Mathematics

1-Modern Algebra

Sylow`s theorems, Sylow p-subgroups, Direct product of groups. Structure theorem for finitely generated Abelian groups. Normal and subnormal series. Composition series, Jordan-Holder theorem. solvable groups. Insolvability of s_n for N_5 Nilpotent groups.

Extension fields. Finite,algebraic, and transcendental extensions. Splitting fields. Simple and normal extensions. perfect fields. primitive·elements. Algebraically closed fieds. Automorphisms of extensions. Galois extensions.

Fundamaental theorem of Galois theory. Galsis group over the rationals.

Cyclic modules, simple modules and semi-simple modules.Schur`s lemma Free modules. Noetherian and Artinian modules and rings. Hibert basis theorem.

Solution of polynomial equations by radicals. Insolvability of the general equation of degree 5 by redicals. Finite fields.

Cononical forms: Similarity of Linear transformations. Invariant subspaces. Reduction to triangular forms. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent transformation . The primary decompostion theorem. Jordan blocks and jordan form.

Vector space- Vector spaces, subspaces and linear spans, linear dependence and independence. Finite dimensional vector space. Linear transformations and their matrix representations. Algebra of linear tarnsformations, the rank and nullity theorem. change of basis. Dual spaces, bi dual space and natural isomorphism. Eigen values and eigen vectors of LT. Diagonalization, Cayley-Hamilton theorem.

Inner product spaces, Cauchy-Schwarz inequality orthogonal vectors. Orthonormal basis,Bessel`s inequality, Gram-Schmidt orthogonlization process.

Discrete Mathematics- Lattices and Boolean algebra: Logic: propositional and predicate. Lattices as partially ordered sets and as algebraic systems. Duality, Disributive, Complemented and complete lattices. Lattices and Boolean Algebra. Boolean functions and expressions. Application of Boolean algebra to switching circuits (uring ANDOR and NOT gates) Graphs and Planar Graphs:

Graph, Multigraph, Weighted Graphs. Directed graphs. Paths and circuits. Matrix representaton of graphs. Eulerian Paths and Circuits. Planar graphs.

2-Topology

Definition and examples of topological spaces. Closed sets. Closure. Dense sets. Neighbourhoods, interior, exterior, and boundary Accumulation points and derived sets Bases and sub-bases. Subspaces and relative topology.

Alternative methods of defining a topology in terms of Kuratowski closure operator and neighbourhood systems.

Continuous functions and homeomorphism. First and second countable space. Lindelof spaces. Separable spaces.

The separation axioms T_0, T_1, T_2, $T_{3\,1/2}$, T_4 their characterizations and basic properties. Unysohn`s lemma. Tietz extension theorem.

Compactness. Basic properties of compactness. Compactness and finite intersection property. Sequential, countable and B-W compactness Local compactness One poiont and Stone-Cech compactifications.

Connected spaces and their basic properties. Connectedness of the real line. Components. Locally connected spaces,

Tychonoff product topology in terms of standard sub base and its characterizations. Product topology and separation axioms, connectedness and compactness (incl the Tychonoff's theorem), countability and product spaces.

Nest and filters, their convergence, and interrelation

Hausdorffness and compactness in terms of net/filter convergence.

3- **Advanced Analyisis**

Definition and existence of Riemann-Stieltjes integral, Conditions for R-S integrability. Properties of the R-S integral, R-S integrability of functions of a function. Integration of vector-valued functions, Rectifiable curves.

Series of arbitrary terms. Convergence, divergence and oscillation, Abel's and Dirichilet's tests. Multiplication of series. Rearrangements of terms of a series, Riemann's theorem.

Sequences and series of functions, pointwise and uniform convergence, Cauchy's criterion for uniform convergence. Weierstrass M-test, Abel's and Dirichlet's for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stielties integration, uniform convergence and differentiation. Weierstrass approximation theorem. Power series. Uniqueness theorem for power series, Abel's and Tauber's theorems.

Functions of several variables. Derivative of functions in a open subset of R_m into R_n as a linear transformation. Chain rule. Partial derivatives. Taylor's theorem. Inverse function theorem. Implicit function theorem. Jacobians.

Measures and outer measures. Measure induced by an outer measure, Extension of a measure. Uniqueness of Extension, Completion of a measure. Lebesgue outer measure. Measurable sets. Non-Lebesgue measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability.

Integration of non-negative functions. The general integral. Convergence theorems. Riemann and Lebesgue Integrals.

