ENGINEERING MATHEMATICS

Time : Three hours

Maximum Marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,**
ANY TWO *from* **Group B** *and* ALL *from* **Group C.**

*All parts of a question (a, b, etc.) should be
answered at one place.*

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mented with neat sketches. Unnecessary long answers
may result in loss of marks.*

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giving proper justification.*

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Group A

1. (*a*) Find the eigenvalues and corresponding eigen-
    vectors of the matrix

    $$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$ 8

    (*b*) Evaluate

    $$\int_C [(x+y)\,dx - x^2\,dy + (y+z)\,dz]$$

    where C is $x^2 = 4y$, $z = x$, $0 \leqslant x \leqslant 2$.   6

    (*c*) Find the surface area of the solid generated by
    revolving the curve $x^2 + (y-b)^2 = a^2$ about x-
    axis.   6

2. (a) Find the normal vector and the equation of tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at the point $(3,4,5)$. Also, find the divergence of the vector field $(x^2 y^2 - z^3) i + (2xyz) j + (e^{xyz}) k$.   8

(b) Verify whether the vectors $x_1 = (1\ 3\ 4\ 2)$, $x_2 = (3\ -5\ 2\ 2)$ and $x_3 = (2\ -1\ 3\ 2)$ are linearly-dependent. If so, express one of these vectors as a linear combination of others.   6

(c) Find the total length of the curve $r = a \sin^3(\theta/3)$.   6

3. (a) Discuss the convergence of the following series: 2×4

(i) $1 + \dfrac{2!}{2^2} + \dfrac{3!}{3^3} + \dfrac{4!}{4^4} + \ldots$

(ii) $\displaystyle\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n} + \sqrt{n+1}}$

(b) Apply Green's theorem to evaluate

$$\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$$

where C is the boundary of the area enclosed by the x-axis and upper-half of the circle $x^2 + y^2 = a^2$.   6

(c) Find the value(s) of λ for which the equations
$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$
$(\lambda - 1)x + 2(2\lambda - 1)y + (\lambda + 3)z = 0$
$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

have non-trivial solution(s). Find the ratios $x : y : z$ when λ has the smallest of these values. What happens when λ has the greatest of these values?   6

4. (a) Find the shortest distance between the line $2x + y = 10$ and the ellipse $9x^2 + 4y^2 = 36$.   10

(b) Evaluate the integral

$$\iint_R e^{x^2} \, dx\, dy$$

where the region R is given by $R : 2y \leq x \leq 2$ and $0 \leq y \leq 1$.   10

Group B

5. (a) Solve the following differential equations: 2×7

(i) $(1 + y^2)dx = (\tan^{-1} y - x)dy$

(ii) $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + y = xe^x \sin x$

(b) Use difference operator technique to get missing values in the following table: 6

x	45	50	55	60	65
y	3.0	—	2.0	—	-2.4

6. (a) Solve

$\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$, $y = 0$ when $x = 0$.   6

(b) Solve the equation

$$y'' + 3y' + 2y = 2e^x$$

by using the method of variation of parameters. 8

(c) Find the Fourier series expansion of the periodic function

$$f(x) = x, \ -\pi \le x \le \pi, \ f(x + 2\pi) = f(x).$$ 6

7. (a) Find the cubic polynomial which takes the following values:

x	0	1	2	3
f(x)	1	2	1	10

Hence or otherwise, evaluate $f(4)$. 8

(b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.
[Given: $P(z \ge 0.5) = 0.31$, $P(z \le 1.4) = 0.92$, where z is standard normal variate.] 6

(c) Given that

x	1.1	1.2	1.3	1.4	1.5	1.6
y	8.403	8.781	9.129	9.451	9.750	10.031

Find dy/dx at $x = 1.1$. 6

8. (a) The data given below show the test scores made by 10 salesmen on an intelligence test and their weekly sales:

Salesman	1	2	3	4	5	6	7	8	9	10
Test Score	40	70	50	60	80	50	90	40	60	60
Sales (in '000)	2.5	6.0	4.5	5.0	4.5	2.0	5.5	3.0	4.5	3.0

Calculate the regression line of sales on test scores and estimate the most probable weekly sales if a salesman makes a score of 70. 8

(b) Given the values:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Evaluate $f(9)$ using Lagrange's formula. 6

(c) Evaluate

$$\int_0^6 dx/(1 + x^2)$$

by using Trapezoidal rule. Take $h = 1$. 6

Group C

9. Answer the following: 10 × 2

(i) A binomial distribution has mean 20 and standard deviation 4. Find the parameters of the distribution.

(ii) Verify whether the vector field

$$v = xyz \, (yzi + xzj + xyk)$$

is conservative.

(iii) Find the value(s) of k for which the vectors $(1, k, 5), (1, -3, 2), (2, -1, 1)$ will form a basis in $\mathbb{R}^3$.

(iv) Verify whether the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ is diagonalizable.

(v) Consider the function $f(x, y) = \tan^{-1}(y/x)$. Find the value of $x \dfrac{\partial f}{\partial x} + y \dfrac{\partial f}{\partial y}$.

(*vi*) Evaluate $\lim\limits_{x \to 0} [\log_e (1 + x)]/\sin x$.

(*vii*) Find the coefficient of x^3 in the expansion of $e^{\sin x}$.

(*viii*) Find the Laplace transform of the function $e^{-2t} . t^3$.

(*ix*) Obtain the function whose first difference is $9x^2 + 11x + 5$.

(*x*) Find complementary function of the differential equation

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = \sec 2x.$$

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Group A

1. (a) If $x = \cosh(\log y/m)$, prove that

    $$(x^2 - 1)y_2 + xy_1 = m^2 y. \qquad 8$$

    (b) If $y = J^{a \sin^{-1} x}$, prove that

    $$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0. \qquad 6$$

    (c) Find the area of the region lying above x-axis, and
    included between the circle $x^2 + y^2 = 2ax$ and the
    parabola $y^2 = ax$. $\qquad$ 6

2. (a) Test for the convergence of series

$$\frac{1}{2\sqrt{2}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^5}{5\sqrt{4}} + \ldots + \infty$$

6

(b) Find the volume of solid obtained by revolving the cardiod $r = (1 + \cos\theta)$ about the initial line. 8

(c) Show that maximum and minimum values of $r^2 = x^2 + y^2$, where $ax^2 + 2hxy + by^2 = 1$, are given by the roots of the equation

$$[a - (1/r^2)][b - (1/r^2)] = h^2$$

6

3. (a) Show that all the points on the curve

$$y^2 = 4a[x + a\sin(x/a)]$$

at which the tangent is parallel to x-axis be on a certain parabola. 6

(b) Apply Green's theorem in the plane to evaluate

$$\int_C [(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy]$$

where C is boundary of the curve enclosed by x-axis and the semi-circle $y = (1 - x^2)^{1/2}$. 6

(c) Evaluate the integral $\int \vec{F} \cdot d\vec{r}$, along the curve $x^2 + y^2 = 1$, $z = 1$ in the positive direction from $(0, 1, 1)$ to $(1, 0, 1)$ for $\vec{F}$ given by

$$(2x + yz)\,\vec{i} + xz\,\vec{j} + (xy + 2z)\,\vec{k}.$$

8

4. (a) Solve the following system of linear equations by Gauss elimination method:

$$x - 2y - 3z = -1$$
$$2x - y - 2z = 2$$
$$3x - y - 3z = 3$$

6

(b) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

8

(c) A vector function $\vec{F}$ is a product of a scalar function and the gradient of a scalar function. Show that

$$\vec{F} \cdot \text{curl}(\vec{F}) = 0.$$

6

Group B

5. (a) In a population, the probability of selecting (i) a male or a smoker is $7/10$; (ii) a male smoker is $2/5$; and (iii) a male if a smoker is already selected is $2/3$. Find the probability of selecting a male. 6

(b) Solve the following differential equations: 3+3

(i) $y - x\dfrac{dy}{dx} = a\left(y^2 + \dfrac{dy}{dx}\right)$

(ii) $\dfrac{dy}{dx} = \dfrac{2y - x - 4}{y - 3x + 3}.$

(c) A survey of 800 families, with four children each, revealed the following distribution:

No. of boys:	0	1	2	3	4
No. of girls:	4	3	2	1	0
No. of families:	32	178	290	236	64

Is this result consistent with the hypothesis that male and female births equal probable ? Assume that the tabulated $\chi^2_{0.5}$ for 4 degrees of freedom (d.f.) is 9.488. 8

6. (a) In a Poisson frequency distribution, the frequency corresponding to three successes is 2/3 times the frequency corresponding to four successes. Find the mean and standard deviation (s.d.) of the distribution. 6

(b) The following is a table of values of a polynomial of degree 5. It is given that $f(3)$ is in error. Correct the error. 8

x	0	1	2	3	4	5	6
$y = f(x)$	1	2	33	254	1025	3126	7777

(c) Solve the following differential equation:

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log_e x.$$ 6

7. (a) Using a Lagrange's interpolation formula, find a polynomial which passes through the points $(0, -12)$, $(1, 0)$, $(3, 6)$ and $(4, 12)$. 6

(b) Solve the differential equation

$$\frac{d^2 y}{dx^2} + 4y = x \sin x$$ 6

(c) Use Simpson's one-third rule to find the value of the integral

$$\int_0^{\pi/3} e^{\sin x}\, dx$$

correct up to four decimal places. Compare the results with that obtained by Trapezoidal rule of integration. 8

8. (a) From the following table of values of x and y, obtain dy/dx for $x = 1.2$: 8

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.025

(b) Solve the following equation:

$$\frac{dy}{dx} + \frac{xy}{1 - x^2} = xy^{1/2}$$ 6

(c) Solve the differential equation

$$x\, dx + y\, dy + (x\, dy - y\, dx)/(x^2 + y^2) = 0$$ 6

Group C

9. Answer the following: 10×2

(i) Prove that between any two real roots of $e^x \sin x = 1$, there exists at least one root of $e^x \cos x + 1 = 0$.

