



2011-GP

Test Paper Code: GP

Time: 3 Hours

Maximum Marks: 300

**INSTRUCTIONS**

1. This question-cum-answer booklet has **36** pages and has **66** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions in the **Answer Table for Objective Questions**, provided on Page Number **12**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
  - (a) For each correct answer, you will be awarded **3 (Three)** marks.
  - (b) For each wrong answer, you will be awarded **-1 (Negative one)** mark.
  - (c) Multiple answers to a question will be treated as a wrong answer.
  - (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
  - (e) Negative marks for objective part(s) will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, calculator, cellular phone and electronic gadgets in any form are NOT allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.



2011-GP

**READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**

REGISTRATION NUMBER						
Name:						
Test Centre:						
Sections Attempted (Tick <input type="checkbox"/> below)						
Geology	Physics	Mathematics				

**Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.**

I have read all the instructions and shall abide by them.

.....

Signature of the Candidate

I have verified the information filled by the Candidate above.

.....

Signature of the Invigilator

**Special Instructions/ Useful Data**

1.  $\mathbb{R}$  denotes the set of real numbers.
2.  $\mathbb{Z}$  denotes the set of integers.

**IMPORTANT NOTE FOR CANDIDATES**

- **Geology Section** : Q. Nos. 1-15 (Objective Questions) and Q. Nos. 46-52 (Subjective Questions).
- **Physics Section** : Q. Nos. 16-30 (Objective Questions) and Q. Nos. 53-59 (Subjective Questions).
- **Mathematics Section**: Q. Nos. 31-45 (Objective Questions) and Q. Nos. 60-66 (Subjective Questions).
- Select any ***TWO*** Sections.
- Attempt objective and subjective questions of the selected ***TWO*** sections.
- Questions 1-45 (objective questions) carry ***three*** marks each and questions 46-66 (subjective questions) carry ***fifteen*** marks each.
- Write the answers to the objective questions in the **Answer Table for Objective Questions** provided on page 11 only.

**GEOLOGY SECTION**

- Q.1 Which one of the following minerals undergoes chemical weathering most readily?
- (A) olivine  
(B) quartz  
(C) muscovite  
(D) K-feldspar
- Q.2 The Himalayas represent an orogeny along a
- (A) continental-continental plate boundary  
(B) oceanic-oceanic plate boundary  
(C) oceanic-continental plate boundary  
(D) intra-continental rift
- Q.3 An example of a rock produced by dynamic metamorphism is
- (A) quartzite  
(B) greenstone  
(C) mylonite  
(D) hornfels
- Q.4 The intensity of an earthquake is measured by
- (A) amplitude  
(B) extent of damage  
(C) depth and location  
(D) time difference between arrivals of P- and S-waves
- Q.5 What is common to these four ore minerals- chalcocite, smithsonite, covellite and tenorite?
- (A) all are ores of copper  
(B) all are sulfide in composition  
(C) all are of same colour  
(D) all are formed by supergene enrichment process



- Q.6 An isolated mass of rock that has undergone thrusting over several kilometers and is disconnected from its roots, is most appropriately described as a
- (A) nappe
  - (B) klippe
  - (C) window
  - (D) monocline
- Q.7 Which one of the following sequences of the various varieties of coal indicates an increasing order of carbon content?
- (A) peat-lignite-bituminous coal-anthracite
  - (B) peat-bituminous coal-lignite-anthracite
  - (C) lignite-peat-anthracite-bituminous coal
  - (D) anthracite-lignite-bituminous coal-peat
- Q.8 Large scale cross-beds are usually characteristic of
- (A) eolian deposits
  - (B) alluvial deposits
  - (C) deltaic deposits
  - (D) glacial deposits
- Q.9 Match the geological agents under Column-I with their associated features under Column-II.
- | <u>Column-I</u>  | <u>Column-II</u>      |
|------------------|-----------------------|
| P. River         | i. Spit               |
| Q. Wind          | ii. Pedestal          |
| R. Ocean current | iii. Alluvial terrace |
| S. Glacier       | iv. U-shaped valley   |
- (A) P-iv, Q-ii, R-iii, S-i
  - (B) P-ii, Q-i, R-iii, S-iv
  - (C) P-i, Q-iii, R-iv, S-ii
  - (D) P-iii, Q-ii, R-i, S-iv
- Q.10 The numerical age of the Permo-Triassic boundary is
- (A) 65 MYBP
  - (B) 145 MYBP
  - (C) 251 MYBP
  - (D) 542 MYBP

