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# AIEEE – 2012 PAPER & SOLUTIONS

## PAPER – 1 : PHYSICS, CHEMISTRY & MATHEMATICS

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet

### Important Instructions:

1. Immediately fill in the particular on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particular carefully.
3. The Test Booklet is of 3 hours duration
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (Four) marks for each correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question.  $\frac{1}{4}$  (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in 6th the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use Blue/Black Point Pen only for writing particularly/markings response on Side-1 and Side – 2 of the Answer sheet. Use of Pencil is strictly prohibited.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit card inside the examination hall/room.
10. Rough work is to be done on the same provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages (Page 21 – 23) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take this Test Booklet with them.
12. The CODE for this booklet is A. Make sure that CODE Printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the answer sheet.
13. Do not fold or make any stray marks the Answer Sheet.

## PART C – MATHEMATICS

61. Let  $X = \{1, 2, 3, 4, 5\}$ . The number of different ordered pairs  $(Y, Z)$  that can be formed such that  $Y \subseteq X, Z \subseteq X$  and  $Y \cap Z$  is empty, is :

- (1)  $3^5$  (2)  $2^5$   
 (3)  $5^3$  (4)  $5^2$

61. (1)  
 Every element has 3 options. Either set Y or set Z or none  
 So number of ordered pairs =  $3^5$

62. The population  $p(t)$  at time  $t$  of a certain mouse species satisfies the differential equation  $\frac{dp(t)}{dt} = 0.5P(t) - 450$ . If  $P(0) = 850$ , then the time at which the population becomes zero is:

- (1)  $\ln 9$  (2)  $\frac{1}{2} \ln 18$   
 (3)  $\ln 18$  (4)  $2 \ln 18$

62. (4)  

$$\frac{dP(t)}{dt} = .5P(t) - 450$$

$$P(0) = 850$$

$$2 \ln |P(t) - 900| = t + c$$

$$2 \ln 50 = c$$

$$2 \ln \left| \frac{P(t) - 900}{50} \right| = t$$

$$P(t) = 0$$

$$\Rightarrow 2 \ln 18 = t$$
 Ans (4)

63. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a function defined by  $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$ , where  $[x]$  denotes the greatest integer function, then  $f$  is :

1. discontinuous only at  $x = 0$ .
2. discontinuous only at non-zero integral values of  $x$ .
3. continuous only at  $x = 0$ .
4. continuous for every real  $x$ .

63.  $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$   
 At  $x = n, n \in \mathbb{I}$   
 $f(n) = 0$   
 $f(n+h) = n \cos\left\{\frac{2(n+h)-1}{2}\right\}\pi$   
 $f(n^+) = 0$

$$f(n-h) = (n-1) \cos \left\{ \frac{2(n-h)-1}{2} \right\} \pi$$

$$f(n^-) = 0$$

$$f(n^+) = f(n^-) = f(n) = 0 \quad \forall n \in I$$

Ans (4)

64. Let P and Q be  $3 \times 3$  matrices with  $P \neq Q$ . If  $P^3 = Q^3$  and  $P^2Q = Q^2P$ , then determinant of  $(P^2 + Q^2)$  is equal to:

- (1) 1 (2) 0  
 (3) -1 (4) -2

64. (2)

$$\text{Subtracting } P^3 - P^2Q = Q^3 - Q^2P$$

$$P^2(P-Q) + Q^2(P-Q) = 0$$

$$(P^2 + Q^2)(P-Q) = 0$$

If  $|P^2 + Q^2| \neq 0$  then  $P^2 + Q^2$  is invertible

$$\Rightarrow P-Q = 0 \text{ contradiction}$$

$$\text{Hence } |P^2 + Q^2| = 0$$

65. If the integral  $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$  then a is equal to:

- (1) -2 (2) 1  
 (3) 2 (4) -1

65. 3

$$\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$$

$$I = \int \frac{5 \sin x}{\sin x - 2 \cos x} dx$$

$$= \int \frac{(\sin x - 2 \cos x) + 2(\cos x + 2 \sin x)}{(\sin x - 2 \cos x)} dx$$

$$= x + 2 \ln |\sin x - 2 \cos x| + k$$

Thus  $a = 2$

Ans (3).

