**M. A. /M.Sc. Mathematics Syllabus for Semester III and Semester IV**
**Credit Based Semester and Grading System**
**To be implemented from the Academic Year 2013-2014**

# SEMESTER III

| Course Code | UNIT | TOPICS | Credits | L/ Week |
|---|---|---|---|---|
| **ALGEBRA III** | | | | |
| **PSMT301 PAMT301** | I | Groups | 6 | 4 |
| | II | Representation of finite groups | | |
| | III | Modules | | |
| | IV | Modules over PID | | |
| **ANALYSIS II** | | | | |
| **PSMT302 PAMT302** | I | Riemann Integration | 6 | 4 |
| | II | Lebesgue Measure | | |
| | III | Lebesgue Integration | | |
| | IV | Limit Theorems | | |
| **DIFFERENTIAL GEOMETRY** | | | | |
| **PSMT303 PAMT303** | I | Geometry of $IR^n$ | 6 | 4 |
| | II | Curves | | |
| | III | Regular Surfaces | | |
| | IV | Curvature | | |
| **OPTIONAL COURSES** | | | | |
| **PSMT304 PAMT304** | | Optional Course I | 3 | 4 |
| **PSMT305 PAMT305** | | Optional Course II | 3 | 4 |

**Optional Courses I and II will be any two of the following courses:**

1. ALGEBRAIC TOPOLOGY- I
2. ADVANCED COMPLEX ANALYSIS
3. COMMUTATIVE ALGEBRA
4. NUMERICAL ANALYSIS-I
5. GRAPH THEORY-I
6. DESIGN THEORY-I
7. OPERATIONS RESEARCH
8. CODING THEORY

# SEMESTER IV

| FIELD THEORY | | | | |
|---|---|---|---|---|
| Course Code | UNIT | TOPICS | Credits | L/ Week |
| **PSMT401 PAMT401** | I | Algebraic Extensions | 6 | 4 |
| | II | Galois Extensions | | |
| | III | Galois Theorem | | |
| | IV | Applications | | |
| **FOURIER ANALYSIS** | | | | |
| **PSMT402 PAMT402** | I | Fourier Series | 6 | 4 |
| | II | Hilbert Spaces | | |
| | III | Riesz Fisher Theorem | | |
| | IV | Dirichlet Problem | | |
| **FUNCTIONAL ANALYSIS** | | | | |
| **PSMT403 PAMT403** | I | Complete Metric Space | 6 | 4 |
| | II | Normed Linear Spaces | | |
| | III | Bounded Linear Transformations | | |
| | IV | Basic Theorems | | |
| **OPTIONAL COURSES** | | | | |
| **PSMT404 PAMT404** | | Optional Course III | 3 | 4 |
| **PSMT405 PAMT405** | | Optional Course IV | 3 | 4 |

**Optional Courses III and IV will be any two of the following:**

1. DIFFERENTIAL TOPOLOGY
2. ALGEBRAIC TOPOLOGY-II
3. ALGEBRAIC NUMBER THEORY
4. NUMERICAL ANALYSIS-II
5. GRAPH THEORY-II
6. INTEGRAL TRANSFORM
7. NONLINEAR OPTIMIZATION
8. DESIGN THEORY-II
9. ADVANCED PROBABILITY THEORY

## Note(s):

1. PSMT301/PAMT301, PSMT302/PAMT302, PSMT303/PAMT303 are compulsory courses for Semester III.
2. PSMT 304 / PAMT 304 and PSMT 305/ PAMT305 are Optional Courses for SemesterIII.

3. PSMT401/PAMT401, PSMT402/PAMT402, PSMT403/PAMT403 are compulsory courses for Semester IV.

2. PSMT 404 / PAMT 404 and PSMT 405/ PAMT 405 are Optional Courses for Semester IV.

**Teaching Pattern:**

1. Four lectures per week per course (1 lecture/period is of 60 minutes duration).

2. In addition, there will be tutorials, seminars, as necessary for each of the five courses.

**M.A./ M. Sc. Mathematics Syllabus for Semester III and Semester IV**
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## Semester III

### PSMT301/PAMT301 : Algebra III

## Unit I. Groups (15 Lectures)

Simple groups, $A_5$ is simple, Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus lemma, Jordan-Holder theorem, Direct and Semi-direct products,
Examples:
(i)The group of affine transformations $X \to AX + B$ as semi-direct product of the group of linear transformations acting on the group of translations.
(ii) Dihedral group $D_{2n}$ as semi-direct product of $Z_2$ and $Z_n$.

## Unit II. Representation of finite groups (15 Lectures)

Linear representations of a finite group over complex numbers, The group ring, Complete reducibility, Characters, Orthogonality, Character tables with emphasis on examples of groups of small order.

