

22121
M

B. Tech. (Sem. - 1st/2nd)
ENGINEERING MATHEMATICS - I
SUBJECT CODE : AM - 101 (2K4)
Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Five** questions from Section - B & C.
- 3) Select at least **Two** Questions from Section - B & C.

Section - A

Q1)

(2 Marks Each)

- a) Find the equation of normal to the surface : $x^2 + y^2 + z^2 = a^2$.
- b) Examine the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$
- c) Define a homogeneous function.
- d) If $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, then what is $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$?
- e) M.I. of rectangular lamina about its side is =?
- f) Name the curve represented by : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$
- g) If $(3+x)^3 - (3-x)^3 = 0$, then prove that $x = 3i \tan \frac{r\pi}{3}$ $r = 0, 1, 2, \dots$
- h) State DeMoivre's theorem.
- i) What is $i^i = ?$
- j) State Ratio test.

Section - B

(8 Marks Each)

- Q2) (a) Use method of Lagrange's to find the minimum value of $x^2 + y^2 + z^2$, given that $xyz = a^3$.
- (b) Expand $e^x \log(1 + y)$ up to six terms of the Taylor series in the neighborhood of $(0,0)$.

Q3) (a) If $u = x + y + z$, $uv = y + z$, $uvw = z$ show that $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$.

(b) if $u = \tan^{-1} \frac{x^3 + y^3}{x + y}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \sin 2x$.

Q4) (a) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

- (b) Find the curvature and radius of curvature of the curve :

$$x = \theta - \sin \theta, y = 1 - \cos \theta.$$

- Q5) (a) Show that the length of an arc of the cycloid :

$$x = a(\theta - \sin \theta), y = a(1 - \cos \theta) \text{ is } 8a.$$

- (b) Find the volume generated by revolving the ellipse : $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about the x-axis.

Section - C

(8 Marks Each)

- Q6) (a) Find the equation of the cone whose vertex is $(1,2,3)$ and which passes through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$.
- (b) Find the centre and radius R of the circle $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z = 15$.

Q7) (a) Change the order of integration in $I = \int_0^{4a^2} \int_{\frac{x^2}{4a}}^{\sqrt{ax}} dy dx$ and hence evaluate it.

- (b) Find the volume of the tetrahedron bounded by the coordinate axes and the plane $x + y + z = a$ by triple integration.

Q8) (a) Sum the series : $\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin(\alpha + \overline{n-1}\beta)$.

(b) If $x + iy = \cosh(u + iv)$ show that $\frac{x^2}{\cosh^2 v} + \frac{y^2}{\cosh^2 u} = 1$.

Q9) (a) Find the interval of convergence of the series $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots \infty$.

(b) Test the convergence of the series :

(i) $\sqrt{x^3 + 1} - x$.

(ii) $\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \dots \infty$.