66. If  $g(x) = \int_0^x \cos 4t dt$ , then  $g(x + \pi)$  equals:

- (1)  $g(x) + g(\pi)$  (2)  $g(x) - g(\pi)$   
 (3)  $g(x) \cdot g(\pi)$  (4)  $g(x)/g(\pi)$

66. 1, 2

$$g(x) = \int_0^x \cos 4t dt = \frac{1}{4} \sin 4x$$

$$g(n + \pi) = \frac{1}{4} \sin 4(n + \pi) = g(x)$$

$$g(\pi) = \frac{1}{4} \sin 4\pi = 0$$

$$\therefore g(n + \pi) = g(x) \pm g(\pi)$$

Ans. (1) and 2 both correct

67. An equation of a plane parallel to the plane  $x + 2y + 2z - 5 = 0$  and at a unit distance from the origin is:

- (1)  $x - 2y + 2z + 1 = 0$                                        (2)  $x - 2y + 2z - 1 = 0$
- (3)  $x - 2y + 2z + 5 = 0$                                        (4)  $x - 2y + 2z - 3 = 0$

67. 4  
 $x - 2y + 2z + k = 0$  .....(1)

$\therefore$  distance of origin from plane (1) is are

$$\therefore \frac{|k|}{3} = 1$$

$$\Rightarrow k = \pm 3$$

$\therefore$  required plane is  $x - 2y + 2z \pm 3 = 0$

Ans (4)

68. A spherical balloon is filled with  $4500\pi$  cubic meters of helium gas. If a leak in the balloon cause the gas to escape at the rate of  $72\pi$  cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is:

- (1)  $7/9$    (2)  $2/9$
- (3)  $9/2$    (4)  $9/7$

68.  $v = \frac{4}{3}\pi r^3 = 4500\pi - 72\pi t$

$$v = \frac{4}{3} = 4500 - 72t$$

$$\frac{dv}{dt} = \frac{4}{3} \times 3\pi r^2 \frac{dr}{dt} = -72\pi$$

$$\frac{dv}{dt} = \frac{4}{3} \times 3r^2 \frac{dr}{dt} = -72 = \frac{dr}{dt} = \frac{-72}{4r^2}$$

At  $t = 49$  minutes

$$r^3 = \frac{(4500 - 72 \times 49) \times 3}{4}$$

$$\Rightarrow r = 9$$

$$\therefore \frac{dr}{dt} = \frac{-18}{9 \times 9} = \frac{-2}{9}$$

69. If the line  $2x + y = k$  passes through the point which divides the line segment joining the points  $(1, 1)$  and  $(2, 4)$  in the ratio  $3 : 2$ , then  $k$  equals:

- (1) 5   (2) 6
- (3)  $11/5$    (4)  $29/5$

69.  $2x + y = k$   
 $(1, 1)$  and  $(2, 4)$   
 $3 : 2$

$$\left(\frac{6 + 2}{5}, \frac{12 + 2}{5}\right) = \left(\frac{8}{5}, \frac{14}{5}\right)$$

$$\frac{16}{5} + \frac{14}{5} = k$$

$$\Rightarrow k = 6$$

Ans. (2)

70. Let  $\hat{a}$  and  $\hat{b}$  be two unit vectors. If the vectors  $\vec{c} = \hat{a} + 2\hat{b}$  and  $\vec{d} = 5\hat{a} - 4\hat{b}$  are perpendicular to each other, then the angle between  $\hat{a}$  and  $\hat{b}$  is :

- (1)  $\frac{\pi}{2}$  (2)  $\frac{\pi}{3}$   
 (3)  $\frac{\pi}{4}$  (4)  $\frac{\pi}{6}$