## Unit III. Modules (15 Lectures)

Modules over rings, Submodules, Quotient modules, Free modules, Homomorphisms, kernels, Images, Cokernels, Noether isomorphism theorems, Matrix representations of homomorphisms between free modules, Abelian groups as modules over the ring of integers, Structure theorem for finitely generated abelian groups.

## Unit IV. Modules over PID (15 Lectures)

Structure theorem for finitely generated modules over a PID, Application to Jordan canonical form and Rational Canonical form.

**Reference Books:**

(1) Dummit and Foote, Abstract Algebra, John Wiley and Sons, 2010.
(2) Serge Lang, Algebra, Springer Verlag, 2004
(3) Jacobson, Basic Algebra, Dover, 1985.
(4) M. Artin, Algebra, Prentice Hall of India, 1994

### PSMT302/PAMT302 : Analysis II

## Unit I. Riemann Integration (15 Lectures)

Riemann Integration over a rectangle in $\mathbb{R}^n$, Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgue's Theorem (statement only), Fubini's Theorem and applications.

## Unit II. Lebesgue Measure (15 Lectures)

Exterior measure in $\mathbb{R}^n$, Construction of Lebesgue measure in $\mathbb{R}^n$, Lebesgue measurable sets in $\mathbb{R}^n$, The sigma algebra of Lebesgue measurable sets, Borel measurable sets, Existence of non-measurable sets.

## Unit III. Lebesgue Integration (15 Lectures)

Measurable functions, Simple functions, Properties of measurable functions, Lebesgue integral of complex valued measurable functions, Lebesgue integrable functions, Approximation of integrable functions by continuous functions with compact support.

## Unit IV. Limit Theorems (15 Lectures)

Monotone convergence theorem, Bounded convergence theorem, Fatou's lemma, Dominated convergence theorem, Completeness of $L^1$.

## Reference Books


(1) Stein and Shakarchi, Measure and Integration, Princeton Lectures in Analysis, Princeton University Press.

(2) Andrew Browder, Mathematical Analysis an Introduction, Springer Undergraduate Texts In Mathematics, 1999.

(3) Walter Rudin, Real and Complex Analysis, McGraw-Hill India, 1974.


# PSMT303/PAMT303 : Differential Geometry

## Unit I. Geometry of $\mathbb{R}^n$ (15 Lectures)

Hyperplanes in $\mathbb{R}^n$, Lines and planes in $\mathbb{R}^3$, Parametric equations, Inner product in $\mathbb{R}^n$, Orthonormal basis, Orthogonal transformations, Orthogonal matrices, The groups $SO(2)$, $SO(3)$, Reflections and rotations, Isometries of $\mathbb{R}^n$.

## Unit II. Curves (15 Lectures)

Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Signed curvature for plane curves, Curvature and torsion for space curves, Serret-Frenet equations, Fundamental theorem for space curves.

## Unit III. Regular Surfaces (15 Lectures)

Regular surfaces in $\mathbb{R}^3$, Examples, Surfaces as level sets, Surfaces as graphs, Surfaces of revolution, Tangent space to a surface at a point, Equivalent definitions, Smooth functions on a surface, Differential of a smooth function defined on a surface, Orientable surfaces.

## Unit IV. Curvature (15 Lectures)

The first fundamental form, The Gauss map, The shape operator of a surface at a point, Self adjointness of the shape operator, The second fundamental form, Principle curvatures and vectors, Euler's formula, Meusnier's Theorem, Normal curvature, Gaussian curvature and mean curvature, Computation of Gaussian curvature, Isometries of surfaces, Gauss's Theorem, Covariant differentiation, Geodesics.

## Reference Books


(1) M. DoCarmo, Differential geometry of curves and surfaces, Princeton University Press, 1976.

(2) S. Montiel and A. Ros, Curves and Surfaces, AMS Graduate Studies in Mathematics, 2009.

(3) A. Pressley, Elementary Differential Geometry, Springer UTM, 2009.

(3) J. Thorpe, Elementary Topics in Differential Geometry, Springer UTM, 2007.


# Algebraic Topology-I

## Unit I. Fundamental Group (15 Lectures)

Homotopy, Path homotopy, The fundamental group, Simply connected spaces, Covering spaces, Path lifting and homotopy lifting lemma, Fundamental group of the circle.

## Unit II. Fundamental group, Applications (15 Lectures)

Deformation retracts and homotopy types, Fundamental group of $S^n$, Fundamental group of the projective space, Brower fixed point theorem, Fundamental theorem of algebra, Borsuk-Ulam theorem.

## Unit III. Van Kampen Theorem (15 Lectures)

Free abelian groups, Free groups, Free product of groups, Seifert-van Kampen Theorem, Fundamental group of wedge of circles, Fundamental group of the torus.

## Unit IV. Covering Spaces (15 Lectures)

Equivalence of covering spaces, The lifting lemma, Universal covering space, Covering transformations and group actions, The classification of covering spaces.

**Reference Books :**


(1) James Munkres, Topology, Prentice Hall of India, 1992.

