Perfect

SSC Higher Order Thinking Skills

HOTS

Fuel Your Brain...

Based on Std. X
Maharashtra Board Syllabus

Mathematics and Science

Target Publications Pvt. Ltd.

Written according to the New Text book (2014-2015) published by the Maharashtra State Board of Secondary and Higher Secondary Education, Pune.

SSC Higher Order Thinking Skills
HOTS

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Salient Features

- Written as per the new textbook.
- Exhaustive coverage of entire syllabus.
- Facilitates complete and thorough preparation of HOTS section.
- Variety of questions provided in order of importance.
- Mark-wise segregation of each lesson.
- Constructions drawn with accurate measurements.
- Board questions of HOTS with solutions updated till the latest year.
- Self evaluative in nature.

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Preface

HOTS stands for Higher Order Thinking Skills. As the name implies, this book is filled to the brim with questions that challenge your Thinking Skills. Unlike the traditional methods that focus on drill and repetition as a mode of education, HOTS brings out the problem solving ability of a child.

Inclusion of HOTS in a Question Paper is to attest if the students factually have an in-depth understanding of the said topic. It is not just how much but how well you understand the subject.

These twisted and brain challenging questions (HOTS) that form a part of the Question Paper demand an out of the box thinking. Students are assessed through this format of Questions in terms of skills pertaining to their analyzing, reasoning, comprehending, application and evaluation of a subject.

This book is full of such thought provoking and curiosity crunching questions across the subjects of Algebra, Geometry and Science. The Questions throughout are framed in an innovative format including flow-charts. The emphasis in this courseware lies not just on the subject knowledge but also its applications in real life.

With a progressive thought we have thoroughly followed the new syllabus pattern while composing this book. There is no doubt that in the long run this book would be a proven aid for students to encounter questions pertaining to Higher Order Thinking Skills.

We wish all the aspirants all the best and hope this book turns out to be a formidable aid in your tryst with success.

Best of luck to all the aspirants!

Yours faithfully

Publisher

Contents

01 Arithmetic Progression

1 Mark Questions

1. Find t_2, if a = 5 and d = –2.

***Solution*:**

Here a = 5, d = –2

$\therefore$ $t_1 = a = 5$

$t_2 = t_1 + d = 5 + (-2)$

$\therefore$ $t_2 = 3$

$\therefore$ $\mathbf{t_2 = 3}$

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2. If $S_n = nP + \frac{1}{2}n(n-1)Q$ where S_n denotes the sum of first n terms of an A.P. Find the common difference of the A.P.

***Solution*:**

Sum of first n terms of an A.P. is given by

$S_n = \frac{n}{2}[2a + (n-1)d] = na + \frac{1}{2}n(n-1)d$

Compare with $S_n = nP + \frac{1}{2}n(n-1)Q$

$\therefore$ **d = Q**

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3. Find t_n of the following A.P.

$\mathbf{\frac{5}{6}, 1, 1\frac{1}{6}, \ldots, t_n}$

***Solution*:**

The given A.P. is $\frac{5}{6}, 1, 1\frac{1}{6} \ldots\ldots$

Here, $a = \frac{5}{6}$, $d = \frac{1}{6}$ ....[$\because$ $d = t_2 - t_1$]

$\therefore$ $t_n = a + (n-1)d$

$= \frac{5}{6} + (n-1) \times \frac{1}{6}$

$= \frac{5}{6} + \frac{n-1}{6}$

$\therefore$ $\mathbf{t_n = \frac{n+4}{6}}$

4. Find the first term of the following sequence if

$$\mathbf{S_n = \frac{(4n-3)3^n + 3}{2}}$$

***Solution*:**

$S_n = \frac{(4n-3)3^n + 3}{2}$

$\therefore$ When n = 1,

$S_1 = \frac{(4 \times 1 - 3) \times 3^1 + 3}{2}$

$= \frac{(4-3) \times 3 + 3}{2} = \frac{1 \times 3 + 3}{2} = \frac{6}{2}$

$\therefore$ $S_1 = 3$

$\therefore$ **The first term is 3.**

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5. Find the value of d for an A.P. if $t_5 = 11$ and $t_6 = 13$.

