NOTE : MA040 - This subject code has been changed to MA1252.

T 8230

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2006.

Fourth Semester

Computer Science and Engineering

MA 1252 — PROBABILITY AND QUEUEING THEORY (Present)

MA 040 – PROBABILITY AND QUEUEING THEORY (Formerly)

(Common to B.E. Part-Time Third Semester R 2005)

(Regulation 2004)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The odds in favour of A solving a mathematical problem are 3 to 4 and the odds against B solving the problems are 5 to 7. Find the probability that the problem will be solved by at least one of them.

2. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find $P(G)$, where G is the event that a number greater than 3 occurs on a single roll of the die.

3. Define a continuous random variable. Give an example.

4. Find the value of (a) C and (b) mean of the following distribution given

$$f(x) = \begin{cases} C(x - x^2) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

5. If the probability is 0.40 that a child exposed to a certain contagious will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it?

6. If X is uniformly distributed over (0, 10) calculate the probability that (a) $X > 6$ (b) $3 < X < 8$.

7. Find the moment generating function for the distribution where

$$f(x)=\begin{cases}2/3 & \text{at } x=1\\ 1/3 & \text{at } x=2\\ 0 & \text{otherwise}\end{cases}$$

8. State central limit theorem.

9. Define random process and its classification.

10. What are the basic characteristics of Queueing process?

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the probability density of X is given by

$$f(x)=\begin{cases}2(1-x) & \text{for } 0<x<1\\ 0 & \text{otherwise}\end{cases}$$

(1) Show that $E[X^r]=\dfrac{2}{(r+1)(r+2)}$

(2) Use this result to evaluate $E[(2X+1)^2]$. (8)

(ii) Given a binary communication channel, where A is the input and B is the output, let $P(A)=0.4, P(B/A)=0.9$ and $P[\overline{B}/\overline{A}]=0.6$. Find

(1) $P(A/B)$

(2) $P(A/\overline{B})$. (8)

Or

(b) (i) A random variable X has density function given by $f(x)=\begin{cases}\dfrac{1}{k} & \text{for } 0<x<k\\ 0 & \text{otherwise}\end{cases}$

Find (1) m.g.f.

(2) r th moment

(3) mean

(4) variance. (8)

(ii) Given that a student studied, the probability of passing a certain quiz is 0.99. Given that a student did not study. The probability of passing the quiz is 0.05. Assume that the probability of studying is 0.7. A student flunks the quiz. What is the probability that he or she did not study? (8)

12. (a) (i) Let the random variable X follows binomial distribution with parameter n and p. Find

(1) probability mass function of X

(2) moment generating function

(3) mean and variance of X. (8)

(ii) The number of personal computer (PC) sold daily at a compuWorld is uniformly distributed with a minimum of 2000 PC and a maximum of 5000 PC. Find

(1) the probability that daily sales will fall between 2,500 and 3,000 PC.

(2) What is the probability that the CompuWorld will sell at least 4,000 PC's?

(3) What is the probability that the CompuWorld will exactly sell 2,500 PC's? (8)

Or

(b) (i) Define the probability density function of normal distribution and standard normal distribution. Write down the important properties of its distribution. (8)

(ii) An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find

(1) the probability that a bulb burns more than 834 hours

(2) the probability that bulb burns between 778 and 834 hours. (8)

13. (a) (i) In producing gallium - arsenide microchips, it is known that the ratio between gallium and arsenide is independent of producing a high percentage of workable wafer, which are main components of microchips. Let X denote the ratio of gallium to arsenide and Y denote the percentage of workable microwafers retrieved during a 1-hour period. X and Y are independent random variables with the joint density being known as

$$f(x,y)=\begin{cases}\dfrac{x(1+3y^2)}{4}, & 0<x<2, 0<y<1\\ 0 & , \text{otherwise}\end{cases}$$

Show that $E(XY)=E(X).E(Y)$. (8)

(ii) If the joint density of X_1 and X_2 is given by

$$f(x_1,x_2)=\begin{cases}6\cdot e^{-3x_1-2x_2} & \text{for } x_1>0, x_2>0\\ 0, & \text{otherwise}\end{cases}$$

find the probability density of $Y=X_1+x_2$. (8)

Or

(b) (i) Two random variables X and Y have joint density function

$$f_{XY}(x,y)=x^2+\frac{xy}{3}; 0\le x\le 1, 0\le y\le 2$$

Find the conditional density functions. Check whether the conditional density functions are valid. (8)

(ii) If the joint probability density of X_1 and X_2 is given by

$$f(x_1,x_2)=\begin{cases}e^{-(x_1+x_2)}; & \text{for } x_1>0, x_2>0\\ 0 & ; \text{elsewhere}\end{cases}$$

find the probability of $Y=\dfrac{X_1}{X_1+X_2}$. (8)

14. (a) (i) Find the coefficient of correlation and obtain the lines of regression from the data given below : (8)

X	50	55	50	60	65	65	65	60	60	50
Y	11	14	13	16	16	15	15	14	13	13

(ii) Let z be a random variable with probability density $f(z)=\dfrac{1}{2}$ in the range $-1\le z\le 1$. Let the random variable $X=z$ and the random variable $Y=z^2$. Obviously X and Y are not independent since $X^2=Y$. Show, none the less, that X and Y are uncorrelated. (8)

Or

(b) (i) Two random variables X and Y are defined as $Y = 4X + 9$. Find the correlation coefficient between X and Y. (8)

(ii) A stochastic process is described by $X(t) = A\sin t + B\cos t$ where A and B are independent random variables with zero means and equal standard deviations show that the process is stationary of the second order. (8)

15. (a) (i) A raining process is considered as two state Markov chain. If it rains, it is considered to be state 0 and if it does not rain, the chain is in state 1. The transition probability of the Markov chain is defined as

$$P\begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

Find the probability that it will rain for 3 days from today assuming that it will rain after 3 days. Assume the initial probabilities of state 0 and state 1 as 0.4 and 0.6 respectively. (8)

(ii) A person owning a scooter has the option to switch over to scooter, bike or a car next time with the probability of (0.3, 0.5, 0.2). If the transition probability matrix is

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

What are the probabilities vehicles related to his fourth purchase? (8)

Or

(b) (i) Define Kendall's notation. What are the assumptions are made for simplest queuing model. (8)

(ii) Arrival rate of telephone calls at telephone booth are according to Poisson distribution with an average time of 12 minutes between two consecutive calls arrival. The length of telephone call is assumed to be exponentially distributed with mean 4 minutes.

(1) Determine the probability that person arriving at the booth will have to wait.

(2) Find the average queue length that is formed from time to time.

(3) The telephone company will install second booth when convinced that an arrival would expect to have to wait at least 5 minutes for the phone. Find the increase in flows of arrivals which will justify a second booth.

(4) What is the probability that an arrival will have to wait for more than 15 min before the phone is free! (8)