(2) Alan Hatcher, Algebraic Topology, Cambridge University Press, 2002.

(3) John Lee, Introduction to Topological Manifolds, Springer GTM, 2000.


# Advanced Complex Analysis

## Unit I. Monodromy (15 Lectures)

Holomorphic functions of one variable, Germs of functions, Analytic continuation along a path, Examples including $z^{\frac{1}{n}}$ and $\log(z)$, Homotopies between paths, The monodromy theorem.

## Unit II. Riemann Mapping Theorem (15 Lectures)

Uniform convergence, Ascoli's theorem, Riemann mapping theorem.

## Unit III. Elliptic Functions (15 Lectures)

Lattices in $\mathbb{C}$, Elliptic functions (doubly periodic meromorphic functions) with respect to a lattice, Sum of residues in a fundamental parallelogram is zero and the sum of zeros and poles (counting multiplicities) in a fundamental parallelogram is zero, Weierstrass $\wp$ function, Relation between $\wp$ and $\wp'$, Theorem that $\wp$ and $\wp'$ generate the field of elliptic functions.

## Unit IV. Zeta Function (15 Lectures)

Gamma and Riemann Zeta functions, Analytic continuation, Functional equation for the Zeta function.

## Reference Books


(1) S. Lang, Complex Analysis, Springer Paperback, 2005.

(2) John Conway, Functions of one complex variable, Narosa India, 1973.

(3) Stein and Shakarchi, Complex Analysis, Princeton Lectures in Analysis, Princeton University Press, 2003.


# Commutative Algebra

## Unit I. Basics of rings and modules (15 Lectures)

Basic operations with commutative rings and modules, Polynomial and power series rings, Prime and maximal ideals, Extension and contractions, Nil and Jacobson radicals, Chain conditions, Hilbert basis theorem, Localization, Local rings, Nakayama's lemma, Tensor products.

## Unit II. Primary decomposition (15 Lectures)

Associated primes, Primary decomposition.

## Unit III. Integral Extensions (15 Lectures)

Integral extensions, Going up and going down theorems, The ring of integers in a quadratic extension of rationals, Noether normalization, Hilbert's nullstellensatz.

## Unit IV. Dedekind Domains (15 Lectures)

Artinian rings, Discrete valuation rings, Alternative characterizations of discrete valuation rings, Dedekind domains, Fractional ideals, Factorization of ideals in a Dedekind domain, Examples.

### Reference Books


(1) Dummit and Foote, Abstract Algebra, John Wiley and Sons, 2010.

(2) S. Lang, Algebra, Springer GTM, 2004.

(3) Atiyah and McDonald, Introduction to Commutative Algebra, Addison Wesley, 1969.

## Numerical Analysis-I

### Unit I. Basics of Numerical Analysis (15 Lectures )

Error in numerical computations, Absolute, Relative and percentage errors, Round off errors, Truncation errors, Inherent errors, Representation of numbers: Binary, Octal, Decimal, Hexadecimal.

#### Unit II. Solution of Algebraic & Transcendental Equations (15 Lectures )

Iteration method, Newton-Rhapson method, Muller's method, Ramanujan's method, Chebyshev method, Rate of convergence, Solution of polynomial equations, Solutions of nonlinear equations: Seidel iteration, Newton-Rhapson method.

### Unit III. System of linear equations and solutions (15 Lectures)

Gaussian elimination, Gauss-Jordan method, Triangularization method: Crout's method, Cholesky method, Iteration methods: Gauss-Jacobi, Gauss-Seidel, Eigen value problem for matrices: Power method, Inverse power method, Jacobi or Given's method for real symmetric matrices, Singular value decomposition.

### Unit IV. Interpolation (15 Lectures)

Difference operators, Lagrangian interpolation formula, Divided difference formula, Newton's forward and backward difference interpolation formulae, Error in interpolating polynomial, Spline interpolation, Numerical differentiation, Maxima and minima of interpolating polynomial.

**Note:** Pre knowledge of $C$ or $C^{++}$ is essential.

#### Reference Books


(1) H.M.Antia, Numerical Analysis for Scientists and Engineers, TMH 1991.

(2) Jain, Iyengar, Numerical methods for Scientific and Engineering problems, New Age International, 2007.

(3) S.S.Sastry, Introductory methods of Numerical Analysis, Prentice-Hall India, 1977.

(4) K.E. Atkinson, An introduction to Numerical Analysis, John Wiley and sons, 1978.


## Graph Theory-I

### Unit I. Connectivity (15 Lectures)

Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa^{'} \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

### Unit II. Trees (15 Lectures)

Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS algorithms.

## Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)

Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

## Unit IV. Matching and Ramsey Theory (15 Lectures)

Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Hall's theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering- $\alpha + \beta = p$, Gallai's theorem, Ramsey theorem-Existence of $r(k, l)$, Upper bounds of $r(k, l)$, Lower bound for $r(k, l) \geq 2^{\frac{m}{2}}$ where $m = \min\{k, l\}$, Generalize Ramsey numbers-$r(k_1, k_2, \ . \ . \ . \ , k_n)$, Graph Ramsey theorem, Evaluation of $r(G, H)$ when for simple graphs $G = P_3$, $H = C_4$.

## Reference Books


(1) J A Bondy and U. S R Murty, Graph Theory with Applications, The Macmillan Press Springer, 1976.

(2) J A Bondy and U. S R Murty, Graph Theory, GTM 244 Springer, 2008.

(3) Mehdi Behzad and ary Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.

(4) Kenneth Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.

(5) D.B.West, Introduction to Graph Theory, PHI, 2009.


# Design Theory-I

## Unit I. Introduction to Balanced Incomplete Block Designs (15 Lectures)

What Is Design Theory? Basic Definitions and Properties, Incidence Matrices,

Isomorphisms and Automorphisms, Constructing BIBDs with Specified Automorphisms, New BIBDs from Old, Fishers Inequality.

## Unit II. Symmetric BIBDs (15 Lectures)

An Intersection Property, Residual and Derived BIBDs, Projective Planes and Geometries, The Bruck-Ryser-Chowla Theorem. Finite affine and and projective planes.

## Unit III. Difference Sets and Automorphisms (15 Lectures)

Difference Sets and Automorphisms, Quadratic Residue Difference Sets, Singer Difference Sets, The Multiplier Theorem, Multipliers of Difference Sets, The Group Ring, Proof of the Multiplier Theorem, Difference Families, A Construction for Difference Families.

**Unit IV. Hadamard Matrices and Designs (15 Lectures)**

Hadamard Matrices, An Equivalence Between Hadamard Matrices and BIBDs, Conference Matrices and Hadamard Matrices, A Product Construction, Williamsons Method, Existence Results for Hadamard Matrices of Small Orders, Regular Hadamard Matrices, Excess of Hadamard Matrices, Bent Functions.

**Reference Books**


(1) D. R. Stinson, Combinatorial Designs: Constructions and Analysis, Springer, 2004.

(2) W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall.

(3) D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cambridge, 1985.

(4) V.N. Bhat-Nayak, Design Theory Notes.

(5) T. Beth, D. Jungnickel and H. Lenz, Design Theory, Volume 1 (Second Edi- tion), Cambridge University Press, Cambridge, 1999.


## Operations Research

**Unit I. Linear Programming (15 Lectures)**

Operations research and its scope, Necessity of operations research in industry, Linear programming problems, Convex sets, Simplex method, Theory of simplex method, Duality theory and sensitivity analysis, Dual simplex method.

**Unit II.Transportation Problems (15 Lectures)**

Transportation and Assignment problems of linear programming, Sequencing theory and Travelling salesperson problem.

**Unit III. Network Analysis (15 Lectures)**

Network analysis: Shortest-path problem, Minimum spanning tree problem, Maximum flow problem, Minimum cost flow problem, Network simplex method, Project planning and control with PERT/CPM.

**Unit IV. Queuing Theory (15 Lectures)**

Queuing theory: Steady state solution of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, Game theory: Two person zero-sum games,

Games with mixed strategies, Graphical solutions, Solutions by linear programming.

**Reference Books**


(1) H.A. Taha, Operations Research- An introduction, Macmillan Publishing Co. Inc., NY.

(2) Kanti Swarup, P. K. Gupta and Man Mohan, Operations Research, S Chand and sons, New Delhi.

(3) S.S. Rao, Optimization Theory and Applications, Wiley Eastern Ltd, New Delhi.

(4) G. Hadley, Linear Programming, Narosa Publishing House, 1995.

(5) F.S. Hillier and G.J. Lieberman, Introduction to Operations Research (Sixth Edition), McGraw Hill International Edition, Industrial Engineering Series, 1995.


## Coding Theory

**Unit I. Error detection, Correction and Decoding (15 Lectures)**

Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

**Unit II. Linear codes (15 Lectures)**

Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbor decoding for linear codes, Syndrome decoding.

**Unit III. Cyclic codes (15 Lectures)**

Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

**Unit IV. Some special cyclic codes (15 Lectures)**

Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

**Reference Books**


(2) San Ling and Chaoing xing, Coding Theory- A First Course.

(5) Lid and Pilz , Applied Abstract Algebra, 2nd Edition.

## Semester IV

### PSMT401/PAMT401: Field Theory

**Unit I. Algebraic Extensions (15 Lectures)**

Definition of field extensions, Algebraic elements, Algebraic extensions, Finite extensions, Degree of an algebraic element, Minimal polynomial, Degree of a field extension, Extension of a field obtained by adjoining one algebraic element, Splitting field for a set of polynomials, Composition of two sub extensions of an extension, Existence of algebraic closure.