(ii) If x, y, z are connected to equations $\phi(x, y, z) = 0$ and $\psi(x, y, z) = 0$, find dy/dx.

(iii) Solve $(x + y)\, dx + dy = 0$.

(iv) Solve $(x^2 + y^2)\,dx - 2xy\,dy = 0$.

(v) Find the inverse Laplace transform of

$$(s-1)/(s^2 - 6s + 25).$$

(vi) Find one vector in R^3 which generates the inter-section of V and W, where

$$V = \{(a, b, 0\ ) \mid a, b \in R\}$$

and W be the space generated by the vectors

$$(1, 2, 3) \text{ and } (1, -1, 1).$$

(vii) Let $A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}$ be the matrix over the real

field R. Find eigenvalues and linearly independent eigenvectors.

(viii) If $\vec{f} = f(x, y, z, t)$, show that

$$d\vec{f} = (d\vec{r} \cdot \nabla)\,\vec{f} + (\partial \vec{F}/\partial t)\,dt$$

(ix) Use Green's theorem to evaluate

$$\int_C (x^2 + xy)\,dx + (x^2 + y^2)\,dy$$

where C is the square formed by the lines $x = \pm 1$ and $y = \pm 1$.

(x) Find the shortest distance from the point $(3, 0)$ to the parabola $y = x^2$.

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Group A

1. (a) If $y(x) = \sin(\alpha \sin^{-1} x)$, then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - \alpha^2)y_n = 0$$

   where y_r represents r^{th} order derivative of $y(x)$. 8

   (b) Is the mean-value theorem valid for the function
   $f(x) = x^2 + 3x + 2$ in $1 \leq x \leq 2$? Find a point
   $x = c$ in $[1, 2]$, if the theorem is applicable. 6

   (c) Find the maximum and minimum values of the
   function $f(x) = xy(a - x - y)$ at the point $(0, a)$. 6

2. (a) Compute the area of the figure bounded by the
   parabolas $x = -2y^2$ and $x = 1 - 3y^2$. 8

(b) Find the volume of the solid generated by revolving the figure bounded by the parabola $y^2 = 8x$ and the straight line $x = 2$, about the line $y = -4$.   6

(c) Find the centre of gravity of a uniform lamina in the form of a quadrant of the ellipse $x^2/a^2 + y^2/b^2 = 1$.   6

3. (a) Prove that the series $\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + \dots$ is divergent.   6

(b) Investigate the value of x for which the ratio-test fails while examining the convergence of the series

$$\frac{1}{1^p} + \frac{x}{3^p} + \frac{x^2}{5^p} + \dots + \frac{x^{n-1}}{(2n-1)^p} + \dots$$ 6

(c) (i) Find the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ when $f(x, y) = \tan^{-1}(y/x)$.   4

(ii) If $u = e^{xyz}$, examine that the result
$$\partial^3 u/\partial x \, \partial y \, \partial z = (1 + 3xyz + x^2 y^2 z^2)\, e^{xyz}$$
is correct.   4

4. (a) Obtain the reduction formulae for

(i) $\displaystyle\int_0^1 x^m (1-x)^n \, dx,$ and

(ii) $\displaystyle\int_0^{\pi/2} \sin^m x \cos^n x \, dx,$ where m and n are positive integers.   8

(b) If $u = xf(y/x) + g(y/x)$, then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$ 8

(c) If $u = f(x-y, y-z, z-x)$, then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$ 4

Group B

5. (a) Divide a given positive number N into three parts (positive) such that their sum is N and product is maximum.   6

(b) Find a point within a triangle such that the sum of squares of its distances from three vertices is minimum.   6

(c) At what point on the curve $xy = 20$ is the tangent parallel to the line $5x + y = 1$?   8

6. (a) Verify whether the two integrals

$$I_1 \equiv \int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} \, dy$$

and

$$I_2 \equiv \int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} \, dx$$

are equal.   8

(b) Evaluate $\displaystyle\iint_R \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx \, dy$

where R consists of points in the positive quadrant of the ellipse $x^2/a^2 + y^2/b^2 = 1$.   8

(c) If $u = x^2 - y^2 - z^2 - 2$, find the gradient ∇u at the point $(1, -1, 2)$.   4

7. (a) Show that $f = (2xy + z^3)i + x^2 j + 3xz^2 k$ is a conservative force field. Also, find the (i) scalar potential ϕ, and (ii) work done by the force f in moving an object from $(1, -2, 1)$ to $(3, 1, 4)$. 10

(b) Use Green's theorem to evaluate

$$\oint_C (x^2 y\, dx + y^3\, dy),$$

where C is the closed path formed by $y = x$ and $y = x^3$ from $(0, 0)$ to $(1, 1)$. 10

8. (a) Use Stokes' theorem to evaluate

$$\oint_C (e^x dx + 2y\, dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$. 6

(b) Find the eigenvalues and eigenvectors of the matrix 6

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

(c) Determine for what values of k the system $x + 2y = 1$; $5x + ky = 5$ has (i) unique solution, and (ii) infinite number of solutions. 8

Group C

9. Answer the following: 10 × 2

(i) Evaluate $\displaystyle\int_0^{\pi/2} \int_0^{\pi} \cos(x + y)\, dx\, dy$.

(ii) Find the sum of the eigenvalues of

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 2 & 4 \\ 1 & 2 & 7 \end{bmatrix}$$

(iii) If $f = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find Curl f.

(iv) If $\nabla\phi$ is solenoidal, find the value of $\nabla^2\phi$.

(v) Find the volume generated by revolving about OX, the area bounded by $y = x^3$ between $x = 0$ and $x = 2$.

(vi) Let $y(x) = (2 - 3x)^{10}$, then evaluate $d^9 y/dx^9$.

(vii) Write a reduction formula for $\int x^n e^{2x}\, dx$.

(viii) The infinite series $\left(\dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4} + ...\right)$ is divergent. Is this statement true or false.

(ix) If Curl $F = 0$, then F is called a solenoidal vector. Is this statement true or false?

(x) When a system of linear algebraic equations is said to be consistent for solution?

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Group A

1. (a) If $x = \tan(\log y)$, prove that

$$(1 + x^2)\, y_n = \{1 - 2(n - 1)\, x\}\, y_{n-1}$$
$$- (n - 1)(n - 2)\, y_{n-2} \quad 6$$

(b) Obtain, by Maclaurin's theorem, the first four terms
of the expansion of $e^{x\cos x}$. Hence or otherwise find
the value of

$$\lim_{x \to 0} \left(\frac{e^x - e^{x\cos x}}{x - \sin x} \right). \qquad 8$$

(c) If $f(x, y) = (1 - 2xy + y^2)^{-1/2}$, show that

$$\frac{\partial}{\partial x}\left\{(1 - x^2)\frac{\partial f}{\partial x}\right\} + \frac{\partial}{\partial y}\left(y^2 \frac{\partial f}{\partial y}\right) = 0 \qquad 6$$

2. (a) Find the maximum or minimum value of the function
$$f(x, y) = (x^2 + y^2) e^{-(x^2 + y^2)}$$
for points not on the circle $x^2 + y^2 = 1$.   6

(b) If $I_{m,n} = \int \cos^m x \sin nx \, dx$, prove that
$$I_{m,n} = -\frac{\cos^m x \cos nx}{m+n} + \frac{m}{m+n} I_{m-1, n-1}$$
Hence or otherwise, evaluate
$$\int_0^{\pi/2} \cos^5 x \sin 3x \, dx$$
    8

(c) Find the inner products $< \vec{x}, \vec{y} >, < \vec{y}, \vec{x} >$, the

norms $\| \vec{x} \|, \| \vec{y} \|$, the normalized $\vec{x}, \vec{y}$ and the

distance between $\vec{x}$ and $\vec{y}$, where
$$\vec{x} = [2, i, 3]^T, \quad \vec{y} = [1 + 2i, 2i, 1]^T$$
    6

3. (a) Find the volume generated by revolving about the axis of x. The area bounded by the curves
$$x^2 + y^2 = 25, \; 3x - 4y = 0, \; y = 0$$
lying in the first quadrant.   5

(b) Find a linear transformation $T: R^2 \to R^2$ such that $T(1,0) = (1,1)$ and $T(0,1) = (-1,2)$. Also, prove that T maps the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$ into a parallelogram.   5

(c) Using matrix method, show that the equations
$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$2x_1 + 5x_2 + 5x_3 - 18x_4 = 9$$
$$-4x_1 - x_2 - x_3 + 8x_4 = 7$$
are consistent and hence solve them.   5

(d) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that

(i) div $(r^n \vec{r}) = (n+3) r^n$, and

(ii) $\vec{r} / r^3$ is solenoidal.   5

4. (a) Solve the following system of equations by Gauss elimination method: 6
$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

(b) The eigenvectors of a 3×3 matrix A corresponding to the eigen values $1, 2, 3$ are $[-1, -1, 1]^T, [0, 1, 0]^T$ and $[0, -1, 1]^T$, respectively. Find the matrix A.   6

(c) Show that the line integral
$$\int_C (3x^2 \, dx + 2yz \, dy + y^2 \, dz)$$
is independent of the path in any domain in space and find its value, if C has the initial point $A(0, 1, 2)$ and terminal point $B(1, -1, 7)$.   8

Group B

5. (a) Solve the differential equations:
$$\frac{dy}{dx} = (4x + y + 1)^2, \text{ if } y(0) = 1.$$
    6

