1) $\frac{2}{3\sqrt{3}}$

2) $\frac{3\sqrt{3}}{2}$

3) $\frac{2}{\sqrt{3}}$

4) $\frac{2}{3}$

Ans:

Direct substitution of $x = a$, answer is 0, which is not given in the option

15. If $f(x) = \begin{cases} \log x & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$ is continuous at $x = 1$, then the value of k is

1) e

2) 1

3) -1

4) 0

Ans: (2)

$$k = \lim_{x \rightarrow 1} \frac{\log x}{x-1} = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{1} \quad [\text{using L'Hospital Rule}]$$

16. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then $A \cdot A'$ is

- 1) A^2 2) $-A$ 3) A 4) I

Ans: (4)

$$AA' = I \quad [\text{Standard Result}]$$

17. If $\begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$ is singular, then the value of x is

- 1) 0 2) 1 3) 3 4) 2

Ans: (4)

$$\text{Expanding: } x - 2 - 1 - 2(1 - x) - 1(1 - x^2 + 2x) = 0$$

$$x - 3 - 2 + 2x - 1 + x^2 - 2x = 0$$

$$x^2 + x - 6 = 0$$

$$x = 2 \text{ satisfies the above equation}$$

18. If A and B are symmetric matrices of the same order, then which one of the following is NOT true?

- 1) $AB - BA$ is symmetric 2) $AB + BA$ is symmetric
3) $A - B$ is symmetric 4) $A + B$ is symmetric

Ans: (1)

$$AB - BA \text{ is skew symmetric}$$

19. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix} 1 & \omega^2 & 1 - \omega^4 \\ \omega & 1 & 1 + \omega^5 \\ 1 & \omega & \omega^2 \end{vmatrix}$ is

- 1) 4 2) ω^2 3) $\omega^2 - 4$ 4) -4

Ans: (3)

$$1[\omega^2 - \omega - 1] - \omega^2[1 - 1 - \omega^2] + (1 - \omega)[\omega^2 - 1]$$

$$= \omega^2 - \omega - 1 + \omega + \omega^2 - 1 - 1 + \omega$$

$$= \omega^2 - 3 + \omega + \omega^2 = \omega^2 - 4$$

20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then angle between $\vec{a}$ & $\vec{b}$ is

- 1) π 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: (2)

$$|\vec{a} + \vec{b}|^2 = |\vec{c}|^2$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow \vec{a} \cdot \vec{b} = -\frac{1}{2} \Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

21. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are noncoplanar, then the value of $\vec{a} \cdot \left\{ \frac{\vec{b} \times \vec{c}}{3\vec{b} \cdot (\vec{c} \times \vec{a})} \right\} - \vec{b} \cdot$

$$\left\{ \frac{\vec{c} \times \vec{a}}{3\vec{c} \cdot (\vec{a} \times \vec{b})} \right\} \text{ is}$$

- 1) $\frac{1}{6}$ 2) $-\frac{1}{6}$ 3) $-\frac{1}{3}$ 4) $-\frac{1}{2}$

Ans: (2)

$$3 \left(\frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{\vec{a} \cdot (\vec{b} \times \vec{c})} \right) - \frac{1}{2} \left(\frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{\vec{b} \cdot (\vec{c} \times \vec{a})} \right) = \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$$

22. If $2\hat{i} + 3\hat{j}$, $\hat{i} + \hat{j} + \hat{k}$ and $\lambda\hat{i} + 4\hat{j} + 2\hat{k}$ taken in an order are conterminous edges of a parallelepiped of volume 2 Cu units, then value of λ is

- 1) 4 2) 3 3) 2 4) -4

Ans: (1)

$$\begin{vmatrix} 2 & 3 & 0 \\ 1 & 1 & 1 \\ \lambda & 4 & 2 \end{vmatrix} = 2$$

$$2(2 - 4) - 3(2 - \lambda) = 2$$

$$-4 - 6 + 3\lambda = 2 \Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$$

23. A unit vector perpendicular to both $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$ is

- 1) $\frac{3\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ 2) $2\hat{i} + \hat{j} + \hat{k}$ 3) $\frac{(2\hat{i} - \hat{j} - \hat{k})}{\sqrt{6}}$ 4) $(2\hat{i} - \hat{j} + \hat{k}) \sqrt{6}$