Solution:

$d = t_6 - t_5 = 13 - 11 = 2$

$\therefore$ **d = 2**

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2 Marks Questions

6. Find the sum of all natural numbers lying between 50 and 500, which are multiple of 5.

***Solution*:**

Required numbers are 55, 60, 65, 70, ..., 495

This is an A.P with

a = 55, d = 60 – 55 = 5

Now, $t_n = a + (n-1)d = 495$

$\therefore$ $55 + (n-1) \times 5 = 495$

$\therefore$ $5(n-1) = 495 - 55$

$\therefore$ $n - 1 = \frac{440}{5}$

$\therefore$ $n - 1 = 88$

$\therefore$ $n = 89$

Here, $t_1 = 55$, $t_n = 495$, n = 89

$S_n = \frac{n}{2}(t_1 + t_n) = \frac{89}{2}(55 + 495) = \frac{89}{2} \times 550$

$\therefore$ $\mathbf{S_n = 24475}$

7. The first, second and last terms of an A.P. are a, b and 2a. Find the number of terms in A.P.

***Solution*:**

a, b, ..., 2a are in A.P.

Since, $t_n = a + (n - 1)d$

$\therefore$ $2a = a + (n - 1)(b - a)$

$\therefore$ $2a - a = (n - 1)(b - a)$

$\therefore$ $a = (n - 1)(b - a)$

$\therefore$ $n - 1 = \frac{a}{b - a}$

$\therefore$ $n = \frac{a}{b-a} + 1 = \frac{a + b - a}{b - a}$

$\therefore$ $\mathbf{n = \frac{b}{b - a}}$

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8. Find the sum of all three digit numbers which leave the remainder 3 when divided by 5.

***Solution*:**

The sequence is 103, 108, 113, …., 998

The sequence is an A.P. with

$a = 103$, $d = 5$, $t_n = 998$

Since, $t_n = a + (n - 1)d$

$\therefore$ $998 = 103 + (n - 1)5$

$\therefore$ $998 = 103 + 5n - 5$

$\therefore$ $5n = 900$

$\therefore$ $n = 180$

$\therefore$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{180}{2}[2 \times 103 + (180 - 1)5]$

$= 90[206 + 179 \times 5] = 90 \times 1101$

$\therefore$ $\mathbf{S_n = 99090}$

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9. If for an A.P., $S_n = 0.02 (2^n - 1)$, find t_n.

***Solution*:**

$S_n = 0.02 (2^n - 1)$

We know that,

$t_n = S_n - S_{n-1}$

$t_n = 0.02(2^n - 1) - [0.02 (2^{n-1} - 1)]$

$= 0.02 (2^n - 1) - [0.02 (2^n \cdot 2^{-1} - 1)]$

$= 0.02 \times 2^n - 0.02 - \frac{0.02 \times 2^n}{2} + 0.02$

$= 0.02 \times 2^n - 0.01 \times 2^n$

$\therefore$ $\mathbf{t_n = 0.01 \times 2^n}$

10. The sum of the first ten terms of an A.P. is three times the sum of the first five terms, then find ratio of the first term to the common difference.

***Solution*:**

We know that,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$\therefore$ $S_{10} = 5(2a + 9d)$

$S_5 = \frac{5}{2}[2a + (5 - 1)d]$

$\therefore$ $S_5 = \frac{5}{2}(2a + 4d)$

According to given condition,

$S_{10} = 3S_5$

$5(2a + 9d) = 3 \times \frac{5}{2}(2a + 4d)$

$\therefore$ $10(2a + 9d) = 15(2a + 4d)$

$\therefore$ $2(2a + 9d) = 3(2a + 4d)$

$\therefore$ $4a + 18d = 6a + 12d$

$\therefore$ $2a = 6d$

$\therefore$ $\mathbf{\frac{a}{d} = \frac{3}{1}}$

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3 Marks Questions

11. How many terms of an A.P. $-6, \frac{-11}{2}, -5, \ldots$ are needed to give the sum -25? Explain the reason for getting two answers.

***Solution*:**

Here, $t_1 = a = -6$, $d = \frac{1}{2}$ ….[$\because$ $d = t_2 - t_1$]

Let -25 be the sum of n terms of this A.P. where $n \in N$.