(b) Using Laplace transform method, solve the initial value problem
$$\frac{d^2 x}{dt^2} - 3\frac{dx}{dt} + 2x = 4e^{2t}, \; x = -3 \text{ and } \frac{dx}{dt} = 5 \text{ at } t = 0.$$
    6

(c) The following data are taken from the steam table :

Temperature, °C:	140	150	160	170	180
Pressure, kg/cm²:	3·685	4·854	6·302	8·076	10·225

Find the pressure at temperature $t = 142$ °C.   8

6. (a) Find general solution of the differential equation

$$\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} - \frac{dy}{dx} - y = \sin (2x - 3)$$

   6

(b) A curve passes through the points
$(1, 2)$, $(1·5, 2·4)$, $(2, 2·7)$, $(2·5, 2·8)$, $(3, 3)$, $(3·5, 2·6)$, $(4, 2·1)$ and $(4·5, 1·9)$
Use Simpson's one–third rule to obtain the volume of the solid of revolution got by revolving this area about x–axis.   6

(c) Find the Fourier sine transform of $e^{-|x|}$. Hence, show that

$$\int_0^\infty \frac{x \sin x}{1 + x^2} dx = \frac{\pi}{2} e^{-m}, \ m > 0.$$

   8

7. (a) Using Lagrange's interpolation formula, find the missing term in the table :

x:	2	5	8	14
y:	94·8	87·9		68·7

   6

(b) If the proportions of a brand of television set requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models sold this year will service during the first year of operation ?   6

(c) The finished inside diameter of a piston ring is normally distributed with a mean of 10 cm and a standard deviation of 0·03 cm. (i) What proportion of ring will have inside diameters exceeding 10·075 cm ? (ii) What is the probability that a piston ring will have an inside diameter between 9·97 cm and 10·03 cm ?
(Given : $\phi(2·5) = 0·9938$, $\phi(1) = 0·8413$, where

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx.$$

   8

8. (a) Find the general solution of the partial differential equation

$$(x^2 - yz)\frac{\partial z}{\partial x} + (y^2 - zx)\frac{\partial z}{\partial y} = z^2 - xy$$

   6

(b) Find the temperature distribution $u (x, t)$ in a laterally insulated bar 80 cm long, if the initial temperature is $100 \sin (\pi x / l)$ °C and the ends are kept at 0°C. How long will it take for the maximum temperature in the bar to drop to 50°C ? Given that

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $c^2 = 1·158$ cm²/sec.   8

(c) For determining the ash contents of six samples of coal, each sample was divided into two parts and a part was given to each of two analysts A and B. The percentage ash contents found by them are given below sample-wise :

Sample no.:	1	2	3	4	5	6
Analyst A:	8	10	11	9	12	13
Analyst B:	9	13	12	10	11	12

Do the two analysts differ significantly ? Given 5% calculated value of t for d.f. 5 is 2·23.   6

Group C

9. Answer the following: 10×2

(i) If $f(x) = f(0) + xf'(0) + \dfrac{x^2}{2!} f''(\theta x)$, find the value of θ as x tends to 1, $f(x)$ being $(1 - x)^{5/2}$.

(ii) Find the magnitude of the vector drawn perpendicular to the surface $x^2 + 2y^2 + z^2 = 7$ at the point $(1, -1, 2)$.

(iii) Two eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

are equal to 1 each. Find the eigenvalues of A^{-1}.

(iv) If $u = x^2 \tan^{-1}(y/x) - y^2 \sin^{-1}(x/y)$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(v) Find the inverse Laplace transform of $\log[(1 + s) / s]$.

(vi) Evaluate

$$\iint xy \, dx dy$$

over the area between $y = x^2$ and $y = x$.

(vii) Prove that

$$\left(\frac{\Delta^2}{E}\right) u_x \neq \frac{\Delta^2 u_x}{E u_x}$$

(viii) If the regression equations of two variables x and y are $x = 3 - y/4$ and $y = 5 - 2x$, find the correlation coefficient between them.

(ix) If the probabilities of happening of some events are $p_1, p_2, ..., p_n$, then find the probability of happening of at least one of them.

(x) If a random variable X has a Poisson distribution such that $P(X = 1) = P(X = 2)$, then find $P(X = 4)$.

ENGINEERING MATHEMATICS

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Group A

1. (*a*) Show that if $0 < u < v$, then

    $$(v-u)/(1+v^2) < \tan^{-1} v - \tan^{-1} u < (v-u)/(1+u^2)$$

    and deduce that

    $$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}(4/3) < \frac{\pi}{4} + \frac{1}{6}.$$

    8

    (*b*) Examine the function $(x-3)^5 (x+1)^4$ for extreme
    values. 8

(c) If $y = (\tan^{-1} x)^2$, prove that

$$(1 + x^2)^2 \frac{d^2y}{dx^2} + 2x(1 + x^2) \frac{dy}{dx} = 2.$$ **4**

2. (a) If the normal to the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

makes an angle θ with x-axis, show that its equation is

$$y \cos \theta - x \sin \theta = a \cos 2\theta.$$ **8**

(b) Solve the following system of linear equations

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

by Gauss elimination method. **6**

(c) Find the unit tangent vector, $\vec{t}$, for the curve

$$\vec{r} = t^2 \vec{i} + t \vec{j} + (2t + 3) \vec{k}$$ **6**

3. (a) Prove that the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

is $8abc / 3\sqrt{3}$. **8**

(b) Evaluate $\iint\limits_{R} f(x, y) \, dxdy$ over the rectangle $R = [0, 1 ; 0, 1]$, where

$$f(x, y) = \begin{cases} x + y, & \text{if } x^2 < y < 2x^2 \\ \\ 0, & \text{elsewhere} \end{cases}$$ **6**

(c) Show that

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

for $-1 < x \leq 1$. **6**

4. (a) Prove that the line integral

$$\int\limits_{C} \frac{xdy - ydx}{x^2 + y^2}$$

taken in the positive direction over any closed contour with the origin inside it is equal to 2π. **8**

(b) Find the equation of the tangent plane and normal line to the surface $x^2y + xz^2 = z - 1$ at the point $(1, -3, 2)$. **6**

(c) Find the unit tangent vector $\vec{t}$ for the curve

$$r = t^2 \vec{i} + t \vec{j} + (2t + 3) \vec{k}.$$ **6**

Group B

5. (a) Solve the differential equation

$$\frac{d^2x}{dt^2} = 3x + 4y$$

$$\frac{d^2y}{dt^2} = -x - y \qquad 8$$

(b) Solve the differential equation

$$y(2xy + e^x)\, dx - e^x\, dy = 0. \qquad 6$$

(c) The table below gives values of $\tan x$ for $0.1 \leqslant x \leqslant 0.3$:

0.1	0.15	0.2	0.25	0.3
0.1003	0.1511	0.2027	0.2553	0.3093

Use Newton's forward difference interpolation formula to obtain the values of (i) $\tan 0.12$, and (ii) $\tan 0.5$. 6

6. (a) Evaluate the value of the integral

$$I = \int_0^1 \frac{1}{1+x}\, dx$$

correct up to three decimal places by using Simpson's one-third rule with step size $h = 0.25$. 8

(b) Solve the differential equation

$$(D^4 - 2D^3 + D^2)\, y = x^3$$

where $D = d/dx$. 6

(c) It has been claimed that in 60% of all solar heat installations the utility bill is reduced by at least one-third. Accordingly, what are the probabilities that the utility bill will be reduced by at least one-third in (i) four of five installations; and (ii) at least four of five installations? 6

7. (a) Find the mean and standard deviations of the following data: 6

19.7	21.5	22.5	22.2	22.6
21.9	20.5	19.3	19.9	21.7
22.8	23.2	21.4	20.8	19.4
22.0	23.0	21.1	20.9	21.3

(b) Use Lagrange's interpolation formula to find the unique polynomial of degree 2 or less such that

$$f(0) = 1,\ f(1) = 3\ \text{and}\ f(3) = 55. \qquad 6$$

(c) Solve the equation by using Laplace transform

$$x' + 2x - 3y = t$$
$$y' - 3x - 2y = e^{2t}$$

which satisfies the conditions $x(0) = y(0) = 0$. 8

8. (a) If $D = \partial/\partial x$ and $D' = \partial/\partial y$, then solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 0 \qquad 6$$

(b) Solve the differential equation

$$(D^4 + 2D^2 + 1)\, y = x^2 \cos x. \qquad 6$$

(c) The following are measurements of the air velocity and evaporation coefficients of burning fuel droplets in an impulse engine:

Air Velocity (cm/s)		Evaporation Coefficients (mm²/s)	
x	x	y	y
20	220	0·18	0·75
60	260	0·37	1·18
100	300	0·35	1·36
140	340	0·78	1·17
180	380	0·56	1·65

Construct a 95% confidence interval for the mean evaporation coefficient, when the air velocity is 190 cm/s.　8

Group C

9. Answer the following:　2 × 10

(i) Obtain the inverse transform of

$$f(p) = (p+2)/(p-1)^2 \, p^3.$$

(ii) Solve $(dy/dx) = (x+y+4)/(x-y-6)$.

(iii) If $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, find the value of grad $(\log | \vec{r} |)$.

(iv) Find the constants a, b and c so that

$$\vec{F} = (x+2y+az)\,\vec{i} + (bx-3y-z)\,\vec{j} + (4x+cy-2z)\,\vec{k}$$

is irrotational.

(v) Find the particular integral of
$$(D^2+4)\,y = x \sin x.$$

(vi) Find the directional derivative of
$$\varphi = x^2yz + 4xz^2$$
at $(1,-2,-1)$ in the direction of $2\,\vec{i} - \vec{j} - 2\,\vec{k}$.

