Test Paper : **II**

Test Subject : **MATHEMATICAL SCIENCE**

Test Subject Code : **K-2613**

Test Booklet Serial No. : ______________________

OMR Sheet No. : ______________________

Roll No. | | | | | | | | |

(Figures as per admission card)

Name & Signature of Invigilator/s

Signature: ______________________ Signature : ______________________

Name : ______________________ Name : ______________________

Paper : II

Subject : MATHEMATICAL SCIENCE

Time : 1 Hour 15 Minutes Maximum Marks : 100

Number of Pages in this Booklet : **16** Number of Questions in this Booklet : **50**

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

1. ಈ ಪುಟದ ಮೇಲ್ತುದಿಯಲ್ಲಿ ಒದಗಿಸಿದ ಸ್ಥಳದಲ್ಲಿ ನಿಮ್ಮ ರೋಲ್ ನಂಬರನ್ನು ಬರೆಯಿರಿ.
2. ಈ ಪತ್ರಿಕೆಯು ಬಹು ಆಯ್ಕೆ ವಿಧದ ಐವತ್ತು ಪ್ರಶ್ನೆಗಳನ್ನು ಒಳಗೊಂಡಿದೆ.
3. ಪರೀಕ್ಷೆಯ ಪ್ರಾರಂಭದಲ್ಲಿ, ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆಯನ್ನು ನಿಮಗೆ ನೀಡಲಾಗುವುದು. ಮೊದಲ 5 ನಿಮಿಷಗಳಲ್ಲಿ ನೀವು ಪುಸ್ತಿಕೆಯನ್ನು ತೆರೆಯಲು ಮತ್ತು ಕೆಳಗಿನಂತೆ ಕಡ್ಡಾಯವಾಗಿ ಪರೀಕ್ಷಿಸಲು ಕೋರಲಾಗಿದೆ.
   (i) ಪ್ರಶ್ನೆ ಪುಸ್ತಿಕೆಗೆ ಪ್ರವೇಶಾವಕಾಶ ಪಡೆಯಲು, ಈ ಹೊದಿಕೆ ಪುಟದ ಅಂಚಿನ ಮೇಲಿರುವ ಪೇಪರ್ ಸೀಲನ್ನು ಹರಿಯಿರಿ. ಸ್ಟಿಕ್ಕರ್ ಸೀಲ್ ಇಲ್ಲದ ಪ್ರಶ್ನೆಪುಸ್ತಿಕೆ ಸ್ವೀಕರಿಸಬೇಡಿ. ತೆರೆದ ಪುಸ್ತಿಕೆಯನ್ನು ಸ್ವೀಕರಿಸಬೇಡಿ.
   (ii) **ಪುಸ್ತಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ ಮತ್ತು ಪುಟಗಳ ಸಂಖ್ಯೆಯನ್ನು ಮುಖಪುಟದ ಮೇಲೆ ಮುದ್ರಿಸಿದ ಮಾಹಿತಿಯೊಂದಿಗೆ ತಾಳೆ ನೋಡಿರಿ. ಪುಟಗಳು/ಪ್ರಶ್ನೆಗಳು ಕಾಣೆಯಾದ, ಅಥವಾ ದ್ವಿಪ್ರತಿ ಅಥವಾ ಅನುಕ್ರಮವಾಗಿಲ್ಲದ ಅಥವಾ ಇತರ ಯಾವುದೇ ವ್ಯತ್ಯಾಸದ ದೋಷಪೂರಿತ ಪುಸ್ತಿಕೆಯನ್ನು ಕೂಡಲೆ 5 ನಿಮಿಷದ ಅವಧಿ ಒಳಗೆ, ಸಂವೀಕ್ಷಕರಿಂದ ಸರಿ ಇರುವ ಪುಸ್ತಿಕೆಗೆ ಬದಲಾಯಿಸಿಕೊಳ್ಳಬೇಕು. ಆ ಬಳಿಕ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಲಾಗುವುದಿಲ್ಲ, ಯಾವುದೇ ಹೆಚ್ಚು ಸಮಯವನ್ನೂ ಕೊಡಲಾಗುವುದಿಲ್ಲ.**
4. ಪ್ರತಿಯೊಂದು ಪ್ರಶ್ನೆಗೂ (A), (B), (C) ಮತ್ತು (D) ಎಂದು ಗುರುತಿಸಿದ ನಾಲ್ಕು ಪರ್ಯಾಯ ಉತ್ತರಗಳಿವೆ. ನೀವು ಪ್ರಶ್ನೆಯ ಎದುರು ಸರಿಯಾದ ಉತ್ತರದ ಮೇಲೆ, ಕೆಳಗೆ ಕಾಣಿಸಿದಂತೆ ಅಂಡಾಕೃತಿಯನ್ನು ಕಪ್ಪಾಗಿಸಬೇಕು.
   ಉದಾಹರಣೆ: (A) (B) ● (D)
   (C) ಸರಿಯಾದ ಉತ್ತರವಾಗಿದ್ದಾಗ.
5. ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆ I ರಲ್ಲಿ ಕೊಟ್ಟಿರುವ OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ, **ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆ I ಮತ್ತು ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆ II ರಲ್ಲಿ ಇರುವ ಪ್ರಶ್ನೆಗಳಿಗೆ ನಿಮ್ಮ ಉತ್ತರಗಳನ್ನು ಸೂಚಿಸತಕ್ಕದ್ದು.** OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಅಂಡಾಕೃತಿಯಲ್ಲದೆ ಬೇರೆ ಯಾವುದೇ ಸ್ಥಳದಲ್ಲಿ ಉತ್ತರವನ್ನು ಗುರುತಿಸಿದರೆ, ಅದರ ಮೌಲ್ಯಮಾಪನ ಮಾಡಲಾಗುವುದಿಲ್ಲ.
6. OMR ಉತ್ತರ ಹಾಳೆಯಲ್ಲಿ ಕೊಟ್ಟ ಸೂಚನೆಗಳನ್ನು ಜಾಗರೂಕತೆಯಿಂದ ಓದಿರಿ.
7. ಎಲ್ಲಾ ಕರಡು ಕೆಲಸವನ್ನು ಪುಸ್ತಿಕೆಯ ಕೊನೆಯಲ್ಲಿ ಮಾಡತಕ್ಕದ್ದು.
8. ನಿಮ್ಮ ಗುರುತನ್ನು ಬಹಿರಂಗಪಡಿಸಬಹುದಾದ ನಿಮ್ಮ ಹೆಸರು ಅಥವಾ ಯಾವುದೇ ಚಿಹ್ನೆಯನ್ನು, ಸಂಗತವಾದ ಸ್ಥಳ ಹೊರತು ಪಡಿಸಿ, OMR ಉತ್ತರ ಹಾಳೆಯ ಯಾವುದೇ ಭಾಗದಲ್ಲಿ ಬರೆದರೆ, ನೀವು ಅನರ್ಹತೆಗೆ ಬಾಧ್ಯರಾಗಿರುತ್ತೀರಿ.
9. ಪರೀಕ್ಷೆಯು ಮುಗಿದನಂತರ, ಕಡ್ಡಾಯವಾಗಿ OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ಸಂವೀಕ್ಷಕರಿಗೆ ನೀವು ಹಿಂತಿರುಗಿಸಬೇಕು ಮತ್ತು ಪರೀಕ್ಷಾ ಕೊಠಡಿಯ ಹೊರಗೆ OMR ನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ಕೊಂಡೊಯ್ಯ ಕೂಡದು.
10. ಪರೀಕ್ಷೆಯ ನಂತರ, ಪರೀಕ್ಷಾ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯನ್ನು ಮತ್ತು ನಕಲು OMR ಉತ್ತರ ಹಾಳೆಯನ್ನು ನಿಮ್ಮೊಂದಿಗೆ ತೆಗೆದುಕೊಂಡು ಹೋಗಬಹುದು.
11. **ನೀಲಿ/ಕಪ್ಪು ಬಾಲ್‌ಪಾಯಿಂಟ್ ಪೆನ್ ಮಾತ್ರವೇ ಉಪಯೋಗಿಸಿರಿ.**
12. **ಕ್ಯಾಲ್ಕುಲೇಟರ್ ಅಥವಾ ಲಾಗ್ ಟೇಬಲ್ ಇತ್ಯಾದಿಯ ಉಪಯೋಗವನ್ನು ನಿಷೇಧಿಸಲಾಗಿದೆ.**
13. **ಸರಿ ಅಲ್ಲದ ಉತ್ತರಗಳಿಗೆ ಋಣ ಅಂಕ ಇರುವುದಿಲ್ಲ.**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the questions are to be indicated in the **OMR Sheet kept inside the Paper I Booklet only**. If you mark at any place other than in the ovals in the Answer Sheet, it will not be evaluated.
6. Read the instructions given in OMR carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away question booklet and carbon copy of OMR Answer Sheet soon after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator or log table etc., is prohibited.**
13. **There is no negative marks for incorrect answers.**

MATHEMATICAL SCIENCE
Paper – II

Note : This paper contains **fifty (50)** objective type questions. **Each** question carries **two (2)** marks. **All** questions are **compulsory**.

