(Pages : 3) **2576**

Reg. No. : ....

Name : ....

Combined First and Second Semester B. Tech. Degree Examination, May 2009
ENGINEERING MATHEMATICS – I
(2003 Scheme)

Time: 3 Hours Max. Marks: 100

Instructions: *1) Part **A** is **Compulsory**.*
*2) From Part **B** answer **six** questions choosing **two** from **each** Module.*

PART – A

(Answer **all** questions. **Each** question carries **4** marks.)

1. If x = a(t – sin t), y = a(1 + cos t) find $\frac{d^2y}{dx^2}$.

2. Evaluate $\lim\limits_{x \to 0} \frac{x - \sin x}{x^3}$.

3. Find the maxima and minima of $x^3y^2(12 - x - y)$ if $x > 0$, $y > 0$.

4. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

5. Find the Laplace transform of $\frac{1 - \cos 2t}{t}$.

6. Show that perpendicular tangents of a parabola intersect on the diretrix.

P.T.O.

7. Find the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1).

8. Show that the vectors $X_1 = (1, -1, 0)$, $X_2 = (0, 1, -1)$, $X_3 = (0, 2, 1)$ and $X_4 = (1, 0, 3)$ are linearly dependent and find the relation between them.

9. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & 7 \\ 1 & 2 & 4 \\ 4 & -1 & 7 \\ 2 & 1 & 5 \end{bmatrix}$.

10. Show that the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. **(10×4=40 Marks)**

PART – B

(Answer **two** questions form **each** Module. **Each** question carries **10** marks.)

MODULE – I

11. a) If $\cos^{-1}\left(\frac{y}{b}\right) = n \log\left(\frac{x}{n}\right)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

b) If $\log u = \frac{x^3 + y^3}{3x + 4y}$ show that $x - \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$.

12. Show that the equation of the evolute of the Parabola $x^2 = 4ay$ is $4(y - 2a)^3 = 27ax^2$.

13. a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^P}$ is convergent for $P > 1$ and divergent for $P \leqq 1$.

b) Show that every absolutely convergent series is convergent.

MODULE – II

14. Using Laplace transform solve the differential equation

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=5\sin t, y(0)=y'(0)=0.$$

15. a) Using convolution theorem evaluate $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$.

b) Show that the normal to the rectanguar hyperbola $xy = c^2$ at the point $P\left(ct,\frac{c}{t}\right)$ meets the curve again at the point $Q\left(\frac{-c}{t^3},-ct^3\right)$

16. Find the centre, eccentricity, foci and directrices of the hyperbola.

$9x^2 - 16y^2 + 72x - 32y - 16 = 0.$

MODULE – III

17. Show that the system of equations

$3x + 3y + 2z = 1$

$x + 2y = 4$

$10y + 3z = -2$

$2x - 3y - z = 5$ are consistent and solve them.

18. Find the eigen values and eigen vectos of the matrix

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

19. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into Canonical form. Specify the matrix also.

(6×10=60 Marks)