Ans: (3)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\hat{n} = \frac{(2\hat{i} - \hat{j} - \hat{k})}{\sqrt{6}}$$

24. The digit in the unit's place of $7^{171} + (177)!$ is

- 1) 0 2) 1 3) 1 4) 3

Ans: (4)

$$7^2 \equiv -1 \pmod{10}$$

$$(7^2)^{85} \equiv -1 \pmod{10}$$

$$7^{170} \cdot 7 \equiv -7 \pmod{10} \equiv 3 \pmod{10}$$

unit digit = 3

but unit digit in $(177)! = 0$

25. The sum of all positive divisors of 242 except 1 and itself is

- 1) 399 2) 342 3) 242 4) 156

Ans: (4)

$$2 \mid 242$$

$$11 \mid 121$$

$$242 = 11^2 \cdot 2^1$$

$$S(242) = \frac{11^3 - 1}{10} \cdot \frac{2^2 - 1}{1} = (133)(3) = 399$$

$$\text{Req. sum} = 399 - (242 + 1) = 156$$

26. On the set of all nonzero reals, an operation $*$ is defined as $a * b = \frac{3ab}{2}$. In this

group, a solution of $(2 * x) * 3^{-1} = 4^{-1}$ is

- 1) $\frac{3}{2}$ 2) $\frac{1}{6}$ 3) 1 4) 6

Ans: (2)

$$4^{-1} = \left(\frac{4}{9}\right) = \frac{1}{9}$$

$$\text{GE} \Rightarrow 2 * x = 4^{-1} * 3 \Rightarrow \frac{3 \cdot 2x}{2} = \frac{1}{9} * 3 \Rightarrow 3x = \frac{1}{2} \Rightarrow x = \frac{1}{6}$$

27. $G = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix}, x \text{ is a nonzero real number} \right\}$ is a group with respect to matrix

multiplication. In this group, the inverse of $\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$ is

- 1) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$ 3) $\begin{bmatrix} 3/4 & 3/4 \\ 3/4 & 3/4 \end{bmatrix}$ 4) $\begin{bmatrix} 4/3 & 4/3 \\ 4/3 & 4/3 \end{bmatrix}$

Ans: (3)

$$\begin{pmatrix} x & x \\ x & x \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{pmatrix} \therefore \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix}$$

28. If $2x^2 + 2y^2 + 4x + 5y + 1 = 0$ and $3x^2 + 3y^2 + 6x - 7y + 3k = 0$ are orthogonal, then value of k is

- 1) $-\frac{17}{12}$ 2) $-\frac{12}{17}$ 3) $\frac{12}{17}$ 4) $\frac{17}{12}$

Ans: (1)

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2 \Rightarrow 2(1) + \frac{5}{2} \left(-\frac{7}{6} \right) = \frac{1}{2} + k \Rightarrow k = -\frac{17}{12}$$

29. The total number of common tangents of $x^2 + y^2 - 6x - 8y + 9 = 0$ and $x^2 + y^2 = 1$ is

- 1) 1 2) 3 3) 2 4) 4

Ans: (2)

$$c_1 = (3, 4) \quad c_2 = (0, 0)$$

$$r_1 = 4 \quad r_2 = 1$$

$$c_1 c_2 = 5 \quad r_1 + r_2 = 5$$

circles touch externally

$\therefore$ No of common tangents 3

30. The center of a circle which cuts $x^2 + y^2 + 6x - 1 = 0$, $x^2 + y^2 - 3y + 2 = 0$ and $x^2 + y^2 + x + y - 3 = 0$ orthogonally is

- 1) $\left(-\frac{1}{7}, \frac{9}{7} \right)$ 2) $\left(\frac{1}{7}, -\frac{9}{7} \right)$ 3) $\left(-\frac{1}{7}, -\frac{9}{7} \right)$ 4) $\left(\frac{1}{7}, \frac{9}{7} \right)$

Ans: (1)