(vii) Find the mean and median of 15, 14, 2, 27, 13.

(viii) If a random variable has the probability density
$$f(x) = \begin{cases} 2e^{-2x}, & \text{for } x > 0 \\ 0, & \text{for } x \leq 0 \end{cases}$$
find the probability that it will take on a value between 1 and 3.

(ix) Show that
$$\mu = \sqrt{1 + \delta^2/4}$$
where μ and δ are usual notation.

(x) Solve the partial differential equation
$$\frac{\partial^3 z}{\partial x^3} - 7 \frac{\partial^3 z}{\partial x \partial y^2} + 6 \frac{\partial^3 z}{\partial y^3} = 0.$$

ENGINEERING MATHEMATICS

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Group A

1. (*a*) If $y = \tan^{-1} x$, show that

$$(1 + x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0. \qquad 5$$

(*b*) Is the mean value theorem valid for the function
$f(x) = x^2 + 3x + 2$ in $1 \leq x \leq 2$. Find ξ, if the
theorem is applicable. $\qquad$ 4

(*c*) Obtain the Taylor's polynomial expansion of the
function $f(x) = \sin x$ about the point $x = \pi/4$.
Further, show that the error tends to zero as $n \to \infty$. $\qquad$ 6

(d) Is the series

$$\sum_{1}^{\infty} \frac{1}{(2n+1)\sqrt{n}}$$

convergent ? Name the series with which you compare the given series to test its convergence.   5

2. (a) Suppose that u and v are two functions of x and y and satisfy the relations

$$\left.\begin{array}{c} u^2 - v^2 = x \\ \\ 2uv = y \end{array}\right\}$$

Then compute $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}$ and $\dfrac{\partial v}{\partial y}$, assuming that they exist.   6

(b) Let $r = \sqrt{x^2 + y^2}$ be the radius vector from the origin to the point (x, y) in the xy-plane and θ be the angle which r makes with x-axis. Show that

(i) $\dfrac{\partial^2 r}{\partial y^2} = \dfrac{\cos^2 \theta}{r}$ 3

(ii) $\dfrac{\partial^2 r}{\partial x^2} + \dfrac{\partial^2 r}{\partial y^2} = \dfrac{1}{r}\left\{\left(\dfrac{\partial r}{\partial x}\right)^2 + \left(\dfrac{\partial r}{\partial y}\right)^2\right\}$ 3

(c) Find the directional derivative of the function $f = x^4 + x^3 y + z^3$ at the point $P(1, 2, 5)$ towards the point $(5, 0, 8)$.   8

3. (a) The cylinder $x^2 + z^2 = 1$ is cut by the planes $y = 0$, $z = 0$ and $x = y$. Find the volume of the region in the first octant.   6

(b) Find the minimum value of $(x^2 + y^2 + z^2)$ subject to the condition $xyz = a^3$.   6

(c) Verify the Green's theorem for $f(x, y) = e^{-x} \sin y$, $g(x, y) = e^{-x} \cos y$ and the contour C is the square with vertices at $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$, $(0, \pi/2)$.   8

4. (a) Show that the vectors $\vec{v_1} = (1, -1, 0)$, $\vec{v_2} = (0, 1, -1)$, $\vec{v_3} = (0, 2, 1)$ and $\vec{v_4} = (1, 0, 3)$ are linearly dependent.   5

(b) If $u = xf(y/x) + g(y/x)$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$ 5

(c) Reduce the matrix

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$$

to row echelon form and determine its rank.   5

(d) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$ 5

Group B

5. (*a*) Solve the differential equation

$$(5x^3 + 12x^2 + 6y^2)\,dx + 6xy\,dy = 0.$$ 5

(*b*) Solve the differential equation

$$\cot 3x \frac{dy}{dx} - 3y = \cos 3x + \sin 3x\,;\ 0 < x < \pi/2.$$ 5

(*c*) Solve the first order partial differential equation

$$x(y^2 - z^2)\frac{\partial z}{\partial x} + y(z^2 - x^2)\frac{\partial z}{\partial y} = z(x^2 - y^2).$$ 5

(*d*) Find the Laplace transform of the function $(t \sin t)$

and use the result to evaluate $\int_0^{\infty} t\,e^{-3t} \sin t\,dt.$ 5

6. (*a*) Using $F(w) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\,e^{-iwx}\,dx$ as the

definition of Fourier transform of $f(x)$, show that

Fourier transform of $e^{-4ix} f(x)$ equals $F(w+4)$. 6

(*b*) Solve the differential equation

$$y''(x) + 3y'(x) + 2y(x) = e^x$$

for its general solution, using the method of variation of parameters. 6

(*c*) Solve the initial value problem

$$y''(t) + y(t) = 2\cos t\,;\ y(0) = 0,\ y'(0) = -1$$

using Laplace transform procedure. 8

7. (*a*) Write down the (*i*) probability distribution function (PDF), (*ii*) mean, and (*iii*) variance for the following selected distributions: (*i*) Binomial (discrete), (*ii*) Poisson (discrete) and the Gaussian (continuous). 8

(*b*) (*i*) If S^2 is the variance of a random sample of size n taken from a normal population having the variance σ^2, then write down the expression for the random variable having the chi-square distribution with a parameter $v = n - 1$. 3

(*ii*) State the central limit theorem. 3

(*c*) If a one gallon can of paint covers on the average 513·3 square feet with a standard deviation of 31·5 square feet, what is the probability that the sample mean area covered by a sample of 40 of these one gallon cans will be anywhere from 510·0 square feet to 520·0 square feet. 6

8. (*a*) Using Lagrange polynomial, find the value of $f(x)$ at $x = 4$, given the table of x versus $f(x)$ as

x	:	1·5	3	6
$f(x)$	:	−0·25	2	20

6

(b) Consider the function $f(x)$ given by

x	:	0.1	0.2	0.3	0.4	0.5	0.6

$f(x)$: 0.425 0.475 0.400 0.450 0.575 0.675

Use composite Trapezoidal rule to evaluate the integral

$$\int_{0.1}^{0.6} [f(x)]\, dx.$$

6

(c) Evaluate the integral

$$\int_{-1}^{1} x^2 e^{-x}\, dx$$

by composite Simpson's one-third rule with spacing $h = 0.25$. You may use any other known numerical method to evaluate the given integral.

8

Group C

9. Answer the following : 2 × 10

(i) If $y = \sin(2\sin^{-1}x)$, then find the value of $(1 - x^2)\, y_2 - xy_1$ in terms of y.

(ii) Determine the region, in which the function

$$z = \sqrt{(1 - x^2 - y^2)}$$ is defined.

(iii) If $f(x, y) = \dfrac{x + y - 1}{x + y + 1}$, then evaluate $\left(\dfrac{\partial f}{\partial x}\right)$ at the point (2, 1).

(iv) Find the gradient of the scalar field

$$f(x, y) = y^2 - 4xy$$ at the point (1, 2).

(v) Find the solution of the differential equation $y\, dx - x\, dy + e^{1/x}\, dx = 0$ by suitably choosing an integrating factor.

(vi) Write down the Laplace transform of

$$f(t) = e^{-3t} \cos 3t.$$

(vii) What is the total number of arithmetic operations required to interpolate by Lagrangian form ?

(viii) A bag contains 10 white balls and 15 black balls. Two balls are drawn in succession. What is the probability that one of them is black and the other white ?

(ix) Distinguish between 'round-off error' and 'truncation error'.

(x) $X_1, \ldots, X_n$ are independent normal random variables, each of which has mean μ and variance σ^2. What is the mean and variance of the random variable

$$Z = \sqrt{n}\ \frac{\bar{X} - \mu}{\sigma}$$

where the symbols have their usual meanings.

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Group A

1. (a) Prove that

$$\vec{F} = (2xy + z^3)\ \vec{i} + x^2\ \vec{j} + 3xz^2\ \vec{k}$$

is a conservative force field. Also, find the scalar
potential. 6

(b) Using Stoke's theorem, evaluate

$$\int_S \vec{n} \cdot \text{curl}\ \vec{F}\ ds \qquad .$$

where $\vec{F} = y\ \vec{i} + z\ \vec{j} + x\ \vec{k}$ and S is a portion
of the surface $x^2 + y^2 - 2ax + a^2 = 0$ above the plane
$Z = 0$. 8

(c) Evaluate the integral

$$\int_S \left(yz \ \vec{i} + zx \ \vec{j} + xy \ \vec{k} \right) \cdot ds$$

over the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.   6

2. (a) Show that

$$f(x, y, z) = (x + y + z)^3 - 3(x + y + z)$$
$$- 24xyz + a^3$$

has a minima at $(1, 1, 1)$ and a maxima at $(-1, -1, -1)$.   6

(b) If the functions $f(x)$, $\phi(x)$ and $\psi(x)$ satisfy the conditions of mean value theorem in $[a, b]$, show that these exist at least one point ξ such that

$$\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(\xi) & \phi'(\xi) & \psi'(\xi) \end{vmatrix} = 0.$$ 6

(c) Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$ 8

3. (a) For what values of λ and μ, the following system of equations has (i) no solution, (ii) a unique solution, and (iii) an infinite number of solutions: 6

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

(b) Prove that

$$\int\int_R \sqrt{|y - x^2|} \, dx \, dy = (3\pi + 8)/6$$

where $R = [-1, 1, 0, 2]$.   8

(c) Find the equations of the tangent line and normal plane to the curve

$$x^2 + 2y^2 + 3z^2 = 3$$
$$2x + 3y + 4z = 5$$

at $(1, 1, 0)$.   6

4. (a) Find the volume of the solid enclosed between the surfaces $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.   6

