NIMCET 2010

MATHEMATICS

1. How many proper subsets of {1, 2, 3, 4, 5, 6, 7} contain the numbers 1 and 7?
(1) 7 (2) 31 (3) 32 (4) 62

2. Identify the wrong statement from the following:
(1) If A and B are two sets, then $A - B = A \cap \overline{B}$
(2) If A, B and C are sets, then $(A - B) - C = (A - C) - (B - C)$
(3) If A and B are two sets, then $\overline{A} \cup \overline{B} = \overline{A} \cap \overline{B}$
(4) If A, B and C are sets, then $A \cap B \cap \overline{C} \subseteq A \cap B$

3. A survey shows that 63% of the Americans like cheese where as 76% like apples. If x% of the Americans lie both cheese and apples, then we have
(1) $x \geq 39$ (2) $x \leq 63$ (3) $39 \leq x \leq 63$ (4) None of these

4. Set A has 3 elements and set B has 4 elements. The number of injection that can be defined from A to B is
(1) 144 (2) 12 (3) 24 (4) 64

5. If $(1 + x)^n = a_0 + a_1x + a_2 x^2 + \ldots . a_n x^n$, then $\left(1+\frac{a_1}{a_0}\right)\left(1+\frac{a_2}{a_1}\right)\left(1+\frac{a_3}{a_2}\right) \ldots . \left(1+\frac{a_n}{a_{n-1}}\right)$
(1) $\frac{n^n}{n!}$ (2) $\frac{(n+1)^n}{n!}$ (3) $\frac{n^{n+1}}{(n+1)!}$ (4) $\frac{(n-1)^n}{n!}$

6. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of india getting at least 7 points is
(1) 0.8750 (2) 0.0875 (3) 0.0625 (4) 0.0250

7. A coin is tossed three times. The probabilities of getting head and tail alternatively is
(1) $\frac{1}{11}$ (2) $\frac{2}{3}$ (3) $\frac{3}{4}$ (4) $\frac{1}{4}$

8. One hundred identical coins, each with probability p of showing up a head, are tossed. If $0 < p < 1$ and if the probability of heads on exactly 50 coins is equal to that of heads on exactly 51 coins then the value of p, is
(1) $\frac{1}{2}$ (2) $\frac{49}{101}$ (3) $\frac{50}{101}$ (4) $\frac{51}{101}$

9. In a Poisson distribution if $P[X = 3] = \frac{1}{4} P[X = 4]$ then $P[X = 5] = kP[X = 7]$ where k equals to
(1) $\frac{1}{7}$ (2) $\frac{21}{128}$ (3) $\frac{128}{21}$ (4) $\frac{21}{256}$

10. The average marks per student in a class of 30 students were 45. On rechecking it was found that marks had been entered wrongly in two cases. After correction these marks were increased by 24 and 34 in the two cases. The correct average marks per student are
(1) 75 (2) 60 (3) 56 (4) 47

11. The value of 'a' for which the system of equations
$a^3 x + (a + 1)^3 y + (a + 2)^3 z = 0$
$ax + (a + 1) y + (a + 2) z = 0$
$x + y + z = 0$
has a non zero solution, is
(1) 1 (2) 0 (3) –1 (4) none of these

12. The value of $X^4 + 9X^3 + 35X^2 - X + 4$ for $X = -5 + 2\sqrt{-4}$ is

(1) 0 (2) –160 (3) 160 (4) –164

13. If $y = a \log x + bx^2 + x$ has its extremum value at $x = -1$ and $x = 2$, then

(1) $a = 2, b = -1$ (2) $a = -2, b = \frac{1}{2}$ (3) $a = 2, b = -\frac{1}{2}$ (4) $a = 1, b = -\frac{1}{2}$

14. If a, b, c are in A.P., p, q, r are in H.P. and ap, bq, cr in G.P., then $\frac{p}{r} + \frac{r}{p}$ is equal to

(1) $\frac{a}{c} - \frac{c}{a}$ (2) $\frac{a}{c} + \frac{c}{a}$ (3) $\frac{b}{q} - \frac{a}{p}$ (4) $\frac{b}{q} + \frac{a}{p}$

15. If $a \neq p, b \neq q, c \neq r$ and $\begin{bmatrix} p & b & c \\ a & q & c \\ a & b & r \end{bmatrix} = 0$, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is

(1) 0 (2) 1 (3) –1 (4) 2

16. Let $\omega \neq 1$ be a cube root of unity and $i = \sqrt{-1}$. The value of the determinant

$\begin{vmatrix} 1 & 1+i+\omega^2 & \omega \\ 1-i & -1 & \omega^2 - 1 \\ -i & -i+\omega-1 & -\omega^3 \end{vmatrix}$ is

(1) 0 (2) ω (3) ω^2 (4) $1 + \omega^2$

17. The point (4, 1) undergoes the following three transformation successively:
(i) Reflection about the line $y = x$
(ii) Transformation through a distance 2 unit along the positive direction of x-axis
(iii) Rotation through an angle of $\frac{\pi}{4}$ about the origin in the anticlockwise direction. The final position of the point is given by the coordinates.