1. Which one of the following statements is not true in a metric space X ?

(A) Every neighbourhood is an open set in X

(B) If p is a limit point of a set $E \subseteq X$, then every neighbourhood of p contains only finitely many points of E

(C) A set $E \subseteq X$ is open if and only if its complement is closed

(D) Compact subsets in X are closed

OR

For sequence $\{A_n\}$ of sets, which of the following statements is correct ?

(A) It has always a limit

(B) lim inf A_n and lim sup A_n always exist

(C) $\liminf A_n \supseteq \limsup A_n$

(D) lim inf A_n and lim sup A_n are always different

2. Which one of the following is true ?

(A) $\sum\frac{1}{n}$ and $\sum\frac{1}{n\log n}$ both converge

(B) $\sum\frac{1}{n}$ diverges but $\sum\frac{1}{n\log n}$ converges

(C) $\sum\frac{1}{n}$ and $\sum\frac{1}{n\log n}$ both diverge

(D) $\sum\frac{1}{n}$ converges and $\sum\frac{1}{n\log n}$ diverges

OR

If $a_n(x) = \left(1+\frac{x}{n}\right)^n$, n = 1, 2, .., which of the following statements is true ?

(A) $\lim\limits_{n\to\infty} a_n(x) = 0$ for all x

(B) $\lim\limits_{n\to\infty} a_n(x) = 1$ for all x

(C) $\lim\limits_{n\to\infty} a_n(x) = e^x$ for all x

(D) $\lim\limits_{n\to\infty} a_n(x)$ does not exist

3. Which one of the following statements is true ?

(A) A function of bounded variation is continuous

(B) A continuous function is of bounded variation

(C) A function of bounded variation is bounded

(D) A bounded function is of bounded variation

OR

If A is a 6 × 6 matrix of rank 4 then its nullity is

(A) 10

(B) 4

(C) 2

(D) 6

4. If for all x > 0, $e^x > x^t$, then

(A) t > e (B) t = e

(C) $t = e^{-1}$ (D) t < e

OR

If A and B are idempotent matrices, then AB is idempotent if

(A) $(AB)^T = B^T A^T$

(B) $AB \neq BA$

(C) $A^2 = B^2$

(D) $AB = BA$

5. Which one of the following series diverges ?

(A) $\sum_{n=1}^{\infty} \frac{(2n)!\,(3n)!}{n!\,(4n)!}$ (B) $\sum_{n=1}^{\infty} \frac{1}{n^{n+\frac{1}{n}}}$

(C) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+1}$ (D) $\sum_{n=0}^{\infty} \frac{n^3+1}{2^n+1}$

OR

If $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational in } [0, 1] \\ -1 & \text{if } x \text{ is irrational in } [0, 1] \end{cases}$,

then in [0, 1] we have

(A) f(x) is continuous only at the irrationals

(B) f(x) is not Riemann integrable

(C) f(x) is continuous everywhere

(D) f(x) is Riemann integrable

6. Following is an example of a function which is differentiable at x = 0 but f^1 is not continuous at x = 0

(A) $f(x) = |x|, x \in \mathbb{R}$

(B) $f(x) = x^3 + 2x + 1, x \in \mathbb{R}$

(C) $f(x) = \begin{cases} x^2 \ \text{Sin}\left(1/x\right), & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$

(D) $f(x) = \begin{cases} x \ \text{Sin}\left(1/x\right), & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$

OR

The minimum possible value of $|Z|^2 + |Z-3|^2 + |Z-6i|^2$, where Z is a complex number is

(A) 15

(B) 45

(C) 30

(D) 20

7. The set $\left\{x : \left|x + \frac{1}{x}\right| > 4\right\}$ is

(A) $\left(0, 2-\sqrt{3}\right) \cup \left(2+\sqrt{3}, +\infty\right)$

(B) $\left(-\infty, -2-\sqrt{3}\right) \cup \left(-2+\sqrt{3}, +\infty\right)$

(C) $\left(-\infty, 2-\sqrt{3}\right) \cup \left(2+\sqrt{3}, +\infty\right)$

(D) $\left(-\infty, -2-\sqrt{3}\right) \cup \left(-2+\sqrt{3}, 2-\sqrt{3}\right) \cup \left(2+\sqrt{3}, +\infty\right) - \{0\}$

OR

Let $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right), & x > 0 \\ 0, & x \leq 0 \end{cases}$

Then in [0, 1], which one of the following is true ?