(b) Find the absolute maximum and minimum values of the function $f(x, y) = 3x^2 + y^2 - x$ over the region $2x^2 + y^2 \leq 1$.   8

(c) The part of the laminscate $r^2 = a^2 \cos 2\theta, 0 \leq \theta \leq \pi/4$ is revolved about the x-axis. Find the surface area of the solid generated.   6

Group B

5. (a) By constructing a difference table and taking second order differences as constants, find the sixth term of the series

$$8, 12, 19, 29, 42, \dots$$ 8

(b) Find the value of

$$\int_1^5 \log_{10} x \, dx$$

taking 8 sub-intervals correct to 4 decimal places by Trapezoidal rule of integration.   6

(c) From the values given in the table, compute dy/dx for $x = 1$:

x	1	2	3	4	5	6
y	1	8	27	64	125	216

6

6. (a) Using Lagrange's interpolation formula, find a polynomial which passes through the points $(0, -12)$, $(1, 0)$, $(3, 6)$, $(4, 12)$. 6

(b) Solve

$$xy\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}(1 + x + x^2)$$

6

(c) Let a random sample of 10 boys has the following IQ:

70, 120, 110, 101, 88, 83, 95, 98, 107, 100

Do these data support the assumption of a population mean IQ of 100? 8

7. (a) Let T be a linear transformation on R_4 defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_2 + x_3 \\ x_1 + x_2 \\ x_1 \\ x_1 - x_2 \end{pmatrix}$$

Describe the image of T, denoted by $V(T)$, and determine its rank. 6

(b) If $Z = ax + by$ and r is the correlation coefficient between x and y, show that

$$\sigma_z^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2ab\gamma\sigma_x\sigma_y.$$

Hence, prove that

$$\gamma = \left(\sigma_x^2 + \sigma_y^2 - \sigma_{xy}^2\right)/2\sigma_x\sigma_y.$$

8

(c) If $x = 4y + 5$ and $y = 1\,kx + 4$ are the regression lines of x on y and y on x respectively, show that 6
$$0 \le k \le 1/4$$

8. (a) Find the general solution of the differential equation

$$(D^2 + 1)y = \operatorname{cosec} x.$$

6

(b) Solve
$$y'' - ty' + y = 1$$
$$y(0) = 1, \; y'(0) = 2.$$

6

(c) Solve the system of equations

$$\frac{d^2x}{dt^2} + \frac{dy}{dt} = e^{2t}$$

$$\frac{dx}{dt} + \frac{dy}{dt} - x - y = 0.$$

8

Group C

9. Answer the following: 2×10

(i) Suppose T_1 and T_2 are two rotation linear transformations which rotate vectors counter clockwise through angles θ_1 and θ_2 about the origin respectively. Find the matrices of $T_1 T_2$ and $T_2 T_1$.

(ii) Obtain the expansion of
$$f(x) = 1/(x-2)(x-3)$$
in powers of $(x-1)$.

(iii) Determine the eigenvalues of the matrix A and then diagonalize it:

$$A = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

(*iv*) Solve

$$x^2 y'' + 2xy' = 0.$$

(*v*) Test whether the series is convergent or divergent :

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

(*vi*) If Δ be the forward difference operator ; δ, the central difference operator ; and E, the shift operator, then show that

$$E^{1/2} \delta = \Delta$$

(*vii*) Find the directional derivative of

$$u(x, y) = x^2 + y^2$$

at $(1, 1)$ in the direction making an angle $30°$ with the positive direction of x-axis.

(*viii*) If $\vec{r}$ is a position vector of any point in three-dimensional space, find ∇r^n.

(*ix*) Test the linear dependence or independence of the given vectors

$$\{(4, 0, 2), (2, 2, 0), (1, 1, 0)\}.$$

(*x*) Show that in the curve $y = be^{-(a/x)}$, the subtangent varies as the square of the abscissa.

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Group A

1. (a) If $y = \{x + \sqrt{(1 + x^2)}\}^m$, prove that

   $$(1 + x^2) y_{n+2} + (2n + 1) xy_{n+1} + (n^2 - m^2) y_n = 0.$$

   5

   (b) If $r^2 = x^2 + y^2 + z^2$ and $v = r^m$, prove that

   $$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m (m + 1) \cdot r^{m-2}.$$

   5

(c) Are the following vectors linearly dependent?

$$\vec{x_1} = (1, 2, 4), \quad \vec{x_2} = (2, -1, 3), \quad \vec{x_3} = (0, 1, 2),$$
$$\vec{x_4} = (-3, 7, 2)$$

If so, find the relation between them. **4**

(d) Examine the convergence of the series

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \ldots + \frac{2^{n+1} - 2}{2^{n+1} + 1}x^n + \ldots$$

where $x > 0$. **6**

2. (a) Use Maclaurin's theorem to show that

$$e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + \ldots$$

5

(b) If $I_n = \int_0^{\pi/2} x \sin^n x \, dx$ and $n > 1$,
prove that

$$I_n = \frac{n-1}{n} I_{n-2} + \frac{1}{n^2}$$

Hence deduce that $I_5 = \frac{149}{225}$ **6**

(c) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. **4**

(d) Test for consistency and solve:

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$2x_1 + 5x_2 + 5x_3 - 18x_4 = 9$$
$$4x_1 - x_2 - x_3 + 8x_4 = 7$$

5

3. (a) The smaller segment of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cut off by the chord $x = \frac{a}{2}$ revolves completely about this chord. Find the volume generated. **5**

(b) If $f(x, y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2 + y^2}$, prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -2f(x, y) \text{ and}$$

$$x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = 6f(x, y)$$

5

(c) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

and verify that it is satisfied by A and hence find the inverse of A. **5**

(d) Divide 24 into three parts such that the continued product of the first, the square of the second and the cube of the third may be a maximum. **5**

4. (a) Reduce the matrix

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

to the diagonal form and find the modal matrix. **5**

(b) A vector field is given by

$$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$$

Show that the field is irrotational, and find the scalar potential such that $\vec{F} = \text{grad } \phi$. **5**
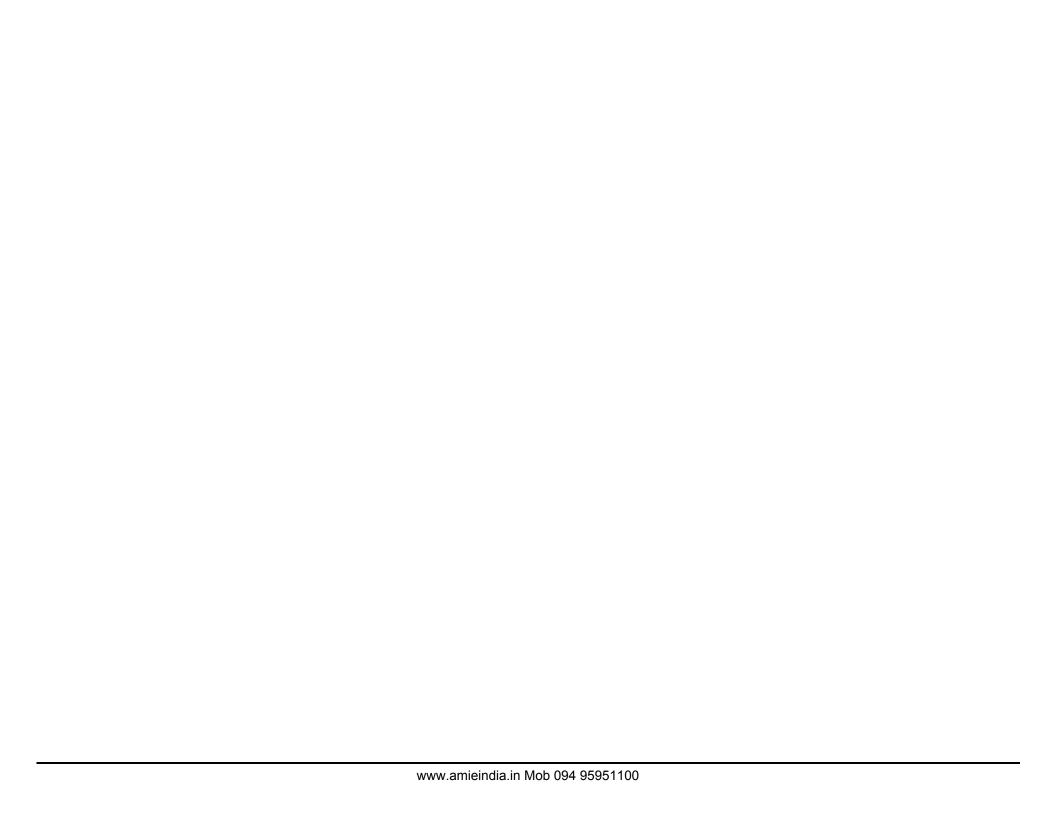
(c) Use Stokes' theorem to evaluate

$$\int_C (e^x\, dx + 2y\, dy - dz)$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$.   4