$\therefore$ $S_n = -25$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore$ $-25 = \frac{n}{2}\left[2 \times -6 + (n - 1)\frac{1}{2}\right]$

$\therefore$ $-25 = \frac{n}{2}\left[\frac{n - 25}{2}\right]$

$\therefore$ $-25 = \frac{n^2 - 25n}{4}$

$\therefore$ $-25 \times 4 = n^2 - 25n$

$\therefore$ $-100 = n^2 - 25n$

$\therefore$ $n^2 - 25n + 100 = 0$

$\therefore$ $(n - 5)(n - 20) = 0$

$\therefore$ $n - 5 = 0$ or $n - 20 = 0$

$\therefore$ **$n = 5$ or $n = 20$**

Both the values of n are natural numbers and hence two answers are obtained.

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12. Divide 45 into three terms which are in A.P. in such a way that the product of the last two terms is 255.

***Solution*:**

Let the three terms in A.P. be $a - d$, a, $a + d$.

$\therefore$ $a - d + a + a + d = 45$

$\therefore$ $3a = 45$

$\therefore$ $a = 15$

$\therefore$ The three terms are $15 - d$, 15, $15 + d$

It is given that the product of last two terms is 255.

$\therefore$ $a \times (a + d) = 255$

$\therefore$ $a + d = \dfrac{255}{a}$

$\therefore$ $15 + d = \dfrac{255}{15}$ ....[$\because$ $a = 15$]

$\therefore$ $15 + d = 17$

$\therefore$ $d = 2$

$\therefore$ **Three terms are 13, 15 and 17.**

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13. If the p^{th} term of an A.P. is q and the q^{th} term is p, prove that n^{th} term is $(p + q - n)$.

***Solution*:**

Let 'a' be the first term and 'd' be the common difference of the given A.P.

$t_p = q$ ....[Given]

$\therefore$ $a + (p - 1)d = q$ ....(i) [$\because$ $t_n = a + (n - 1) d$]

and $t_q = p$ ....[Given]

$\therefore$ $a + (q - 1)d = p$ ....(ii) [$\because$ $t_n = a + (n - 1) d$]

Subtracting equation (ii) from (i), we get

$(a - a) + (p - 1)d - d(q - 1) = q - p$

$\therefore$ $d(p - 1 - q + 1) = q - p$

$\therefore$ $d(p - q) = -(p - q)$

$\therefore$ $d = -1$

Substituting value of d in equation (i), we get

$a + (p - 1)(-1) = q$

$\therefore$ $a - p + 1 = q$

$\therefore$ $a = p + q - 1$

$t_n = a + (n - 1)d$

$= (p + q - 1) + (n - 1)(-1)$

$= p + q - 1 - n + 1$

$\therefore$ **$t_n = p + q - n$**

14. Find 31^{st} term of A.P. whose 11^{th} term is 38 and 16^{th} term is 73.

Solution:

Let 'a' be the first term and 'd' be the common difference.

$\therefore$ $t_{11} = a + 10d = 38$ ....(i)

and $t_{16} = a + 15d = 73$ ....(ii)

Subtracting equation (ii) from (i), we get

$$\begin{array}{r} a + 10d = 38 \\ a + 15d = 73 \\ - \quad - \qquad - \\ \hline -5d = -35 \end{array}$$

$\therefore$ $d = 7$

Substituting $d = 7$ in equation (i), we get

$t_{11} = a + 10d = 38$

$\therefore$ $a + 10 \times 7 = 38$

$\therefore$ $a = 38 - 70$

$\therefore$ $a = -32$

$t_{31} = a + 30d$

Substituting the value of a and d, we get

$t_{31} = -32 + 30 \times 7$

$\therefore$ $t_{31} = -32 + 210$

$\therefore$ $t_{31} = 178$

$\therefore$ **31^{st} term = 178**

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15. Which term of A.P. 3, 15, 27, 39, ... will be 132 more than its 54^{th} term?

Solution:

Given series 3, 15, 27, 39, ...

$a = 3$, $d = 12$ [$\because$ $d = t_2 - t_1 = 15 - 3 = 12$]

$\because$ $t_n = a + (n - 1)d$

$\therefore$ $t_{54} = 3 + (54 - 1)12$

$= 3 + (53 \times 12)$

$\therefore$ $t_{54} = 639$

Let us consider the required term as t_x.