70.  $\vec{c} \cdot \vec{d} = 0$   
 $\Rightarrow (\hat{a} + 2\hat{b}) \cdot (5\hat{a} - 4\hat{b}) = 0$   
 $\Rightarrow 5 - 8 + 6\hat{a} \cdot \hat{b} = 0$   
 $\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$   
 $\Rightarrow \cos \theta = \frac{1}{2}$   
 $\Rightarrow \theta = \frac{\pi}{3}$

Ans. (2)

71. Statement 1 : An equation of a common tangent to the parabola  $y^2 = 16\sqrt{3}x$  and the ellipse  $2x^2 + y^2 = 4$  is  $y = 2x + 2\sqrt{3}$ .

Statement 2: If the line  $y = mx + \frac{4\sqrt{3}}{m}$ , ( $m \neq 0$ ) is a common tangent to the parabola  $y^2 = 16\sqrt{3}x$  and the ellipse  $2x^2 + y^2 = 4$ , then  $m$  satisfies  $m^4 + 2m^2 = 24$ .

1. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
2. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
3. Statement 1 is true, Statement 2 is false
4. Statement 1 is false, Statement is true.

71.  $y = mx + \frac{4\sqrt{3}}{m}$  is touching the ellipse

$$2x^2 + y^2 = 4$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$$

$$\frac{48}{m^2} = 2m^2 + 4$$

$$\Rightarrow m^4 + 2m^2 = 24$$

Ans. (1)

72. Three numbers are chosen at random without replacement from  $\{1, 2, 3, \dots, 8\}$ . The probability that their minimum is 3, given that their maximum is 6, is:

- (1)  $\frac{1}{5}$  (2)  $\frac{1}{4}$   
 (3)  $\frac{2}{5}$  (4)  $\frac{3}{8}$

$$72. \frac{2c_1}{5c_2} = \frac{2}{10} = \frac{1}{5}$$

Ans (1)

73. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, the slope of the line PQ is :

- |     |      |     |      |
|-----|------|-----|------|
| (1) | - 4  | (2) | -2   |
| (3) | -1/2 | (4) | -1/4 |

$$73. \frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{1}{a} + \frac{2}{b} = 1 \quad \Rightarrow \frac{2}{b} = \frac{a-1}{a} \quad \Rightarrow b = \frac{2a}{a-1}$$

$$\text{Ar.} = \frac{ab}{2} = \frac{a^2}{a-1}$$

$$\frac{d \text{ Ar.}}{da} = 0$$

$$\Rightarrow \frac{(a-1) \cdot 2a - a^2}{(a-1)^2} = 0 \quad \Rightarrow a^2 - 2a = 0 \quad \Rightarrow a = 2$$

$$\therefore b = 4$$

$$\frac{x}{2} + \frac{y}{4} = 1 \quad \Rightarrow 2x + y = 4$$

Thus slope is -2.

Ans (2).

74. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is:

- |     |     |     |     |
|-----|-----|-----|-----|
| (1) | 629 | (2) | 630 |
| (3) | 879 | (4) | 880 |

$$74. (n_1 + 1)(n_2 + 1)(n_3 + 1) - 1$$

$$880 - 1 = 879$$

Ans (3).

75. Statement 1 : The sum of the series  $1 = (1 + 2 + 4) + (4 + 6 + 0) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$  is 8000.

Statement 2 :  $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$ , for any natural number n.

- (1) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
- (2) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
- (3) Statement 1 is true, Statement 2 is false.
- (4) Statement 1 is false, Statement 2 is true.