Req. point = radical center

$$S_1 - S_2 = 0 \Rightarrow 6x + 3y - 3 = 0$$

$$S_2 - S_3 = 0 \Rightarrow -x - 4y + 5 = 0$$

$$\Rightarrow -6x - 24y + 30 = 0$$

$$\Rightarrow -21y + 27 = 0$$

$$y = \frac{27}{21} = \frac{9}{7}$$

$$\therefore x = 5 - 4 \left(\frac{9}{7} \right)$$

$$= -\frac{1}{7}$$

$$\Rightarrow (x, y) = \left(-\frac{1}{7}, \frac{9}{7} \right)$$

31. The length of the latus rectum of $3x^2 - 4y + 6x - 3 = 0$ is

1) 3

2) 2

3) $\frac{4}{3}$

4) $\frac{3}{4}$

Ans: (3)

$$3x^2 + 6x = 4y + 3 \Rightarrow 3(x + 1)^2 = 4y + 6 \Rightarrow (x + 1)^2 = \frac{4}{3} \left(y + \frac{6}{4} \right)$$

$$\text{LLR} = \frac{4}{3}$$

32. The sum of the reciprocals of focal distances of a focal chord PQ of $y^2 = 4ax$ is

1) $\frac{1}{2a}$

2) $2a$

3) a

4) $\frac{1}{a}$

Ans: (4)

consider latus rectum, with ends $(a, 2a), (a, -2a)$.

$$\text{Sum of reciprocals of focal distances is } \frac{1}{2a} + \frac{1}{2a} = \frac{1}{a}$$

33. If the foci of $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{3} = 1$ coincide, then value of a is **(question incorrect)**

1) 1

2) 2

3) $\frac{1}{\sqrt{3}}$

4) $\sqrt{3}$

Ans: (answer not found)

$$\text{Foci of } \frac{x^2}{16} + \frac{y^2}{4} = 1 \text{ is}$$

$$(\pm \sqrt{12}, 0)$$

$$\text{Foci of } \frac{x^2}{a^2} - \frac{y^2}{3} = 1 \text{ is}$$

$$(\pm \sqrt{a^2 + 3}, 0)$$

$$\text{Given } a^2 + 3 = 12 \Rightarrow a^2 = 9 \Rightarrow a = 3$$

34. The equation of a hyperbola whose asymptotes are $3x \pm 5y = 0$ and vertices are $(\pm 5, 0)$ is

1) $9x^2 - 25y^2 = 225$

2) $25x^2 - 9y^2 = 225$

3) $5x^2 - 3y^2 = 225$

4) $3x^2 - 5y^2 = 25$

Ans: (1)

The asymptotes are $\frac{x}{5} + \frac{y}{3} = 0$ and $\frac{x}{5} - \frac{y}{3} = 0$

The equation of hyperbola is $\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1 \Rightarrow 9x^2 - 25y^2 = 225$

OR

Only option (1) has its vertices as $(\pm 5, 0)$

35. The domain of $f(x) = \sin^{-1} \left[\log_2 \left(\frac{x}{2} \right) \right]$ is

1) $4 \leq x \leq 6$

2) $1 \leq x \leq 4$

3) $0 \leq x \leq 4$

4) $0 \leq x \leq 1$

Ans: (2)

$$f(x) = \sin^{-1} \left[\log_2 \left(\frac{x}{2} \right) \right]$$

$$-1 \leq \log_2 \frac{x}{2} \leq 1$$

$$\log_2 \frac{x}{2} \leq 1 \Rightarrow \frac{x}{2} \leq 2$$

$$x \leq 4$$

$$\Rightarrow 1 \leq x \leq 4$$

$$\log_2 \frac{x}{2} \geq -1$$

$$\frac{x}{2} \geq 2^{-1}$$

$$\frac{x}{2} \geq \frac{1}{2}$$

$$x \geq 1$$

$$\text{Also, } \frac{x}{2} > 0$$

$$x > 0$$

$$x \geq 1$$

36. If $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3} \right)$, then x is

1) $\frac{1}{6}$

2) $\frac{1}{4}$

3) $\frac{1}{2}$

4) $\frac{1}{3}$

Ans: (3)

$$\tan^{-1} x + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$$

$$\text{Clearly, } x = \frac{3-1}{3+1} = \frac{1}{2}$$

37. A value of θ satisfying $\sin 5\theta - \sin 3\theta + \sin \theta = 0$ such that $0 < \theta < \frac{\pi}{2}$ is

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$

Ans: (3)

$$\sin \left(\frac{5\pi}{6} \right) - \sin \frac{\pi}{2} + \sin \frac{\pi}{6} = \frac{1}{2} - 1 + \frac{1}{2} = 0$$