$t_x = 132 + t_{54}$ ...[Given]

$\therefore$ $t_x = 132 + 639$

$\therefore$ $t_x = 771$

$\because$ $t_x = a + (x - 1)d$

$\therefore$ $771 = 3 + (x - 1)12$

$\therefore$ $771 = 3 + 12x - 12$

$\therefore$ $771 = 12x - 9$

$771 + 9 = 12x$

$\therefore$ $780 = 12x$

$\therefore$ $x = 65$

$\therefore$ **Thus, the 65^{th} term of A.P. 3, 15, 27 ... will be 132 more than its 54^{th} term.**

16. There are 20 rows of seat in a concert hall with 20 seats in first row, 21 seats in second row, 22 seats in third row and so on. Calculate the total number of seats in that concert hall?

Solution:

As per given condition, the sequence of number of seats in a hall is an A.P. with $t_1 = 20$, $n = 20$ and $d = 1$

$\therefore$ $S_n = \frac{n}{2} \times [2a + (n-1)d]$

$= \frac{20}{2} \times [2 \times 20 + (20-1) \times 1]$

$= 10 \times [40 + 19]$

$= 10 \times 59$

$\therefore$ $S_n = 590$

$\therefore$ **There are 590 seats in hall.**

4 Marks Questions

17. A man repays a loan of ₹ 3250 by paying ₹ 305 in the first month and then decreases the payment by ₹ 15 every month. How long will it take to clear his loan?

***Solution*:**

Here, $a = 305$, $d = -15$, $S_n = 3250$

Let time required to clear the loan be n months.

$S_n = \frac{n}{2} [2a + (n - 1)d]$

$\therefore$ $3250 = \frac{n}{2} [2 \times 305 + (n - 1)(-15)]$

$\therefore$ $6500 = n(610 - 15n + 15)$

$\therefore$ $6500 = n(625 - 15n)$

$\therefore$ $6500 = 625n - 15n^2$

$\therefore$ $15n^2 - 625n + 6500 = 0$

$\therefore$ $3n^2 - 125n + 1300 = 0$

$\therefore$ $3n^2 - 60n - 65n + 1300 = 0$

$\therefore$ $3n(n - 20) - 65(n - 20) = 0$

$\therefore$ $n - 20 = 0$ or $3n - 65 = 0$

$\therefore$ $n = 20$ or $n = \frac{65}{3}$

Since n is a natural number.

$\therefore$ $n \neq \frac{65}{3}$

$\therefore$ $n = 20$

$\therefore$ **The time required to clear the loan is 20 months.**

18. If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then show that sum of (m + n) terms is –(m + n).

***Solution*:**

Let 'a' be the first term and 'd' be the common difference of A.P.

$S_n = \frac{n}{2} [2a + (n - 1) d]$

$\therefore$ According to the given condition,

$S_m = n$

$\therefore$ $n = \frac{m}{2} [2a + (m - 1)d]$

$\therefore$ $2n = m[2a + md - d]$

$\therefore$ $2n = 2am + m^2d - md$ ….(i)

Also, $S_n = m$

$\therefore$ $m = \frac{n}{2} [2a + (n - 1)d]$

$\therefore$ $2m = n[2a + nd - d]$

$\therefore$ $2m = 2an + n^2d - nd$ ….(ii)

Subtracting (ii) from (i), we get

$2am - 2an + m^2d - n^2d - md + nd = 2n - 2m$

$\therefore$ $2a(m - n) + d(m^2 - n^2) - d(m - n) = 2(n - m)$

$\therefore$ $(m - n)[2a + (m + n)d - d] = -2(m - n)$

$\therefore$ $(m - n)[2a + (m + n - 1)d] = -2(m - n)$

$\therefore$ $[2a + (m + n - 1)d] = -2$ ….(iii)

$S_{m+n} = \frac{m+n}{2} [2a + (m + n - 1)d]$

$= \frac{m+n}{2} \times (-2)$ ….[From (iii)]

$\therefore$ $\mathbf{S_{m+n} = -(m + n)}$

19. A contract on construction job specifies a penalty for delay of completion beyond a certain limit as follows:
₹ 200 for first day,
₹ 250 for second day,
₹ 300 for third day, etc.
If the contractor pays ₹ 27,750 as penalty, find the numbers of days for which the construction work is delayed.

