UNIVERSITY OF CALCUTTA

SYLLABI

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THREE-YEAR HONOURS & GENERAL
DEGREE COURSES OF STUDIES

MATHEMATICS
2010

UNIVERSITY OF CALCUTTA

Syllabi of three-year B.Sc.(Hons. & Genl.) Courses in Mathematics, 2010

<u>MATHEMATICS HONOURS</u>

<u>PAPER-WISE DISTRIBUTION:</u>

Paper I   : Module I   and   Module II

Paper II   : Module III   and   Module IV

Paper III   : Module V   and   Module VI

Paper IV   : Module VII   and   Module VIII

Paper V   : Module IX   and   Module X

Paper VI   : Module XI   and   Module XII

Paper VII   : Module XIII   and   Module XIV

Paper VIII   : Module XV   and   Module XVI

<u>MATHEMATICS HONOURS</u>

DISTRIBUTION OF MARKS

MODULE I   **: Group A :** Classical Algebra (35 marks)
              Group B : Modern Algebra I (15 marks)

MODULE II   **: Group A :** Analytical Geometry of Two Dimensions (20 marks)
               Group B : Analytical Geometry of Three Dimensions I (15 marks)

               Group C : Vector Algebra (15 marks)

MODULE III   **: Group A :** Analysis I (40 marks)
              Group B : Evaluation of Integrals (10 marks)

MODULE IV   **: Group A :** Linear Algebra (35 marks)
              Group B : Vector Calculus I (15 marks)

MODULE V   **: Group A :** Modern Algebra II (15 marks)
                Group B : Linear Programming and Game Theory (35 marks)

MODULE VI   **: Group A :** Analysis II (15 marks)
                Group B : Differential Equations I (35 marks)

MODULE VII **: Group A :** Real-Valued Functions of Several Real Variables (30 marks)

                Group B : Application of Calculus (20 marks)

MODULE VIII **: Group A :** Analytical Geometry of Three Dimensions II (15 marks)

                Group B : Analytical Statics I (10 marks)
                Group C : Analytical Dynamics of A Particle I (25 marks)

MODULE IX   **: Group A :**   Analysis III (50 marks)

MODULE X   **: Group A :** Linear Algebra II and Modern Algebra II (20 marks)
                Group B : Tensor Calculus (15 marks)
                Group C : Differential Equation II (15 marks)
                          Or
                Group C : Graph Theory (15 marks)

MODULE XI   **: Group A :** Vector calculus II (10 marks)
                Group B : Analytical Statics II (20 marks)
                Group C : Analytical Dynamics of A Particle II (20 marks)

MODULE XII   **: Group A :** Hydrostatics (25 marks)
                Group B : Rigid Dynamics (25 marks)

MODULE XIII   **: Group A :** Analysis IV (20 marks)
                Group B : Metric Space (15 marks)
                Group C : Complex Analysis (15 marks)

MODULE XIV   **: Group A :** Probability (30 marks)
                Group B : Statistics (20 marks)

MODULE XV   **: Group A :** Numerical Analysis (25 marks)
                Group B : Computer Programming (25 marks)

MODULE XVI   **:** Practical (50 marks)   Problem : 30
                                            Sessional Work : 10

Module I

Group A (35 marks)

<u>Classical Algebra</u>

1. Statements of well ordering principle, first principle of mathematical induction, second principle of mathematical induction. Proofs of some simple mathematical results by induction. Divisibility of integers. The division algorithm (a = gb + r, b ≠ 0, 0 ≤ r < b). The greatest common divisor (g.c.d.) of two integers a and b. [This number is denoted by the symbol (a,b)]. Existence and uniqueness of (a,b). Relatively prime integers. The equation ax + by = c has integral solution iff (a,b) divides c. (a , b, c are integers).
Prime integers. Euclid's first theorem: If some prime p divides ab, then p divides either a or b.
Euclid's second theorem: There are infinitely many prime integers.
Unique factorization theorem. Congruences, Linear Congruences.
Statement of Chinese Remainder Theorem and simple problems. Theorem of Fermat. Multiplicative function ø (n).
[15]

2. Complex Numbers : De-Moivre's Theorem and its applications, Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z (a≠0). Inverse circular and Hyperbolic functions.
[8]

3. Polynomials with real co-efficients: Fundamental theorem of Classical Algebra (statement only). The n-th degree polynomial equation has exactly n roots. Nature of roots of an equation (surd or complex roots occur in pairs). Statements of Descartes' rule of signs and of Sturm's Theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots. Transformation of equations.   [8]

4. Polynomial equations with real co-efficients : Reciprocal equations.
Cardan's method of solving a cubic equation. Ferrari's method of solving a biquadratic equation. Binomial equation. Special roots.
[7]

5. Inequalities AM $\geq$ GM $\geq$ HM and their generalizations : the theorem of weighted means and m-th. Power theorem. Cauchy's inequality (statement only) and its direct applications. [8]

Group B (15 marks)

<u>Modern Algebra I</u>

1. **Set, mapping and algebraic structure:** Basic properties of sets including De Morgan's Laws. Cartesian product of sets, Binary relation, Equivalence relation, Relation between equivalence relation and partition. Congruence of integers, Congruence Classes.

   Mapping: Injection, surjection, bijection, identity and inverse mappings. Composition of
   mappings and its associativity.
   Binary operations: Commutative and Associative binary operations.
   Algebraic structure: Concept of algebraic structure, definition (only) of group, ring and
   field – Real numbers with usual operations as an example.
   [10]

2. **Group Theory:** Semigroup, Group, Abelian Group. Examples of groups from number system, root of unity, matrices, symmetries of squares, triangles etc. Groups of congruence classes. Klein's 4 group.
   Properties deducible from definition of group including solvability of equations like ax = b, ya = b. Any finite semigroup having both cancelation laws is a group.
   Integral power of elements and laws of indices in a group. Order of an element of a group, Order of a group.
   Subgroups: Necessary and sufficient condition for a subset of group to be a subgroup. Intersection and union of subgroups. Necessary and sufficient condition for union of two subgroups to a subgroup.
   [10]

Module II

Group A (20 marks)

Analytical Geometry of Two Dimensions

1. **(a)** Transformation of Rectangular axes : Translation, Rotation and their combinations.
    Theory of Invariants.
    [2]
    (b) General Equation of second degree in two variables : Reduction into canonical form.
    Classification of conics, Lengths and position of the axes.
    [2]

2. Pair of straight lines : Condition that the general equation of second degree in two variables may represent two straight lines. Point of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Angle bisector. Equation of two lines joining the origin to the points in which a line meets a conic.   [8]

3. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equations of tangent, normal, chord of contact.
    [5]

4. Circle, Parabola, Ellipse and Hyperbola : Equations of pair of tangents from an external point, chord of contact, poles and polars, conjugate points and conjugate lines.   [4]

Note: Euclid's Axiom and its Consequences.

Group B (15 marks)

Analytical Geometry of Three Dimensions I

1. Rectangular Cartesian co-ordinates in space. Halves and Octants. Concept of a geometric vector (directed line segment). Projection of a vector on a co-ordinate axis. Inclination of a vector with an axis. Co-ordinates of a vector. Direction cosines of a vector. Distance between two points. Division of a directed line segment in a given ratio.   [4]

2. Equation of Plane: General form, Intercept and Normal form. The sides of a plane. Signed distance of a point from a plane. Equation of a plane passing through the intersection of two planes.
Angle between two intersecting planes. Bisectors of angles between two intersecting planes. Parallelism and perpendicularity of two planes. [8]

3. Straight lines in space: Equation (Symmetric & Parametric form). Direction ratio and Direction cosines. Canonical equation of the line of intersection of two intersecting planes. Angle between two lines. Distance of a point from a line. Condition of coplanarity of two lines. Equations of skew-lines. Shortest distance between two skew lines. [10]

Group C (15 marks)

Vector Algebra

Vector Algebra : Vector (directed line segment) Equality of two free vectors. Addition of Vectors. Multiplication by a Scalar.

Position vector, Point of division, Conditions of collinearity of three points and co-planarity of four points.

Rectangular components of a vector in two and three dimensions.

Product of two or more vectors. Scalar and vector products, scalar triple products and Vector triple products. Product of four vectors.

Direct application of Vector Algebra in (i) Geometrical and Trigonometrical problems (ii) Work done by a force, Moment of a force about a point.

Vector equations of straight lines and planes. Volume of a tetrahedron. Shortest distance between two skew lines. [15]

Module III

Group A (40 marks)

Analysis I

1. Real number system :

(a) Intuitive idea of numbers. Mathematical operations revisited with their

properties
(closure, commutative, associative, identity, inverse, distributive). Sets and functions - definition and properties (union, intersection, complementation, injection, surjection, bijection). [3]

(b) Field Axioms. Concept of ordered field. Bounded set, L.U.B. (supremum) and G.L.B.
(infimum) of a set. Properties of L.U.B. and G.L.B. of sum of two sets and scalar multiple of a set. Least upper bound axiom or completeness axiom. Characterization of IR as a complete ordered field. Definition of an Archimedean ordered field. Archimedean property of IR. Q is Archimedean ordered field but not ordered complete. Linear continuum. [6]

2. Sets in IR :

(a)   Intervals.
[1]

(b) Neighbourhood of a point. Interior point. Open set. Union, intersection of open sets.
Every open set can be expressed as disjoint union of open intervals (statement only).

[2]
(c) Limit point and isolated point of a set. Criteria for L.U.B. and G.L.B. of a bounded set
to be limit point of the set. Bolzano-Weierstrass theorem on limit point. Definition of
derived set.Derived set of a bounded set A is contained in the closed interval [inf A, sup
A]. Closed set. Complement of open set and closed set. Union and intersection of closed
sets as a consequence. No nonempty proper subset of IR is both open and closed.   [3]

(d) Dense set in IR as a set having non-empty intersection with every open interval. Q
and   IR   -   Q   are   dense   in   IR.
[2]

3. Sequences of real numbers :

(a) Definition of a sequence as function from IN to IR. Bounded sequence. Convergence
(formalization of the concept of limit as an operation in IR) and non-convergence. Examples. Every convergent sequence is bounded and limit is unique.   Algebra   of   limits.
[4]

(b) Relation between the limit point of a set and the limit of a convergent sequence of
distinct elements. Monotone sequences and their convergence. Sandwich rule. Nested
interval theorem.