The Lp - space. Convex functions. Jensen's inequality. Holder and Minkowski inequalities. Completeness of Lp. Convergence in measure, Almost uniform convergence.

Normed linear spaces: Banach spaces and examples. Quotient space of normed linear space and its completeness. Equivalent norms. Riesz lemma. Basic properties of finite dimensional normed linear space and compactness.

Signed measure. Hahn and Jordan decomposition theorems. Absolutely continuous and singular measures. Radon Nikodyn theorem. Labesgue decomposition. Riesz representation theorem. Extension theorem (Carathedory). Lebesgue-Stieltjes integral.

Product measures. Fubini's theorem. Baire sets. Baire measure. Continuous functions with compact support. Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Reisz-Markoff theorem.

Set theory : Countable and uncountable sets. Infinite sets and the Axiom of choice. Cardinal numbers and its arithmetic. Schroder-Bernstein theorem. Well-ordering theorem.

Complex Analysis : Complex Integration. Cauchy-Goursat Theorem. Cauchy's integral formula. Higher order derivatives. Morera's theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra. Taylor's Theorem. Maximum modulus Principle, Schwarz lemma.

Lalurent's Series. Isolated singularities. Casporati-Weierstress theorem. Meromorphic functions. The argument principle. Rouche's theorem. Inverse function theorem.

Residues. Cauchy's residue theorem. Evaluation of integrals. Branches of many valued functions with special reference to arg Z, Log Z, and Z^a.

Analytic continuation. Uniqueness of direct analytic continuation. Uniquencess of analytic continuation along a curve. Power series method of analytic continuation. Schwarz reflection Principle. Monodromy theorem and its consequences.

Functional Analysis- Bounded Linear transformations. B (X, Y) as a normed linear space. Open mapping and closed graph theorems.

Uniform boundedness Prnciple and its consequences. Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces. Application of Hahn-Banach theorem. Dual spaces with examples. Separability. Reflexive. Spaces. Stone-Weierstrass theorem. Weak convergence. Weak sequential compactness. Compact operators.

Inner product spaces, Hilbert spaces. Orthonormal sets. Bessel'sinequality. Complete orthonormal sets and Parseval's identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Riesz Fischer theorem, Adjoint of an operator on a Hilbert space.

4- Differential equations, Integral equation and calculus of variation

Differential equations: **Ordinary Differential Equations-** Linear differential equations, Selfadjoint differential equations, Lagrange identity, The existence and uniqueness theorem. Green's Function for ordinary differential initial and boundary value problem.

Series solution of differential equations- Power series method, Bessel, Legendry and Hypergeometric equations. Bessel, Legendre and Hypergeometric functions and their properties. Convergence, recurrence and generating relations. Orthogonally of functions. Sturn-Liouville problem. Eigen-functions and eigen-values. Orthogonally of Bessel functions and Legendre polynomial.

Linear systems in R^2, System of first order linear differential equations, Fundamental theorem for linear systems, Fundamental matrices, Initial value problems, Autonomous systems and stability, Almost linear system.

Partial differential equations of the first order. Lagrange's solutions. Some special types of equations which can be solved easily by methods other than the general method. Charpit's general method of solution. Partial dfferential equations of second and higher orders. Classification of linear partial differential equations of second order. Homogeneous and non-homogeneous equations with constant coefficients. Monge's methods.

Integral Equation: Classification of integral equations of Volterra and Fredholm types. Conversion of initial and boundary value problem into integral equations. Conversion of integral equations into differential equations (when it is possible) Volterra and Fredholm integral operators and their interacted kernels. Resolvent kernels and Neumann series method for solution of integral equations. Banach contraction principle, its application in solving integral equations of second kind by the method of successive iteration and basic existence theorems.

Abel's integral equations and tantochrone problem.

Fredholm-alternative for Fredholm integral equation of second kind with degenerated kernels.

Use of Laplace and Fourier transforms to solve integral equations.

Calculus of variations: Variational problems with fixed boundaries. Euler's equation for functionals containing first order and higher order total derivatives. Functionals containing first order partial derivatives. Variational problems in parametric form. Invariance of Euler's equations under coordinates transformation.

Variational problems with Moving Boundaries-Functional dependent on one and two functions. Onde sided variations. Sufficient conditions for an extremum-Jacobi and Legendre conditions.