***Solution*:**

Here, $a = 200$, $d = 50$, $S_n = 27750$

$S_n = \frac{n}{2} [2a + (n - 1)d]$

$\therefore$ $27750 = \frac{n}{2} [2 \times 200 + (n - 1)50]$

$\therefore$ $27750 = \frac{n}{2} [350 + 50n]$

$\therefore$ $27750 = 175n + 25n^2$

$\therefore$ $25n^2 + 175n - 27750 = 06$

$\therefore$ $n^2 + 7n - 1110 = 0$

$\therefore$ $n^2 + 37 - 30n - 1110 = 0$

$\therefore$ $n(n + 37) - 30(n + 37) = 0$

$\therefore$ $(n + 37)(n - 30) = 0$

$\therefore$ $n \neq -37$ or $n = +30$

$n \neq -37$ as no. of days can not be negative.

$\therefore$ $n = +30$

$\therefore$ **Number of days for which the construction work is delayed = 30 days**

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20. The interior angles of a polygon are in arithmetic progression. The smallest angle is 52° and the common difference is 8°. Find the number of sides of the polygon.

Solution:

Let 'n' be the number of sides of the polygon.

Sum of all interior angles of polygon

$= (n - 2) \times 180°$

Here, $a = 52$, $d = 8$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ $(n - 2) \times 180 = \frac{n}{2}[2 \times 52 + (n - 1)8]$

$\therefore$ $180n - 360 = \frac{n}{2}[104 + 8n - 8]$

$\therefore$ $180n - 360 = \frac{n}{2}[96 + 8n]$

$\therefore$ $360n - 720 = 96n + 8n^2$

$\therefore$ $8n^2 + 96n - 360n + 720 = 0$

$\therefore$ $8n^2 - 264n + 720 = 0$

$\therefore$ $n^2 - 33n + 90 = 0$

$\therefore$ $(n - 30)(n - 3) = 0$

$\therefore$ $n - 30 = 0$ or $n - 3 = 0$

$\therefore$ $n = 30$ or $n = 3$

But, when $n = 30$, the last angle is

$t_n = a + (n - 1)d$

$\therefore$ $52° + (30 - 1)8° = 284°$, which is not possible as interior angle of a polygon cannot be more than 180°.

$\therefore$ **Number of sides of given polygon is 3.**

21. A man set out on a cycle ride of 50 km. He covers 5 km in the first hour and during each successive hour his speed falls by $\frac{1}{4}$ km/hr. How many hours will he take to finish his ride?

***Solution*:**

Here, $a = 5$, $S_n = 50$, $d = -\frac{1}{4}$

Let number of hours required to finish the ride be 'n'.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\therefore$ $50 = \frac{n}{2}\left[2 \times 5 + (n - 1)\left(-\frac{1}{4}\right)\right]$

$\therefore$ $50 = \frac{n}{2}\left[10 + \frac{1}{4} - \frac{n}{4}\right]$

$\therefore$ $100 = n\left[\frac{41}{4} - \frac{n}{4}\right]$

$\therefore$ $100 = n \times \left(\frac{41 - n}{4}\right)$

$\therefore$ $400 = 41n - n^2$

$\therefore$ $n^2 - 41n + 400 = 0$

$\therefore$ $(n - 25)(n - 16) = 0$

$\therefore$ $n - 25 = 0$ or $n - 16 = 0$

$\therefore$ $n = 25$ or $n = 16$

If $n = 25$, speed would be negative.