Limit of some important sequences : $\{n^{1/n}\}$. $\{x^n\}_n$. $\{x^{1/n}\}_n$. $\{x_n\}_n$ with

$$\frac{x_{n+1}}{x_n} \to 1$$

and $|l| < 1$. $\{(1+1/n)^n\}$, $\left\{1+\frac{1}{1!}+\frac{1}{2!}+.......+\frac{1}{n!}\right\}_n$, $\{a^{x_n}\}_n$, $(a > 0)$.

Cauchy's   first   and   second   limit   theorems.
[7]

(c) Subsequence. Subsequential limits. Lim sup upper (limit) and lim inf (lower limit) of
a sequence using inequalities. Alternative definitions of lim sup and lim inf of a
sequence $\{x^n\}_n$ using L.U.B. and G.L.B. of the set containing all the subsequential
limits or by the properties of the set $\{x_n, x_{n+1}, ...\}$ (Equivalence between these
definitions are assumed). A bounded sequence $\{x_n\}$ is convergent if lim sup $x_n =$
lim inf x_n (statement only). Every sequence has a monotone subsequence. Bolzano-
Weierstrass theorem. Cauchy's general principle of convergence
[5]

4. **Countability of sets :** Countability (finite and infinite) and uncountability of a set. Subset of a countable set is countable. Every infinite set has a countably infinite subset. Cartesian product of two countable sets is countable. Q is countable. Non-trivial intervals are uncountable. IR is uncountable. [4]

5. Continuity of real-valued functions of a real variable :

(a) Limit of a function at a point (the point must be a limit point of the domain set of the function). Sequential criteria for the existence of finite and infinite limit of a function at a point. Algebra of limits. Sandwich rule. Important limits like
$\frac{\sin x}{x}$, $\frac{\log(1+x)}{x}$, $\frac{a^x - 1}{x}$ (a>0) as $x \to 0$.
[3]

(b) Continuity of a function at a point. Continuity of a function on an interval and at an isolated point. Familiarity with the figures of some well known functions : $y = x^a (a = 2, 3, \frac{1}{2}, -1), |x|, \sin x, \cos x, \tan x, \log x, e^x$. Algebra of continuous functions as a consequence of algebra of limits. Continuity of composite functions. Examples of continuous functions. Continuity of a function at a point does not necessarily imply the continuity in some neighbourhood of that point.   [4]

(c) Bounded functions. Neighbourhood properties of continuous functions regarding boundedness and maintenance of same sign. Continuous function on [a,b] is bounded and attains its bounds. Intermediate value theorem. [4]

(d) Discontinuity of function, type of discontinuity. Step function. Piecewise continuity. Monotone function. Monotone function can have only jump discontinuity. Set of points of discontinuity of a monotone function is at most countable. Monotone bijective function from an interval to an interval is continuous and its inverse is also continuous.[3]

(e) Definition of uniform continuity and examples. Lipschitz condition and uniform continuity. Functions continuous on a closed and bounded interval is uniformly continuous. A necessary and sufficient condition under which a continuous function on a bounded open interval *I* will be uniformly continuous on *I*. A sufficient condition under which a continuous function on an unbounded open interval *I* will be uniformly continuous on *I* (statement only).

Group B (10 marks)

Evaluation of Integrals

Evaluation of Integrals : Indefinite and suitable corresponding definite integrals for the functions $\dfrac{1}{(a + b \cos x)^n}$, $\dfrac{l \cos x + m \sin x}{p \cos x + q \sin x}$, $\dfrac{1}{(x^2 + a^2)^n}$, $\cos^m x$, $\sin^n x$ etc. where *l, m, p, q, n* are integers. Simple problems on definite integral as the limit of a sum.   [5]

Module IV

Group A (35 marks)

Linear Algebra

1. Matrices of real and complex numbers : Algebra of matrices. Symmetric and skew-symmetric matrices. Hermitian and skew-Hermitian matrices. Orthogonal matrices.

[4]

2. Determinants: Definition, Basic properties of determinants, Minors and cofactors. Laplace's method. Vandermonde's determinant. Symmetric and skew-symmetric determinants. (No proof of theorems)(problems of determinants of order >4 will not be asked).
Adjoint of a square matrix. For a square matrix A, A.adjA – adjA.A = (detA)I_n. Invertible matrix, Non-singular matrix. A square matrix is invertible if and only if it is non-singular. Inverse of an orthogonal. Matrix.
[13]

3. Elementary operations on matrices. Echelon matrix. Rank of a matrix. Determination of rank of a matrix (relevant results are to be state only). Normal forms. Elementary matrices. Statements and application of results on elementary matrices. [4]

4. Vector / Linear space : Definitions and examples, Subspace, Union and intersection of subspaces. Linear sum of two subspaces. Linear combination, independence and dependence. Linear span. Generators of vector space. Finite dimensional vector space. Replacement Theorem, Extension theorem, Statement of the result that any two bases of a finite dimensional vector space have same number of elements. Dimension of a vector space. Extraction of basis, formation of basis with special emphasis on IR^n (n $\leq$ 4).
Row space and column space of matrix. Row rank and column rank of matrix. Equality of row rank, column rank and rank of a matrix.
Linear homogeneous system of equations : Solution space. For a homogeneous system AX = 0 in n unknowns, Rank X(A) + Rank A = n; AX = 0 contains non-trivial solution if Rank A < n. Necessary and sufficient condition for consistency of a linear non-homogeneous system of equations. Solution of system of equations (Matrix method).
Eigenvalues and eigenvectors of matrices, Caley Hamilton Theorem. Simple properties of eigenvalues and eigenvectors.
[25]

5. Congruence of matrices : Statement of applications of relevant results, Normal form of a matrix under congruence, Real Quadratic Form involving three variables. Reduction to Normal Form (Statements of relevant theorems and applications). [5]

6. Inner Product Space : Definition and examples, Norm, Euclidean Vector Space, Triangle inequality and Cauchy-Schwarz Inequality in Euclidean Vector Space, Orthogonality of vectors, Orthonormal basis, Gram-Schmidt Process of orthonomalization.
[5]

Vector Calculus I

1. Vector differentiation with respect to a scalar variable, Vector functions of one scalar variable. Derivative of a vector. Second derivative of a vector. Derivatives of sums and products, Velocity and Acceleration as derivative. [5]

2. Concepts of scalar and vector fields. Direction derivative. Gradient, Divergence and curl, Laplacian and their physical significance. [5]

Module V

Group A (15 marks)

Modern Algebra II

1. Cosets and Lagrange's theorem. Cyclic groups. Generator, Subgroups of cyclic groups. Necessary and sufficient condition for a finite group to be cyclic.

2. Rings and Fields: Properties of Rings directly following from the definition, Unitary and commutative rings. Divisors of zero, Integral domain, Every field is an integral domain, every finite integrals domain is a field. Definitions of Sub-ring and sub-field. Statement of Necessary of sufficient condition for a subset of a ring (field) to be sub-ring (resp. subfield). Characteristic of ring and integral domain.
Permutation : Cycle, transposition, Statement of the result that every permutation can be expressed as a product of disjoint cycles. Even and odd permutations, Permutation Group. Symmetric group. Alternating Group. Order of an alternating group.   [20]

Group B (35 marks)

Linear Programming and Game Theory

1. Definition of L.P.P. Formation of L.P.P. from daily life involving

inequations. Graphical solution of L.P.P. Basic solutions and Basic Feasible Solution (BFS) with reference to L.P.P. Matrix formulation of L.P.P. Degenerate and Non-degenerate B.F.S. [8]

2. Hyperplane, Convex set, Cone, extreme points, convex hull and convex polyhedron. Supporting and Separating hyperplane. The collection of all feasible solutions of an L.P.P. constitutes a convex set. The extreme points of the convex set of feasible solutions correspond to its B.F.S. and conversely. The objective function has its optimal value at an extreme point of the convex polyhedron generated by the set of feasible solutions. (the convex polyhedron may also be unbounded). In the absence of degeneracy, if the L.P.P. admits of an optimal solution then at least one B.F.S. must be optimal. Reduction of a F.S. to a B.F.S. [6]

3. Slack and surplus variables. Standard from of L.P.P. theory of simplex method. Feasibility and optimality conditions. [6]

4. The algorithm. Two phase method. Degeneracy in L.P.P. and its resolution. [6]

5. Duality theory : The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Relation between their optimal values. Complementary slackness, Duality and simplex method and their applications. [6]

6. Transportation and Assignment problems. Mathematical justification for optimality criterion. Hungarian method. Traveling Salesman problem. [8]

7. Concept of Game problem. Rectangular games. Pure strategy and Mixed strategy. Saddle point and its existence. Optimal strategy and value of the game. Necessary and sufficient condition for a given strategy to be optimal in a game. Concept of Dominance. Fundamental Theorem of Rectangular games. Algebraic method. Graphical method and Dominance method of solving Rectangular games. Inter-relation between the theory of Games and L.P.P. [10]

Module VI

Group A (15 marks)

<u>Analysis II</u>

1. **Infinite Series of real numbers :**

   a) Convergence, Cauchy's criterion of convergence. [1]
   b) Series of non-negative real numbers : Tests of convergence – Cauchy's condensation test. Comparison test (ordinary form and upper limit and lower limit criteria), Kummer's test. Statements and applications of : Abel – Pringsheim's Test, Ratio Test , Root test, Raabe's test, Bertrand's test, Logarithmic test and Gauss's test. [3]
   c) Series of arbitrary terms : Absolute and conditional convergence [1]
   d) Alternating series : Leibnitz test (proof needed).
   e) Non-absolute convergence : Abel's and Dirichlet's test (statements and applicatins). Riemann's rearrangement theorem (statement only) and rearrangement of absolutely convergent series (statement only). [3]