(A) f(x) is discontinous at $x = \frac{2}{\pi}$

(B) f(x) is of bounded variation and $f'\left(\frac{1}{2}\right)$ does not exist

(C) f(x) is not of bounded variation and f(x) is continous

(D) f(x) is of bounded variation and $f'(x)$ exists

8. Which one of the following improper integral converges ?

(A) $\int_1^\infty \frac{x \ \arctan x}{\sqrt[3]{1+x^4}} dx$

(B) $\int_0^\infty \left(\frac{1}{x} - \frac{1}{\text{Sin hx}}\right)\frac{dx}{x}$

(C) $\int_2^\infty \frac{x^3}{\sqrt{x^7+1}} dx$

(D) $\int_0^\infty \frac{dx}{1+x}$

OR

Which of the following is true of the system of linear equations ?

$2x_2 - x_3 = 1,\ x_1 - x_2 + 3x_2 = 2,$
$x_1 + x_2 + 2x_3 = 5$

(A) The system is consistent

(B) The system has no solution

(C) The system has a unique solution

(D) The system has many solutions

9. If y > 0, then $\int_0^\infty e^{-xy} \frac{\text{Sinx}}{x} dx =$

(A) $\frac{\pi}{4}$ + arc tan y

(B) $\frac{\pi}{4}$ – arc tan y

(C) $\frac{\pi}{2}$ + arc tan y

(D) $\frac{\pi}{2}$ – arc tan y

OR

The matrix $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ is a

(A) Symmetric matrix

(B) Skew- symmetric matrix

(C) Skew- Hermitian matrix

(D) Hermitian matrix

10. If $f(x, y) = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $x \neq y$ then

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} =$

(A) 2 tan(f(x, y))

(B) Sin(f(x, y))

(C) Sec^2(f(x, y))

(D) Sin(2f(x, y))

OR

The dimension of the vector space consisting of all n × n matrices with real entries and having trace 0 is

(A) n

(B) n^2

(C) $n^2 + 1$

(D) $n^2 - 1$

11. The dimension of T($\mathbb{R}^n$, $\mathbb{R}^m$), the set of all transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$ as a real vector space, is

(A) n + m (B) n^m

(C) m^n (D) mn

OR

A lottery contains 100 tickets and there is only one ticket bearing the price Rs. 10,000/-. The rest are zero. If you buy 2 tickets, the expected gain in Rs. will be

(A) 0.01 (B) 0.02

(C) 100 (D) 200

12. The vectors (p, q) and (r, s) in $\mathbb{R}^2$ are linearly dependent if

(A) ps = qr

(B) pq = rs

(C) pr = qs

(D) pr = – qp

OR

Let X be a positive integer valued random variable. Then $\sum_{n=1}^{\infty} p(X \geq n)$ stands for

(A) Expectation

(B) Variance

(C) Survival function

(D) Hazard function

13. Let f : $\mathbb{R}^4 \rightarrow \mathbb{R}^4$ be a linear mapping. If the minimal polynomial of f is $x^2 + 1$, then the characteristic polynomial of f can be

(A) $X^2 + 1$

(B) $(x^2 + 1)^2$

(C) $x^4 + 1$

(D) $(x + 1)^4$

OR

If X and Y are random variables with equal variance, U = X + Y and V = X – Y, then which of the following is true ?

(A) Corr (U, V) = 1

(B) Corr (U, V) = 0

(C) Corr (U, V) = $^-1$

(D) U and V are independent

14. Let T : U $\rightarrow$ V be a surjective linear mapping and dim U = 6, dim V = 3. Then

(A) dim Ker T > 4

(B) dim Ker T = 4

(C) dim Ker T > 3

(D) dim Ker T = 3

OR

If $X_1, X_2, \ldots, X_n$ are independent random variables with common mean μ and variances $\sigma_1^2, \sigma_2^2, \ldots \sigma_n^2$, and $\overline{X} = \frac{1}{n}\sum_{1=1}^{n} X_i$, then an unbiased estimator of Var ($\overline{X}$) is

(A) $\frac{\sum_i X_i}{n^2}$

(B) $\frac{\sum_i (X_i - \overline{X})^2}{n-1}$

(C) $\frac{\sum_i (X_i - \overline{X})^2}{n}$

(D) $\frac{\sum_i (X_i - \overline{X})^2}{n(n-1)}$

15. The dimension of the subspace of $\mathbb{R}^3$ spanned by (–3, 0, 1), (1, 2, 1) and (3, 0, –1) is

(A) 0

(B) 1

(C) 3

(D) 2

OR

Each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man radomly selects a hat. What is the probability that none of three men selects his own hat ?