Q.11 Match the formations under Column-I with their respective ages under Column-II.

Column-I

P. Karewa  
Q. Ariyalur  
R. Barail  
S. Malani

Column-II

i. Oligocene  
ii. Plio-Pleistocene  
iii. Cretaceous  
iv. Triassic

- (A) P-i, Q-ii, R-iii, S-iv  
(B) P-iii, Q-iv, R-ii, S-i  
(C) P-ii, Q-iii, R-i, S-iv  
(D) P-iv, Q-i, R-iii, S-ii

Q.12 Which one of the following statements is correct with regard to explosive volcanism?

- (A) It is usually associated with basaltic magma  
(B) It produces extensive lava flows  
(C) It has a high volatile content  
(D) It has a low viscosity

Q.13 A fold with limbs dipping in the same direction but at different angles is best described as

- (A) symmetric  
(B) recumbent  
(C) upright  
(D) overturned

Q.14 Match the minerals under Column-I with their respective crystal structure under Column-II.

Column-I

P. Olivine  
Q. Pyroxene  
R. Amphibole  
S. Mica

Column-II

i. Single chain silicate  
ii. Double chain silicate  
iii. Phyllosilicate  
iv. Nesosilicate

- (A) P-i, Q-ii, R-iii, S-iv  
(B) P-iv, Q-i, R-ii, S-iii  
(C) P-iii, Q-iv, R-ii, S-i  
(D) P-ii, Q-iii, R-iv, S-i

Q.15 Earth's lithosphere is composed of

- (A) crust only  
(B) mantle only  
(C) lower mantle and outer core  
(D) upper mantle and crust

**PHYSICS SECTION**

- Q.16 Kepler's second law of motion states that the rate  $\left(\frac{dS}{dt}\right)$ , at which the area ( $S$ ) is swept out by the line from the sun to the planet of mass  $m$ , is constant and its value in terms of the angular momentum  $L$  of the planet is given by
- (A)  $\frac{L}{m}$   
(B)  $\frac{L^2}{2m}$   
(C)  $\frac{L}{2}$   
(D)  $\frac{L}{2m}$
- Q.17 A manometer tube (U-shaped) is partially filled with water. A non-mixing oil having density 20% less than water density is poured in one of the arms until the oil-water interface comes to the middle of the tube. If the height of water column is 20cm, the height of the oil column will be
- (A) 16 cm  
(B) 20 cm  
(C) 22 cm  
(D) 25 cm
- Q.18 Consider the speed of gas molecules in a container. The ratio of speeds of the gas molecules at  $27^\circ C$  to that at  $-73^\circ C$  is
- (A)  $\sqrt{\frac{3}{2}}$   
(B)  $\sqrt{\frac{2}{3}}$   
(C)  $\sqrt{3}$   
(D)  $\sqrt{\frac{1}{3}}$

Q.19 An electromagnetic wave in free space has an electric field given by

$$E(y, t) = -(3.0 \times 10^5 \text{ V/m}) \hat{k} \sin [ky - (4\pi \times 10^{12} \text{ rad/s})t].$$

The wavelength ( $\lambda$ ) and the magnetic induction vector (B) of the wave is given by

(A)  $\lambda = 300 \mu\text{m}$  and  $B(y, t) = -(1.0 \times 10^{-3} \text{ T}) \hat{i} \sin [ky - (4\pi \times 10^{12} \text{ rad/s})t]$