2. **Derivatives of real –valued functions of a real variable :**

   a) Definition of derivability. Meaning of sign of derivative. Chain rule. [2]
   b) Successive derivative : Leibnitz theorem. [1]
   c) Theorems on derivatives : Darboux theorem, Rolle's theorem, Mean value theorems of Lagrange and Cauchy – as an application of Rolle's theorem. Taylor's theorem on closed and bounded interval with Lagrange's and Cauchy's form of remainder deduced from Lagrange's and Cauchy's mean value theorem respectively. Maclaurin's theorem as a consequence of Taylor's theorem. Statement of Maclaurin's

   Theorem on infinite series expansion. Expansion of e^x, $\log(1+x), (1+x)^m, \sin x, \cos x$ with their range of validity. [10]
   d) Statement of L' Hospital's rule and its consequences. Point of local extremum (maximum, minimum) of a function in an interval. Sufficient condition for the existence of a local maximum/minimum of a function at a point (statement only). Determination of local extremum using first order derivative. Application of the principle of maximum/minimum in geometrical problems. [4]

Group B (35 marks)

Differential Equation I

1. Significance of ordinary differential equation. Geometrical and physical consideration. Formation of differential equation by elimination of arbitrary

constant. Meaning of the solution of ordinary differential equation.
Concept of linear and non-linear differential equations.
[2]

2. Equations of first order and first degree : Statement of existence theorem. Separable, Homogeneous and Exact equation. Condition of exactness, Integrating factor. Rules of finding integrating factor, (statement of relevant results only). [5]

3. First order linear equations : Integrating factor (Statement of relevant results only). Equations reducible to first order linear equations. [2]

4. Equations of first order but not of first degree. Clairaut's equation. Singular solution.[3]

5. Applications : Geometric applications, Orthogonal trajectories. [2]

6. Higher order linear equations with constant co-efficients : Complementary function, Particular Integral. Method of undetermined co-efficients, Symbolic operator D. Method of variation of parameters. Exact Equation. Euler's homogeneous equation and Reduction to an equation of constant co-efficients.

   [8]

7. Second order linear equations with variable co-efficients :
$\dfrac{d^2y}{dx^2} + P(x)\dfrac{dy}{dx} + Q(x)y = F(x)$. Reduction of order when one solution of
the homogeneous part is known. Complete solution. Method of variation of parameters. Reduction to Normal form. Change of independent variable. Operational Factors. [10]

8. Simple eigenvalue problems. [2]

9. Simultaneous linear differential equations. Total differential equation : Condition of integrability. [3]

10. Partial differential equation (PDE) : Introduction. Formation of P.D.E., Solution of PDE by Lagrange's method of solution and by Charpit's method. [5]

Module VII

Group A (30 marks)

Real-Valued Functions of Several Real Variables

1. Point sets in two and three dimensions: Concept only of neighbourhood of a point, interior point, limit point, open set, closed set. [2]

2. Concept of functions on IR^n. [1]

3. Function of two and three variables : Limit and continuity. Partial derivatives. Sufficient condition for continuity. Relevant results regarding repeated limits and double limits. [3]

4. Functions $IR^2 \rightarrow IR$: Differentiability and its sufficient condition, differential as a map, chain rule, Euler's theorem and its converse. Commutativity of the second order mixed partial derivatives : Theorems of Young and Schwarz. [10]

5. Jacobian of two and three variables, simple properties including function dependence. Concept of Implicit function : Statement and simple application of implicit function theorem for two variables Differentiation of Implicit function. [8]

6. Taylor's theorem for functions two variables. Lagrange's method of undetermined multipliers for function of two variables (problems only). [3]

Group B (20 marks)

Application of Calculus

1. Tangents and normals : Sub-tangent and sub-normals. Angle of intersection of curves. Pedal equation of a curve, pedal of a curve. [4]

2. Rectilinear asymptotes of a curve (Cartesian, parametric and polar form).

[3]

3. Curvature-Radius of curvature, centre of curvature, chord of curvature, evolute of a curve.
[4]

4. Envelopes of families of straight lines and curves (Cartesian and parametric equations only).
[4]

5. Concavity, convexity, singular points, nodes, cusps, points of inflexion, simple problems on species of cusps of a curve (Cartesian curves only).
[5]

6. Familiarity with the figure of following curves : Periodic curves with suitable scaling, Cycloid, Catenary, Lemniscate of Bernoulli, Astroid, Cardiode, Folium of Descartes, equiangular spiral.
[1]

7. Area enclosed by a curve, determination of C.G., moments and products of inertia (Simple problems only).
[3]

Module VIII

Group A (15 marks)

<u>Analytical Geometry of 3 Dimensions II</u>

1. (a) Sphere : General Equation. Circle, Sphere through the intersection of two spheres. Radical Plane, Tangent, Normal.
[3]
 (b) Cone : Right circular cone. General homogeneous second degree equation. Section of cone by a plane as a conic and as a pair of lines. Condition for three perpendicular generators. Reciprocal cone.
[5]
 (c) Cylinder : Generators parallel to either of the axes, general form of equation. Right-circular cylinder.
[2]
 (d) Ellipsoid, Hyperboloid, Paraboloid : Canonical equations only.
[1]

2. Tangent planes, Normals, Enveloping cone.
[5]

3. Surface of Revolution (about axes of reference only). Ruled surface.

Generating lines
  of   hyperboloid   of   one   sheet   and   hyperbolic   paraboloid.
[10]

4. Transformation of rectangular axes by translation, rotation and their combinations. [2]

5. Knowledge of Cylindrical, Polar and Spherical polar co-ordinates, their relations (No
  deduction   required).
[2]

Group B (10 marks)

Analytical Statics I

1. **Friction :** Law of Friction, Angle of friction, Cone of friction, To find the positions of equilibrium of a particle lying on a (i) rough plane curve, (ii) rough   surface   under   the   action   of   any   given   forces.
[4]

2. Astatic Equilibrium, Astatic Centre, Positions of equilibrium of a particle lying on a smooth plane curve under action of given force. Action at a joint in a frame work.   [4]

Group C (25 marks)

Analytical Dynamics of A Particle I

Recapitulations of Newton's laws. Applications of Newton's laws to elementary problems of simple harmonic motion, inverse square law and composition of two simple harmonic motions. Centre of mass. Basic kinematic quantities : momentum, angular momentum and kinetic energy. Principle of energy and momentum. Work and poser. Simple examples on their applications.

Impact of elastic bodies. Direct and oblique impact of elastic spheres. Losses of kinetic energy. Angle of deflection.

Tangent and normal accelerations. Circular motion. Radial and cross-radial accelerations.

Damped harmonic oscillator. Motion under gravity with resistance proportional to some integral power of velocity. Terminal velocity. Simple cases of a constrained motion of a particle.

Motion of a particle in a plane under different laws of resistance. Motion of a projectile in a resisting medium in which the resistance varies as the velocity. Trajectories in a resisting medium where resistance varies as some integral power of the velocity.

[25]

Module IX

Analysis III (50 marks)

1. Compactness in IR : Open cover of a set. Compact set in IR, a set is compact iff it is closed and bounded. [2]
2. Function of bounded variation (BV) : Definition and examples. Monotone function is of BV. If f is on BV on [a,b] then f is bounded on [a,b]. Examples of functions of BV which are not continuous and continuous functions not of BV. Definition of variation function. Necessary and sufficient condition for a function f to be of BV on [a,b] is that f can be written as the difference of two monotonic increasing functions on[a,b]. Definition of rectifiable curve. A plane curve $\gamma = (f,g)$ is rectifiable if f and g both are of bounded variation (statement only). Length of a curve (simple problems only). [8]

3. Riemann integration :
   (a) Partition and refinement of partition of a closed and bounded interval. Upper Darboux sum U (P,f) and lower Darboux sum L(P,f) and associated results. Upper integral and lower integral. Darboux's theorem. Darboux's definition of integration over a closed and bounded interval. Riemann's definition of integrability. Equivalence with Darboux definition of integrability (statement only). Necessary and sufficient condition for Riemann intergrability.   [6]
   (b) Continuous functions are Riemann integrable. Definition of a set of measure zero (or negligible set or zero set) as a set covered by countable number of open intervals sum of whose lengths is arbitrary small. Examples of sets of measure zero : any subset of a set of measure zero, countable set, countable union of sets of measure zero.

Concept of oscillation of a function at a point. A function is continuous at x if its oscillation at x is zero. A bounded function on a closed and bounded interval is Riemann integrable if the set of points of discontinuity is a set of measure zero (Lebesgue's theorem on Riemann integrable function). Problems on Riemann integrability of functions with sets of points of discontinuity having measure zero. [5]

(c) Integrability of sum, scalar multiple, product, quotient, modulus of Riemann integrable functions. Properties of Riemann integrable functions arising from the above results. [3]

(d) Function defined by definite integral $\int\limits_{a}^{x} f(t)dt$ and its properties. Antiderivative (primitive or indefinite integral). [4]

(e) Fundamental theorem of integral calculus. First mean value theorem of integral calculus. Statement of second mean value theorems of integrals calculus (both Bonnet's and Weierstrass' form). [2]

4. Sequence and Series of functions of a real variable :

(a) Sequence of functions defined on a set : Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence (statement only). Weierstrass's M-test. [4]

(b) Limit function : Boundedness, Repeated limits, Continuity, Integrability and differentiability of the limit function of sequence of functions in case of uniform convergence. [5]

(c) Series of functions defined on a set : Pointwise and uniform convergence. Cauchy criterion of uniform convergence. Dini's theorem on uniform convergence (statement only), Tests of uniform convergence – Weierstrass' M-test. Statement of Abel's and Dirichlet's test and their applications. Passage to the limit term by term. [5]

(d) Sum function : boundedness, continuity, integrability, differentiability of a series of functions in case of uniform convergence. [2]

(e) Power series: Cauchy-Hadamard theorem. Determination of radius of convergence. Uniform and absolute convergence of power series. Properties of sum function. Abel's limit theorems. Uniqueness of power series having sum function. [8]

Module X

Group A (20 marks)

Linear Algebra II   &   Modern Algebra III

Section – I : Linear Algebra II (10 marks)

1. Linear Transformation (L.T.) on Vector Spaces : Definition of L.T., Null space, range space of an L.T., Rank and Nullity, Sylvester's Law of Nullity. [Rank (T) + Nullity (T) = dim (V)]. Determination of rank (T), Nullity (T) of linear transformation $T : IR^n \rightarrow IR^m$. Inverse of Linear Transformation. Non-singular Linear Transformation.
2. Linear Transformation and Matrices : Matrix of a linear transformation relative to ordered bases of finite-dimensional vector spaces. Correspondence between Linear Transformations and Matrices. Linear Transformation is non-singular if its representative matrix be non-singular. Rank of L.T. = Rank of the corresponding matrix.
[5]

Section – II : Modern Algebra III (10 marks)

3. Normal sub-groups of a Group : Definition and examples. Intersection, union of normal sub-groups. Prefect of a normal sub-group and a sub-group. Quotient Group of a Group by a normal sub-group.
[5]
4. Homomorphism and Isomorphism of Groups. Kernel of a Homomorphism. First Isomorphism Theorem. Properties deducible from definition of morphism. An infinite cyclic group is isomorphic to (Z,+) and a finite cyclic group of order *n* is isomorphic to the group of residue classes modulo *n*.   [5]

Group B (25 marks)

Tensor Calculus

A tensor as a generalized concept of a vector in an Euclidean space E^3. To generalize the idea in an n-dimensional space. Definition of E^n. Transformation of co-ordinates in E^n (n = 2, 3 as example). Summation convention.