**Unit II. Normal and Separable Extensions (15 Lectures)**

Separable elements, Separable extensions, In characteristic 0 all extensions are separable, Separable extensions form a distinguished class, Existence of separable closure, Normal extensions, Finite fields : existence and uniqueness.

**Unit III. Galois Theory (15 Lectures)**

Galois extensions, Galois groups, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory, Galois theory - Sylow theory proof that the field of complex numbers is algebraically closed.

**Unit IV. Applications (15 Lectures)**

Straight edge and compass constructions, Impossibility of trisection of angle $\frac{\pi}{3}$, Solvability by radicals in terms of Galois group, Insolvability of a general quintic.

**Recommended books:**

(1) Dummit and Foote, Algebra: John Wiley and Sons, 2010.

(2) Serge Lang, Algebra, 2004.

(3) N. Jacobson, Algebra, Dover, 1985.

(3) M. Artin, Algebra, Prentice Hall 1994.

### PSMT402/PAMT402 : Fourier Analysis

**Unit I. Fourier Series (15 Lectures)**

Periodic Functions, Fourier series of $L^1$ functions, Riemann Lebesgue Lemma, Fourier series of periodic continuous functions, Uniqueness of Theorem, Dirichlet Kernel, Dirichlet Theorem on point wise convergence of Fourier series, Fejer Kernel, Fejer's Theorem, Denseness of trigonometric polynomials in $L^2[-\pi, \pi]$.

**Unit II. Hilbert Spaces (15 Lectures)**

Hilbert spaces, Separable Hilbert spaces, Examples, Cauchy-Schwartz inequality, Gram Schmidt, Orthonormal bases, Existence of orthonormal bases, Equivalent characterizations, Bessel's inequality, Parseval's Identity, Orthogonal decomposition.

**Unit III. Riesz Fischer Theorem (15 Lectures)**

The Hilbert space $L^2[-\pi, \pi]$, Orthonormal basis for $L^2[-\pi, \pi]$, Separability of $L^2[-\pi, \pi]$, Convergence of Fourier series in the $L^2$ norm, Best Approximation, Bessel's inequality for $L^2$ functions, The sequence space $\ell^2$, Unitary isomorphism from $L^2[-\pi, \pi]$ onto the sequence space of square summable complex sequences.

**Unit IV. Dirichlet Problem (15 Lectures)**

Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The Poisson kernel, Abel summability, Abel summability of periodic continuous functions, Weierstrass Approximation Theorem as application, Solution of Dirichlet problem.

**Reference Books**


(1) Stein and Shakarchi, Fourier Analysis an Introduction, Princeton Lectures in Analysis: Princeton University Press, 2003.

(1) Stein and Shakarchi, Real , Measure and Integration, Princeton Lectures in Analysis: Princeton University Press, 2003.

(2) Richard Beals, Analysis an Introduction:, Cambridge University Press, 2004.

(4) R.E.Edwards, Fourier Series, A Modern Introduction (Volume I):, Springer GTM, 1982.


## PSMT403/PAMT403 : Functional Analysis

**Unit I. Complete Metric Spaces (15 Lectures)**

Review of complete metric spaces, Examples, Completion of a metric space, Equicontinuity, Ascoli-Arzela Theorem, Baire spaces, Baire's theorem for compact Hausdorff and complete metric spaces, Application to a sequence of continuous functions converging point wise to a limit function.

**Unit II. Normed Linear Spaces (15 Lectures)**

Normed Linear spaces, Banach spaces, Quotient space of a normed linear space, Examples such as the sequence spaces $\ell^p$, Space of linear transformations, Function spaces $L^p$, Holder and Mankowski inequalities, Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to infinite dimensional normed linear spaces.

**Unit III. Bounded Linear Transformations (15 Lectures)**

Bounded linear transformations, Equivalent characterizations, Examples, The space $B(X,Y)$, Completeness of $B(X,Y)$ when $Y$ is complete, The dual space, Dual spaces of $\ell^1$, $\ell^p$ and $C[a,b]$.

**Unit IV. Basic Theorems (15 Lectures)**

Open mapping theorem, Closed graph theorem, Uniform boundedness Principle, Hahn-Banach Theorem and applications.

**Recommended Books**


(1) E. Keryszig, Introductory Functional Analysis with Applications, Wiely India, 2010.

(2) G. F. Simmons, Introduction to Topology and Modern Analysis, Tata Mac Grahill , 2004.

(3) W. Rudin, Real and Complex Analysis, Mcgraw hill, India, 1966.

(4) Pedersen W. K., Analysis Now, Springer GTM, 1989.

(5) Royden, Real Analysis, Macmillian, 1968.


## Differential Topology

**Unit I. Manifolds basics (15 Lectures)**

Manifolds, Sub manifolds in $\mathbb{R}^n$, Manifolds with boundary, Smooth functions, Examples of manifolds, Tangent vectors, Tangent spaces, Orientations, Oriented manifolds.