75. (1)

$$\sum_{k=1}^n \{k^3 - (k-1)^3\} = n^3$$

$$1^3 + (2^3 - 1^3) + (3^3 - 2^3) + \dots + (20^3 - 19^3) = 20^3$$

$$\Rightarrow 1 + (1 + 2 + 2^2) + (2^2 + 2 \cdot 3 + 3^2) + \dots + (19^2 + 19 \cdot 20 + 20^2) = 8000$$

$$\Rightarrow 1 + (1 + 2 + 4) + (4 + 6 + 9) + \dots + (361 + 380 + 400) = 8000$$

76. Let  $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ . If  $u_1$  and  $u_2$  are column matrices such that  $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  and  $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ , then

$u_1 + u_2$  is equal to:

- |    |   |    |   |
|----|---|----|---|
| 1. | $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ | 2. | $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ |
| 3. | $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ | 4. | $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$  |

76.  $Au_1 + Au_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \Rightarrow u_1 + u_2 = A^{-1} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\Rightarrow u_1 + u_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

Ans. (3)

77. The area bounded between the parabolas  $x^2 = \frac{y}{4}$  and  $x^2 = 9y$ , and the straight line  $y=2$  is :

- |    |                        |    |                        |
|----|------------------------|----|------------------------|
| 1. | $\frac{10\sqrt{2}}{3}$ | 2. | $\frac{20\sqrt{2}}{3}$ |
| 3. | $10\sqrt{2}$           | 4. | $20\sqrt{2}$           |

77.  $2 \cdot \int_0^2 \left| \frac{\sqrt{y}}{2} - 3\sqrt{y} \right| dy = 2 \cdot \frac{5}{2} \cdot \frac{2}{3} y^{3/2} \Big|_0^2 = 2 \cdot \frac{5}{3} \cdot 2\sqrt{2} = \frac{20\sqrt{2}}{3}$

Ans. (2)

78. Let  $x_1, x_2, \dots, x_n$  be  $n$  observations, and let  $\bar{x}$  be their arithmetic mean and  $\sigma^2$  be their variance.

Statement 1: Variance of  $2x_1, 2x_2, \dots, 2x_n$  is  $4\sigma^2$ .

Statement 2: Arithmetic mean of  $2x_1, 2x_2, \dots, 2x_n$  is  $4\bar{x}$ .

1. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.
2. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
3. Statement 1 is true, Statement 2 is false.
4. Statement 1 is false, Statement 2 is true.

78. Ans. (3)

79. If  $n$  is a positive integer, then  $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$  is :
1. an odd positive integer
  2. an even positive integer
  3. a rational number other than positive integers
  4. an irrational number

79. (4)

$$(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n} = 2 \left[ {}^{2n}C_1 (\sqrt{3})^{2n-1} + {}^{2n}C_3 (\sqrt{3})^{2n-3} + \dots \right]$$

Which is an irrational number

Ans. (4)

80. If 100 times the 100<sup>th</sup> term of an AP with non zero common difference equals the 50 times its 50<sup>th</sup> term, then the 150<sup>th</sup> term of this AP is:

1. 150 times its 50<sup>th</sup> term
2. 150
3. zero
4. -150

80.  $100 t_{100} = 50 t_{50} \Rightarrow 2[a + 99d] = a + 49d$   
 $\Rightarrow a = -149d$

$$t_{150} = a + 149d = 0$$

Ans. [3]

81. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is:

1.  $3/5$
2.  $6/5$
3.  $5/3$
4.  $10/3$

81.  $a = \sqrt{1 + (3 - a)^2}$   
 $a^2 = 10 - 6a + a^2$   
 $6a = 10$   
 $a = 5/3$   
 $d = 2a = 10/3$   
 Ans. (4)

82. Let  $a, b \in \mathbb{R}$  be such that the function  $f$  given by  $f(x) = \ln|x| + bx^2 + ax$ ,  $x \neq 0$  has extreme values at  $x = -1$  and  $x = 2$ .

Statement 1:  $f$  has local maximum at  $x = -1$  and at  $x = 2$ .

Statement 2:  $a = \frac{1}{2}$  and  $b = \frac{-1}{4}$ .

1. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
2. Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
3. Statement 1 is true, Statement 2 is false.
4. Statement 1 is false, Statement 2 is true.

82. (1)

$$f(x) = \ln|x| + bx^2 + ax$$

$$f'(x) = \frac{1}{x} + 2bx + a$$

$$f'(-1) = 0 \Rightarrow -1 - 2b + a = 0$$

$$f'(2) = 0 \Rightarrow \frac{1}{2} + 4b + a = 0$$



$$\frac{3}{2} + 6b = 0 \quad \Rightarrow \quad b = -\frac{1}{4}$$

$$\therefore a = \frac{1}{2}$$

$$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2}$$

$$f''(-1) = -1 - \frac{1}{2} < 0 \quad \therefore \text{maximum at } x = -1$$

$$f''(2) = -\frac{1}{4} - \frac{1}{2} < 0 \quad \therefore \text{maximum at } x = 2$$

Ans (2)

83. Let ABCD be a parallelogram such that  $\overline{AB} = \vec{q}$ ,  $\overline{AD} = \vec{p}$  and  $\angle BAD$  be an acute angle. If  $\vec{r}$  is the vector that coincides with the altitude directed from the vertex B to the side AD, then  $\vec{r}$  is given by:

- |  |   |
|--|---|
| 1. $\vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right) \vec{p}$ | 2. $\vec{r} = \vec{q} - \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right) \vec{p}$ |
| 3. $\vec{r} = -3\vec{q} + \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$        | 4. $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$        |

83.  $\vec{r} = -\vec{q} + \lambda \vec{p}$  ... (i)

$$\because \vec{r} \perp \vec{p}$$

$$\vec{r} \cdot \vec{p} = 0$$

$$\Rightarrow 0 = -\vec{q} \cdot \vec{p} + \lambda \vec{p} \cdot \vec{p}$$

$$\Rightarrow \lambda = \frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}$$

$$\therefore \vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}}\right) \vec{p}$$

Ans. (1)

84. If the lines  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$  and  $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$  intersect, then k is equal to:

- |                  |                  |
|------------------|------------------|
| 1. $\frac{2}{9}$ | 2. $\frac{9}{2}$ |
| 3. 0             | 4. -1            |

84.  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda_1$

$$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \lambda_2$$

$$2\lambda_1 + 1 = \lambda_2 + 3 \quad \dots(1)$$

$$3\lambda_1 - 1 = 2\lambda_2 + k \quad \dots(2)$$

$$4\lambda_1 + 1 = \lambda_2 \quad \dots(3)$$

From (1) & (3)  $2\lambda_1 + 1 = 4\lambda_1 + 4$

$$\Rightarrow \lambda_1 = -3/2$$

$$\therefore \lambda_2 = -5$$

From (2)  $k = \frac{-9}{2} - 1 + 10 = \frac{9}{2}$

Ans. (2)

85. An ellipse is drawn by taking a diameter of the circle  $(x-1)^2 + y^2 = 1$  as its semi-minor axis and a diameter of the circle  $x^2 + (y-2)^2 = 4$  as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

1.  $x^2 + 4y^2 = 8$
2.  $4x^2 + y^2 = 8$
3.  $x^2 + 4y^2 = 16$
4.  $4x^2 + y^2 = 4$

85.  $b = 2, a = 4$

$\therefore$  equation of the required ellipse is  $\frac{x^2}{16} + \frac{y^2}{4} = 1$

$\Rightarrow x^2 + 4y^2 = 16$

Ans. (3)

86. The negation of the statement

“If I become a teacher, then I will open a school”, is:

1. Either I will not become a teacher or I will not open a school.
2. Neither I will become a teacher nor I will open a school.
3. I will not become a teacher or I will open a school.
4. I will become a teacher and I will not open a school.

86. Ans. (4)

Negation of  $p \rightarrow q$  is  $\sim (p \rightarrow q) = p \wedge \sim q$

i.e., I will become a teacher and I will not open a school.