$\therefore$ $n = 16$

$\therefore$ **16 hours are required to finish the ride.**

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5 Marks Questions

22. Prove that the sequence $S_n = 2n^2 + 5n$ is in A.P. Hence find t_n.

***Solution*:**

$S_n = 2n^2 + 5n$

For $n = 1$,

$S_1 = 2(1)^2 + 5(1) = 2 + 5 = 7$

$S_1 = 7 = t_1$

For $n = 2$,

$S_2 = 2(2)^2 + 5(2) = 2 \times 4 + 10 = 18$

$\therefore$ $S_2 = 18$

$t_2 = S_2 - S_1 = 18 - 7 = 11$

For $n = 3$,

$S_3 = 2(3)^2 + 5(3)$

$= 2 \times 9 + 15$

$= 33$

$\therefore$ $S_3 = 33$

$t_3 = S_3 - S_2 = 33 - 18 = 15$

Now,

$t_2 - t_1 = 11 - 7 = 4$

$t_3 - t_2 = 15 - 11 = 4$

$\therefore$ The sequence is an A.P.

Here $a = t_1 = S_1 = 7$ and $d = 4$

$\therefore$ $t_n = a + (n - 1)d = 7 + (n - 1)4 = 7 + 4n - 4$

$\therefore$ $\mathbf{t_n = 4n + 3}$

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23. If the ratio of the sum of m terms and n terms of an A.P. be $m^2 : n^2$, prove that the ratio of m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

***Solution*:**

It is given that $\dfrac{S_m}{S_n} = \dfrac{m^2}{n^2}$

$$\frac{\left(\frac{m}{2}\right)[2a+(m-1)d]}{\left(\frac{n}{2}\right)[2a+(n-1)d]} = \frac{m^2}{n^2}$$

$\therefore$ $\dfrac{2a+(m-1)d}{2a+(n-1)d} = \dfrac{m}{n}$

$\therefore$ $\left.\begin{array}{ll} 2a + (m - 1)d = km & \ldots.(i) \\ 2a + (n - 1)d = kn & \ldots.(ii) \end{array}\right\}$[k is constant]

Subtracting equation (ii) from (i), we get

$(m - 1)d - d(n - 1) = km - kn$

$\therefore$ $md - d - nd + d = k(m - n)$

$\therefore$ $d(m - n) = k(m - n)$

$\therefore$ $d = k$

Substituting value of d in equation (i), we get

$2a + (m - 1)k = km$

$\therefore$ $2a + mk - k = km$

$\therefore$ $2a = k$

$\therefore$ $a = \dfrac{k}{2}$

Now, $t_m = a + (m - 1)d$ and $t_n = a + (n - 1)d$

$\therefore$ $\dfrac{t_m}{t_n} = \dfrac{a+(m-1)d}{a+(n-1)d}$

Substituting value of a and d, we get

$$\frac{t_m}{t_n} = \frac{\left(\frac{k}{2}\right)+(m-1)k}{\left(\frac{k}{2}\right)+(n-1)k}$$

$$= \frac{k\left(\frac{1}{2}+m-1\right)}{k\left(\frac{1}{2}+n-1\right)}$$

$$= \frac{\frac{2m-1}{2}}{\frac{2n-1}{2}}$$

$\therefore$ $\mathbf{\dfrac{t_m}{t_n} = \dfrac{2m-1}{2n-1}}$

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24. Find the number of terms common to A.P. 3, 7, 11, …., 407 and A.P. 2, 9, 16, …., 709.

Solution:

Let the number of terms in A.P. 3,7,11,….,407 be m and the number of terms in A.P. 2,9,16,….,709 be n.

$\therefore$ $t_m = 407$ and $t_n = 709$

As $t_m = a + (m - 1)d$

$\therefore$ $407 = 3 + (m - 1)4$

$\therefore$ $407 = 3 + 4m - 4$

$407 = 4m - 1$

$408 = 4m$

$\therefore$ $m = 102$

$t_n = a + (n - 1)d$

$\therefore$ $709 = 2 + (n - 1)7$

$\therefore$ $709 = 2 + 7n - 7$

$\therefore$ $709 = 7n - 5$

$\therefore$ $714 = 7n$

$\therefore$ $n = 102$

$\therefore$ Each A.P. consists of 102 terms.

Let p^{th} term of 1^{st} A.P. be equal to q^{th} term of the 2^{nd} A.P.