**Unit II. Multilinear Algebra (15 Lectures)**

Partitions of unity, Multilinear algebra, Vectors and tensors, Alternating tensors, The exterior product.

**Unit III. Differential Forms (15 Lectures)**

Vector fields on manifolds, Tensor fields, Differential forms, The exterior derivative, Closed and exact forms, Pull back of forms, Integration of forms, Change of variables, Poincare's lemma.

### Unit IV. Stokes Theorem (15 Lectures)

Integration of forms on manifolds, Stokes theorem for manifolds, Volume element, Integration of functions on a manifold, Classical theorems: Green's theorem, Stokes theorem for surfaces, Divergence theorem, Green's identities.

### References:


(1) Andrew Browder, Mathematical Analysis, Springer International edition, 1996.

(2) Victor Guillemin and Alan Pollack, Differential Topology, AMS Chelsea Publishing, 2010.

(3) Spivak M., Calculus on Manifolds, Cambridge University Press, 2008.

(4) James Munkres, Analysis on Manifolds, Addision Wesley, 1991 .


## Algebraic Topology-II

### Unit I. Homotopy (15 Lectures)

Homotopy, Homotopy equivalence, Deformation retracts, Criteria for homotopy equivalence, Cellular complexes, Examples.

### Unit II. Singular Homology (15 Lectures)

$\Delta$-Complexes, Simplicial Homology, Singular Homology, Homotopy Invariance.

### Unit III. Excision (15 Lectures)

Exact sequences, Excision Theorem, Equivalence of simplicial and singular homology.

### Unit IV. Computation (15 Lectures)

Mayer-Vietoris sequence, Homology computations, Applications.

### Reference Books


(1) Allen Hatche , Algebraic Topology, Cambridge University Press, 2002.

(2) James Munkres . Elements of Algebraic Topology, Addision Wesley, 1984.

(3) Joseph Rotman , An Introduction to Algebraic Topology, Springer, 1988.

## Algebraic Number Theory

### Unit I. Number Fields (15 Lectures)

Field extensions, Number fields, Algebraic numbers, Integral extensions, Ring of integers in a number field, Fractional ideals, Prime factorization of ideals, Norm of an ideal, Ideal classes, The class group, The group of units.

### Unit II. Quadratic Reciprocity (15 Lectures) .

The Legendre symbol, Jacobi symbols, The laws of quadratic reciprocity.

### Unit III. Quadratic Fields: Factorization (15 Lectures)

Quadratic fields, Real and imaginary quadratic fields, Ring of integers in a quadratic field, The group of units, Ideal Factorization in a quadratic field, Examples: The ring of Gaussian integers, The ring $\mathbb{Z}[\sqrt{-5}]$, Factorization of rational primes in quadratic fields.

### Unit IV. Imaginary Quadratic Fields: The Class Group (15 Lectures)

The Ideal class group of a quadratic field, Class groups of imaginary quadratic fields, The Minkowski lemma, The finiteness of the class group, Computation of class groups, Application to Diophantine equations.

### Reference Books:


(1) Algebraic Number Theory: T.I.F.R. Lecture Notes, 1966.

(2) P. Samuel, Algebraic Theory of Numbers, Dover, 1977.

(3) M. Artin, Algebra, Prentice-Hall, India, 2000.

(3) Marcus , Number Fields, Springer.


## Numerical Analysis-II

### Unit I. Numerical Integration (15 Lectures)

Numerical Integration: Newton-Cotes quadrature formula, Trapezoidal rule, Simpson's one third and three eightth rules, Errors in trapezoidal and Simpson's rules, Romberg's method, Gaussian quadrature, Multiple integrals.

### Unit II. Approximation of functions (15 Lectures)

Least squares approximation, Weighted least squares method, Gram-Schimdt orthogonalizing process, Least squares approximation by Chebyshev polynomials, Discrete Fourier transforms, Fast Fourier Transforms.

**Unit III. Differential Equations (15 Lectures)**

Differential equations: Solutions of linear differential equations with constant coefficients, Series solutions, Euler's modified method, Runge-Kutta methods, Predictor corrector Methods, Stability of numerical methods.

**Unit IV. Numerical Solutions of partial differential Equations (15 Lectures)**

Classification, Finite difference approximations to derivatives, Numerical methods of solving elliptic, Parabolic and hyperbolic equations.

**Reference Books**


(1) H.M.Antia , Numerical Analysis for scientists and engineers, TMH 1991.

(2) Jain, Iyengar, Numerical methods for scientific and engineering problems, New Age International, 2007.

(3) S.S.Sastry, Introductory methods of numerical analysis, Prentice-Hall India, 1977.

(3) K.E. Atkinson, An introduction to numerical analysis, John Wiley and sons, 1978.