(B)  $\lambda = 150 \mu\text{m}$  and  $B(y, t) = (1.0 \times 10^{-3} \text{ T}) \hat{i} \sin [ky - (4\pi \times 10^{12} \text{ rad/s})t]$

(C)  $\lambda = 150 \mu\text{m}$  and  $B(y, t) = -(1.0 \times 10^{-3} \text{ T}) \hat{i} \sin [ky - (4\pi \times 10^{12} \text{ rad/s})t]$

(D)  $\lambda = 300 \mu\text{m}$  and  $B(y, t) = (1.0 \times 10^{-3} \text{ T}) \hat{i} \sin [ky - (4\pi \times 10^{12} \text{ rad/s})t]$

Q.20 The total thermal energy of one litre of oxygen gas at normal temperature and pressure is nearly

(A)  $\approx 10^{23} kT$

(B)  $\approx 0.5 \times 10^{23} kT$

(C)  $\approx \frac{1}{1.5} \times 10^{23} kT$

(D)  $\approx \frac{1}{2.5} \times 10^{23} kT$

Q.21 The total nuclear binding energy of nuclei (except for light mass nuclei) having mass number  $A$  is generally proportional to

(A)  $A^{1/3}$

(B)  $A^{2/3}$

(C)  $A$

(D)  $A^2$

Q.22 The average energy per oscillator for blackbody radiation is given by

(A)  $kT$

(B)  $h\nu$

(C)  $\frac{h\nu}{e^{h\nu/kT} + 1}$

(D)  $\frac{h\nu}{e^{h\nu/kT} - 1}$

Q.23 A radioactive sample has  $27 \times 10^{20}$  atoms at time  $t = 0$ . It reduces to  $1/3$  in number in  $10 \text{ sec}$ . How many atoms of the sample will decay in the time interval  $20 < t < 30 \text{ sec}$ ?

(A)  $10^{20}$  atoms

(B)  $2 \times 10^{20}$  atoms

(C)  $3 \times 10^{20}$  atoms

(D)  $9 \times 10^{20}$  atoms

Q.24 If a crystal plane makes intercepts of 2, 3 and  $\infty$  units on the three crystallographic axes  $a$ ,  $b$  and  $c$ . The Miller indices of the plane are

- (A) (2 3  $\infty$ )
- (B) (3 2 0)
- (C) (0 0 6)
- (D) (6 6 0)

Q.25  $I$ - $V$  characteristics of a p-n junction diode can be represented by

- (A)  $I + I_s = I_s e^{eV/kT}$
- (B)  $I - I_s = I_s e^{-eV/kT}$
- (C)  $I_s - I = I_s e^{eV/kT}$
- (D)  $I + I_s = I_s e^{-eV/kT}$

where  $I_s$  is the saturation current,  $k$  is the Boltzmann constant and  $T$  is the absolute temperature.

Q.26 A diffraction-limited light beam of wavelength  $\lambda$  and width  $2r$  at source, falls on a screen placed at a distance  $D$  from the source. The full-width of the spot illuminated on the screen is

- (A)  $D \frac{\lambda}{2r}$
- (B)  $r \frac{\lambda}{D}$
- (C)  $r \frac{\lambda}{2D}$
- (D)  $D \frac{\lambda}{r}$

Q.27 Which one of the following is a circularly polarized standing wave?