(A) 1

(B) $1/2$

(C) $2/3$

(D) $1/3$

16. Let V_1 and V_2 be two subspaces of a vector space V with $V_1 \cap V_2 = \{0\}$ and with bases B_1 and B_2 respectively. Then a basis of $V_1 + V_2$ is

(A) $B_1 \cup B_2$

(B) $B_1 \cap B_2$

(C) $\{x + y : x \in B_1, y \in B_2\}$

(D) $\{x - y : x \in B_1, y \in B_2\}$

OR

For any two events A and B, $P(AB^c \cup BA^c)$ is

(A) P(A) + P(B) – 2P(AB)

(B) $P(A) + P(B) - P(A \cap B)$

(C) P(A) + P(B) – 3 P(AB)

(D) P(A) + P(B)

17. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and a + d = 1 = ad – bc

Then $A^3 =$

(A) 0

(B) $-I_2$

(C) $3I_2$

(D) I_2

OR

Which of the following is the mode of convergence in Central Limit Theorem ?

(A) Almost sure convergence

(B) Convergence in distribution

(C) Convergence in mean

(D) Convergence in probability

18. If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, then A (adj A) is

(A) $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 10 \\ 10 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & -14 \\ 4 & -2 \end{pmatrix}$

OR

Let $X \sim B(1, \theta)$. Then for any integer n > 1, the distribution of X^n is

(A) $B(1, \theta)$

(B) $B(n, \theta)$

(C) Poisson (θ)

(D) Negative Binomial $(1, \theta)$

19. If V is a n-dimensional vector space and T is a linear transformation on V such that the rank of T and the nullity of T are identical then

(A) n is even

(B) n is odd

(C) dim V = 3 rank (T)

(D) n = 5 rank (T)

OR

Let Q denote the set of rationals in $\mathbb{R}$. Then Lebesgue measure of Q is

(A) 0

(B) 1

(C) ∞

(D) indeterminate

20. Let $T : \mathbb{R}^6 \to \mathbb{R}^6$ be a linear mapping such that $T^2 = 0$. Then,

(A) the rank of $T \leq 3$

(B) the rank of $T > 3$

(C) the rank of $T = 5$

(D) the rank of $T = 6$

OR

A continuous random variable X with distribution function F is said to be symmetric about zero if, for all x

(A) $F(x) = 1 - F(-x)$

(B) $F(x) = F(-x)$

(C) $F(x) = 1 - F(x)$

(D) $F(x) = -F(x)$

21. If $f(z) = \frac{1-e^z}{1+e^z}$, then at $z = \infty$, f(z) has

(A) a zero

(B) a removable singularity

(C) an isolated singularity

(D) a non-isolated essential singularity

OR

Let $X_1, X_2, ...X_n$ be a random sample from Poisson (θ) distribution. Then the MLE of $P[X = 0]$ is

(A) $\overline{X}$

(B) $\frac{1}{\overline{X}}$

(C) $e^{\overline{X}}$

(D) $e^{-\overline{X}}$

22. The zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$ are respectively

(A) 1 and i, – i

(B) 0 and i, –1

(C) –1 and i, – i

(D) i, –i and –1

OR

If X has a Cauchy distribution, then the distribution of $Y = -X$ is

(A) Normal

(B) Weibull

(C) Cauchy

(D) Pareto

23. If $T_1(z) = \frac{z+2}{z+3}$ and $T_2(z) = \frac{z}{z+1}$ then $T_2(T_1(z)) =$

(A) $z + 2$

(B) $\frac{2}{2z+5}$

(C) $\frac{z+2}{2z+5}$

(D) $\frac{(z+2)z}{(z+1)(z+3)}$

OR

If $\{E_n\}_{n=1}^{\infty}$ is a disjoint sequence of sets, then $\lim_{n\to\infty} E_n$ is

(A) $\bigcup_{i=1}^{\infty} E_i$

(B) $\bigcup_{i=1}^{\infty} E_i^c$

(C) ϕ

(D) Does not exist

24. If C is the unit circle, then the value of $\int_C \frac{z^2 - z + 1}{z - 1} dz$ is equal to

(A) πi
(B) $2\pi i$
(C) $3\pi i$
(D) 0

OR

If E(Y/x) is the conditional expectation of Y given X = x, then E(XY) is

(A) E(x) . E(y/x)
(B) E(x E(y/x))
(C) E(y) .E(y/x)
(D) E(x) .E(y)

25. The value of the integral $\int_C \frac{dz}{z^2 + 9}$, where C is the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ and the integral is taken in positive sense, is