Contravariant and covariant vectors. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction. Outer and Inner products of tensors. Quotient law. Reciprocal Tensor. Riemannian space. Line element and metric tensor. Reciprocal metric tensor. Raising and lowering of indices with the help of metric tensor. Associated tensor. Magnitude of a vector. Inclination of two vectors. Orthogonal vectors. Christoffel symbols and their laws of transformations. Covariant differentiation of vectors and tensors. [15]

Group C (15 marks)

Differential Equations II

1. Laplace Transformation and its application in ordinary differential equations : Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Laplace Transform of derivatives. Laplace transform of integrals. Convolution theorem (Statement only). Application to the solution of ordinary differential equations of second order with constant coefficients.

2. Series solution at an ordinary point : Power Series solution of ordinary differential equations. Simple problems only.

Or

Group C (15 marks)

Graph Theory

1. Graphs : Undirected graphs. Directed graphs. Basic properties. Walk. Path. Cycles. Connected graphs. Components of a graph. Complete graph. Complement of a graph. Bipartite graphs. Necessary and sufficient condition for a Bipartite graph.   [7]

2. Euler graphs : Necessary and Sufficient condition for a Euler graph. Königsberg   Bridge   Problem. [3]

3. Planar graphs : Face-size equation, Euler's formula for a planar graph. To show : the graphs K_5 and $K_{3,3}$ are non-planar. [3]

4. Tree : Basic properties, Spanning tree, Minimal Spanning tree, Kruskal's algorithm, Prim's algorithm, Rooted tree, Binary tree. [5]

Module XI

Group A (10 marks)

<u>Vector Calculus II</u>

Line integrals as integrals of vectors, circulation, irrotational vector, work done, conservative force, , potential orientation. Statements and verification of Green's theorem, Stokes' theorem and Divergence theorem. [8]

Group B (20 marks)

<u>Analytical Statics II</u>

1. Centre of Gravity: General formula for the determination of C.G. Determination of position of C.G. of any arc, area of solid of known shape by method of integration.   [3]

2. Virtual work: Principle of virtual work for a single particle. Deduction of the conditions of equilibrium of a particle under coplanar forces from the principle of virtual work. The principle of virtual work for a rigid body. Forces which do not appear in the equation of virtual work. Forces which appear in the equation of virtual work. The principle of virtual work for any system of coplanar forces acting on a rigid body. Converse of the principle of virtual work. [8]

3. Stable and unstable equilibrium. Coordinates of a body and of a system of bodies. Field of forces. Conservative field. Potential energy of a system. The energy test of stability. Condition of stability of equilibrium of a perfectly rough heavy body lying on fixed body. Rocking stones. [6]

4. Forces in the three dimensions. Moment of a force about a line. Axis of a couple. Resultant of any two couples acting on a body. Resultant of any number of couples acting on a rigid body. Reduction of a system of forces acting on a rigid body. Resultant force is an invariant of the system but the resultant couple is not an invariant.
Conditions of equilibrium of a system of forces acting on a body. Deductions of the conditions of equilibrium of a system of forces acting on a rigid body from the principle of virtual work. Poisnot's central axis. A given system of forces can have only one central axis. Wrench, Pitch, Intensity and Screw. Condition that a given system of forces may have a single resultant. Invariants of a given system of forces. Equation of the central axis of a given system of forces. [12]

Group C (20 marks)

<u>Analytical Dynamics of A Particle II</u>

1. Central forces and central orbits. Typical features of central orbits. Stability of nearly circular orbits.

2. Planetary motion and Kepler's laws. Time of describing an arc of the orbit. Orbital energy. Relationship between period and semi-major axis. Motion of an artificial satellite.

3. Motion of a smooth cure under resistance. Motion of a rough curve under gravity e.g., circle, parabola, ellipse, cycloid etc.

4. Varying mass problems. Examples of falling raindrops and projected rockets.

5. Linear dynamical systems, preliminary notions: solutions, phase portraits, fixed or critical points. Plane autonomous systems. Concept of Poincare phase plane. Simple examples of damped oscillator and a simple pendulum. The two-variable case of a linear plane autonomous system. Characteristic polynomial. Focal, nodal and saddle points.

[20]

Module XII

Group A (25 marks)

<u>Hydrostatics</u>

1. Definition of Fluid. Perfect Fluid, Pressure. To prove that the pressure at a point in fluid in equilibrium is the same in every direction. Transmissibility of liquid pressure. Pressure of heavy fluids. To prove

   (i) In a fluid at rest under gravity the pressure is the same at all points in the same horizontal plane.

   (ii) In a homogeneous fluid at rest under gravity the difference between the pressures at two points is proportional to the difference of their depths.

   (iii) In a fluid at rest under gravity horizontal planes re surfaces of equal density.

   (iv) When two fluids of different densities at rest under gravity do not mix, their surface of separation is a horizontal plane.

   Pressure in heavy homogeneous liquid. Thrust of heavy homogeneous liquid on plane surface.

2. Definition of centre of pressure. Formula for the depth of the centre of pressure of a plane area. Position of the centre of pressure. Centre of pressure

of a triangular area whose angular points are at different depths. Centre of pressure of a circular area. Position of the centre of pressure referred to co-ordinate axes through the centroid of the area. Centre of pressure of an elliptical area when its major axis is vertical or along the line of greatest slope. Effect of additional depth on centre of pressure.

3. Equilibrium of fluids in given fields of force : Definition of field of force, line of force. Pressure derivative in terms of force. Surface of equi-pressure. To find the necessary and sufficient conditions of equilibrium of a fluid under the action of a force whose components are *X, Y, Z* along the co-ordinate axes. To prove (i) that surfaces of equal pressure are the surfaces intersecting orthogonally the lines of force, (ii) when the force system is conservative, the surfaces of equal pressure are equi-potential surfaces and are also surfaces of equal density. To find the differential equations of the surfaces of equal pressure and density.

4. Rotating fluids. To determine the pressure at any point and the surfaces of equal pressure when a mass of homogeneous liquid contained in a vessel, revolves uniformly about a vertical axis.

5. The stability of the equilibrium of floating bodies. Definition, stability of equilibrium of a floating body, metacentre, plane of floatation, surface of buoyancy. General propositions about small rotational displacements. To derive the condition for stability.

6. Pressure of gases. The atmosphere. Relation between pressure, density and temperature. Pressure in an isothermal atmosphere. Atmosphere in convective equilibrium.

[30]

Group B (25 marks)

Rigid Dynamics

1. Momental ellipsoid, Equimomental system. Principal axis. D'Alembert's principle. D'Alembert's equations of motion. Principles of moments. Principles of conservations of linear and angular momentum. Independence of the motion of centre of inertia and the motion relative to the centre of inertia. Principle of energy. Principle of conservation of energy.

2. Equation of motion of a rigid body about a fixed axis. Expression for kinetic energy and moment of momentum of a rigid body moving about a fixed axis. Compound pendulum. Interchangeability of the points of a suspension and centre of oscillation. Minimum time of oscillation.

3. Equations of motion of a rigid body moving in two dimensions. Expression for kinetic energy and angular momentum about the origin of rigid body moving in two dimensions. Necessary and sufficient condition for pure rolling. Two dimensional motion of a solid of revolution moving on a rough horizontal plane.

4. Equations of motion under impulsive forces. Equation of motion about a fixed axis under impulsive forces. To show that (i) of there is a definite straight line such that the sum of the moments of the external impulses acting on a system of particles about it vanishes, then the total angular momentum of the system about that line remains unaltered,   (ii) the change of K.E. of a system of particles moving in any manner under the application of impulsive forces is equal to the work done by the impulsive forces.   [30]

Module XIII

Group A (20 marks)

<u>Analysis IV</u>

   1.   Improper Integral :

     (a) Range of integration, finite or infinite. Necessary and sufficient condition for   convergence   of   improper   integral   in   both   cases. [2]

     (b) Tests of convergence : Comparison and μ-test. Absolute and non-absolute convergence and interrelations. Abel's and Dirichlet's test for convergence

of the integral of a product (statement only). [3]

(c) Convergence and working knowledge of Beta and Gamma function and their interrelation $(\Gamma(n)\Gamma(1-n) = \dfrac{\pi}{\sin n\pi}, 0 < n < 1,$ to be assumed). Computation of the integrals $\displaystyle\int_0^{\pi/2} \sin^n x\,dx, \int_0^{\pi/2} \cos^n x\,dx, \int_0^{\pi/2} \tan^n x\,dx$ when they exist (using Beta and Gamma function). [5]

2. Fourier series : Trigonometric series. Statement of sufficient condition for a trigonometric series to be a Fourier series. Fourier co-efficients for periodic functions defined on $[-\pi, \pi]$. Statement of Dirichlet's conditions convergence. Statement of theorem of sum of Fourier series. [5]