## Graph Theory-II

**Unit I. Graph Coloring (15 Lectures )**

Line Graphs, Edge coloring-edge chromatic number, Vizing theorem, Timetabling problem, Vertex coloring- Vertex chromatic number, Critical graphs, Brook's theorem, Chromatic polynomial of a graph- $\pi_k(G) = \pi_k(G - e) - \pi_k(G \centerdot e)$ properties of chromatic polynomial of a graph, Existence of a triangle free graph with high vertex chromatic number, Mycieleski's construction.

**Unit II. Planner Graph (15 Lectures)**

Planar graph, Plane embedding of a graph, Stereographic projection, Dual of a plane graph, Euler formula, Non planarity of $K_5$ and $K_{3,3}$, Outer planar graph, Five color theorem, Sub-division, Kuratowski's theorem(Without Proof).

**Unit III. Flow Theory (15 Lectures)**

Directed graphs, Directed paths and directed cycle, Tournament, Networks, Max flow min cut theorem, Ford- Fulkerson Theorem and Algorithm.

**Unit IV. Characteristic Polynomials (15 Lectures)**

Spectrum of a graph, Characteristic polynomial of a graph, Coefficients of characteristic polynomial of a graph, Adjacency algebra $A(G)$ of a graph $G$, Dimension of $A(G) \geq diam(G) + 1$, A connected graph with diameter $d$ has at least $d + 1$ eigen values, Circulant matrix, Determination of spectrum of graphs.

**Reference Books**


(1) J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, The Macmillan Press, 1976.

(1) J. A. Bondy and U. S. R. Murty, Graph Theory GTM Springer, 2008.

(2) M. Behzad and G. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.

(3) K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.

(4) D.B.West, Introduction to Graph Theory, Prentice-Hall, India, 2009.

(5) N. Biggs, Algebraic Graph Theory, Prentice-Hall, India.


## Integral Transforms

**Unit I. Laplace Transform (15 Lectures)**

Definition of Laplace Transform, Laplace transforms of some elementary functions, Properties of Laplace transform, Laplace transform of the derivative of a function, Inverse Laplace Transform, Properties of Inverse Laplace Transform, Inverse Laplace Transform of derivatives, Convolution Theorem, Heaviside's expansion theorem, Application of Laplace transform to solutions of ODEs and PDEs.

**Unit II. Fourier Transform (15 Lectures)**

Fourier Integral theorem, Properties of Fourier Transform, Inverse Fourier Transform, Convolution Theorem, Fourier Transform of the derivatives of functions, Parseval's Identity, Relationship of Fourier and Laplace Transform, Application of Fourier transforms to the solution of initial and boundary value problems.

**Unit III. Mellin Transform (15 Lectures)**

Properties and evaluation of Mellin transforms, Convolution theorem for Mellin transform, Complex variable method and applications.

### Unit IV, Z- Transform (15 Lectures)

Definition of Z-transform, Inversion of the Z-transform, Solutions of difference equations using Z-transform.

### Reference Books


(1) Brian Davies, Integral transforms and their Applications, Springer, 1978.

(2) Larry Andrews and Bhimsen Shivamogg, Integral Transforms for Engineers, Prentice Hall of India.

(3) I.N.Sneddon, Use of Integral Transforms, Tata-McGraw Hill, 1972.

(3) Bracemell R., Fourier Transform and its Applications, MacDraw hill, 1965.


## Nonlinear Optimization

### Unit I. Unconstrained Optimization (15 Lectures)

First and second order conditions for local optima, One-Dimensional Search Methods: Golden Section Search, Fibonacci Search, Newton's Method, Secant Method, Gradient Methods, Steepest Descent Methods.

### Unit II. Algorithms for Unconstrained Optimization Problems (15 Lectures)

Newton's Method, Conjugate Direction Algorithm, Conjugate Gradient Algorithm, Quasi-Newton methods: Quasi-Newton Algorithm, Rank One Algorithm, the DFP algorithm, the BFGS algorithm.

### Unit III. Constrained Optimization Problems (15 Lectures)

Problems with equality constraints, Tangent and normal spaces, Lagrange Multiplier Theorem, Second order conditions for equality constraints problems, Problems with inequality constraints, Karush-Kuhn-Tucker Theorem, Second order necessary conditions for inequality constraint problems.

### Unit IV. Convex Optimization Problems (15 Lectures)

Convex sets, Convex and Strictly Convex Functions, Characterization of Convex Functions, Differentiable Convex Functions, Characterization of Differentiable Convex Functions, Optimization of Convex Functions.

**Reference Books**

(1) Chong and Zak, Introduction to Optimization, Wiley-Interscience, 1996.

(2) Rangarajan and K. Sundaram, A First Course in Optimization Theory, Cambridge University Press.