(d) Change the order of integration in the double integral

$$\int_0^a \int_0^x \frac{\phi'(y)\, dy\, dx}{\sqrt{(a-x)(x-y)}}$$

and hence find its value (in terms of ϕ).   6

Group B

5. (a) Solve the differential equation

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$ 4

(b) Solve the differential equation

$$(D^3 - 2D^2 + 3)\, y = \cos x + x^{-x}$$ 6

(c) Solve

$$x(y^2 - z^2)\, p + y(z^2 - x^2)\, q = z(x^2 - y^2)$$

where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$ 4

(d) Using Laplace transforms, find the solution of the equation

$$\frac{d^2 x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-t}$$

with $x = \dfrac{dx}{dt} = 1$ at $t = 0$.   6

6. (a) Solve the differential equation using the method of variation of parameters

$$\frac{d^2 y}{dx^2} + y = x \sin x$$ 6

(b) Solve the differential equation:

$$x^2\frac{d^2 y}{dx^2} + 7x\frac{dy}{dx} + 13y = \log x$$ 6

(c) A bar with insulated sides is initially at temperature 0 °C throughout. The end $x = 0$ is kept at 0 °C and heat is suddenly applied at the end $x = l$ so that $\dfrac{\partial u}{\partial x} = A$ for $x = l$ where A is a constant. Find the temperature function $u(x, t)$:   8
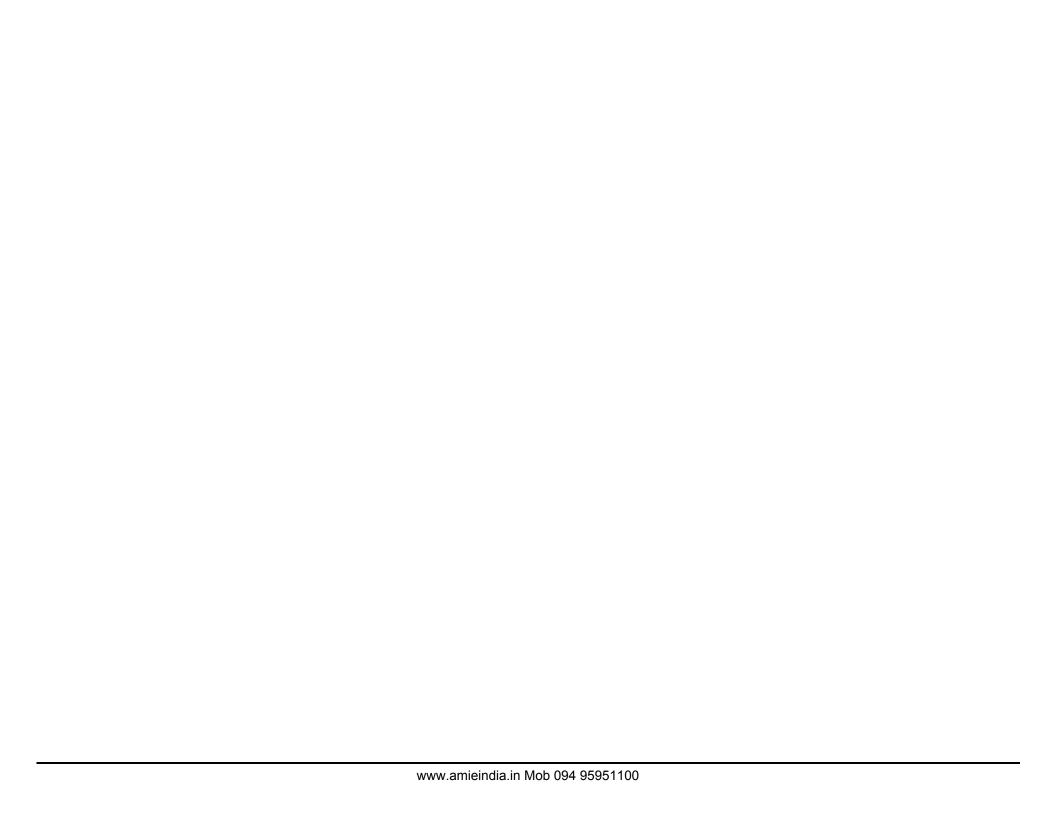
7. (a) The following data are taken from the steam table:

Temp. °C	140	160	170	190
Pressure kgf/cm²	3·685	6·302	8·076	12·575

Use Lagrange's interpolation formula to compute the pressure at temperature 165 °C.   5

(b) A company generally purchases large lots of a certain kind of electronic device. A method is used that rejects a lot if two or more defective units are found in a random sample of 100 units.

(i) What is the probability of rejecting a lot that is 1% defective?

(ii) What is the probability of accepting a lot that is 5% defective?   8

(c) In an explosive factory cordite sticks are manufactured by two processes A and B. After inspection the results found were:

	Defective	Nondefective
Process A	100	900
Process B	60	440

Do these data indicate that process A and process B are of same quality? (5% values of χ^2 for d.f. 1, 2 and 3 are $3 \cdot 84$, $5 \cdot 99$ and $7 \cdot 82$ respectively). 7

. (a) Find the first and second derivatives of the functions tabulated below at $x = 0 \cdot 40$:

x : 0·40   0·50   0·60   0·70   0·80
y : 1·5836 1·7974 2·0442 2·3275 2·6511

5

(b) Find an approximate value of $\log_e 5$ up to 4 decimal places, by evaluating the integral

$$\int_0^5 \frac{dx}{4x + 5},$$

by Simpson's one–third rule, dividing the range into ten equal parts. 5

(c) In an industrial process the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be $3 \cdot 0 \pm 0 \cdot 01$ cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $3 \cdot 0$ cm and standard deviation $0 \cdot 005$ cm. On the average, what percentage of manufactured ball bearings will be scrapped? (Given that $\Phi(2) = 0 \cdot 9772$ where

$$\Phi(z) = \frac{1}{\sqrt{}} \int_0^z e^{-\frac{1}{2}x^2} dx).$$

(d) For determining the ash contents of six samples of coal, each sample was divided into two equal parts and a part given to each of two analysts A and B. The percentage of ash contents found by them are given below samplewise:

Sample No.	1	2	3	4	5	6
Analyst A	8	10	11	9	12	13
Analyst B	9	13	12	10	11	12

Do the two analysts differ significantly? ($t_{0 \cdot 05}$ for d.f. $5 = 2 \cdot 57$). 5

Group C

9. Select most appropriate and unique choice for the following: 1×20

(i) If $f(x) = (x - a)^{5/2}$ and $f(x + h) = f(x) + h f'(x) + \frac{h^2}{\underline{|2}} f''(x + \theta h),$ then the value of θ as x tends to a is

(a) $\frac{8}{225}$

(b) $\frac{16}{225}$

(c) $\frac{32}{225}$

(d) $\frac{64}{225}$

(ii) If $u = \log(x^3 + y^3 - x^2y - xy^2)$, then the value of $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$ is

(a) $\dfrac{2}{x-y}$

(b) $\dfrac{2}{x+y}$

(c) $\dfrac{2}{x^2-y^2}$

(d) $\dfrac{2}{x^2+y^2}$

(iii) If $z = f(u,v)$ where $u = x^2 - y^2$ and $v = 2xy$, with $(x,y) \neq (0,0)$ then the differential equation

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0 \text{ is equivalent to}$$

(a) $\dfrac{\partial z}{\partial u} = 0$

(b) $\dfrac{\partial z}{\partial v} = 0$

(c) $\dfrac{\partial z}{\partial u} - \dfrac{\partial z}{\partial v} = 0$

(d) $\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v} = 0$

(iv) If A be a 2×2 matrix over a field, the $\det(I+A) = 1 + \text{Trace } A$ when A is

(a) singular

(b) non-singular

(c) symmetric

(d) orthogonal

(v) If two of the eigenvalues of a 3×3 matrix, whose determinant equals 4, are -1 and 2. Then the third eigenvalue of the matrix is

(a) 2

(b) -2

(c) 3

(d) -3

(vi) The angle between the tangent vectors to the curve
$$x = t^2 + 1, \ y = 4t - 3, \ z = 2t^2 - 6t$$
at the points $t = 1$ and $t = 2$ is

(a) $\cos^{-1}\left(\dfrac{5}{\sqrt{6}}\right)$

(b) $\cos^{-1}\left(\dfrac{5}{2\sqrt{6}}\right)$

(c) $\cos^{-1}\left(\dfrac{5}{3\sqrt{6}}\right)$

(d) $\cos^{-1}\left(\dfrac{5}{4\sqrt{6}}\right)$

(vii) The value of n for which the vector $r^n \vec{r}$ is solenoidal is

(a) -3

(b) 3

(c) -2

(d) 2

(*viii*) The value of the line integral

$$\int_C [\sin y\, dx + x\,(1 + \cos y)\, dy]$$

taken over the circular path given by $x^2 + y^2 = a^2$, $z = 0$ is

(*a*) πa^2

(*b*) $2\pi a^2$

(*c*) $3\pi a^2$

(*d*) $4\pi a^2$

(*ix*) The solution of the differential equation

$$\cos^2 x \frac{d^2 y}{dx^2} = 1 \text{ such that } y = \frac{dy}{dx} = 0 \text{ when } x = 0 \text{ is}$$

(*a*) $y = \tan x$

(*b*) $y = \sin x$

(*c*) $\log (\tan x + 1)$

(*d*) $y = \log \sec x$

(*x*) The particular integral of the equation

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = e^x$$

is

(*a*) $\frac{1}{2} e^x$

(*b*) $\frac{1}{2} x e^x$

(*c*) $\frac{1}{2} x^2 e^x$

(*d*) $\frac{1}{2} x^3 e^x$

(*xi*) If $f(t)$ is the staircase function such that $f(t) = b$ when $0 < t < a$, $f(t) = 2b$ when $a < t < 2a$, and so forth, then Laplace transform of $f(t)$ is

