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ENTRANCE EXAMINATION, 2013

Pre-Ph.D./Ph.D.
PHYSICAL SCIENCES

[Field of Study Code : PHYP (161)]

Time Allowed : 3 hours

Maximum Marks : 70

INSTRUCTIONS FOR CANDIDATES

(i) This Question Paper consists of two Parts, i.e., Part—A and Part—B.

(ii) All questions are compulsory. Answers should be written in the space following each question.

(iii) Use of calculators is permitted.

(iv) Extra pages are attached at the end of the Question Paper for Rough Work.

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ENTRANCE EXAMINATION, 2013

Pre-Ph.D./Ph.D.
PHYSICAL SCIENCES

SUBJECT . . . .
(Field of Study/Language)

FIELD OF STUDY CODE . . . .

NAME OF THE CANDIDATE . . . .

. . . .

REGISTRATION NO. | | | | | |

CENTRE OF EXAMINATION. . . . .

. . . .

DATE. . . . .

....
(Signature of Candidate)

....
(Signature of Invigilator)

....
(Signature and Seal of Presiding Officer)

FOR OFFICE USE ONLY

Question No.	Marks	Question No.	Marks
A1		**B1**	
A2		**B2**	
A3		**B3**	
A4		**B4**	
A5		**B5**	
Total [A]		**B6**	
		B7	
		B8	
		B9	
		B10	
		Total [B]	

Total Marks [A + B] = ...

Useful Physical Constants :

Acceleration due to gravity, $g = 9{\cdot}81\ \text{m/s}^2$

Avogadro number, $N_A = 6{\cdot}022 \times 10^{23}/\text{mol}$

Boltzmann constant, $k_B = 1{\cdot}38 \times 10^{-23}$ J/K

Charge of electron, $e = 1{\cdot}6 \times 10^{-19}$ C

Gravitational constant, $G = 6{\cdot}67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$

Mean radius of the earth, $R_e = 6{\cdot}37 \times 10^6$ m

Permittivity of vacuum, $\varepsilon_0 = 8{\cdot}85 \times 10^{-12}$ F/m

Planck constant, $h = 6{\cdot}63 \times 10^{-34}$ J-s

Rest mass of electron, $m_e = 9{\cdot}11 \times 10^{-31}$ kg

Rest mass of neutron, $m_n = 1{\cdot}67 \times 10^{-27}$ kg

Speed of light in vacuum, $c = 3 \times 10^8$ m/s

Stefan-Boltzmann constant, $\sigma = 5{\cdot}67 \times 10^{-8}\ \text{W/m}^2\text{-K}^4$

Universal gas constant, $R = 8{\cdot}31$ J/mol-K

Conversion Factors :

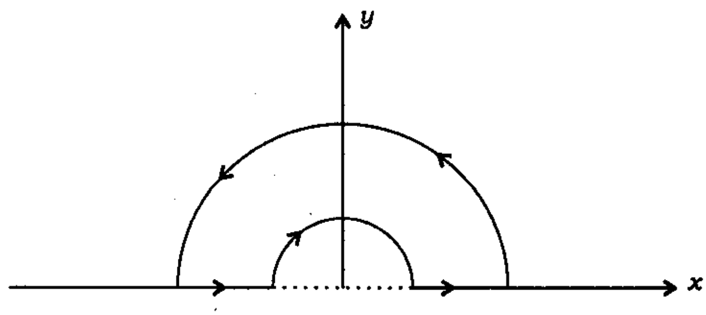
1 calorie = $4 \cdot 2$ J

1 eV = $1{\cdot}6 \times 10^{-19}$ J

PART—A

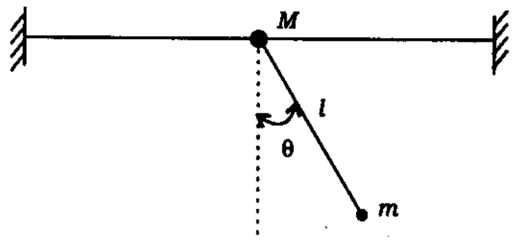
Note : Answer **all** questions. Each question carries 6 marks.

A1. Using the contour of integration in the complex plane (shown below)



calculate the integral $\int_0^{\infty} \frac{\sin(2x)}{x}\, dx$.

A2. Consider a pendulum consisting of a point mass m and massless string of length l. The string is supported from a bead of mass M which slides freely (without friction) along a horizontal wire.



(a) Write the Lagrangian for the system.

(b) Find the frequency of the pendulum for the small-amplitude oscillations.

A3. The free energy of formation of a cluster consisting of i molecules has contributions from both volume and surface terms as follows :

$$\Delta G = \Delta_0 V + \gamma S$$

where Δ_0 is the free energy (per unit volume) cost for bringing the particles together and γ is the surface tension. V and S denote the volume and surface area of the cluster respectively. Assume that the cluster is spherical in shape and each molecule is associated with a volume v_m, so that $V = i v_m$.