$\therefore$ $3 + (p - 1)4 = 2 + (q - 1)7$

$3 + 4p - 4 = 2 + 7q - 7$

$\therefore$ $4p - 1 = 7q - 5$

$\therefore$ $4p + 4 = 7q$

$4(p + 1) = 7q$

$\therefore$ $\dfrac{p+1}{7} = \dfrac{q}{4}$

Let $\dfrac{p+1}{7} = \dfrac{q}{4} = k$ $\quad (k \neq 0)$

$\therefore$ $p = 7k - 1$ and $q = 4k$

Since each A.P. consists of 102 terms.

$p \leq 102$ and $q \leq 102$

$\therefore$ $7k - 1 \leq 102$ and $4k \leq 102$

$\therefore$ $k \leq 14\frac{5}{7}$ and $k \leq 25\frac{1}{2}$

$\therefore$ $k \leq 14$

$\therefore$ $k = 1, 2, 3, 4, \ldots, 14$

For each value of k, we get a pair of identical terms.

$\therefore$ **There are 14 common terms.**

25. Insert five number between 4 and 8 so that the resulting sequence is an A.P.

Solution:

Let the required numbers be t_2, t_3, t_4, t_5 and t_6

Thus 4, t_2, t_3, t_4, t_5, t_6, 8 are in A.P.

In this case $t_7 = 8$

$t_1 = a = 4$, $t_n = 8$, $n = 7$

We know that

$t_n = a + (n - 1)d$

$8 = 4 + (7 - 1)d$

$8 = 4 + 6d$

$4 = 6d$

$\therefore \quad d = \frac{2}{3}$

$t_2 = a + (2 - 1)d$

$= 4 + (1) \times \frac{2}{3} = 4 + \frac{2}{3}$

$\therefore \quad t_2 = \frac{14}{3}$

$t_3 = 4 + (3 - 1) \times \frac{2}{3}$

$= 4 + 2 \times \frac{2}{3} = 4 + \frac{4}{3}$

$\therefore \quad t_3 = \frac{16}{3}$

$t_4 = 4 + (4 - 1) \times \frac{2}{3} = 4 + 3 \times \frac{2}{3} = 4 + 2$

$\therefore \quad t_4 = 6$

$t_5 = 4 + (5 - 1) \times \frac{2}{3}$

$= 4 + 4 \times \frac{2}{3} = 4 + \frac{8}{3}$

$\therefore \quad t_5 = \frac{20}{3}$

$t_6 = 4 + (6 - 1) \times \frac{2}{3}$

$= 4 + 5 \times \frac{2}{3} = 4 + \frac{10}{3}$

$= \frac{12 + 10}{3} = \frac{22}{3}$

$\therefore \quad t_6 = \frac{22}{3}$

$\therefore$ **The required numbers are $\frac{14}{3}$, $\frac{16}{3}$, 6, $\frac{20}{3}$, $\frac{22}{3}$**

26. If the sum of first m terms of an A. P. is equal to the sum of first n terms then show that the sum of its first (m + n) terms is zero where m ≠ n. [March 2014]

Solution:

The sum of first n terms of an A.P. is given by

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_m = \frac{m}{2}[2a + (m - 1)d]$

But $S_m = S_n$ …. [Given]

$\therefore \quad \frac{m}{2}[2a + (m - 1)d] = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad \frac{m}{2}[2a + md - d] = \frac{n}{2}[2a + nd - d]$

$\therefore \quad \frac{m}{2}[2a + md - d] - \frac{n}{2}[2a + nd - d] = 0$

$\therefore \quad \frac{1}{2}[2am + m^2d - md - 2an - n^2d + nd] = 0$

$\therefore \quad 2am + m^2d - md - 2an - n^2d + nd = 0$

$\therefore \quad (2am - 2an) + (m^2d - n^2d) - (md - nd) = 0$

$\therefore \quad 2a(m - n) + d(m^2 - n^2) - d(m - n) = 0$

$\therefore \quad 2a + d(m + n) - d = 0$ …. [Dividing by (m – n)]

$\therefore \quad 2a + d(m + n - 1) = 0$

Multiplying by $\frac{m+n}{2}$ on both sides, we get

$\frac{m+n}{2}[2a + (m + n - 1)d] = 0$ …. (i)

But, $S_{m+n} = \frac{m+n}{2}[2a + (m + n - 1)d]$ …. (ii)

From (i) and (ii), we get

$\mathbf{S_{m+n} = 0}$