5- **Integral Transform**

Laplace Transform- Existence theorem for Laplace transform, properties of Laplace transform, Laplace transform of derivatives and integrals, invernse Laplace transform, convolution theorem, Heavi-side expansion formula, complex invention formula, evaluation of integrals, solution of initial and boundary-value problems of ordinary and partial differential equations.

Fourier Transform- Infinite Fourier transform, Fourier cosine and sine transform, finite Fourier transform, finite Fourier cosine and sine transform,. Solution of initial and boundary value problems of ordinary and partial differential equation.

6. **Numerical Analysis**

Error Solution of Equations: Bisection, Secant, Regular False, Newton's Method, Roots of Polynomials.

Interpolation: Langrange and Hermite Interpolation. Divided differences. Differences schemes. Interpolation formula using differences.

Difference operators. Δ, E, S and μ.

Numerical Quadrature: Newton-Cote's formulas, Gauss quadrature formulas. Chebychev's formulas.

Linear Equations: Direct methods for solving systems of Linear Equations(Gauss Elimination, LU Decomposition, Chalesky Decomposition) Iterative Methods (Jacobi, Gauss-Seidel, Relaxation Methods) Matrix norm, ill conditioned system.

Algebraic Eigenvalue Problem: Jacobi's method, Given's method, Householder's method, power method, Lancozos method. QR decomposition, singular value decomposition.

Ordinary Differential Equation: Euler method, Single-step methods, Runge-Kuita's method, Multi-step methods, Milne-Simpson method, Methods based on numerical integration. Methods based on numerical differentiation. Boundary value problems. Eigenvalue problems. Difference Equations.

Approximation: Different type of approximation, Least square polynomial approximation, polynomial approximation using orthogonal polynomials. Approximation with trigonometric functions, Exponential functions, Chebychey polynomials. Rational Functions. Boundary value problems of ordinary and partial differential equations, finite, difference Galerkins method, collocation method.

7. **Mechanics** Rotation of a vector in two and three dimensional fixed frame of reference. Kinetic energy and angular momentum of rigid body rotating about its fixed point.

Euler dynamics and geometrical equations of motion.

Generalized coordinates, momentum and force components. Lagrange equations of motion under finite forces, cyclic coordinates and conservation of energy.

Lagrangian approach to some known problems-motions of simple, double, spherical and cycloidal pendulums, motion of a particle in polar system, motion of a particle in a rotating plane, motion of a particle inside a paraboloid, motion of an insect crawling on a rod rotating about its one end, motion of masses hung by light strings passing over pulleys, motion of a sphere on the top of a fixed sphere and Euler dynamic equations.

Lagrange equations for constrained motion under finite forces. Lagrange equations of motion under impulses, motion of parallelogram about its centre and done of its particular cases.

Small oscillations for longitudinal and transverse vibrations.

Equations of motion in Hamiltonian approach and its application on known problems as given above. Conservation of energy. Legendre dual transformation.

Hamilton principle and principle of least action. Hamilton-Jacobi equation of motion, Hamilton-Jacobi theorem and its verification on the motions of a projectile under gravity in two dimensions and motion of a particle describing a central orbit.

Phase space, canonical transformations, conditions of canonically, cyclic relations, generating functions invariance of elementary phase space, canonical transformations form a group and Liouville theorem.

Poission brackets, Poisson first and second theorems, Poisson. Jacobi identity and invariance of Poisson bracket.

8. Tensor Differential Manifold

Tensors-Contravariant and Covariant vectors, Transformation formulae, Tensor product of two vector spaces. Tensor of type(r, s) Symmetric and Skew-symmetric properties. Contraction of tensors, Quotient law. Inner product of vectors.

Geometry General equations of second degree-System of conics, Confocal conics, Polar equation of a conic.

Plane-The straight line and the plane, Sphere.

Cone, Cylinder, Central conicoids. Tangent plane at a point of central concoid, Normal, Polar planes and polar lines.

Paraboloids plane sections of Concoids, Generating lines confocal concoids.

Differential geometry of Manifold- Topological groups. Lie groups and Lie algebras. Product of two Lie groups. One parameter subgroups and exponential maps. Examples of Lie groups. Hamomorphism and isomorphism. Lie transformation groups. General linear groups.

Principal fiber bundle. Linear frame bundle. Associated fiber bundle. Vector bundle. Tangent bundle. Induced bundle. Bundle homeomorphisms.

Almost complex and almost contact structures. Nijendhuis tensor. Contravariant and covariant almost analytic vector fields in almost complex manifold. F-Connexion.

Almost complex and almost contact submanifolds and hypersurfaces.