38. The value of $\left| \frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)^2} \right|$ is

- 1) $\frac{4}{5}$ 2) $\frac{5}{4}$ 3) 9 4) 20

Ans: (1)

$$\left| \frac{1+i\sqrt{3}}{\left(1+\frac{1}{i+1}\right)^2} \right| = \left| \frac{1+i\sqrt{3}}{(i+2)^2} \right| = \frac{\sqrt{1+3}}{1+4} \times (1+1) = \frac{2 \times 2}{5}$$

39. If ω is an imaginary cube root of unity, then the value of

$(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8) \dots (2n \text{ factors})$ is

- 1) 0 2) 1 3) 2^n 4) 2^{2n}

Ans: (4)

Put $n = 1$

$$\begin{aligned} GE &= (1 - \omega + \omega^2)(1 - \omega^2 + \omega) \\ &= (-2\omega)(-2\omega^2) \\ &= 4\omega^3 = 4 \end{aligned}$$

40. If $P(x, y)$ denotes $z = x + iy$ in Argand's plane and $\left| \frac{z-1}{z+2i} \right| = 1$, then the locus of P is

a/an

- 1) straight line 2) circle 3) ellipse 4) hyperbola

Ans: (1)

$$|z - 1|^2 = |z + 2i|^2$$

$$(x + iy - 1)(x - iy - 1) = (x + iy + 2i)(x - iy + 2i)$$

41. If $\sqrt{r} = ae^{\theta \cot \alpha}$ where a and α are real numbers, then $\frac{d^2r}{d\theta^2} - 4r \cot^2 \alpha$ is

1) 0

2) 1

3) $\frac{1}{r}$

4) r

Ans: (1)

$$\frac{1}{2} \log r = \log a + \theta \cot \alpha$$

$$\frac{1}{2} \cdot \frac{1}{r} \frac{dr}{d\theta} = 0 + \cot \alpha$$

$$\frac{dr}{d\theta} = 2r \cot \alpha$$

$$\frac{d^2r}{d\theta^2} = 2 \cot \alpha \frac{dr}{d\theta} = 2 \cot \alpha \cdot 2r \cot \alpha$$

$$\frac{d^2r}{d\theta^2} = 4r \cot^2 \alpha = 0$$

42. The derivative of $\tan^{-1} \left[\frac{\sin x}{1 + \cos x} \right]$ with respect to $\tan^{-1} \left[\frac{\cos x}{1 + \sin x} \right]$ is

1) -2

2) 0

3) -1

4) 2

Ans: (3)

$$u = \tan^{-1} \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= \tan^{-1} \left(\tan \frac{x}{2} \right)$$

$$\frac{du}{dv} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$$

$$v = \tan^{-1} \left(\frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)^2} \right)$$

$$= \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right)$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

43. $\frac{d}{dx} \left[\cos^2 \left(\cot^{-1} \sqrt{\frac{2+x}{2-x}} \right) \right]$ is

1) $-\frac{3}{4}$

2) $-\frac{1}{2}$

3) $\frac{1}{2}$

4) $\frac{1}{4}$

Ans: (4)

$$x = 2 \cos \theta$$

$$\frac{2+2\cos\theta}{2-2\cos\theta} = \frac{1+\cos\theta}{1-\cos\theta} = \frac{2\cos^2 \frac{\theta}{2}}{2\sin^2 \frac{\theta}{2}} = \cot^2 \frac{\theta}{2}$$

$$y = \cos^2 \left(\frac{\theta}{2} \right) = \frac{1 + \cos \theta}{2} = \frac{1 + \frac{x}{2}}{2} = \frac{1}{2} + \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{4}$$