3. Multiple integral : Concept of upper sum, lower sum, upper integral, lower-integral and double integral (no rigorous treatment is needed). Statement of existence theorem for continuous functions. Change of order of integration. Triple integral. Transformation of double and triple integrals (Problem only). Determination of volume and surface area by multiple integrals (Problem only). [5]

Group B (15 marks)

Metric Space

1. Definition and examples of metric spaces. Open ball. Open set. Closed set defined as complement of open set. Interior point and interior of a set. Limit point, derived set and closure of a set. Boundary point and boundary of a set. Properties of interior, closure and boundary of a set. Diameter of a set and bounded set. Distance between a point and a set. [7]

2. Subspace of a metric space. Convergent sequence. Cauchy sequence. Every Cauchy sequence is bounded. Every convergent sequence is Cauchy, not the converse. Completeness : definition and examples. Cantor intersection theorem. IR is a complete metric space. Q is not complete. [4]

Group C (15 marks)

Complex Analysis

1. Extended complex plane. Stereographic projection. [2]

2. Complex function : Limit , continuity and differentiability of complex functions. Cauchy-Riemann equations. Sufficient condition for differentiability of a complex function. Analytic functions. Harmonic functions. Conjugate harmonic functions. Relation between analytic function and harmonic function. [8]

Module XIV

Group A (30 marks)

Probability

Mathematical Theory of Probability :

Random experiments. Simple and compound events. Event space. Classical and frequency definitions of probability and their drawbacks. Axioms of Probability. Statistical regularity. Multiplication rule of probabilities. Bayes' theorem. Independent events. Independent random experiments. Independent trials. Bernouli trials and binomial law. Poisson trials. Random variables. Probability distribution. distribution function. Discrete and continuous distributions. Binomial, Poisson, Gamma, Uniform and Normal distribution. Poisson Process (only definition). Transformation of random variables. Two dimensional probability distributions. Discrete and continuous distributions in two dimensions. Uniform distribution and two dimensional normal distribution. Conditional distributions. transformation of random variables in two dimensions. Mathematical expectation. Mean, variance, moments, central moments. Measures of location, dispersion, skewness and kurtosis. Median, mode, quartiles. Moment-generating function. Characteristic function. Two-dimensional expectation. Covariance, Correlation co-efficient, Joint characteristic function. Multiplication rule for expectations. Conditional expectation. Regression curves, least square regression lines and parabolas. Chi-square and t-distributions and their important properties (Statements only). Tchebycheff's inequality. Convergence in probability. Statements of : Bernoulli's limit theorem. Law of large numbers. Poissons's approximation to binomial distribution and normal approximation to binomial distribution. Concepts of asymptotically normal distribution. Statement of central limit theorem in the case of equal components and of limit theorem for characteristic functions (Stress should be more on the distribution function theory than on combinatorial problems. Difficult combinatorial problems should be avoided).

[40]

Group B (20 marks)

Statistics

Random sample. Concept of sampling and various types of sampling. Sample and population. Collection, tabulation and graphical representation. Grouping of data. Sample characteristic and their computation. Sampling distribution of a statistic. Estimates of a population characteristic or parameter. Unbiased and consistent estimates. Sample characteristics as estimates of the corresponding population characteristics. Sampling distributions of the sample mean and variance. Exact sampling distributions for the normal populations.

Bivariate samples. Scatter diagram. Sample correlation co-efficient. Least square regression lines and parabolas. Estimation of parameters. Method of maximum likelihood. Applications to binomial, Poisson and normal population.

Confidence intervals. Interval estimation for parameters of normal population. Statistical hypothesis. Simple and composite hypothesis. Best critical region of a test. Neyman-Pearson theorem (Statement only) and its application to normal population. Likelihood ratio testing and its application to normal population. Simple applications of hypothesis testing. [35]

Module XV

Group A (25 marks)

Numerical Analysis

What is Numerical Analysis ?

Errors in Numerical computation : Gross error, Round off error, Truncation error. Approximate numbers. Significant figures. Absolute, relative and percentage error.

Operators : Δ, ∇, E, μ, δ (Definitions and simple relations among them).

Interpolation : Problems of interpolation, Weierstrass' approximation theorem only statement). Polynomial interpolation. Equispaced arguments. Difference table. Deduction of Newton's forward and backward interpolation formulae. Statements of Stirling's and Bessel's interpolation formulae. error terms. General interpolation formulae : Deduction of Lagrange's interpolation formula. Divided difference. Newton's General Interpolation formula (only statement). Inverse interpolation.

Interpolation formulae using the values of both $f(x)$ and its derivative $f(x)$: Idea of Hermite interpolation formula (only the basic concepts).

Numerical Differentiation based on Newton's forward & backward and Lagrange's formulae.

Numerical Integration : Integration of Newton's interpolation formula. Newton-Cote's formula. Basic Trapezoidal and Simpson's ⅓rd. formulae. Their composite forms. Weddle's rule (only statement). Statement of the error terms associated with these formulae. Degree of precision (only definition).

Numerical solution of non-linear equations : Location of a real root by tabular method. Bisection method. Secant/Regula-Falsi and Newton-Raphson methods, their geometrical significance. Fixed point iteration method.

Numerical solution of a system of linear equations : Gauss elimination method. Iterative method – Gauss-Seidal method. Matrix inversion by Gauss elimination method (only problems – up to 3×3 order).

Eigenvalue Problems : Power method for numerically extreme eigenvalues.

Numerical solution or Ordinary Differential Equation : Basic ideas, nature of the problem. Picard, Euler and Runge-Kutta (4th order) methods (emphasis on the problems   only).
[30]

Group B (25 marks)

Computer Programming

Fundamentals of Computer Science and Computer Programming :

Computer fundamentals : Historical evolution, computer generations, functional description, operating system, hardware & software.

Positional number system : binary, octal, decimal, hexadecimal system. Binary arithmetic.

Storing of data in a computer : BIT, BYTE, Word. Coding of data – ASCII, EBCDIC, etc.

Algorithm and Glow Chart : Important features, Ideas about the complexities of algorithm. Application in simple problems.

Programming languages : General concepts, Machine language, Assembly Language, High Level Languages, Compiler and Interpreter. Object and Source Program. Ideas about some major HLL.

Students are required to opt for any one of the following two programming languages :
(i)   Programming with FORTRAM 77/90.
                      Or
(ii)   Introduction to ANSI C.

Programming with FORTRAN 77/90 :

Introduction, Keywords, Constants and Variables – integer, real, complex, logical, character, double precision, subscripted. Fortran expressions. I/O statements-formatted and unformatted. Program execution control-logical if, if-then-else, etc. Arrays-Dimension statement. Repetitive computations – Do. Nested Do, etc. Sub-programs : Function sub program and Subroutine sub program.

Application to simple problems : Evaluation of functional values, solution of quadratic equations, approximate sum of convergent infinite series, sorting of real numbers, numerical integration, numerical solution of non-linear equations, numerical solution of ordinary differential equations, etc.

Introduction to ANSI C :

Character set in ANSI C. Key words : if, while, do, for, int, char, float etc.

Data type : character, integer, floating point, etc. Variables, Operators : =, = =, !!<, >, etc. (arithmetic, assignment, relational, logical, increment, etc.). Expresions : e.g. (a = = b) !! (b = = c), Statements : e.g. if (a>b) small = a; else small = b. Standard input/output. Use of while, if…. Else, for, do…while, switch, continue, etc. Arrays, strings. Function definition. Running simple C Programs. Header File.   [30]

Boolean Algebra : Huntington Postulates for Boolean Algebra. Algebra of sets and Switching Algebra as examples of Boolean Algebra. Statement of principle of duality. Disjunctive normal and Conjunctive normal forms of Boolean Expressions.   Design   of   simple   switching   circuits. [10]

Module XVI

Practical
(Problem:30, Sessional Work:10, Viva:10)

(A) Using Calculator

(1) INTERPOLATION :
   Newton's forward & Backward Interpolation.
   Stirling & Bessel's Interpolation.
   Lagrange's Interpolation & Newton's Divided Difference Interpolation.
   Inverse Interpolation.
(2) Numerical Differentiation based on Newton's Forward & Backward
   Interpolation Formulae.
(3) Numerical Integration : Trapezoidal Rule, Simpson's ⅓ Rule and Weddle's
   Formula.
(4) Solution of Equations : Bisection Method, Regula Falsi, Fixed Point Iteration.
   Newton-Raphson formula (including modified form for repeated roots and
   complex roots).
(5) Solution of System of Linear Equations : Gauss' Elimination Method with
   partial pivoting, Gauss-Seidel/Jordon Iterative Method, Matrix Inversion.
(6) Dominant Eigenpair of a (4×4) real symmetric matrix and lease eigen value of
   a (3×3) real symmetric matrix by Power Method.
(7) Numerical Solution of first order ordinary Differential Equation (given the
   initial condition) by :
                    Picard's Method, Euler Method, Heun's Method,
                    Modified Euler's Method, 4th order Runge-Kutta Method.
(8) Problems of Curve Fitting : To fit curves of the form y=a+bx, y=a+bx+cx^2,
   exponential curve of the form y=ab^x, geometric curve y=ax^b by Least Square
   Method.

(B) ON COMPUTER :

   The following problems should be done on computer using either FORTRAN
or C language :

(i)   To find a real root of an equation by Newton-Raphson Method.
(ii)   Dominant eigenpair by Power Method.
(iii)   Numerical Integration by Simpson's ⅓ Rule.
(iv)   To solve numerically Initial Value Problem by Euler's and RK$_4$ Method.

LIST OF BOOKS FOR REFERRENCE

Module I   Group A :

1.   The Theory of Equations (Vol. I) – Burnside and Panton.
2.   Higher Algebra – Barnard and Child.
3.   Higher Algebra – Kurosh (Mir).

Module I   Group B &   Module V   Group A :

1.   Modern Algebra – Surjeet Singh & Zameruddin.
2.   First Course in Abstract Algebra – Fraleigh.
3.   Topics in Algebra – Hernstein.
4.   Test book of algebra – Leadership Project Committee (University of Bombay).
5.   Elements of Abstract Algebra – Sharma, Gokhroo, saini (Jaipur Publishing House, S.M.S. Highway, Jaipur - 3).
6.   Abstract Algebra – N. P. Chaudhuri (Tata Mc.Graw Hill).