(*a*) $\dfrac{b}{s\,(1 - e^{as})}$

(*b*) $\dfrac{b}{s\,(1 + e^{-as})}$

(*c*) $\dfrac{b}{s\,(1 + e^{as})}$

(*d*) $\dfrac{b}{s\,(1 - e^{-as})}$

(*xii*) The inverse Laplace transform of $\dfrac{e^{-as}}{s^n}$ is

(*a*) $\dfrac{(t - a)^{n - 1}}{\lfloor n - 1} u\,(t - a)$

(*b*) $\dfrac{(t - a)^{n - 1}}{\lfloor n} u\,(t - a)$

(*c*) $\dfrac{(t - a)^n}{\lfloor n - 1} u\,(t - a)$

(*d*) $\dfrac{(t - a)^n}{\lfloor n} u\,(t - a)$

(*viii*) The value of the line integral

$$\int_C [\sin y \, dx + x \, (1 + \cos y) \, dy]$$

taken over the circular path given by $x^2 + y^2 = a^2$, $z = 0$ is

(*a*) πa^2

(*b*) $2\pi a^2$

(*c*) $3\pi a^2$

(*d*) $4\pi a^2$

(*ix*) The solution of the differential equation

$$\cos^2 x \frac{d^2 y}{dx^2} = 1 \text{ such that } y = \frac{dy}{dx} = 0 \text{ when } x = 0 \text{ is}$$

(*a*) $y = \tan x$

(*b*) $y = \sin x$

(*c*) $\log (\tan x + 1)$

(*d*) $y = \log \sec x$

(*x*) The particular integral of the equation

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = e^x$$

is

(*a*) $\frac{1}{2} e^x$

(*b*) $\frac{1}{2} x e^x$

(*c*) $\frac{1}{2} x^2 e^x$

(*d*) $\frac{1}{2} x^3 e^x$

(*xi*) If $f(t)$ is the staircase function such that $f(t) = b$ when $0 < t < a$, $f(t) = 2b$ when $a < t < 2a$, and so forth, then Laplace transform of $f(t)$ is

(*a*) $\dfrac{b}{s(1 - e^{as})}$

(*b*) $\dfrac{b}{s(1 + e^{-as})}$

(*c*) $\dfrac{b}{s(1 + e^{as})}$

(*d*) $\dfrac{b}{s(1 - e^{-as})}$

(*xii*) The inverse Laplace transform of $\dfrac{e^{-as}}{s^n}$ is

(*a*) $\dfrac{(t-a)^{n-1}}{\lfloor n-1} u(t-a)$

(*b*) $\dfrac{(t-a)^{n-1}}{\lfloor n} u(t-a)$

(*c*) $\dfrac{(t-a)^n}{\lfloor n-1} u(t-a)$

(*d*) $\dfrac{(t-a)^n}{\lfloor n} u(t-a)$

(*xiii*) The inverse Laplace transform of $\log \dfrac{s+1}{s-1}$ is

(a) $\dfrac{\sinh t}{t}$

(b) $\dfrac{2 \sinh t}{t}$

(c) $\dfrac{\sin t}{t}$

(d) $\dfrac{2 \sin t}{t}$

(*xiv*) The value of $\Delta^3 (1-x)(1-2x)(1-3x)$ with unit interval of differencing is

(a) -24

(b) 24

(c) -36

(d) 36

(*xv*) If $y_0 = 2$, $y_1 = 6$, $y_2 = 8$, $y_3 = 9$ and $y_4 = 17$, then $\Delta^4 y_0$ is

(a) 5

(b) 6

(c) 7

(d) 8

(*xvi*) In Simpson's one-third rule, $y(x)$ is a polynomial of degree

(a) one

(b) two

(c) three

(d) four

(*xvii*) If $x = 4y + 5$, $y = kx + 4$ are the regression lines of x and y and y on x respectively, then

(a) $0 < k \leq \dfrac{1}{4}$

(b) $-\dfrac{1}{4} \leq k < 0$

(c) $k \geq \dfrac{1}{4}$

(d) $k \leq -\dfrac{1}{4}$

(*xviii*) Two events A and B have probabilities $0 \cdot 25$ and $0 \cdot 50$ respectively. The probability that both A and B occur simultaneously is $0 \cdot 14$. Then the probability that neither A nor B occurs is

(a) $0 \cdot 36$

(b) $0 \cdot 37$

(c) $0 \cdot 38$

(d) $0 \cdot 39$

(*xix*) If the function

$$f(x) = ae^{-cx}, \quad \text{when } x \geq 0$$
$$= 0 \quad , \quad \text{when } x < 0$$

is the probability density function of a continuous distribution, then the value of C is

(*a*) a

(*b*) $-a$

(*c*) $2a$

(*d*) $-2a$

(*xx*) The life-lengths of two electronic devices D_1 and D_2 are normally distributed with means 44 hours and 48 hours, and standard deviation of 6 hours and 3 hours respectively. In a particular situation if one of these devices is to be used for a 52-hour period, then

(*a*) D_1 is preferable

(*b*) D_2 is preferable

(*c*) D_1 and D_2 are equally good

(*d*) None of these

ENGINEERING MATHEMATICS

Time : Three hours

Maximum marks : 100

Answer FIVE *questions, taking* ANY TWO *from* **Group A,**
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answered at one place.*

*Answer should be brief and to-the-point and be supple-
mented with neat sketches. Unnecessary long answers
may result in loss of marks.*

*Any missing data or wrong data may be assumed suitably
giving proper justification.*

Figures on the right-hand side margin indicate full marks.

Group A

1. (a) A twice differential function f is such that
$f(a) = f(b) = 0$ and $f(c) > 0$ for $a < c < b$. Prove
that there is at least one value ξ between a and b
for which $f''(\xi) < 0$.   6

(b) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$ then
evaluate

$$\int_C \vec{F} \cdot d\vec{r}$$

where C is the straight line joining $(0,0,0)$ to
$(1,1,1)$.   6

(c) Evaluate by Green's theorem in plane

$$\int_C e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$$

where C is the rectangle with vertices $(0,0)$, $(\pi,0)$, $(\pi,\pi/2)$ and $(0,\pi/2)$.   8

2. (a) Find the greatest value of the directional derivative of the function

$$2x^2 - y - z^4 \text{ at the point } (2, -1, 1).$$ 6

(b) If $P = \dfrac{d^n}{dx^n}\{(x^2-1)^n\}$ show that

$$\frac{d}{dx}\left\{(1-x^2)\frac{dP}{dx}\right\} + n(n+1)P = 0.$$ 8

(c) Prove that $\dfrac{x}{a} + \dfrac{y}{b} = 1$ touches the curve $y = b\,e^{-x/a}$ at the point where the curve crosses the y-axis.   6

3. (a) By using the methods of Langrange's undetermined multipliers. Find the maxima and minima of $x^2 + y^2 + z^2$ subject to the conditions

$$ax + by + cz = 1, \text{ and}$$
$$a'x + b'y + c'z = 1$$ 8

(b) Evaluate

$$\iint_A xy\,dx\,dy$$

where A is the domain bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$.   6

(c) Find the shortest distance from the origin to the hyperbola

$$x^2 + 8xy + 7y^2 = 225,\ z = 0$$ 6

4. (a) Find the basis of eigenvectors and then diagonalize the matrix, A

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$ 8

(b) Find the interpolating polynomial to interpolate the following data :

x	2	2·1	2·2	2·3	2·4
$f(x)$	1·414214	1·449138	1·48324	1·516575	1·549193

Use it to compute the value of $f(2\cdot15)$.   6

(c) Examine the consistency of the system of equations

$$2x - 3y + 5z = 1$$
$$3x + y - z = 2$$
$$x + 4y - 6z = 1.$$ 2

(d) Find the inverse of the matrix

$$A = \begin{pmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{pmatrix}$$ 4

Group B

5. (a) Solve the differential equation

$$y'' + 2y' + y = e^{-x}\cos x$$

by using the method of variation of parameters.   8

(b) Use Laplace's transform to solve the differential equation

$$y'' + 2y' + 2y = 2$$
$$y(0) = 0, \quad y'(0) = 1.$$
6

(c) Solve the differential equation

$$\frac{dy}{dx}(x^2 y^3 + xy) = 1.$$
6

6. (a) Find the surface passing through the lines
$$z = x = 0 \text{ and } z - 1 = x - y = 0$$
and satisfying the differential equation

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0.$$
8

(b) Solve the differential equation

$$(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy.$$
6

(c) If δ is the central difference operator and μ be the average operator then show that

$$\mu = \sqrt{1 + \frac{1}{4}\delta^2}$$
6

7. (a) A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x = 0$ and $x = 1$, and a curve through the points with the following coordinates:

x : 0·00 0·25 0·50 0·75 1·0
y : 1·000 0·9896 0·9589 0·9089 0·8415

Estimate the volume of the solid formed, giving the answer to three decimal places.
6

(b) Use Lagrange's interpolation formula to find the value of $\log_{10} 301$ from the following data values

x : 300 304 305 307
$\log_{10} x$: 2·4771 2·4829 2·4843 2·4871
8

(c) Solve the differential equation

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right).$$
6

8. (a) An incomplete frequency function values distribution is given as follows:

Variable	Frequency	Variable	Frequency
10-20	12	50-60	?
20-30	30	60-70	25
30-40	?	70-80	18
40-50	65	Total:	229

Given that the median value is 46, determine the missing frequencies using the median formula.
8

(b) A car hire firm has two cars which it hires out day by day. The number of cars demanded on each day is distributed as Poisson variate with mean 1·5. Calculate the proportion of days on which (i) neither car is used, (ii) some demand is refused.
6

(c) The theory predicts the proportion of beans in the four groups A, B, C, and D should be 9:3:3:1. In an experiment among 1600 beans, the number in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory?
6

Group C

). Answer the following :   2 × 10

(*i*) Test the convergence of the series

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \dots + \frac{1}{(\log n)^p} + \dots$$

where $p > 0$.

(*ii*) If $z = e^{xy^2}$, $x = t \cos t$ and $y = t \sin t$ then compute

$\dfrac{dz}{dt}$ at $t = \dfrac{\pi}{2}$.