44. If $f(x) = \frac{\sin^2 x}{1 + \cot x} + \frac{\cos^2 x}{1 + \tan x}$, then $f' \left(\frac{\pi}{4} \right)$ is

1) $-\sqrt{3}$

2) 0

3) $\frac{1}{\sqrt{3}}$

4) $\sqrt{3}$

Ans: (2)

$$f(x) = \frac{\sin^3 x}{\sin x + \cos x} + \frac{\cos^3 x}{\cos x + \sin x}$$

$$f(x) = \sin^2 x - \sin x \cos x + \cos^2 x$$

$$= 1 - \sin x \cos x = 1 - \frac{\sin 2x}{2}$$

$$f'(x) = 0 - \frac{1}{2} \cdot 2 \cos 2x = -\cos 2x$$

$$f' \left(\frac{\pi}{4} \right) = 0$$

45. If $\cos^{-1} \left(\frac{y}{b} \right) = n \log \left(\frac{x}{n} \right)$, then

1) $xy_1 - \sqrt{b^2 - y^2} = 0$

2) $y_1 = x \sqrt{b^2 - y^2}$

3) $xy_1 + n \sqrt{b^2 - y^2} = 0$

4) $xy_1 = n \sqrt{b^2 - y^2}$

Ans: (3)

$$\cos^{-1} \left(\frac{y}{b} \right) = 2 \log \left(\frac{x}{2} \right)$$

$$\text{diff. w.r.t } x = -\frac{1}{\sqrt{1 - \frac{y^2}{b^2}}} \cdot \frac{1}{b} \cdot y_1 = 2 \cdot \frac{1}{x} \cdot \frac{1}{2}$$

$$= -\frac{1}{\sqrt{b^2 - y^2}} \cdot y_1 = \frac{2}{x} \Rightarrow -xy_1 = 2\sqrt{b^2 - y^2}$$

$$xy_1 + 2\sqrt{b^2 - y^2} = 0$$

46. Area of a triangle formed by tangent and normal to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at P

$\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ with x-axis is

- 1) $\frac{b(a^2 + b^2)}{4a}$ 2) $\frac{ab\sqrt{a^2 - b^2}}{4}$ 3) $\frac{ab\sqrt{a^2 + b^2}}{4}$ 4) $4ab$

Ans: (1)

Put $a = b$

$$\frac{xx_1}{a^2} + \frac{yy_1}{a^2} = 1$$

$$\frac{x}{\sqrt{2}a} + \frac{y}{\sqrt{2}a} = 1$$

$$\text{area} = \frac{1}{2} \sqrt{2} a \times \frac{a}{\sqrt{2}} = \frac{a^2}{2}$$

$\therefore$ option (2), (3) and (4) are incorrect

hence option (1) is correct

47. The angle between $y^2 = 4x$ and $x^2 + y^2 = 12$ at a point of their intersection is

- 1) $\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\tan^{-1} 2\sqrt{2}$ 3) $\tan^{-1} 2$ 4) $\tan^{-1} \sqrt{2}$

Ans: (2)

$$2yy_1 = 4$$

$$2x + 2yy_1 = 0$$

$$y_1 = \frac{4}{2y}$$

$$yy_1 = -x$$

$$y_1 = \frac{2}{y}$$

$$y_1 = -\frac{x}{y}$$

$$m_1 = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$m_2 = -\frac{2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\tan \theta = \left| \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{1 + \left(-\frac{1}{2}\right)} \right| = \left(\frac{\sqrt{2}}{\frac{1}{2}} \right) \cdot 2\sqrt{2}$$

48. A sphere increases its volume at the rate of π cc/s. The rate at which its surface area increases when the radius is 1 cm is

- 1) $\frac{\pi}{2}$ sq. cm/s 2) $\frac{3\pi}{2}$ sq. cm/s 3) π sq. cm/s 4) 2π sq. cm/s

Ans: (4)

$$V = \frac{4}{3} \pi r^3 \quad S = 4\pi r^2$$

$$\frac{dV}{dt} = 4\pi \cdot r^2 \frac{dr}{dt} \quad \frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt}$$

$$\pi = 4\pi \frac{dr}{dt} = 8\pi r \cdot \frac{1}{4} = 2\pi$$

$$\Rightarrow \frac{dr}{dt} = \frac{1}{4}$$

49. The value of $\int_0^4 |x-1| dx$ is

1) 1

2) 4

3) 5

4) $\frac{5}{2}$

Ans: (3)

$$\int_0^4 |x-1| dx = \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= + \int_0^1 (1-x) dx + \int_1^4 (x-1) dx$$

$$= x - \frac{x^2}{2} \Big|_0^1 + \frac{x^2}{2} - x \Big|_1^4$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 = 5$$