## Design Theory-II

### Unit I. Resolvable BIBDs (15 Lectures)

Introduction, Affine Planes and Geometries, Resolvability of Affine Planes, Projective and Affine Planes, Affine Geometries, Boses Inequality and Affine Resolvable BIBDs, Symmetric BIBDs from Affine Resolvable BIBDs, Orthogonal Resolutions.

### Unit II. Latin Squares (15 Lectures)

Latin Squares and Quasigroups, Steiner Triple Systems, The Bose Construction, The Skolem Construction, Orthogonal Latin Squares, Mutually Orthogonal Latin Squares, MOLS and Affine Planes, MacNeish's Theorem, Orthogonal Arrays, Orthogonal Arrays and MOLS, Some Constructions for Orthogonal Arrays, Transversal Designs, Wilsons Construction, Disproof of the Euler Conjecture.

### Unit III. Pairwise Balanced Designs (15 Lectures)

Definitions and Basic Results, Necessary Conditions and PBD-Closure, Steiner Triple Systems, $(v, 4, 1)$-BIBDs, Kirkman Triple Systems, The Stanton-Kalbfleisch Bound, The Erdös-de Bruijn Theorem, Improved Bounds, PBDs and Projective Planes, Minimal PBDs with $\lambda > 1$.

### Unit III. $t$-Designs and $t$-wise Balanced Designs (15 Lectures)

Basic Definitions and Properties of $t$-Designs, Some Constructions for $t$-Designs with $t \geq 3$, Inversive Planes, Some 5-Designs, $t$-wise Balanced Designs, Holes and Subdesigns.

**Reference Books**

(1) Douglas R. Stinson , Combinatorial Designs: Constructions and Analysis, Springer, 2004.

(2) W.D. Wallis, Introduction to Combinatorial Designs, (2nd Ed), Chapman & Hall

(3) D. R. Hughes and F. C. Piper, Design Theory, Cambridge University Press, Cambridge, 1985.

(4) V.N. Bhat-Nayak, Design Theory Notes.

(5) T. Beth, D. Jungnickel and H. Lenz , Design Theory, Volume 1 (Second Edi- tion), Cambridge University Press, Cambridge, 1999.

## Advanced Probability Theory

**Unit I. Probability Basics (15 Lectures)**
Probability spaces, Random variables and their moments, Distribution functions and the measures they induce on the real line, Binomial, Poisson and Normal distributions, Conditional probability and independence, Borel-Cantelli Lemmas, Kolmogorov zero-one law, Formulation and statement (without proof) of Kolmogorov consistency theorem.

**Unit II. Laws of Large Numbers (15 Lectures)**
Various types of convergence of random variables, Chebyshev inequality and weak law of large numbers, Kolmogorov inequality and the strong law of large numbers, Conditional expectation and its distance minimizing property.

**Unit III. Characteristics Functions, Weak Convergences (15 Lectures)**
Weak convergence and equivalent definitions, Helley's Lemma, Characteristic functions and moment generating functions, Independence and convolution of distributions, Bochner's theorem.

**Unit IV. Central Limit Theorem (15 Lectures)**
Levy's continuity theorem and his inversion formula, Proof of the Central Limit Theorem, Cramer's theorem on large deviations, Definition and examples of martingales, Statement (without proof) of the Martingale Convergence Theorem.

**Reference Books**

(1) R.M.Dudley, Real Analysis and Probability: Cambridge studies in advanced mathematics.

(2) Patrick Billingsley, Probability and Measure: John Wiley and sons.

(3) Jeffrey Rosenthal , A first look at rigorous probability theory: World Scientific

(4) Marek Capinski , Measure Integral and Probability: Peter E. Kopp, Springer

The scheme of examination for the revised course in the subject of Mathematics at the M.A./M.Sc. Part II Programme (Semesters III & IV) will be as follows.

**Scheme of Examination**

In each semester, the performance of the learners shall be evaluated into two parts. The learners performance in each course shall be assessed by a mid- semester test of 30 marks, active participation 05 marks and attendance 05 marks in the first part, by conducting the Semester End Examinations with 60 marks in the second part.

**External Theory examination of 60 marks:**

1. Duration: - Theory examination shall be of $2\frac{1}{2}$ Hours duration.
2. Theory Question Paper Pattern:-
   (a) There shall be five questions each of 12 marks.
   (b) On each unit there will be one question and the fifth one will be based on entire syllabus.
   (c) All questions shall be compulsory with internal choice within each question.
   (d) Each question may be subdivided into sub-questions a, b, c, and the allocations of marks depend on the weightage of the topic.
   (e) Each question will be of 18 marks when marks of all the subquestions are added (including the options) in that question.

| **Questions** | | **Marks** |
|---|---|---|
| Q1 | Based on Unit I | 12 |
| Q2 | Based on Unit II | 12 |
| Q3 | Based on Unit III | 12 |
| Q4 | Based on Unit IV | 12 |
| Q4 | Based on Unit I, II, III, IV | 12 |
| | Total Marks | 60 |