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A bar with insulated sides is initially at a temperature 0°C throughout. The end $x = 0$ is kept at 0°C , and heat is suddenly applied at the end $x = l$ so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is a constant. Find the temperature $u(x, t)$.

5. a) Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point $P(1, 1, 1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$
- b) Prove that vector $f(r)\vec{r}$ is irrotational.
- c) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2, z = 0$.
- d) Verify Stoke's theorem for the vector $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy -plane.

OR

Evaluate $\int_S \vec{A} \cdot \hat{n} \, dS$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

Roll No ...

BE - 301

B.E. III Semester

Examination, December 2015

Mathematics - II

(Common for all Branches)

Time : Three Hours

Maximum Marks : 70

- Note:** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.
 ii) All parts of each question are to be attempted at one place.
 iii) All questions carry equal marks, out of which part A and B (Max. 50 words) carry 2 marks, part C (Max. 100 words) carry 3 marks, part D (Max. 400 words) carry 7 marks.
 iv) Except numericals, Derivation, Design and Drawing etc.
1. a) Expand $\pi x - x^2$ in a half-range sine series in the interval $(0, \pi)$ upto the first three terms.
 b) Find the Fourier transform of Dirac Delta Function $\delta(t-a)$.
 c) Find the function whose sine transform is $\frac{e^{-s}}{s}$.
 d) Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = \begin{cases} 0 & \text{for } -\pi \leq x \leq 0 \\ \sin x & \text{for } 0 \leq x \leq \pi \end{cases}$$

Deducé that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi-2}{4}$.

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OR

Develop $\sin\left(\frac{\pi x}{l}\right)$ in half-range cosine series in the range $0 < x < l$.

2. a) Find $L\{F(t)\}$ if $F(t) = \begin{cases} \sin\left(t - \frac{\pi}{3}\right), & t > \frac{\pi}{3} \\ 0, & t < \frac{\pi}{3} \end{cases}$

b) Define Unit Step Function and Find its Laplace Transform.

c) Prove that: $L^{-1}\left(\frac{s}{s^4 + s^2 + 1}\right) = \frac{2}{\sqrt{3}} \sin h \frac{t}{2} \sin \frac{\sqrt{3}}{2} t$.

d) State convolution theorem and hence evaluate

$$L^{-1}\left\{\frac{p}{(p^2 + 1)(p^2 + 4)}\right\}$$

OR

Solve the simultaneous equations using Laplace Transform:

$$\frac{dx}{dt} - y = e^t, \quad \frac{dy}{dt} + x = \sin t, \quad \text{given } x(0) = 1, y(0) = 0.$$

3. a) Solve the differential equation by Removal of first derivative method.

$$\frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$$

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b) Solve by changing the independent variable

$$(1+x^2)^2 \frac{d^2 y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} + 4y = 0$$

c) Solve by the method of variation of parameters:

$$\frac{d^2 y}{dx^2} - y = \frac{2}{1+e^x}$$

d) Solve in series the equation

$$2x(1-x) \frac{d^2 y}{dx^2} + (5-7x) \frac{dy}{dx} - 3y = 0$$

OR

Solve in series the equation

$$(1-x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0.$$

4. a) Solve the differential equation

$$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx.$$

b) Solve $p^2 - q^2 = x - y$.

c) Solve $(D^2 + 5DD' + 6D'^2)z = \frac{1}{y-2x}$.

d) If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity

$$\left(\frac{dy}{dt}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}, \quad \text{find the displacement } y(x, t).$$

OR