50. If $I_n = \int_0^{\pi/4} \tan^n x dx$, where n is a positive integer, then $I_{10} + I_8$ is

1) 9

2) $\frac{1}{7}$

3) $\frac{1}{8}$

4) $\frac{1}{9}$

Ans: (4)

$$I_n = \int_0^{\pi/4} \tan^n x dx = \int_0^{\pi/4} \tan^{n-2} x \cdot \tan^2 x dx = \int_0^{\pi/4} \tan^{n-2}(x) [\sec^2 x - 1] dx$$

$$I_n = \int_0^{\pi/4} \tan^{n-2}(x) dx - I_{n-2}$$

$$I_n + I_{n-2} = \frac{\tan^{(n-1)}(x)}{n-1} \Big|_0^{\pi/4}$$

$$I_n + I_{n-2} = \frac{1}{n-1} \quad \therefore I_{10} + I_8 = \frac{1}{9}$$

51. $\int e^x \left[\frac{\sin x + \cos x}{1 - \sin^2 x} \right] dx$ is

1) $e^x \tan x + c$

2) $(e^x \cdot \sec x) + c$

3) $e^x \cot x + c$

4) $(e^x \operatorname{cosec} x) + c$

Ans: (2)

$$\int e^x \left(\frac{\sin x + \cos x}{1 - \sin^2 x} \right) dx$$

$$= \int e^x (\tan x \cdot \sec x + \sec x) dx = e^x \sec x + C$$

52. When $x > 0$, then $\int \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx$ is

1) $2x \tan^{-1} x - \log(1+x^2) + c$

2) $2x \tan^{-1} x + \log(1+x^2) + c$

3) $2 [x \tan^{-1} x + \log(1+x^2)] + c$

4) $2 [x \tan^{-1} x - \log(1+x^2)] + c$

Ans: (1)

$$\int \cos^{-1} \left[\frac{1-x^2}{1+x^2} \right] dx = \int \cos^{-1} \left[\frac{1-\tan^2 \theta}{1+\tan^2 \theta} \right] \quad \text{Put } x = \tan \theta$$

$$= \int \cos^{-1}(\cos 2\theta) dx = \int 2\theta dx = 2 \int \tan^{-1} x dx = 2 \left[\tan^{-1} x \cdot x - \int \frac{1}{1+x^2} dx \right]$$

$$= 2 \left[x \tan^{-1} x - \frac{1}{2} \log(1+x^2) \right] + C$$

$$= 2x \tan^{-1} x - \log(1+x^2) + C$$

53. If the area between $y = mx^2$ and $x = my^2$ ($m > 0$) is $\frac{1}{4}$ sq units, then value of m is

1) $\sqrt{3}$

2) $\sqrt{2}$

3) $\pm 2\sqrt{3}$

4) $\pm 3\sqrt{2}$

Ans: (answer not found)

$$y = mx^2 \quad x = my^2$$

$$x^2 = \frac{1}{m} y \quad y^2 = \frac{1}{m} x$$

$$4a = m, \quad a = \frac{1}{4} m$$

$$\text{Area} = \frac{16a^2}{3} = \frac{1}{4} \Rightarrow 16 \cdot \frac{1}{3} \cdot \frac{16m^2}{3} = \frac{1}{4} \Rightarrow \frac{1}{3m^2} = \frac{1}{4} \Rightarrow m^2 = \frac{4}{3}$$

$$\Rightarrow m = \pm \frac{2}{\sqrt{3}}$$

54. If m and n are degree and order of $(1 + y_1^2)^{2/3} = y_2$, then the value of $\frac{m+n}{m-n}$ is

- 1) 2 2) 5 3) 4 4) 3

Ans: (2)

$$(1 + y_1^2)^{2/3} = y_2$$

$$(1 + y_1^2)^2 = (y_2)^3$$

$$\therefore m = \text{deg} = 3 \quad n = \text{order} = 2$$

$$\therefore \frac{m+n}{m-n} = \frac{3+2}{3-2} = 5$$

55. The general solution of $\frac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2$ is

1) $2\sin^{-1}y = x\sqrt{1-y^2} + c$

2) $\sin^{-1}y = \frac{1}{2}\sin^{-1}x + c$

3) $\cos^{-1}y = x\cos^{-1}x + c$

4) $2\sin^{-1}y = x\sqrt{1-x^2} + \sin^{-1}x + c$

Ans: (answer not found)