- (A)  $\Psi(z, t) = (\hat{i}A_1 + \hat{j}A_2) \sin(kz) \cos(\omega t)$
- (B)  $\Psi(z, t) = (\hat{i}A_1 + \hat{j}A_2) \cos(\omega t - kz)$
- (C)  $\Psi(z, t) = [\hat{i} \cos(\omega t) + \hat{j} \cos(\omega t - \pi/2)] A \sin(kz)$
- (D)  $\Psi(z, t) = A \{ \hat{i} \cos(\omega t - kz) + \hat{j} \cos[(\omega t - \pi/2) - kz] \}$

Q.28 Young's double slit experiment measures

- (A) temporal coherence of light
- (B) spatial coherence of light
- (C) polarization of light source
- (D) beat frequency of light source



Q.29 A quarter-wave plate is inserted in between two crossed polarizers. The polarizer pass axes make an angle  $45^\circ$  with the fast and slow axes of the wave plate. If an unpolarized light beam is passed through the system, the transmission would be

- (A) 0%
- (B) 25%
- (C) 50%
- (D) 100%

Q.30 Two long straight parallel wires are carrying current of same magnitude ' $I$ ', but in opposite direction. If the wires are separated by a distance ' $d$ ', then the magnetic flux at the middle of the separation is

- (A)  $\frac{\mu_0 I}{2\pi d}$
- (B)  $\frac{2\pi\mu_0 I}{d}$
- (C)  $\frac{\mu_0 I}{\pi d}$
- (D) Zero

**MATHEMATICS SECTION**

- Q.31 What is the set of all the limit points of  $\left\{\frac{1}{2n} + \frac{1}{m} : n, m \in \mathbb{N}\right\}$  in  $\mathbb{R}$ ?
- (A)  $\left\{\frac{1}{2n} + \frac{1}{m} : n, m \in \mathbb{N}\right\}$   
 (B)  $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$   
 (C)  $\{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$   
 (D)  $\left\{\frac{1}{2n} : n \in \mathbb{N}\right\}$
- Q.32  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{2n}\right)^{n+5}$  is
- (A)  $e^{\frac{5}{2}}$   
 (B)  $e^5$   
 (C)  $e^{\frac{11}{2}}$   
 (D)  $e^{\frac{1}{2}}$
- Q.33  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} =$
- (A)  $\infty$   
 (B)  $\sum_{n=1}^{\infty} 2^{-n}$   
 (C) 0  
 (D)  $\sum_{n=1}^{\infty} \frac{1}{n^2}$
- Q.34 Consider the sequences of functions  $\{f_n\}_{n=1}^{\infty}$  on  $[0, 1]$ , where  $f_n(x) = x^n$ . Then the sequence  $\{f_n\}_{n=1}^{\infty}$ :
- (A) converges uniformly to function  $f$  where  $f(x) = \begin{cases} 0 & \text{if } x \in [0, 1) \\ 1 & \text{if } x = 1 \end{cases}$   
 (B) does not converge uniformly  
 (C) converges uniformly to 0  
 (D) converges pointwise to 0

Q.35 If the solution of the differential equation  $(e^x + axy)dx + (xb + e^y)dy = 0$  exists, then its solution is:

- (A)  $e^x + x^2y + e^y = c_1$
- (B)  $e^x + xy^2 + e^y = c_2$
- (C)  $e^x + 2x^2y + e^y = c_3$
- (D)  $e^x + 2xy^2 + e^y = c_4$

Q.36  $\frac{d}{dx} \int_0^{\sin x} e^{t^2} dt$  is equal to

- (A)  $e^{\sin^2 x} \cos x$
- (B)  $e^{\sin^2 x}$
- (C)  $(2 \sin x)e^{\sin^2 x}$
- (D)  $e^{2\sin x}$

Q.37 The solution of the initial value problem

$$\frac{dy}{dx} = (y + y^2) \cot x, \quad y\left(\frac{\pi}{2}\right) = 1, \text{ is:}$$

- (A)  $y + 2(1 + y)|\sin x| = 0$
- (B)  $y(1 + y) = |\cos x|$
- (C)  $y(1 - y) = |\cos x|$
- (D)  $2y - (1 + y)|\sin x| = 0$

Q.38 In a kindergarten school, 6% of the boys and 2% of the girls are taller than 50 cm. Moreover, 55% of the kids in this kindergarten school are girls. Suppose a randomly selected kid is taller than 50 cm. What is the probability that the kid is a boy?