Module IV   Group A :

1.   Linear Algebra – Hadley
2.   Test Book of Matrix – B. S. Vaatsa

Module II   Group A , Group B & Module VIII Group A :

1.   Co-ordinate Geometry – S. L. Loney.
2.   Co-ordinate Geometry of Three Dimensions – Robert J. T. Bell.
3.   Elementary Treatise on Conic sections – C. Smith.
4.   Solid Analytic Geometry – C. smith.
5.   Higher Geometry – Efimov.

Module III   Group A , Module VI Group A Module VII Group A, Module IX & Module XIII Group A :

1.   Basic Real & Abstract Analysis – Randolph J. P. (Academic Press).
2.   A First Course in Real Analysis – M. H. Protter & G. B. Morrey (Springer Verlag, NBHM).
3.   A Course of Analysis – Phillips.
4.   Problems in Mathematical Analysis – B. P. Demidovich (Mir).
5.   Problems in Mathematical Analysis – Berman (Mir).
6.   Differential & Integral Calculus (Vol. I & II) – Courant & John.
7.   Calculus of One Variable – Maron (CBS Publication).
8.   Introduction to Real Analysis – Bartle & Sherbert (John Wiley & Sons.)
9.   Mathematical Analysis – Parzynski.
10.   Introduction to Real Variable Theory – Saxena & Shah (Prentice Hall Publication).

11. Real Analysis – Ravi Prakash & Siri Wasan (Tata McGraw Hill).
12. Mathematical Analysis – Shantinarayan (S. Chand & Co.).
13. Theory & Applications of Infinite Series – Dr. K. Knopp.
14. Advanced Calculus – David Widder (Prentice Hall).
15. Charles Chapman Pugh: Real mathematical analysis; Springer;
    New York; 2002
16. Sterling K. Berberian: A First Course in Real Analysis; Springer;
    New York; 1994
17. Steven G. Krantz: Real Analysis and Foundations; Chapman and Hall/CRC; .
    .   2004
18. Stephen Abbott: Understanding Analysis; Springer; New York, 2002
19. T. M. Apostol: Mathematical Analysis, Addison-Wesley Publishing Co. 1957
20. W. Ruddin: Principles of Mathematical Analysis, McGraw-Hill,
     New York, 1976
21. J. F. Randolhp: Basic Real and Abstract Analysis, Academic Press;
    New York, 1968
22. Robert G Bartle, Donald R Sherbert: Introduction to real analysis;
    John Wiley Singapore; 1994
23. Integral Calculus – Shanti Narayan & P. K. Mittal (S. Chand & Co. Ltd.)
24. Integral Calculus – H. S. Dhami (New Age International)
25. Integral Calculus – B. C. Das & B. N. Mukherjee (U. N. Dhur)
26. Differential & Integral Calculus (Vols. I & II) – Courant & John.
27. Differential & Integral Calculus (Vol. I) – N. Piskunov

    (CBS Publishers & Distributors)

Module VII :

1. Differential Calculus – Shantinarayan.
2. Integral Calculus – Shantinarayan.
3. An elementary treatise on the Differential Calculus – J. Edwards (Radha
   Publishing House).
4. Advanced Calculus – David V. Widder (Prentice Hall)
5. Real Analysis – Ravi Prakash & Siri Wasan (Tata McGraw Hill)
6. A Course of Analysis – E. G. Phillips (Cambridge University Press)
7. Differential Calculus – Shanti Naryaan (S. Chand & Co. Ltd.)
8. An elementary treatise on the Differential Calculus – J. Edwards
   (Radha Publishing House)
9. Differential Calculus – H. S. Dhami (New Age International)
10. Differential & Integral Calculus (Vols. I & II) – Courant & John.
11. Differential & Integral Calculus (Vol. I) – N. Piskunov
    (CBS Publishers & Distributors)

Module II   Group C , Module IV Group B ,   Module XI Group A :

1. Vector Analysis – Louis Brand.
2. Vector Analysis – Barry Spain.

3.   Vector & Tensor Analysis – Spiegel (Schaum).
4.   Elementary Vector Analysis – C. E. Weatherburn (Vol. I & II).

Module V   Group B :

1.   Linear Programming : Method and Application – S. I. Gass.
2.   Linear Programming – G. Hadley.
3.   An Introduction to Linear Programming & Theory of Games – S. Vajda.

Module VI Group B :

1.   Differential Equations – Lester R. Ford (McGraw Hill).
2.   Differential Equations – S. L. Ross (John Wiley).
3.   Differential Equations – H. T. H. Piaggio.
4.   A Text Book of Ordinary Differential Equations – Kiseleyev, Makarenko & Krasnov (Mir).
5.   Differential Equations – H. B. Phillips (John Wiley & Sons).
6.   Differential Equations with Application & Programs – S. Balachanda Rao, H. R. Anuradha (University Press).
7.   Text Book of Ordinary Differential Equations (2nd Ed.) – S. G. Deo, V. Lakshmikantham & V. Raghavendra (Tata McGraw Hill).
8.   An Elementary Course in Partial Differential Equation – T. Amarnath (Narosa).
9.   An Introductory Course on Ordinary Differential Equation – D. A. Murray.

Module VIII   Group C   &   Module XI Group C :

1.   An Elementary Treatise on the Dynamics of a Particle & of Rigid bodies – S. L. Loney (Macmillan).

Module XV :

1.   The elements of probability theory and some of its applications - H. Cramer.
2.   An introduction to probability theory and its applications (Vol. 1) – W. Feller.
3.   Mathematical methods of statistics – H. Cramer.
4   Theory of probability – B. V. Gnedenko.
5.   Mathematical probability – J. V. Uspensky.
6   Programming with FORTRAN 77 – A Structured approach – R. S. Dhaliwal, S. K. Agarwal, S. K. Gupta (Wiley Eastern Limited/New Age International Ltd.).
7.   Structured FORTRAN 77 for engineers and scientists – D. M. Etter (The Benjamin/Cummings Publishing Co. Inc.).
8.   Programming and Computing with FORTRAN 77/90 – P. S. Grover (Allied Publishers).

9. Programming with FORTRAN including structured FORTRAN - Seymour Lipschutz and Arthur Poe (Schaum's Outline Series).
10. FORTRAN 77 and numerical methods – C. Xavier (Wiley Eastern limited).
11. Numerical methods – E. Balagurusamy (Tata McGraw Hill).
12. Let us C – Y. Kanetkar (BPB Publications).
13 Programming in C – V. Krishnamoorthy and K. R. Radhakrishnan (Tata Mcgraw Hill).
14. C by example : Noel Kalicharan (Cambridge University Press).
15. Programming in ANSI C – E. Balagurusamy (Tata McGraw Hill).
16. Introduction to numerical analysis – F. B. Hilderbrand (TMH Edition).
17. Numerical Analysis – J. Scarborough.
18. Introduction to numerical analysis – Carl Erik Froberg (Addison Wesley Publishing).
19. Numerical methods for science and engineering – R. G. Stanton (Prentice Hall).

Module XII Group A :

1. Vector Analysis – Spiegel (Schaum).
2. Vector Calculus – C. E. Weatherburn.
3. Analytical Statics – S. L. Loney
4. Dynamics of Particle and of Rigid Bodies – S. L. Loney.
5. Hydrostatics – A. S. Ramsay.

Module X :

1. Advanced Calculus – David Widder (Prentice Hall)
2. Elementary Treatise on Laplace Transform – B. Sen (World Press).
3. Operational Methods in Applied Mathematics – H. S. Carslaw.   J. C. Jaeger.
4. Graph Theory and its Applications – Gross, Jonathan and Yellen, Jay (CRC Prss, USA, 1999).
5. Graph Theory with applications to Engineering and Computer Science – Deo, Narsingh (Prentice Hall, 2000).
6. Graph Theory – Harary Grank (Addison-Wesley Publishing Co. 1972).
7. Tensor Calculus – Barry Spain
8. Vector Analysis and Tensor Calculus(Schaum Series) – Spiegel.
9   P. K. Jain and K. Ahmad: Metric Spaces, Narosa Publishing House; New Delhi; 1996
10 R. V. Churchill and J.W.Brown: Complex Variables and Applications; Mcgraw-Hill; New York; 1996
11 J. B. Conway: Functions of One Complex Variables; Narosa Publishing; New Delhi; 1973973
12 S. Ponnusamy: Foundations of Complex Analysis; Narosa; New Delhi; 1995
13 L. V. Ahlfors: Complex Analysis: an introduction to the theory of analytic .
.   .   .   functions of one complex variable; McGraw-Hill; New York;
1966

GENERAL

(Total 8 modules each of 50 marks)

MODULE I : **Group A :** Classical Algebra (20 marks)
            Group B : Analytical Geometry of two dimensions (15 marks)
            Group C : Vector Algebra (15 marks)

MODULE II : **Group A :** Differential Calculus (25 marks)
            Group B : Integral Calculus (10 marks)
            Group C : Differential Equations (15 marks)

MODULE III : **Group A :** Modern Algebra (25 marks)
            Group B : Analytical Geometry of three dimensions (25 marks)

MODULE IV : **Group A :** Differential Calculus (25 marks)
            Group B : Integral Calculus (15 marks)
            Group C : Differential Equations (10 marks)

MODULE V : **Group A :** Numerical Methods (20 marks)
            Group B : Linear Programming (30 marks)

MODULE VI : Any **one** of the following groups **:**

            Group A : Analytical Dynamics (50 marks)
            Group B : Probability & Statistics (50 marks)

MODULE VII : Computer Science & Programming (50 marks)

MODULE V : Any **one** of the following groups **:**

            Group A : A Course of Calculus (50 marks)
            Group B : Discrete Mathematics (50 marks)

MODULE I

Group A (20 marks)

Classical Algebra

01. **Complex Numbers :** De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of a^z, $(a \neq 0)$. Inverse circular and Hyperbolic functions.
02. **Polynomials :** Fundamental Theorem of Classical Algebra (Statement only). Polynomials with real co-efficients: The nth degree polynomial equation has exactly n roots. Nature of roots of an equation (Surd or Complex roots occur in pairs). Statement of Descarte's Rule of signs and its applications.
    Statements of **:**
    - (i)   If the polynomial $f(x)$ has opposite signs for two real values of x, e.g. a and b, the equation $f(x) = 0$ has an odd number of real roots between a and b; if $f(a)$ and $f(b)$ are of same sign, either no real root or an even number of roots lies between a and b.
    - (ii) Rolle's Theorem and its direct applications.