$$\frac{dy}{dx} = 1 - x^2 - y^2 + x^2y^2 = (1 - x^2) - y^2(1 - x^2)$$

$$\frac{dy}{dx} = (1 - x^2)(1 - y^2)$$

$$\frac{dy}{1-y^2} = (1 - x^2) dx$$

$$\int \frac{dy}{1-y^2} = \int (1 - x^2) dx$$

$$\frac{1}{2} \log \left(\frac{1+y}{1-y} \right) = x - \frac{x^3}{3} + C$$

56. If $x \cos \alpha + y \sin \alpha = 4$ is tangent to $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then the value of α is

- 1) $\tan^{-1}(3/\sqrt{7})$ 2) $\tan^{-1}(7/3)$ 3) $\tan^{-1}(\sqrt{3}/7)$ 4) $\tan^{-1}(3/7)$

Ans: (1)

$$x \cos \alpha + y \sin \alpha = 4$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$y \sin \alpha = -x \cos \alpha + 4$$

$$y = (-\cot \alpha)x + 4 \operatorname{cosec} \alpha$$

$$\therefore m = -\cot \alpha, c = 4 \operatorname{cosec} \alpha, a^2 = 25, b^2 = 9$$

$$c^2 = a^2m^2 + b^2$$

$$16 \operatorname{cosec}^2 \alpha = 25 \cot^2 \alpha + 9$$

$$16 (1 + \cot^2 \alpha) = 25 \cot^2 \alpha + 9$$

$$7 = 9 \cot^2 \alpha \Rightarrow \cot \alpha = \frac{\sqrt{7}}{3} \Rightarrow \tan \alpha = \frac{3}{\sqrt{7}} \therefore \alpha = \tan^{-1} (3/\sqrt{7})$$

57. If P is a point on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S and S', then the maximum value of triangle SPS' is

- 1) ab/e 2) abe 3) abe^2 4) ab

Ans: (2)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Maximum area} = \frac{1}{2} \cdot 2ae \cdot b = abe$$

58. In Argand's plane, the point corresponding to $\frac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)}$ lies in

- 1) quadrant IV 2) quadrant III 3) quadrant II 4) quadrant I

Ans: (1)

$$\frac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)} = \frac{1+i-\sqrt{3}i+\sqrt{3}}{\sqrt{3}+i} = \frac{(1+\sqrt{3})+i(1-\sqrt{3})}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{1}{4} [4 - 4i] = 1 - i$$

OR

$$\frac{(1-i\sqrt{3})(1+i)}{(\sqrt{3}+i)} = \frac{2 \operatorname{cis}\left(-\frac{\pi}{3}\right) \cdot \sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)}{2 \operatorname{cis}\left(\frac{\pi}{6}\right)} = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{6}\right) = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$$

$$= \sqrt{2} \left[\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right] = 1 - i$$

59. If $y = \sin x \cdot \sin 2x \cdot \sin 3x \dots \sin nx$; then y' is

- 1) $\sum_{k=1}^n \cot kx$ 2) $y \cdot \sum_{k=1}^n k \tan kx$ 3) $y \cdot \sum_{k=1}^n k \cot kx$ 4) $\sum_{k=1}^n k \tan kx$

Ans: (3)

$$\text{Let } n = 1, y = \sin x, y' = \cos x$$

Method of inspection

Options (1), (2) and (4) are incorrect

Hence option (4) is correct

$$60. \begin{vmatrix} \sin \alpha & \cos \alpha & \sin(\alpha + \delta) \\ \sin \beta & \cos \beta & \sin(\beta + \delta) \\ \sin \gamma & \cos \gamma & \sin(\gamma + \delta) \end{vmatrix} =$$

$$1) 1 - (\sin \alpha - \sin \beta) (\sin \beta - \sin \gamma) (\sin \gamma - \sin \alpha)$$

$$2) 1 + \sin \alpha \sin \beta \sin \gamma$$

$$3) 1$$

$$4) 0$$

Ans: (4)

$$\text{Let } \alpha = \beta = \gamma = \delta = \frac{\pi}{4}$$

$$= \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \end{vmatrix} = 0$$