- (A)  $\frac{27}{38}$
- (B)  $\frac{11}{38}$
- (C)  $\frac{9}{36}$
- (D)  $\frac{27}{110}$

- Q.39 Let  $\bar{u} = x^3\hat{i} + y^2\hat{j}$ . The directional derivative of  $div\bar{u}$  at the point  $(4,2,0)$  in the direction of the vector  $4\hat{i} - 3\hat{j}$  is
- (A) 14  
 (B) 17  
 (C) 18  
 (D) 15
- Q.40 A fair coin is tossed 5 times. What is the probability that (i) exactly 3 heads occur and (ii) at least 4 heads occur, respectively?
- (A)  $\frac{5}{13}, \frac{5}{32}$   
 (B)  $\frac{5}{32}, \frac{1}{32}$   
 (C)  $\frac{5}{16}, \frac{3}{16}$   
 (D)  $\frac{5}{32}, \frac{3}{16}$
- Q.41 The values of constants  $a$  and  $b$  for which the vector  $\bar{v} = (x^2 + y + (a-b)z)\hat{i} + ((a+b)x - y^2 - z)\hat{j} + (2x - y + z^2)\hat{k}$  is irrotational, are respectively,
- (A)  $a = \frac{3}{2}, b = \frac{1}{2}$   
 (B)  $a = \frac{1}{2}, b = \frac{3}{2}$   
 (C)  $a = -\frac{1}{2}, b = \frac{3}{2}$   
 (D)  $a = \frac{3}{2}, b = -\frac{1}{2}$
- Q.42 The standard deviation of the marks obtained by a very large number of students in an entrance examination is 14. Samples of size 150 students are drawn and standard deviations of marks of these samples are obtained. Then the standard deviation of the sampling distribution of standard deviations is
- (A) 0.57  
 (B) 0.81  
 (C) 1.41  
 (D) 1.62

Q.43 Let  $C_1(1)$  denote the circle of radius 1 and centered at 1 in the complex plane. Then the value of  $\int_{C_1(1)} \left(\frac{z}{z-1}\right)^n dz$  is

(A)  $\frac{1}{n+1} \left(\frac{z}{z-1}\right)^{n+1}$

(B)  $(2\pi i)^n$

(C)  $(1 + \ln|z-1|)^n$

(D)  $2n\pi i$

Q.44 The iterative scheme to obtain the  $p^{\text{th}}$  root of a positive number  $Q$  by the Newton-Raphson's method is:

(A)  $x_{n+1} = \frac{1}{p} \left[ \frac{(p-1)x_n^p - Q}{x_n^{p-1}} \right]$

(B)  $x_{n+1} = \frac{1}{p} \left[ \frac{(p-1)x_n^p + Q}{x_n^{p-1}} \right]$

(C)  $x_{n+1} = \frac{x_n^p - Q}{px_n^{p-1}}$

(D)  $x_{n+1} = \frac{Q - x_n^p}{px_n^{p-1}}$

Q.45 For the central difference operator  $\delta$ , the operator  $\sqrt{1 + \frac{\delta^2}{4}}$  is equivalent to:

(A)  $\left(1 - \frac{\Delta}{2}\right) E^{\frac{1}{2}}$

(B)  $\left(1 - \frac{\nabla}{2}\right) (1 + \nabla)^{\frac{1}{2}}$

(C)  $\left(1 + \frac{\Delta}{2}\right) E^{-\frac{1}{2}}$

(D)  $\left(1 + \frac{\nabla}{2}\right) (1 - \nabla)^{\frac{1}{2}}$

***Answer Table for Objective Questions***

**Write the Code of your chosen answer only in the 'Answer' column against each Question Number. Do not write anything else on this page.**