(A) 0
(B) $2\pi i$
(C) πi
(D) $-2\pi i$

OR

If X and Y are iid random variables with distribution function (d.f.) .G, then the d.f. of min (X, Y) is

(A) G(2 – G)
(B) $G(2 - G^2)$
(C) G
(D) G^2

26. Let f be an analytic function such that $f(z) = 1 + 2z + 3z^2 + ...$, for $|z| < 1$. Define a sequence of real numbers a_0, a_1, a_2 ... by $f(z) = \sum_{n=0}^{\infty} a_n (z+2)^n$. Then the radius of convergence of $\sum a_n z^n$ is

(A) 1
(B) 2
(C) 3
(D) ∞

OR

The mean of the two-sample Mann-Whitney-Wilcoxon statistic under the null hypothesis with samples m and n is given by

(A) mn
(B) $\frac{m}{n}$
(C) $\frac{mn}{2}$
(D) $\frac{mn}{4}$

27. The center of a group G is always a

(A) normal subgroup of G
(B) proper subgroup of G
(C) cyclic subgroup of G
(D) nontrivial subgroup of G

OR

Which of the following is a contrast ?

(A) $-3T_1 - T_2 + T_3 + 3T_4$
(B) $3T_1 + T_2 - 3T_3 + T_4$
(C) $T_1 + 3T_2 - 3T_3 + T_4$
(D) $T_1 + T_2 + T_3 - T_4$

28. Let G be an abelian group. Then

(A) atleast one subgroup of G is not normal

(B) no subgroup of G is normal

(C) every subgroup of G is normal

(D) G has a non-abelian subgroup

OR

If the interactions AB and BC are confounded with incomplete blocks in 2^n – factorial experiment, then automatically confounded effect is

(A) A

(B) C

(C) AC

(D) ABC

29. The integral domain in the following, which is not a principal ideal domain, is

(A) $\mathbb{Z}$ (B) $\mathbb{Z}[X]$

(C) $\mathbb{R}[X]$ (D) $\mathbb{C}[X]$

OR

In sampling with probability proportional to size, the units are selected with probability in proportion to

(A) The size of the sample

(B) The size of the unit

(C) The size of the population

(D) Size of the stratum

30. If $F \subseteq K \subseteq L$ are field extensions such that $[K : F] = 3$ and $[L : F] = 6$ and the degree $[L : K]$ is

(A) 2

(B) 3

(C) 1

(D) 4

OR

Let Y_1 and Y_2 be independent random variables with means θ and 2θ respectively. Then the least square estimate of θ is

(A) $\frac{1}{5} Y_1 + \frac{2}{5} Y_2$ (B) $\frac{1}{2} Y_1 + \frac{1}{2} Y_2$

(C) $\frac{1}{2} Y_1 - \frac{1}{2} Y_2$ (D) $\frac{1}{2} Y_1^2 + \frac{1}{2} Y_2^2$

31. Let $\mathbb{Z}[i]$ denote the ring of Gaussian integers. For which of the following value of n is the quotient ring $\frac{\mathbb{Z}[i]}{n\mathbb{Z}[i]}$ a field ?

(A) n = 2 (B) n = 15

(C) n = 13 (D) n = 11

OR

If $P_x(s)$ denotes the probability generating function (p.g.f.) of the random variable X, then the p.g.f. of the random variable $Y = mX + n$, where m and n are integers $(m \neq 0)$ is given by

(A) $s P_x(s)$

(B) $sP_x(s^m)$

(C) $s^m P_x(s^n)$

(D) $s^n P_x(s^m)$

32. If f(x) is any polynomial of degree 19 with real coefficients, then

(A) f(x) has atleast one real root

(B) f(x) has 19 real roots

(C) the number of imaginary roots of f(x) is 19

(D) no real root exists for f(x)

OR

If X is an F(m, n) random variable, where m, n > 2, then E(X) . $E\left(\frac{1}{X}\right)$ equals

(A) $\frac{mn}{(m-2)(n-2)}$

(B) $\frac{m(n-2)}{n(m-2)}$

(C) $\frac{n(n-2)}{m(m-2)}$

(D) $\frac{m(m-2)}{n(n-2)}$

33. Let G be a group of order mp^k with (m, p) = 1 and $k \geq 1$. If H and K are two subgroups of order p^k, then which of the following is always true ?

(A) H = K

(B) G = H K

(C) kh = hk for all $h \in H$ and $k \in K$

(D) gH = Kg for some $g \in G$.

OR

Which of the following distribution is not of exponential family of distributions ?