(*iii*) Find the length of the curve

$$x = a \cos^3 t, \;\; y = a \sin^3 t.$$

(*iv*) Find the directional derivative of

$$f(x, y, z) = 2x^2 + 3y^2 + z^2$$

at the point (2, 1, 3) in the direction of the vector

$\vec{a} = \vec{i} - 2\vec{k}$

(*v*) Let T be the function from R^3 into R^3 defined by

$$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2,$$
$$- x_1 - 2x_2 + 2x_3)$$

Verify whether T is a linear transformation? If it is find the associated matrix.

(*vi*) Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x} \log x$$

(*vii*) Check for the linear independence or dependence of the following set of vectors

$$\{(1, 1, 1), (4, 1, 1), (1, -1, 2)\}$$

(*viii*) If $\vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$ then prove that

$$\text{div}\left(\text{grad}\frac{1}{\vec{v}}\right) = 0$$

(*ix*) Find the mode for the following distribution :

Class interval :	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency :	5	8	7	12	28	20	10	10

(*x*) Form a partial differential equation by eliminating f from $z = e^{mx} f(x + y)$.

W'05 : 5 FN : AN 209 (1409)

ENGINEERING MATHEMATICS

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mented with neat sketches. Unnecessary long answers
may result in loss of marks.*

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Group A

1. (*a*) State the Leibnitz's theorem and find y_n, if

$$y = \frac{x^n}{1 + x}$$

5

   (*b*) State the Lagrange's mean value theorem and show
   that
   $$|\sin b - \sin a| \le |b - a| \quad \text{for all real } a \text{ and } b.$$
   5

   (*c*) Show that the arc of the upper half of the cardiod
   $r = a (1 - \cos \theta)$ is bisected by the line $\theta = 2\pi/3$.
   5

   (Turn Over)

(d) If $x^x y^y z^z = c$, show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = - (x \log ex)^{-1}.$$

5

2. (a) The temperature T at any point (x, y, z) in space is $T = 400 xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

5

(b) In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^2 y^2 z^4$ maximum and what is its magnitude?

5

(c) Test the convergence of the power series

$$\sum_0^\infty \frac{2n!}{n^n} \frac{2^n}{n!} x^n, \quad (x > 0)$$

5

(d) Expand $f(x) = \tan x$ about $x = \pi/4$ up to first three terms by Taylor's series.

5

3. (a) State the Green's theorem and evaluate

$$\int_C [(y - \sin x) \, dx + \cos x \, dy], \quad \text{where } C \text{ plane}$$

triangle enclosed by line $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2}{\pi} x$.

5

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

7

(c) Solve the following system of equation by using Gauss elimination method:

$x - y + 2z = -8$
$x + y + z = -2$
$2x - 2y + 3z = -20$

8

4. (a) State the Stokes's theorem and evaluate

$$\oint_C \vec{F} \cdot d\vec{r}, \quad \text{where} \quad \vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x + z) \hat{k}$$

and C is the boundary of the triangle with vertices at $(0, 0, 0)$, $(1, 0, 0)$ and $(1, 1, 0)$.

6

(b) For what value of λ and r, the system of equations
$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = r$
have (i) no solution, (ii) unique solution, (iii) more than one solution?

8

(c) Use divergence theorem to show

$$\oint_S \nabla r^2 \, d\vec{s} = 6V, \quad \text{where} \quad S \text{ is any closed}$$

surface enclosed by volume V.

6

Group B

5. (a) Define Bernoulli's equation and solve

$$xy(1 + xy^2) \frac{dy}{dx} = 1.$$

6

(b) Solve $(xy^2 - e^{1/x^3}) \, dx - x^2 y \, dy = 0$.

7

(c) Solve $(D^2 + 4) y = x \sin x$.

7

6. **(a)** Solve $(x^3D^3 + 3x^2D^2 + xD)\,y = x^3 \log x$.   7

(b) Solve

$$(D^2 - 6D + 9)\,y = e^{3x}/x^2$$

by using variation of parameter method.   6

(c) If $L\{f(t)\} = \bar{f}(s)$, then prove that

$$L\{f(at)\} = \frac{1}{a}\,\bar{f}(s/a)$$

and evaluate $L(\sin 2t \cdot \cos 3t)$ where L represents the Laplace transform.   7

7. **(a)** Write Newton's backward interpolation formula and evaluate value of $f(21)$ from the following data:

x	0	5	10	15	20
y	1·0	1·6	3·8	8·2	15·4

     7

(b) Define Simpson's one-third rule and estimate the area bounded by the curve $y = f(x)$, x-axis and ordinate $x = 0$ and $x = 5$ from the following data:

x_i	0	0·5	1	1·5	2	2·5	3	3·5	4	4·5	5
$f(x_i)$	1	2·1	3·2	4	5·1	6·2	7·2	8·3	9	10·2	12

     6

(c) A certain screw-making machine produces an average 2 defective screws out of 100, and pack them in boxes of 500. Find the probability that a box contains 15 defective screws.   7

8. **(a)** Construct the difference table from the following value of y, where value of x are (i) 1·05, and (ii) 1·45

x :	1·0	1·1	1·2	1·3	1·4	1·5
y :	0·24197	0·21875	0·19419	0·17137	0·14973	0·12952

     6

(b) Estimate the value of π, correct up to three decimal places, from $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$ by using Simpson's one-third rule when number of sub-intervals is 4.   7

(c) The incidence of occupational disease in an industry is such that the workers have 20% chances of suffering from it. What is the probability that out of six workers chosen at random, four or more will suffer from the disease?   7

Group C

9. Select most appropriate and unique choice for the following: 1×20

(i) In usual notation, for Binomial distribution, npq, is

     (a) $< np$

     (b) np

     (c) $> np$

     (d) none of the above

(ii) The particular integral of $\dfrac{d^2y}{dx^2} + 4y = \cos 2x$ is

     (a) $\dfrac{x \sin 2x}{4}$

     (b) $\dfrac{x \cos 2x}{4}$

     (c) $\dfrac{\sin 2x}{4}$

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(iii) The C.F. of $y'' - 2y' + y = xe^x \sin x$ is

(a) $(c_1 + c_2 x) e^x$

(b) $(c_1 + c_2) e^{-x}$

(c) none of the above

(iv) If n is a positive integer, then $L(t^n)$ is

(a) $(n+1)!/s^n$

(b) $n!/s^{n+1}$

(c) $n!/s^n$

(v) The value of $L^{-1}\left\{\dfrac{1}{s-a}\right\}$ is

(a) e^{at}

(b) e^{-at}

(c) e^{a-t}

(vi) For the Poisson's distribution if $p(x=2) = \dfrac{2}{3} p(x=1)$, then mean of Poisson's distribution will be

(a) 0

(b) 4/13

(c) 3/4

(vii) The differential equation
$$y' + y \cos x = \sin 2x \text{ is}$$

(a) exact

(b) linear

(c) homogeneous

(viii) The rank of the matrix
$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 8 \\ 3 & 7 & 22 \end{bmatrix} \text{ is}$$

(a) 2

(b) 3

(c) 4

(ix) For any closed surface S,
$$\iint\limits_{S} \text{curl } \vec{F} \cdot \hat{n} \, ds \text{ is}$$

(a) V

(b) S

(c) 0

(x) The directional derivative of the function $f = 4xz^3$ at the point $(1, -1, 1)$ along z-axis is

(a) 12

(b) 144

(c) 0

(*xi*) The unit normal vector to the surface $f = xyz$ at $(1, 1, 1)$ is

(*a*) $\dfrac{1}{\sqrt{3}}(i - j - k)$

(*b*) $\dfrac{1}{\sqrt{3}}(i + j + k)$

(*c*) θ

(*xii*) The integrating factor of the differential equation

$(xy^3 + y)\, dx + 2\,(x^2 y^2 + x + y^4)\, dy = 0$ is

(*a*) $1/y$

(*b*) y

(*c*) $4/x$

(*xiii*) The equation $x\, dy - y\, dx = \sqrt{x^2 + y^2}\, dx$ is

(*a*) exact.

(*b*) linear.

(*c*) homogeneous.

(*xiv*) If expected and observed frequencies are equal, then x^2 is equal to

(*a*) 1

(*b*) 0

(*c*) 1·5

(*xv*) For standard normal variate mean μ is

(*a*) 1

(*b*) 0

(*c*) 6

(*d*) none of the above

(*xvi*) By using Green's theorem, value of

$$\oint_C (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy, \quad \text{where } C \text{ is}$$

the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$ is

(*a*) 3/2

(*b*) 3/10

(*c*) 8/3

(*d*) none of the above

(*xvii*) One of the eigenvalues of the matrix

$$\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix} \text{ is}$$

(*a*) 0

(*b*) 1

(*c*) 6

(*d*) none of the above

(*xviii*) Given that

$x:$ 1   2   3   4   5

$y:$ 2   5   10   17   26,   then

the value of $\nabla^2 y_5$ is

(*a*) 0

(*b*) 2

(*c*) 3

(*d*) 12

(*xix*) Consider the difference table

$$x: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$
$$y: \quad 3 \quad 6 \quad 11 \quad 18 \quad 27,$$

then   *y*

(*a*) $3x^2 + 2x + 1$

(*b*) $x^2 + 2x + 3$

(*c*) $2x^2 + 3x + 1$

(*xx*) The equations of two regression lines are

$$8x - 10y + 66 = 0 \quad \text{and}$$
$$40x - 18y = 214, \quad \text{then}$$

mean of *x* and *y* are

(*a*) 13, 17

(*b*) 3, 7

(*c*) 17, 3