    Relation between roots and co-efficients. Symmetric functions of roots, Transformations of equations. Cardan's method of solution of a cubic.
03. **Determination up to the third order :** Properties, Cofactor and Minor. Product of two determinants. Adjoint, Symmetric and Skew-symmetric determinants. Solutions of linear equations with not more than three variables by Cramer's Rule.
04. **Matrices of Real Numbers :** Equality of matrices. Addition of matrices. Multiplication of a matrix by a scalar. Multiplication of matrices – Associative properties. Transpose of matrix – its properties. Inverse of a non-singular square matrix. Symmetric and Skew-symmetric matrices. Scalar matrix. Orthogonal matrix. Elementary operations on matrices.

    Rank of a matrix : Determination of rank either by considering minors or by sweep-out process. Consistency and solution of a system of linear of equations with not more than 3 variables by matrix method.

Group B (15 marks)

Analytical Geometry of 2 Dimensions

01. **Transformations of Rectangular axes :** Translation, Rotation and their combinations. Invariants.
02. **General equation of second degree in x and y :** Reduction to canonical forms. Classification of conic.
03. **Pair of straight lines :** Condition that the general equation of 2nd degree in x and y may represent two straight lines. Points of intersection of two intersecting straight lines. Angle between two lines given by $ax^2 + 2hxy + by^2 = 0$. Equation of bisectors. Equation of two lines joining the origin to the points in which a line meets a conic.

04. **Equations of pair of tangents from an external point, chord of contact, poles and polars in case of General conic :** Particular cases for Parabola, Ellipse, Circle, Hyperbola.

05. Polar equation of straight lines and circles. Polar equation of a conic referred to a focus as pole. Equation of chord joining two points. Equations of tangent and normal.

Group C (15 marks)

Vector Algebra

Addition of Vectors. Multiplication of a Vector by a scalar. Collinear and Coplanar Vectors. Scalar and Vector products of two and three vectors. Simple applications to problems of Geometry. Vector equation of plane and straight line. Volume of Tetrahedron. Application to problems of Mechanics (Work done and Moment).

MODULE II

Group A (25 marks)

Differential Calculus

01. Rational Numbers. Geometrical representation. Irrational number. Real number represented as point on a line – Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included).

02. **Sequence :** Definition of bounds of a sequence and monotone sequence. Limit of a sequence. Statements of limit theorems. Concept of convergence and divergence of monotone sequences – applications of the theorems, in particular, definition of e. Statement of Cauchy's general principle of convergence and its application.

03. **Infinite series of constant terms :** Convergence and Divergence (definitions). Cauchy's principle as applied to infinite series (application only). Series of positive terms **:** Statements of Comparison test, D.Alembert's Ratio test. Cauchy's nth root test and Raabe's test – Applications. Alternating series: Statement of Leibnitz test and its applications.

04. **Real-valued functions defined on an interval :** Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval.

Acquianance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.

05. **Derivative** – its geometrical and physical interpretation. Sign of derivative – Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential – application in finding approximation.

06. Successive derivative – Leibnitz's Theorem and its application.

07. Application of the principle of Maxima and Minima for a function of single variable in geometrical, physical and other problems.

08. Applications of Differential Calculus : Tangents and Normals, Pedal equation and Pedal of a curve. Rectilinear Asymptotes (Cartesian only). Definition and examples of singular points (viz. Node, Cusp, Isolated point).

Group B (10 marks)

Integral Calculus

01. Integration of the form **:**

$$\int \frac{dx}{a+b\cos x} \quad , \quad \int \frac{l\sin x + m\cos x}{n\sin x + p\cos x}dx \quad \text{and Integration of Rational}$$

functions.

02. Evaluation of definite integrals.

03. Integration as the limit of a sum (with equally spaced as well as unequal intervals)

Group C (15 marks)

Differential Equations

01. Order, degree and solution of an ordinary differential equation (ODE) in presence of arbitrary constants. Formation of ODE.

First order equations **:**
(i) Variables separable.
(ii) Homogeneous equations and equations reducible to homogeneous forms.
(iii) Exact equations and those reducible to such equation.
(iv) Euler's and Bernoulli's equations (Linear).
(v) Clairaut's Equations : General and Singular solutions.

02. Simple applications : Orthogonal Trajectories.

MODULE III

Group A (25 marks)

Modern Algebra

01. **Basic concept :** Sets, Sub-sets, Equality of sets, Operations on sets : Union, intersection and complement. Verification of the laws of Algebra of sets and De Morgan's Laws. Cartesian product of two sets.

Mappings, One-One and onto mappings. Composition of Mappings – concept only, Identity and Inverse mappings. Binary Operations in a set. Identity element. Inverse element.

02. **Introduction of Group Theory :** Definition and examples taken from various branches (examples from number system, roots of unity, 2 x 2 real matrices, non-singular real matrices of a fixed order). Elementary properties using definition of Group. Definition and examples of sub-group – Statement of necessary and sufficient condition – its applications.

03. Definitions and examples of (i) Ring, (ii) Field, (iii) Sub-ring, (iv) Sub-field.

04. **Concept of Vector space over a Field :** Examples, Concepts of Linear combinations, Linear dependence and independence of a finite set of vectors, Sup-space. Concepts of generators and basis of a finite-dimensional vector space. Problems on formation of basis of a vector space (No proof required).

05. Real Quadratic Form involving not more than three variables – Problems only.

06. Characteristic equation of a square matrix of order not more than three – determination of Eigen Values and Eigen Vectors – Problems only. Statement and illustration of Cayley-Hamilton Theorem.

Group B (25 marks)

Analytical Geometry of 3 dimensions

01. **Rectangular Cartesian co-ordinates :** Distance between two points. Division of a line segment in a given ratio. Direction cosines and direction ratios of a straight line. Projection of a line segment on another line. Angle between two straight lines.

02. **Equation of a Plane :** General form. Intercept and Normal form. Angle between two planes. Signed distance of a point from a plane. Bisectors of angles between two intersecting planes.

03. **Equations of Straight line :** General and symmetric form. Distance of a point from a line. Coplanarity of two straight lines. Shortest distance between two skew-lines.

04. Sphere and its tangent plane.

05. Right circular cone.

MODULE IV

Group A (25 marks)

Differential Calculus

01. Statement of Rolle's theorem and its geometrical interpretation. Mean Value Theorems of Lagrange and Cauchy. Statements of Taylors and Maclaurin's Theorems with Lagrange's and Cauchy's form of remainders. Taylor's and Maclaurin's Infinite series for functions like

e^x, sin x, cos x. $(1+x)^n$, log(1+x) [with restrictions wherever necessary]

02. Indeterminate Forms : L'Hospital's Rule : Statement and problems only.

03. Functions of two and three variables : Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives : Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Functions of two variables – Successive partial derivatives : Statement of Schwarz's Theorem on commutative property of mixed derivatives. Euler's theorem on homogeneous function of two and three variables. Maxima and minima of functions of not more than three variables – Lagrange's Method of undetermined multiplier – Problems only. Implicit function in case of function of two variables (existence assumed) and derivative.

Group B (15 marks)

Integral Calculus

01. Reduction formulae of $\int \sin^m x \cos^n x \, dx$, $\int \dfrac{\sin^m x}{\cos^n x} dx$, $\int \tan^n x \, dx$ and associated problems (m and n are non-negative integers).

02. Definition of Improper Integrals : Statements of (i) μ-test, (ii) Comparison test (Limit form excluded) – Simple problems only. Use of Beta and Gamma functions (convergence and important relations being assumed).

03. Working knowledge of Double integral.

04. Applications : Rectification, Quadrature, Volume and Surface areas of solids formed by revolution of plane curve and areas – Problems only.

Group C (10 marks)

Differential Equations

01. Second order linear equations : Second order linear differential equations with constant. Coefficients. Euler's Homogeneous equations.

MODULE V
Group A (20 marks)

Numerical Methods

01. Approximate numbers, Significant figures, Rounding off numbers. Error – Absolute, Relative and Percentage.
02. **Operators** - Δ, ∇ and E (Definitions and some relations among them).
03. **Interpolation :** The problem of Interpolation, Equispaced arguments – Difference Tables, Deduction of Newton's Forward Interpolation Formula. Remainder term (expression only). Newton's Backward Interpolation formula (statement only) with remainder term. Unequally – spaced arguments – Lagrange's Interpolation Formula (statement only). Numerical problems on Interpolation with both equi- and unequally-spaced arguments.
04. **Number Integration :** Trapezoidal and Simpson's ⅓rd formula (statement only). Problems on Numerical Integration.
05. **Solution of Numerical Equation :** To find a real root of an algebraic or transcendental equation. Location of root (Tabular method), Bisection method. Newton-Raphson method with geometrical significance. Numerical problems.

(Note : emphasis should be given on problems)

Group B (30 marks)

Linear Programming

01. Motivation of Linear Programming problem. Statement of L.P.P. formulation of L.P.P. Slack and Surplus variables. L.P.P. is matrix form. Convex set, Hyperplane, Extreme points, Convex Polyhedron, Basic solutions and Basic Feasible Solutions (B.F.S.) Degenerate and Non-degenerate B.F.S.

The set of all feasible solutions of an L.P.P. is a convex set. The objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. A B.F.S. to an L.P.P. corresponds to an extreme point of the convex set of feasible solutions.

Fundamental Theorem of L.P.P. (Statement only). Reduction of a feasible solution to a B.F.S. Standard form of an L.P.P. Solution by graphical method (for two variables), by simplex method and method of penalty. Concept of duality. Duality theory. The dual of the dual is the primal. Relation between the objective values of dual and the primal problems. Dual problems with at most one unrestricted variable, one constraint of equality.