(A) Poisson, $P(\lambda)$

(B) Beta, Be (α, β)

(C) Negative Binomial, NB (α, β) with α known

(D) Uniform, $u(\alpha, \beta)$

34. If α = (1 2 3 4 5 6), then α^2 is

(A) (1 3 5)

(B) (2 4 6)

(C) (1 3 5) (2 4 6)

(D) (2 3 5) (1 4 6)

OR

If a SRS of size 100 is drawn without replacement from a population of 1000 units, the probability that a specified unit of the population is included in the sample is

(A) $1/1000$

(B) $1/100$

(C) $1/10$

(D) $1\Big/\binom{1000}{100}$

35. In the ring of integers $\mathbb{Z}$ if I = (15) and J = (21) are two ideals, then the ideals I + J and $I \cap J$ are respectively given by

(A) (36) and (10)

(B) (105) and (3)

(C) (3) and (105)

(D) (3) and (315)

OR

A random sample from $U(-\theta, \theta)$ gives observations 5-6, 0.3, 0, 3.3, – 8.2, – 3. Then the maximum likelihood estimate of θ is

(A) 0

(B) 5.6

(C) 8.2

(D) –3

36. If G is a group of prime order, then G is

(A) non-abelian

(B) abelian but not cyclic

(C) cyclic

(D) isomorphic to (z, +)

OR

Which one of the following statements is correct for a Poisson process ?

(A) It is a stationary process

(B) It is a covariance stationary process

(C) It is an evolutionary process

(D) Its increments are dependent

37. Which one of the following statements always is true in a metric space ?

(A) Separated sets are disjoint

(B) Disjoint sets are separated sets

(C) Arbitrary intersection of open sets is open

(D) Arbitrary union of closed sets is closed

OR

An aperiodic Markov chain with stationary transition probability on the state space {1, 2, 3, 4, 5 } must have

(A) At least one positive recurrent state

(B) At least one null recurrent state

(C) At least one positive recurrent and atleast one null recurrent state

(D) At least one transient state

38. A topological space X is compact if

(A) Every open cover of X has a countable subcover

(B) Every open cover of X has a finite subcover

(C) Every countable open cover of X has a finite subcover

(D) Every cover of X has a finite sub cover

OR

For a Poisson process, the renewal function is

(A) Asymptotically linear

(B) Exactly linear

(C) Non-linear

(D) A constant

39. Which one of the following is always true ?

(A) Continuous image of a compact space is compact

(B) A compact subset of a Housdorff space not closed

(C) The closed interval [a, b] is not compact in $\mathbb{R}$

(D) The set $K = \{0\} \cup \left\{\frac{1}{n} \middle| n \in \mathbb{Z}^{+}\right\}$ is not compact in $\mathbb{R}$

OR

Consider a series system with two components, with the lifetimes of components independently and identically distributed as exponential with mean 0.5 days. Then the expected lifetime of the system in days is

(A) 0.25

(B) 0.50

(C) 1.0

(D) 1.5

40. If (X, T) is any topological space and (Y, T_y) is a subspace of X and $A \subset Y$, then which one of the following is not true ?

(A) $Cl_y(A) = y \cap Cl_x(A)$

(B) $dy(A) = y \cap d_x(A)$

(C) $Int_y(A) = y \cap Int_x(A)$

(D) A subset A of Y is closed in y if and only if $A = C \cap Y$, for some closed set C in X

OR

In an M|M|1 queue with arrival rate λ and service rate μ, the steady state solution exists if

(A) $\lambda = \mu$ (B) $\lambda > \mu$

(C) $\lambda < \mu$ (D) $\lambda + \mu = 0$

41. Which one of the following is always true ?

(A) The family of all singleton point sets forms a basis for any discrete space

(B) The family of all Singleton point sets forms a basis for any topological space

(C) Every basis element is a sub basis element

(D) For any topology T on a set X, T is not a sub basis for T

OR

Which of the following is usually the most difficult cost to determine ?

(A) Waiting cost

(B) Calling cost

(C) Facility cost

(D) Service cost

42. The Eigenvalues of the sturm-Liouville system $y'' + \lambda y = 0$; $0 \le x \le \pi$; y(0) = 0; $y'(\pi) = 0$ are

(A) $\frac{1}{4}n^2$ (B) $\frac{1}{4}(2n-1)^2\pi^2$

(C) $\frac{1}{4}(2n-1)^2$ (D) $\frac{1}{4}n^2\pi^2$

OR

Suppose that customers arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. Then the average number of customers in the system is