Question Number	Answer	Do not write in this column	Question Number	Answer	Do not write in this column
01			31		
02			32		
03			33		
04			34		
05			35		
06			36		
07			37		
08			38		
09			39		
10			40		
11			41		
12			42		
13			43		
14			44		
15			45		
16			<b>FOR EVALUTAION ONLY</b>		
17			Correct		Marks (+)
18			Incorrect		Marks (-)
19			<b>Total Marks in Q. 1-15</b>		( )
20					
21			Correct		Marks (+)
22			Incorrect		Marks (-)
23			<b>Total Marks in Q. Nos. 16-30</b>		( )
24					
25			Correct		Marks (+)
26			Incorrect		Marks (-)
27			<b>Total Marks in Q. Nos. 31-45</b>		( )
28					
29					
30					

**GEOLOGY SECTION**

- Q.46 (a) (i) Explain what is meant by the Earth System approach. (6)  
(ii) What is the main driving force for plate movement? (6)
- (b) (i) What is the source of most of the Earth's internal heat? (3)  
(ii) How do P-waves differ from S-waves? (6)

- Q.47 (a) (i) Explain the main difference between Braided and Meandering streams. Draw a neat sketch of a meandering stream and show the sites of deposition and erosion. (6)  
(ii) Explain the difference between Till and Stratified Drift. (3)
- (b) (i) Draw a sketch of a Barchan Dune and show its relationship to the wind direction. (6)  
(ii) How do depositional sea coasts differ from erosional coasts? (6)



- Q.48 (a) (i) How many crystal faces are found in a trisoctahedron and trapezohedron forms of the cubic system?  
(ii) What is the general symbol (Miller Indices) of a dipyramid form in the orthorhombic system?  
(iii) What is the difference between isomorphism and polymorphism? (9)
- (b) (i) Mention two main criteria to distinguish augite from hornblende.  
(ii) Name the characteristic chemical bond which makes graphite useful for pencil leads. (6)

- Q.49 (a) (i) Give a generalized classification of the stratigraphic succession of the Gondwana Supergroup. Also, state the age range of the Gondwana Supergroup as a whole. (6)
- (ii) Which stratigraphic unit in India represents the early Cretaceous continental flood basalt volcanism? (3)
- (b) State Nicolas Steno's three fundamental principles of stratigraphy. (6)

- Q.50 (a) (i) Name the two main types of orthochemical components of limestone.
- (ii) Identify a terrigenous sedimentary rock which is matrix-poor (<15%) and consists mainly of sand-sized grains of quartz, feldspar and rock fragments.
- (iii) Identify a rock consisting of quartz + alkali feldspar + plagioclase feldspar + hypersthene. (9)
- (b) (i) What is the main difference between panidiomorphic and porphyritic texture?
- (ii) Draw a neat sketch showing a Batholith structure. (6)

- Q.51 (a) (i) What is a Similar fold? Give a supporting sketch.
- (ii) Draw a block diagram showing a dipping bed affected by dextral strike-slip fault.
- (iii) What is the difference between Intersection and Crenulation lineation? (9)
- (b) (i) Explain the difference between Disconformity and Nonconformity.
- (ii) Show the outcrop pattern of two thin beds, one vertical and another horizontal, on a contour map of a hill. (6)

- Q.52 (a) (i) What are Early Magmatic Segregation ore deposits? Give a suitable example.  
(ii) Draw a neat sketch showing Comb Structure formed by hydrothermal open space filling.  
(iii) What is the most important source rock for diamond and what is its common mode of occurrence? (9)
- (b) (i) Name a sedimentary basin of India which is exploited for both petroleum and coal (lignite) resources.  
(ii) What is the most important characteristic of an aquifer? (6)

**PHYSICS SECTION**

Q.53 Two particles (A & B) of charges  $+3.0 \times 10^{-9} C$  and  $+6.0 \times 10^{-9} C$  are kept fixed at a distance of  $3 \text{ cm}$  in the horizontal plane. Another particle 'P' of charge  $(- ) 3.0 \times 10^{-9} C$  and mass  $6.48 \times 10^{-9} \text{ Kg}$  is released from the rest at the middle of the line joining A and B.