Transportation and Assignment problem and their optimal solutions.

MODULE VI
(50 marks)
(Any <u>one</u> of the following groups)

Group A

Analytical Dynamics

01. Velocity and Acceleration of a particle. Expressions for velocity and acceleration in rectangular Cartesian and polar co-ordinates for a particle moving in a plane. Tangential and normal components of velocity and acceleration of a particle moving along a plane curve.

02. Concept of Force : Statement and explanation of Newton's laws of motion. Work, power and energy. Principles of conservation of energy and momentum. Motion under impulsive forces. Equations of motion of a particle (i) moving in a straight line, (ii) moving in a plane.

03. Study of motion of a particle in a straight line under (i) constant forces, (ii) variable forces (S.H.M., Inverse square law, Damped oscillation, Forced and Damped oscillation, Motion in an elastic string). Equation of Energy. Conservative forces.

04. Motion in two dimensions : Projectiles in vacuo and in a medium with resistance varying linearly as velocity. Motion under forces varying as distance from a fixed point.

05. Central orbit. Kepler's laws of motion. Motion under inverse square law.

OR

Group B
Probability and Statistics

01. Elements of Probability Theory : Random experiment, Outcome, Event, Mutually Exclusive Events, Equality like and Exhaustive, Classical definition of Probability, theorems of Total Probability, Conditional Probability and Statistical Independence. Bayes' theorem. Problems. Shortcomings of the classical definition. Axiomatic approach – Problems. Random Variable and its Expectation. Theorems on mathematical expectation. Joint distribution of two random variables.

Theoretical Probability Distribution – Discrete and Continuous (p.m.f. pd.d.f.) Binomial, Poisson and Normal distributions and their properties.

02. Elements of Statistical Methods. Variables, Attributes, Primary data and secondary data. Population and sample. Census and Sample Survey. Tabulation – Chart and Diagram, graph, Bar diagram, Pie diagram etc. Frequency Distribution – Un-grouped and grouped cumulative frequency distribution. Histogram, Frequency curve, Measure of Central Tendencies – Average : AM, GM, HM, Mean, Median and Mode (their advantages and disadvantages). Measures of Dispersions – Range, Quartile Deviation, Mean Deviation, Variance/S.D., Moments, Skewness and Kurtosis.

03. Sampling Theory : Meaning and objects of sampling. Some ideas about the methods of selecting samples. Statistic and Parameter, Sampling Distribution – standard error of a statistic (e.g. sample mean, sample proportion). Four fundamental distributions derived from the normal : (i) Standard Normal Distribution, (ii) Chi-square distribution, (iii) Student's distribution, (iv) Snedecor's F-distribution.

Estimation and Test of Significance. Statistical Inference. Theory of estimation – Point estimation and Interval estimation. Confidence Inter/Confidence Limit. Statistical Hypothesis – Bull Hypothesis and Alternative Hypothesis. Level of significance. Critical Region. Type I and Type II error. Problems.

04. Bivariate Frequency Distribution. Scatter Diagram, Correlation co-efficient – Definition and properties. Regression lines.

05. **Time Series :** Definition. Why to analyze Time series data ? Components. Measurement of Trend – (i) Moving Average Method, (ii) Curve Fittings (linear and quadratic curve). (Ideas of other curves, e.g. exponential curve etc.). Ideas about the measurement of other components.

06. **Index Number :** Meaning of Index Number. Construction of Price Index Number. Consumer Price Index Number. Calculation of Purchasing Power of Rupee.

MODULE VII

(50 marks)

Computer Science & Programming

01. **Boolean algebra –** Basic Postulates and Definition. Tow-element Boolean algebra. Boolean function. Truth table. Standard form of Boolean function – DNF and CNF. Minterms and maxterms. Principle of Duality. Some laws and theorem of Boolean algebra. Simplification of Boolean expressions – Algebraic method and Karnaugh Map method. Application of Boolean algebra – Switching Circuits, Circuit having some specified properties, Logical Gates – AND, NOT, OR, NAND, NOR etc.

02. **Computer Science and Programming :** Historical Development, Computer Generation, Computer Anatomy – Different Components of a Computer System. Operating System, Hardware and Software.

Positional Number System. Binary to Decimal and Decimal to Binary. Other systems. Binary Arithmetic. Octal, Hexadecimal, etc. Storing of data in a Computer – BIT, BYTE, WORD, etc. Coding of a data – ASCII , etc.

Programming Language : Machine Language, Assembly language and High level language. Compiler and Interpreter. Object Programme and Source Programme. Ideas about some HLL – e.g. BASIC, FORTRAN, C, C++, COBOL, PASCAL, etc.

Algorithms and Flow Charts – their utilities and important features, Ideas about the complexities of an algorithm. Application in simple problems. FORTRAN 77/99 **:** Introduction, Data Type **–** Keywords, Constants and Variables – Integer, Real, Complex, Logical, Character, Subscripted Variables, Fortran Expressions.

I/O Statements – formatted and unformatted. Programme execution control – Logical if, if-then-else, etc. Arrays, dimension statement. Repetitive Computation – Do, Bested Do etc.

Sub Programs – (i)   Function Sub Programme
　　　　　　　　(ii) Subroutine Sub Programme

Elements of BASIC Programming Language : Reading, Printing, Branch & Loop, Array, Functions.

Application to Simple Problems. An exposure to M.S. Office, e-mail, Internet (Through Demonstration only).

MODULE VIII
(50 marks)
(Any <u>one</u> of the following groups)

<u>Group A</u>

<u>A Course of Calculus</u>

01. Concept of Point-wise and Uniform convergence of sequence of functions and series of functions with special reference of Power Series. Statement of Weierstrass M-Test for Uniform convergence of sequence of functions and of series of functions. Simple applications. Statement of important properties like boundedness, continuity, differentiability and integrability of the limit function of uniformly convergent sequence of functions and of the sum function of uniformly convergent series of functions. Determination of Radius of convergence of Power Series.

    Statement of properties of continuity of sum function power series. Term by term integration and Term by term differentiation of Power Series. Statements of Abel's Theorems on Power Series. Convergence of Power Series. Expansions of elementary functions such as e^x, sin x, log(1+x), $(1+x)^n$. Simple problems.

02. Fourier series on $(-\pi, \pi)$ **:** Periodic function. Determination of Fourier co-efficients. Statement of Dirichlet's conditions of convergence and statement of the theorem on convergence of Fourier Sine and Cosine series.

03. Third and Fourth order ordinary differential equation with constant co-efficients. Euler's Homogeneous Equation.

04. Second order differential equation **:** (a) Method of variation of parameters. (b) Method of undetermined co-efficients. (c) Simple eigenvalue problem.

05. Simultaneous linear differential equation with constant co-efficients.

06. Laplace Transform and its application to ordinary differential equation. Laplace Transform and Inverse Laplace Transform. Statement of Existence theorem. Elementary properties of Laplace Transform and its Inverse. Application to the solution of ordinary differential equation of second order with constant eo-efficients.

07. Partial Differential Equation (PDE) **:** Introduction, Formation of PDE, Solutions of PDE, Lagrange's method of solution.

OR

<u>Group B</u>

<u>Discrete Mathematics</u>

01. **Integers :** Principle of Mathematical Induction. Division algorithm. Representation of integer in an arbitrary base. Prime integers. Some properties of prime integers. Fundamental theorem of Arithmetic. Euclid's Theorem. Linear Diophantine Equations. (Statement of Principle of Mathematical Induction, Strong form of Mathematical induction. Applications in different problems. Proofs of division algorithm. Representation of an integer uniquely in an arbitrary base, change of an integer from one base to another base. Computer operations with integers – Divisor of an integer, g.c.d. of two positive integers, prime integer, Proof of Fundamental theorem, Proof of Euclid's Theorem. To show how to find all prime numbers less than or equal to a given positive integer. Problems related to prime number. Linear Diophantine equation – when such an equation has solution, some applications).

02. **Congruences :** Congruence relation on integers, Basic properties of this relation. Linear Congruences, Chinese Remainder Theorem. System of Linear Congruences. (Definition of Congruence – to show it is an equivalence relation, to prove the following **:** $a \equiv b \pmod{m}$ implies (i) $(a+c) \equiv (b+c) \pmod{m}$ (ii) $ac \equiv bc \pmod{m}$ (iii) $a^n \equiv b^n \pmod{m}$, for any polynomial $f(x)$ with integral coefficients $f(a) \equiv f(b) \pmod{m}$ etc. Linear Congruence, to show how to solve these congruences, Chinese remainder theorem – Statement and proof and some applications. System of linear congruences, when solution exists – some applications).

03. **Application of Congruences :** Divisibility tests. Computer file, Storage and Hashing functions. Round-Robin Tournaments. Check-digit in an ISBN, in Universal Product Code, in major Credit Cards. Error detecting capability. (Using Congruence, develop divisibility tests for integers base on their expansions with respect to different bases, if d divides $(b-1)$ then $n = (a_k a_{k-1} a_1 b)$ is divisible by d if and only if the sum of the digits is divisible by d etc. Show that congruence can be used to schedule Round-Robin tournaments. A university wishes to store a file for each of its students in its computer. Systematic methods of arranging files have been developed based on Hashing functions $h(k) \equiv k \pmod{m}$. Discuss different properties of this congruence and also problems based on this congruence. Check digits for different identification numbers – International standard book number, universal product code etc. Theorem regarding error detecting capability).

04. **Congruence Classes :** Congruence classes, addition and multiplication of congruence classes. Fermat's little theorem. Euler's Theorem. Wilson's theorem. Some simple applications. (Definition of Congruence Classes, properties of Congruence classes, addition and multiplication, existence of inverse. Fermat's little theorem. Euler's theorem. Wilson's theorem – Statement, proof and some applications).

05. **Recurrence Relations and Generating functions :** Recurrence Relations. The method of Iteration. Linear difference equations with constant coefficients. Counting with generating functions.

06. **Boolean Algebra :** Boolean Algebra, Boolean functions, Logic gates, Minimization of circuits.