(A) 4

(B) 3

(C) 2

(D) 1

43. A non-trivial solution of $x^2y'' + xy' + 4y = 0$, $x > 0$ is

(A) bounded and non-periodic

(B) unbounded and non-periodic

(C) bounded and periodic

(D) unbounded and periodic

OR

In a BIBD, 4 treatments are arranged in 4 blocks of 3 plots each. Each treatment occurs once and only once in 3 blocks and any two treatments occur together in λ blocks. Then the value of λ is

(A) 2

(B) 3

(C) 4

(D) 5

44. Which of the following concerning the solution of the Dirichlet problem for a smooth bounded region is true ?

(A) Solution is unique upto a multiplicative constant

(B) Solution is unique upto a additive constant

(C) Solution is unique

(D) No conclusion can be drawn about uniqueness of solution

OR

The value of the objective function at an optimal solution of the LPP;

$M_m X_1 + X_2$ subject to $x_1 - x_2 = 5$, $x \geq 0$, $x_2 \geq 0$, will be

(A) –5 (B) 0

(C) 5 (D) 10

45. Pick up the region in the following in which the partial differential equation

$$y\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + x\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$ is

hyperbolic

(A) $xy \neq 1$ (B) $xy \neq 0$

(C) $xy > 1$ (D) $xy > 0$

OR

If X and Y are independent Gamma random variables with parameters (α, λ) and (β, λ) respectively, then the distribution of $\frac{X}{\alpha + Y}$ is

(A) Gamma (B) Normal

(C) Beta (D) Cauchy

46. Newton-Rapshon method is applicable only when

(A) $f(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$

(B) $f'(x) \neq 0$ in the neighbourhood of actual root $x = \alpha$

(C) $f'(x) = 0$ in the neighbourhood of actual root $x = \alpha$

(D) $f(x) = 0$ and $f'(x) = 0$ in the neighbourhood of actual root $x = \alpha$

OR

If $X \sim N_p(\mu, \Sigma)$ then which of the following is not correct ?

(A) $\overline{X}$ is an unbiased and consistent estimator of μ

(B) MLE of Σ is an unbiased estimator of Σ

(C) $(\overline{X} - \mu)' \Sigma^{-1} (\overline{X} - \mu) \sim \chi^2 p$

(D) Every linear combination of components of X is univariate normal

47. Let f(x) be a differentiable function such that $M_2 = \max_{a \leq x \leq b} |f''(x)|$. Then the maximum error in linear interpolation in [a, b] is given by

(A) $\frac{h M_2}{8}$ (B) $\frac{h^2 M_2}{8}$

(C) $\frac{h^2 M_2}{2}$ (D) $\frac{h^2 M_2}{4}$

OR

Let $X = (X_1, X_2, X_3)'$ with

$Var[X] = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$, the variance of $X_1 - 2X_2 + X_3$ is

(A) 6 (B) 18

(C) 20 (D) 24

48. Solutions of Euler-Poisson equation are called as

(A) trial solution

(B) externals

(C) stationary solution

(D) functional

OR

In a branching process, let the offspring distribution be $p_K = \frac{1}{3}\left(\frac{2}{3}\right)^K$, K = 0, 1, 2,...

Then the probability of ultimate extinction is

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 1

49. The value of λ for which $y(x) = 1 + \lambda x$ is a solution of the integral equation $x = \int_0^x e^{x-t} y(t)\, dt$ is

(A) $\lambda = 1$ (B) $\lambda = -1$

(C) $\lambda = 0$ (D) $\lambda = 2$

OR

If $X_1, \ldots, X_n$ is a random sample from a Poisson distribution with mean λ, the Cramer-Rao lower bound to the variance of any unbiased estimator of λ^2 is given by

(A) $\frac{\lambda}{n}$

(B) $\frac{2\lambda^2}{n}$

(C) $\frac{\sqrt{\lambda}}{n}$

(D) $e^{-\lambda/n}$

50. For a conservative system, generalised force

(A) has necessarily the dimensions of force

(B) is a dimensionless quantity

(C) cannot have dimensions of force

(D) may have the dimensions of torque

OR

For the SPRT with stopping bounds A and B, A > B and strength (α, β), $0 < \alpha$, $\beta < 1$, which of the following inequality holds ?

(A) $A \le \frac{\alpha}{1-\beta}, B \ge \frac{\beta}{1-\alpha}$

(B) $A \le \frac{1-\beta}{\alpha}, B \le \frac{\beta}{1-\alpha}$

(C) $A \ge \frac{1-\beta}{\alpha}, B \le \frac{\beta}{1-\alpha}$

(D) $A \le \frac{1-\beta}{\alpha}, B \ge \frac{\beta}{1-\alpha}$

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Space for Rough Work

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Space for Rough Work