Department of Economics

Jadavpur University

M.A. Admission Test Examination, 2015

Time: 2 hours 30 minutes **Full marks: 100**

Answer the questions in Group A following the instructions given in Group A. Then answer three questions taking one from each group: B, C and D. Maximum marks are given in the margin.

Group A

Answer all questions 1 to 5 from Section I. Choose any five questions from Section II.

Section I: Choose the correct answer.

1. A publisher from its experience knows that the demand for the forthcoming title of author X would be: p = 1000 – 2Q. The marginal cost of producing the book is Rs. 20. X receives 10% of revenue earned from the book as royalty.

i) The publisher and author will agree about the pricing of the book
ii) The publisher's price choice will be higher than that of the author's
iii) The publisher's price choice will be lower than that of the author's

2. A potential new drug, NoSmak, could cure lip-smacking by one doze, but research and development would cost $80 million. Under patent the profit of the innovating firm would be $10 million per year. After the patent expires the innovating firm will have sufficient brand loyalty to earn $3 million per year for another 10 years. The shortest length of patent required to induce a firm in innovating the drug would be:

i) 5 years
ii) 8 years
iii) 0 years
iv) 10 years

3. Suppose π_c , π_d , π_m represent the profits of a firm in perfectly competitive market, duopoly market and monopoly market:

i) $\pi_c > \pi_d > \pi_m$
ii) $\pi_c < \pi_d < \pi_m$
iii) $\pi_c > \pi_d < \pi_m$
iv) $\pi_c < \pi_d > \pi_m$

4. Consider an exchange economy with two individuals and two commodities. The endowments of the commodities are 1 unit each. Which one of the following allocations are Pareto optimum:

i) {(½, ½), (½, ½)}
ii) {(¼, ¼), (½, ½)}
iii) {(¾, ¾), (½, ½)}
iv) none of the above

5. "Requiring airline travelers to spend at least one Saturday night away from home to qualify for a low fare" is an example of:

i) Uniform pricing
ii) First degree price discrimination
iii) Second degree price discrimination
iv) Third degree price discrimination

2 x 5 =10

Section II: Give short answers

6. Consider a perfectly competitive industry. Let the total cost function facing each of the 5 firms in the industry be given by $TC = 20 + 0.5q^2$ where q is quantity produced by a typical firm. Let the industry demand curve be given by $Q = 60 - p$ where Q is industry demand and p denotes price. Determine equilibrium price and quantity. What is the total producer surplus generated in equilibrium?

7. In a pure exchange economy of 2 persons Ram is endowed with two oranges whereas Lata is endowed with two bananas. Given that their utility functions are identically given by $U = xy$ where x stands for quantity of oranges and y stands for the quantity of bananas, what is the competitive equilibrium price if they are allowed to trade?

8. An individual has an income of Rs. 20 which he spends on buying apples and oranges. When the price of an apple is Rs. 5 and price of an orange is Rs.2 he buys 2 apples and 5 oranges. When the price of an apple is Rs. 6 and price of an orange is Re.1 he buys 2 apples and 8 oranges. Are his purchases consistent with the Weak Axiom of Revealed Preference? Justify your answer.

9. Consider the production function: q = min[l, k], where q is the quantity of the output of good Q, and l and k are the quantities of the two inputs L and K respectively. Let w and r, where w $\in$ (0, ∞) and r $\in$ (0, ∞), be the prices of inputs L and K respectively. Show that the cost function for this production function C(q) is given by:

$$C(q) = (w+r)q.$$

10. Explain briefly why the Cost function of a firm is analogous to the Expenditure function of a consumer.

11. Consider the following two person game. There are two players: Row Player and Column Player. The Row Player has two action choices: T or B; and the Column has two action choices: T or B. The payoff vectors show the payoff to the Row Player first and the payoff to the Column Player second.

Column Player

		L	R
Row Player	T	10, 10	50, 8
	B	8, 50	20, 20

Define: Dominant strategy, and show that the dominant strategy equilibrium in this case is (T, L). 3 X 5=15

Group B

12. Assume the production function for the economy is given by $Y= L^{0.5}K^{0.5}$, where Y denotes output, K denotes the capital stock and L denoteslabour. The evolution of the capital stock is given by

$K_{t+1} = (1-\delta)\, K_t + I_t$

,where δ lies between 0 and 1 and is the rate of depreciation of capital. I represents investment, given by $I_t = sY_t$, where s is the savings rate.Derive the condition for steady state growth, and find out the savings rate that maximizes steady state consumption per capita.

10+15=25

13. An economy has the following Relations:

Production function: $Y = A(5N - 0.0025N^2)$
Labour Supply function: $N = 55 + 5w$
Consumption function: $C = 300 + 0.8(Y-T) - 200r$
Investment function: $I = 258.5 - 250r$
Tax Function: $T = 20 + 0.5Y$
$G=50$
Money demand function: $M^d/P = 0.5Y - 250i$
Rate of inflation = 0.02= constant
Money supply = 9150

Where, C= consumption demand, I = Investment demand, T = Income tax, Y = Output, M^d= Money demand, P= Price level, r = Real Rate of interest, I = Nominal rate of interest, w= Real wage

(i) Find equation of IS curve.
(ii) Find equation of LM curve.
(iii) Determine full employment level of output.
(iv) Determine equilibrium values for real wage, employment and price at the full employment level of output.
(v) Under full flexibility of wage and price, find the changes in the value of consumption and investment when G rises to 72.5. Explain your result.