87. Consider the function,  $f(x) = |x-2| + |x-5|, x \in \mathbb{R}$ .

Statement 1:  $f'(4) = 0$

Statement 2:  $f$  is continuous in  $[2, 5]$ , differentiable in  $(2, 5)$  and  $f(2) = f(5)$ .

1. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
2. Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
3. Statement 1 is true, Statement 2 is false.
4. Statement 1 is false, Statement 2 is true.

87. (3)

$$f(x) = |x-2| + |x-5| \begin{cases} 7-2x & \text{if } x \leq 2 \\ 3 & \text{if } 2 < x < 5 \\ 2x-7 & \text{if } x \geq 5 \end{cases}$$

$\therefore f'(4) = 0$

Ans. (3)

88. If  $z \neq 1$  and  $\frac{z^2}{z-1}$  is real, then the point represented by the complex number  $z$  lies:

1. on a circle with centre at the origin.
2. either on the real axis or on a circle not passing through the origin.
3. on the imaginary axis.
4. either on the real axis or on a circle passing through the origin.

88.  $\frac{z^2}{z-1} = \frac{x^2 - y^2 + 2imy}{(x-1) + iy}$  is real  
 $\Rightarrow -(x^2 - y^2)y + 2xy(x-1) = 0$   
 $\Rightarrow y[-x^2 + y^2 + 2x^2 - 2x] = 0$   
 $\Rightarrow y=0$  or  $x^2 + y^2 - 2x = 0$   $(x-1)^2 + y^2 = 1$   
 $\Rightarrow$  either real axis or circle passing through origin.  
 Ans. (4)

89. The equation  $e^{\sin x} - e^{-\sin x} - 4 = 0$  has :
- |                            |                                   |
|----------------------------|-----------------------------------|
| 1. no real roots.          | 2. exactly one real root.         |
| 3. exactly four real roots | 4. infinite number of real roots. |

89.  $e^{\sin x} - e^{-\sin x} - 4 = 0$   
 $(e^{\sin x})^2 - 4e^{\sin x} - 1 = 0$   
 $e^{\sin x} = \frac{4 \pm 2\sqrt{5}}{2} = 2 + \sqrt{5}$  or  $2 - \sqrt{5}$   
 $e^{\sin x} \neq 2 - \sqrt{5}$  ( $\because e^{\sin x} > 0$ )  
 $\Rightarrow \sin x = \ln(2 + \sqrt{5})$   $\because \ln(2 + \sqrt{5}) > 1$   
 $\therefore \sin x \neq \ln(2 + \sqrt{5})$   
 $\therefore$  no solution  
 Ans. (1)

90. In a  $\Delta PQR$ , if  $3 \sin P + 4 \cos Q = 6$  and  $4 \sin Q + 3 \cos P = 1$ , then the angle R is equal to :

- |                     |                     |
|---------------------|---------------------|
| 1. $\frac{\pi}{6}$  | 2. $\frac{\pi}{4}$  |
| 3. $\frac{3\pi}{4}$ | 4. $\frac{5\pi}{6}$ |

90. (1)  
 $3 \sin P + 4 \cos Q = 6$  ... (1)  
 $4 \sin Q + 3 \cos P = 1$  ... (2)

Squaring and adding (1) & (2)  
 $9 + 16 + 24 \sin(P+Q) = 37$

$$\sin(P+Q) = \frac{1}{2}$$

$$P+Q = 30^\circ \text{ or } 150^\circ$$

$$\therefore R = 150^\circ \text{ or } 30^\circ = \frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

$$\text{If } R = \frac{5\pi}{6} \text{ then } 0 < P, Q < \frac{\pi}{6}$$

$$\Rightarrow \cos Q < 1 \text{ and } \sin P < \frac{1}{2} \quad \Rightarrow \quad 3 \sin P + 4 \cos Q < \frac{11}{2}$$

$$\text{So } R = \frac{\pi}{6}$$