Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-III Remedial Examination March 2010

Subject code: 130001 **Subject Name: Mathematics -3**

Date: 09 / 03 / 2010 **Time: 11.00 am – 02.00 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Find the solution of differential equation $y e^x dx + \left(2y + e^x\right) dy = 0$, where $y(0) = -1$. **02**

(2) Find the solution of differential equation $y'' + 4y = 2\sin 3x$ by method of undetermined coefficient. **02**

(3) Find $L^{-1}\left\{-\frac{s+10}{s^2 - s - 2}\right\}$. **03**

(b) (1) If possible, find the series solution of $y'' = y'$. **03**

(2) Find the Fourier series of $f(x) = x + |x|$, $-\pi < x < \pi$ **04**

Q.2 **(a)** (1) Find the particular solution of $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$. **02**

(2) Evaluate $\int_0^\infty x^m e^{-a x^n} dx$. **02**

(3) Solve the partial differential equation $u_{xy} = -u_x$. **03**

(b) (1) Evaluate $\int_{-1}^{1} (1+x)^m (1-x)^n dx$, where $m > 0$, $n > 0$ are integers. **03**

(2) Find the solution of Wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions **04**

(i) $u(0,t) = 0$, for all t, (ii) $u(1,t) = 0$ for all t,

(iii) $u(x,0) = f(x) = \begin{cases} 2kx & \text{if } 0 < x < 1/2 \\ 2k(1-x) & \text{if } 1/2 < x < 1 \end{cases}$ (iv) $\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x) = 0$.

OR

(b) (1) Find general solution of $y'' + 9y = \sec 3x$ by method of variation of parameter. **03**

(2) Get the Laplacian operator in cylindrical coordinates. **04**

Q.3 **(a)** (1) Find $L^{-1}\left\{\frac{s^3 + 2s^2 + 2}{s^3\left(s^2+1\right)}\right\}$. **03**

(2) State Convolution theorem and use it to evaluate Laplace inverse of $\frac{a}{s^2\left(s^2 + a^2\right)}$. **04**

(b) (1) Find the Laplace transform of half-wave rectification of $\sin \omega t$ defined by $f(t) = \begin{cases} \sin \omega t & \text{if } 0 < t < \frac{\pi}{\omega} \\ 0 & \text{if } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ where $f\left(t + \frac{2n\pi}{\omega}\right) = f(t)$ for all integer n. **03**

(2) Find a series solution of differential equation $x y'' + 2 y' + x y = 0$. **04**

OR

Q.3 **(a)** (1) Find $L^{-1}\left\{\frac{s^3}{s^4 - 81}\right\}$. **03**

(2) By Laplace transform solve, $y'' + a^2 y = K \sin at$. **04**

(b) (1) Find the inverse transform of the function $ln\left(1 + \frac{w^2}{s^2}\right)$. **03**

(2) Find a series solution of differential equation $\left(x^2 - x\right)y'' - x y' + y = 0$. **04**

Q.4 **(a)** (1) Solve the differential equation $y' + y \sin x = e^{\cos x}$. **03**

(2) Solve the Legendre's equation $\left(1 - x^2\right)y'' - 2 x y' + n(n+1)y = 0$ for $n = 0$. **04**

(b) (1) Write the Bessel's function of the first kind. Also derive $J_0(x)$ and $J_1(x)$ from it. **03**

(2) Prove that $J_0'(x) = -J_1(x)$. **04**

OR

Q.4 **(a)** (1) Solve the differential equation $y' + 6x^2 y = \frac{e^{-2x^3}}{x^2}$, where $y(1) = 0$. **03**

(2) Obtain the Legendre's function as a solution of $\left(1 - x^2\right)y'' - 2 x y' + 2 y = 0$. **04**

(b) (1) Discuss the linear independency/dependency of Bessel's functions $J_n(x)$ and $J_{-n}(x)$. **03**

(2) Show that $J_1'(x) = J_0(x) - x^{-1} J_1(x)$. **04**

Q.5 **(a)** (1) Solve $\left(x^2 D^2 - 3 x D + 3\right)y = 3 ln x - 4$. **03**

(2) Find Fourier series expansion of $f(x) = x^2/2$, $(-\pi < x < \pi)$ **04**

(b) (1) Prove that $\int_0^\infty \frac{1 - \cos \pi w}{w} \sin x w \, dw = \begin{cases} \pi/2 & if \quad 0 < x < \pi \\ 0 & if \quad x > \pi \end{cases}$. **03**

(2) Find Fourier sine series of $f(x) = \pi - x$, $(0 < x < \pi)$. **04**

OR

Q.5 **(a)** (1) Solve $\left(D^2 + 4D + 4\right)y = \frac{e^{-2x}}{x^2}$. **03**

(2) Sketch the function $f(x) = x + \pi$, $(-\pi < x < \pi)$ where $f(x + 2\pi) = f(x)$ and find its Fourier series. **04**

(b) (1) Find the Fourier cosine integral of $f(x) = e^{-kx}$, where $x > 0$, $k > 0$. **03**

(2) Find Fourier cosine series of $f(x) = e^x$, $(0 < x < L)$. **04**

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