(a) Calculate the potential at the mid point of  $\overline{AB}$ . (6)

(b) What is the speed of the particle 'P' after moving a distance of  $1 \text{ cm}$ ?

(Given:  $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$ ) (9)

- Q.54 A series electrical circuit consists of a resistance  $1\text{ K}\Omega$ , an inductance  $0.4\text{ mH}$ , a capacitance  $100\text{ pF}$  and an alternating voltage source of  $2\text{ V}$ .
- (a) Calculate the resonance frequency and the rms current at resonance of the circuit. (9)
- (b) What is the rms voltage across each circuit element at resonance? (6)

Q.55 A diffraction grating is to be designed, which will disperse the first order spectrum ( $\lambda_2 - \lambda_1 = \Delta\lambda$ ) through an angular range of  $\Delta\theta$ .

(a) Prove that the number of slits per unit length ( $m$ ) for the grating should be

$$m = \frac{1}{\sqrt{\lambda_1^2 + (\lambda_2 \operatorname{cosec} \Delta\theta - \lambda_1 \cot \Delta\theta)^2}} \quad (9)$$

(b) Determine the condition for which the second order spectra does not overlap with the first order. (6)



Q.56 A straight pipe with a diameter of 1 *cm* and a length of 50 *m* carries oil of density  $\rho = 930 \text{ Kg/m}^3$  and a viscosity coefficient of 0.12 *Pa.s*. The discharge rate is 0.80 *Kg/s* at atmospheric pressure.

(a) Find the gauge pressure at the pipe input. (6)

(b) Determine the maximum stream velocity of the oil. (9)

- Q.57 (a) Write the van der Waals equation of state and explain the two main differences with the ideal gas equation. Obtain an expression for the work done in an isothermal process of a van der Waals gas. (9)
- (b) Using the van der Waals equation, show that the volume at the critical point is  $V_c = 3nb$ , where ' $b$ ' is the usual constant in the van der Waals equation and ' $n$ ' is the number of moles of gas. (6)

- Q.58 (a) Show that the density of nuclear matter ( $\rho$ ) is constant for all nuclei.  
If  $\rho = 2.3 \times 10^{17} \text{ Kg} / \text{m}^3$ , calculate the mass of a nucleon. Given  $r_0 = 1.2 \times 10^{-15} \text{ m}$ . (9)
- (b) A gamma ray photon breaks a deuteron into a proton and a neutron (photo disintegration). Find the minimum possible energy of the gamma ray in MeV. Given  $m_H = 1.00785 \text{ amu}$ ,  $m_n = 1.008665 \text{ amu}$  and  $m_D = 2.014102 \text{ amu}$ . (6)

Q.59 A hypothetical one electron atom, Kaunium, has the following energy levels:

n=4	_____	-3 eV
n=3	_____	-5 eV
n=2	_____	-9 eV
n=1	_____	-21 eV

- (i) What is the energy needed to ionize an electron from its ground state and  $n = 2$  state?
- (ii) If an 18 eV photon is absorbed by the atom in its ground state, what will happen to the atom?
- (iii) Identify the transitions in the atom which fall in the visible spectrum of light and find its wavelength.
- (iv) If the photon emitted in  $n = 3$  to  $n = 2$  transition falls upon a metal and emits a photo electron of energy 0.5 eV, what is the work function of the metal?
- (v) Assuming that Bohr model is valid, what will be the ratio of the orbital radius of  $n=1, 2, 3$  and 4 levels.

(15)

**MATHEMATICS SECTION**

Q.60 Find the eigenvalues and the eigenvectors of  $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ .

(15)

Q.61 (a) Given  $p \geq 1$ , what are the maximum and minimum values of  $f(x) = \frac{(1+x)^p}{1+x^p}$  on  $[0,1]$ ? (9)

(b) Using part (a), show that for  $a \geq 0, b \geq 0$   
$$a^p + b^p \leq (a+b)^p \leq 2^{p-1}(a^p + b^p). \quad (6)$$

Q.62 (a) Find the differential equation of the one parameter family of curves which is orthogonal to the given one parameter family  $x^2 - y^2 + 1 = \lambda xy$ . Obtain the solution of that differential equation. (9)

(b) Obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}(\sin x + x) \quad (6)$$

Q.63 (a) Compute  $\int_{1 \leq x^2 + y^2 \leq 4} \log(x^2 + y^2) dx dy$ .