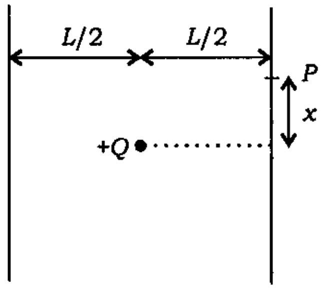
(a) Obtain how ΔG depends on i.

(b) Assuming $\Delta_0 < 0$, obtain the value of i for which ΔG becomes maximum.

(c) Compute the radius of the optimum cluster in *(b)*.

A4. A free particle of mass m is moving inside a sphere of radius R. Assuming that the wall of the sphere is impenetrable, calculate the ground-state energy of the particle.

A5. A point charge $+Q$ is placed midway between the lines joining the centres of two large parallel conducting plates separated by a distance L. The plates are connected by a thin conducting wire. Using the method of images, obtain the surface charge density at a point P on the right plate at a distance x from its centre.



PART—B

Note : Answer **all** questions. Each question carries 4 marks.

B1. Consider the following two systems :

(a) A harmonic spring of spring constant K with a mass m attached to its one end while the other end is held fixed

(b) A harmonic spring of spring constant K with two identical masses (each m) attached to its respective ends

Write the Hamiltonian of the two systems. Which of these two Hamiltonians is invariant under translation in space?

B2. Write the ground-state wave function of the helium atom ($1s^2$) in terms of the hydrogen-like atomic orbitals (ignoring the effects of electron-electron repulsion). Denote the $1s$ orbital as $\psi_{1s}(\mathbf{r})$ and the two spin states of electron as $\alpha\uparrow$ and $\alpha\downarrow$. What is the total spin of the He ground state?

B3. Write the first law of thermodynamics for a system whose thermodynamic variables are temperature T, magnetization M and number of particles N. Identify the Gibbs free energy G which becomes minimum at constant T, H, N (respectively denoting the temperature, magnetic field and number of particles).

B4. Solve the following non-linear equation by introducing a transformation which converts it to a linear equation :

$$2x\frac{du}{dx} + (x - 1)u = \frac{x^2 e^x}{u}$$

B5. A particle of mass m moves in a central force field given by the (repulsive) potential $V(r) = -V_0 r^n$. The particle is set on a radial trajectory at $r = r_0$ (directed to the centre of the force) with just enough energy (that is $E \to 0$) to reach the centre. What is the condition on the value of n that the particle reaches the centre in finite time?

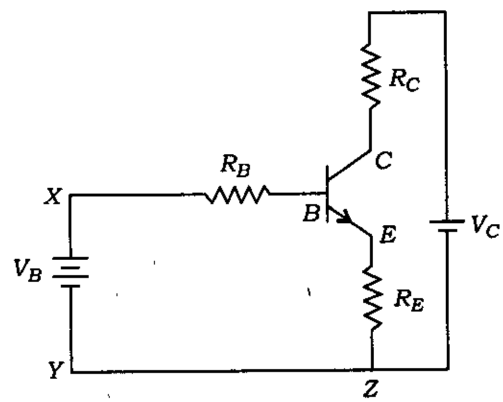
B6. Evaluate the electric and magnetic fields, and the charge and current densities corresponding to the potentials (q is the charge)

$$V(\mathbf{r}, t) = 0; \quad \mathbf{A}(\mathbf{r}, t) = -\frac{1}{4\pi\varepsilon_0}\frac{qt}{r^2}\hat{r}$$

B7. Write down the expression for Maxwell's velocity distribution law. Obtain the most probable value of the velocity of the particle and its ratio with the root-mean-square speed of the particles.

B8. Consider a point charge Q that is moving in a circular orbit of radius a centred at the origin in the xy-plane. Find the Liénard-Wiechert potentials for points in the z-axis. Assume angular velocity ω of the particle to be a constant.

B9. Given below is a transistor circuit based on common-emitter configuration. The *Q*-point here drifts with temperature T as $V_{BE} = V_{BE}(T)$, where V_{BE} equals the junction voltage at the base-emitter *p-n* junction. Suggest a modification so as to make the circuit insensitive to temperature variations. What component(s) is(are) required to be incorporated and in which segment(s)? Why?



B10. A two-dimensional metallic system consists of a monovalent atom in a rectangular primitive cell with $a = 2$ Å and $b = 4$ Å. Sketch the reciprocal lattice and draw the first Brillouin zone. Estimate the radius of free-electron Fermi surface (circle in two dimensions) in cm^{-1}. Sketch the Fermi circle on top of the first Brillouin zone.