5+5+4+6+5 = 25

GROUP C

14. Consider the utility maximization problem:

$$\max_{x,y} U(x,y) \equiv \frac{1}{\alpha}x^{\alpha} + \frac{1}{\beta}y^{\beta} \text{ subject to } \quad px + qy = W$$

a. Find the first-order conditions that characterize an optimum. 2

b. Calculate the comparative statics for x and y with respect to p and W. Are the signs of effects sensible? 4

c. Use envelope theorem to calculate the effect of a change in W on the value function. 3

d. Suppose the amount of utility achieved by optimal choice of consumption in this problem is U^*. Notice that utility cannot be maximized in this problem unless the total amount spent to achieve is minimized. Rewrite the problem as an expenditure minimization problem subject to constraint that utility must be at least U^*; call the multiplier in this problem μ. 3

e. Show that set of first order conditions resulting from the "dual" expenditure minimization problem is equivalent to those resulting from the original "primal" utility maximization problem. 3

f. What is the relation between the multipliers $\lambda \ \ and \ \ \mu$ from the original primal and dual problems? 3

g. Use the envelope theorem to calculate the effect of a change in U^* on the dual problem. Explain your answer. 3

h. This problem can be solved explicitly if $\alpha = \beta = 1/2$. Find the solution of the primal problem for this case. Find an explicit formula for the value function. Calculate its derivative with respect to W and verify that your answer is the same as in (c). 4

15. Check whether the following statements are true or false (You have to show the required derivations to find the result)

i) The level curves of the plane $z = d - (ax + by)$, where a; b; c; $d \neq 0$, are not parallel lines in the xy-plane. 3

ii) The domain of the function $g(x; y) = \ln((x + 1)^2 + (y - 2)^2 - 1)$ consistsof all points (x; y) lying strictly outside of a circle centered at (1; 2) of radius 1. 3

iii) The following function has a limiting value at x=2 and y=-1

$$f(x, y) = \frac{x^2 + 2xy - xy + 2y^2}{x + 2y}$$ 2

iv) The Following function is homogenous and homothetic

$f(x, y) = xy/(1 + xy)$ 2

v) The following function has a relative maxima in the interval $0 < x < .5$

$$f(x) = \frac{500(2x - 1)}{1 + x^2}$$ 5

vi) The following problem does not have a solution:

$\max imise \quad (x^3 + y^3) \quad$ subject to $x + y = 1$ 5

vii) In the following optimization problem , at the optimum first non-negativity constraint will be binding

Max $f(x, y) = x^2 + x + 4y^2$

Subject to $2x + 2y \leq 1$ and $x \geq 0 \quad y \geq 0$ 5

GROUP D

16.

a) The following data represent observations on work experience in years and the salary structure of 32 sample points.

Y= Salary in thousand Rupees

X= Year of experience

Y	**X**	**Y**	**X**	**Y**	**X**	**Y**	**X**
54	32	44	22	45	18	51	12
51	32	43	21	50	17	50	12
39	30	46	20	37	17	62	10
52	26	42	20	61	16	39	10
55	25	56	19	48	16	43	9
41	23	55	19	30	16	40	7
47	22	53	19	51	15	37	6
63	43	55	18	40	13	27	3

(i) Calculate the regression of **Y = a+bX**

(ii) Test the hypothesis b=0, at 5% level of significance and give economic interpretation of the results.

(iii) Find out the goodness of fit of the model and explain the implication of the result.

6+4+3=13

b.

i. In a factory, machines A, B and C manufacture 25%, 35% and 40% respectively of the total of their capacity; the defective items produced by A, B and C are 5%, 4% and 2% respectively. One item (product) is drawn at random and is found to be defective. What is the probability that it is manufactured by machine C?

ii. Show that the correlation between y(observed value) and its predicted value (Y) is non-negative and must be numerically the same as the correlation between y and x.

6 X2=12

17.

a) Justify whether each of the following statement is true or false :

(i) In classical linear regression model the explanatory variables are assumed to be correlated with the disturbance term of the equation.

(ii) Under the presence of heteroscedasticity the estimates of the parameters will have minimum variance.

(iii) In a least square regression of y on x, observations for which x is close to the mean value will have more effect on the estimated slope than the observations for which x is close to its mean value.

(iv) In a least square regression of y on x, the higher is the value of the estimated slope coefficient, higher will be the degree of correlation between y and x.

3x4=12

b)

i) Find the probability distribution of the number of boys in a family with three children, assuming equal probabilities for boys and girls. Find the distribution function F(x) for the random variable, X (viz. number of boys)

ii) If X=4Y+5 and Y=KX +4 are the two regression equations of "X on Y" and "Y on X" respectively, then find the maximum possible value of K.

iii) The mode of a certain frequency curve y=f(x) is attained at x=9.1 and the values of frequency function f(x) for x=8.9, 9.0 and 9.3 are respectively equal to 0.30, 0.35, 0.25. Find the approximate value of f(x) at the mode.

5+4+3=13