(9)

(b) Compute the partial derivatives of the function  $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x, y) = \begin{cases} 0 & \text{if } (x, y) = (0, 0) \\ \frac{xy^2}{x^2 + y^4} & \text{otherwise.} \end{cases}$$

Discuss the continuity of  $f$  at  $(0, 0)$ .

(6)



- Q.64 (a) Let  $S$  be the upper half surface of the unit sphere centered at the origin and let  $C$  be its boundary. For the vector

$$\vec{F} = (x^2 - y)\hat{i} + (y^2 z^3)\hat{j} + (y^3 z^2)\hat{k}$$

evaluate the integral  $\oint_C \vec{F} \cdot d\vec{r}$  using the Stoke's theorem.

(9)

- (b) Find the value of the integral  $\oint_C \frac{dz}{(z^2 + 9)^2}$  around the circle  $C: |z + i| = 3$  in the positive sense in  $z$ -plane.

(6)

- Q.65 (a) A box contains 1000 wrist-watches. It was found that 200 defects were left in those watches. If one watch is taken randomly from the box, what is the probability that:  
(i) it has exactly 2 defects? (ii) it has 2 or more defects? Given:

$e^{-0.5} = 0.607$	$e^{-5} = 0.017$	$e^{-0.2} = 0.819$	$e^{-0.02} = 0.980$
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 (9)

- (b) A box contains 15 bolts and three of these bolts are defective. Two bolts are drawn randomly from the box. Find the probability function  $f(x)$  of the random variable  $X$ , which denotes the number of defective bolts in the sample with replacement. Also determine  $f(0)$ ,  $f(1)$  and  $f(2)$ . (6)

Q.66 (a) Evaluate the integral  $\int_{-1.5}^{0.5} \frac{dx}{|x|+1}$  by the Simpson's 1/3-rule with  $h = 0.5$ . (9)

(b) Using the Stirling's formula, find  $f(3.5)$  from the following tabular values:

$x$	1.0	2.0	3.0	4.0	5.0
$f(x)$	1	7	25	61	121

 (6)



**Space for rough work**



**Space for rough work**



**Space for rough work**

GEOLOGY SECTION			PHYSICS SECTION			MATHEMATICS SECTION		
Q. No.	Marks	Signature	Q. No.	Marks	Signature	Q. No.	Marks	Signature
<b>1-15</b> (Objective)			<b>16-30</b> (Objective)			<b>31-45</b> (Objective)		
Candidates are required to answer Objective and Subjective portions of any two (but same) Sections								
<b>46</b>			<b>53</b>			<b>60</b>		
<b>47</b>			<b>54</b>			<b>61</b>		
<b>48</b>			<b>55</b>			<b>62</b>		
<b>49</b>			<b>56</b>			<b>63</b>		
<b>50</b>			<b>57</b>			<b>64</b>		
<b>51</b>			<b>58</b>			<b>65</b>		
<b>52</b>			<b>59</b>			<b>66</b>		
<b>Total</b> (Subjective)			<b>Total</b> (Subjective)			<b>Total</b> (Subjective)		

<b>Total (Objective Part)</b>	:	
<b>Total (Subjective Part)</b>	:	
<b>Grand Total</b>	:	
<b>Total Marks (in words)</b>	:	
<b>Signature of Examiner(s)</b>	:	
<b>Signature of Head Examiner(s)</b>	:	
<b>Signature of Scrutinizer</b>	:	
<b>Signature of Chief Scrutinizer</b>	:	
<b>Signature of Coordinating Head Examiner</b>	:	