(1) $\left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (2) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (3) $(-2, 7\sqrt{2})$ (4) $(\sqrt{2}, 7\sqrt{2})$

18. If the two pair of lines $X^2 - 2mxy - Y^2 = 0$ and $X^2 - 2nxy - Y^2 = 0$ are such that one of them represent the bisecter of the angles between the other, then

(1) mn+1=0 (2)mn-1=0 (3) $\frac{1}{m} + \frac{1}{n} = 0$ (4) $\frac{1}{m} - \frac{1}{n} = 0$

19. The circle $x^2 + y^2 = 9$ is contained in the circle $x^2 + y^2 - 6x - 8y + 25 = c^2$ if

(1) c = 2 (2) c = 3 (3) c = 5 (4) c = 10

20. If any tangent to the ellipse $\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$ intercepts equal length l on the axes, then l =

(1) $a^2 + b^2$ (2) $\sqrt{a^2 + b^2}$ (3) $(a^2 + b^2)^2$ (4) None of these

21. The angle between the asymptotes of the hyperbola $27x^2 - 9y^2 = 24$ is

(1) 60° (2)120° (3) 30° (4) 150°

22. The angle of intersection of the cardioids $r = a(1 + \cos\theta)$, $r = a(1 - \cos\theta)$ is

(1) $\frac{\pi}{2}$ (2) 0 (3) $\frac{\pi}{4}$ (4) π

23. If f(x) = $\begin{cases} x\sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$ then

(1) f is a continuous function
(2) f'(0+) exits but f'(0 –) does not exist
(3) $f'(0+) \neq f'(0-)$
(4) f'(0 +) and f'(0 –) do not exist

24. If the tangents at the extremities of a focal chord of the parabola $x^2 = 4ay$ meet the tangent at the vertex at points whose abscissa are x_1 and x_2 then x_1x_2 =
(1) a^2 (2) $a^2 - 1$ (3) $a^2 + 1$ (4) $-a^2$

25. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) 2

26. The value of the integral $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{3 + \sin 2x}dx$ is

(1) log 2 (2) log 3 (3) $\frac{1}{4}\log 3$ (4) $\frac{1}{8}\log 3$

27. $\int \log_{10} x dx$ is

(1) $(x - 1) \log_e x + c$
(2) $\log_e 10 . x \log_e\left(\frac{x}{e}\right) + c$
(3) $\log_{10} e . x \log_e\left(\frac{x}{e}\right) + c$
(4) $\frac{1}{x} + c$

28. If $I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2x^3 dx, I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then
(1) $I_3 = I_4$ (2) $I_3 > I_4$ (3) $I_2 > I_1$ (4) $I_1 > I_2$

29. The area between the curves $y = 2 - x^2$ and $y = x^2$ is
(1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{2}{3}$ (4) $\frac{5}{3}$

30. A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If, with respect to the new system, $\vec{a}$ has components p + 1 and 1, then

(1) p = 0 (2) p = 1 or p = $\frac{1}{3}$ (3) p = –1 or p = $\frac{1}{3}$ (4) p = 1 or p = –1

31. The vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are equal in length and taken pairwise make equal angles. If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}$ and $\vec{c}$ make an obtuse angle with the base vector i, then $\vec{c}$ is equal to

(1) $\hat{i} + \hat{k}$ (2) $-\hat{i} + 4\hat{j} - \hat{k}$ (3) $-\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} - \frac{1}{3}\hat{k}$ (4) $\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} - \frac{1}{3}\hat{k}$

32. The position vector of A, B, C and D are $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$, and $\hat{i} - 6\hat{j} - \hat{k}$ then the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is

(1) 0 (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{2}$ (4) π

33. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors, no two of which are collinear and the vector $\vec{a}+\vec{b}$ is collinear with $\vec{c}$, while $\vec{b}+\vec{c}$ is collinear with $\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}$, is equal to

(1) $\vec{a}$ (2) $\vec{b}$ (3) $\vec{c}$ (4) none of these

34. If C is the middle point of AB and P is any point outside AB, then

(1) $\vec{PA}+\vec{PB}=\vec{PC}$ (2) $\vec{PA}+\vec{PB}=2\vec{PC}$

(3) $\vec{PA}+\vec{PB}+\vec{PC}=\vec{O}$ (4) $\vec{PA}+\vec{PB}+2\vec{PC}=\vec{O}$

35. The value of $\sqrt{3}$ cot 20° – 4 cos 20° is

(1)1 (2) –1 (3) 0 (4) none of these

36. If $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$ then x is equal to

(1) a (2) b (3) $\frac{a+b}{1-ab}$ (4) $\frac{a-b}{1+ab}$

37. In a triangle ABC, R is circumradius and $8R^2 = a^2 + b^2 + c^2$. The triangle ABC is

(1) Acute angled (2) Obtuse angled (3) Right angled (4) none of these

38. The rate of increase of length of the shadow of a man 2 meters height, due to a lamp at 10 meters height, when he is moving away from it at the rate of 2m/sec is

(1) $\frac{1}{2}$ m / sec (2) $\frac{2}{5}$ m / sec (3) $\frac{1}{3}$ m / sec (4) 5m / sec

39. A person stands at a point A due south of a tower and observes that its elevation is 60°. He then walks westwards towards B, where the elevation is 45°. At a point C on AB produced, he finds it to be 30°. Then AB/BC is equal to

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) $\frac{5}{2}$

40. The distance between the parallel lines y = 2x + 4 and 6x = 3y + 5

(1) $\frac{17}{\sqrt{3}}$ (2) 1 (3) $\frac{3}{\sqrt{5}}$ (4) $\frac{17\sqrt{5}}{15}$