FLUID MECHANICS

LAMINAR FLOW AND VISCOUS FLOWS

Viscosity plays an important role.
Low velocity flows.
Laminar flow - where each fluid layer glides over the adjacent layer.
Shear stress (τ) = $\mu(du/dy)$.
Ex., Flow of viscous fluid through circular pipe, two parallel plates, bearings etc.,

NO SLIP CONDITION

In ideal fluids, when fluid passes over a boundary, it slips over the boundary and velocity distribution is uniform over the boundary.
In real fluids, due to viscosity, there is no relative motion between the boundary and fluid. The fluid at the boundary has the same velocity as the boundary – This is known as “No Slip Condition”.

In a real fluid flow with stationary boundary, the velocity is zero at the boundary and increases as we go away from the boundary. This change in velocity gives rise to a velocity gradient and hence the viscous shear resistance opposing the motion. Due to this resistance to motion, power is required to maintain flow of real fluids. Hence, in many fluid flow problems, effect of viscosity cannot be neglected near the boundaries.

LAMINAR AND TURBULENT FLOWS

Depending upon the relative magnitudes of the viscous forces and inertia forces, flow can exist in two types- Laminar Flow and Turbulent Flow

REYNOLDS EXPERIMENTS

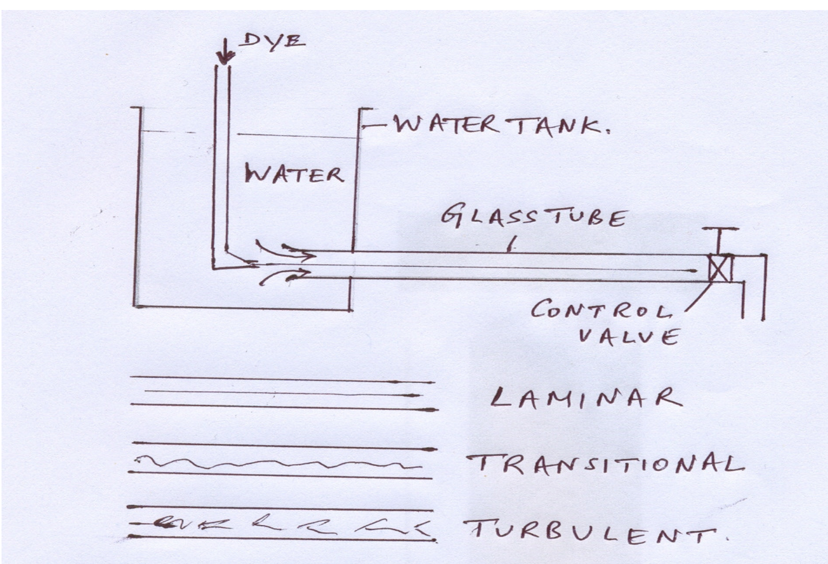


Fig. Reynolds Experiments

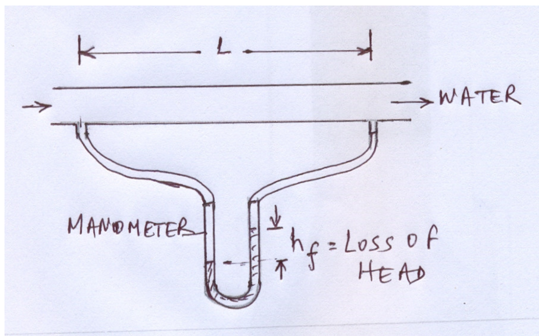
The nature of dye filament was observed at different velocities:

1. At low velocities, the dye remained in the form of straight stable filament parallel to the axis of the tube: (a) – The flow is laminar.
2. At higher velocities, dye filament showed irregularities and wavy nature: (b)- The flow is transitional.
3. With further increase in velocity, the filaments become more and more irregular, and finally dye is diffused over the complete cross section: (c) The flow is turbulent.

At low velocities, flow takes place in number of sheets or laminae. This flow is called Laminar Flow. At high velocities, the flow is disturbed and inter-mixing of particles takes place. The flow is called Turbulent Flow.

TYPES OF FLOW AND LOSS OF HEAD

Loss of head, h_f is measured in a pipe of length (L) for various values of velocity (v) in the pipe and (h_f/L) vs (v) is plotted in a log – log plot



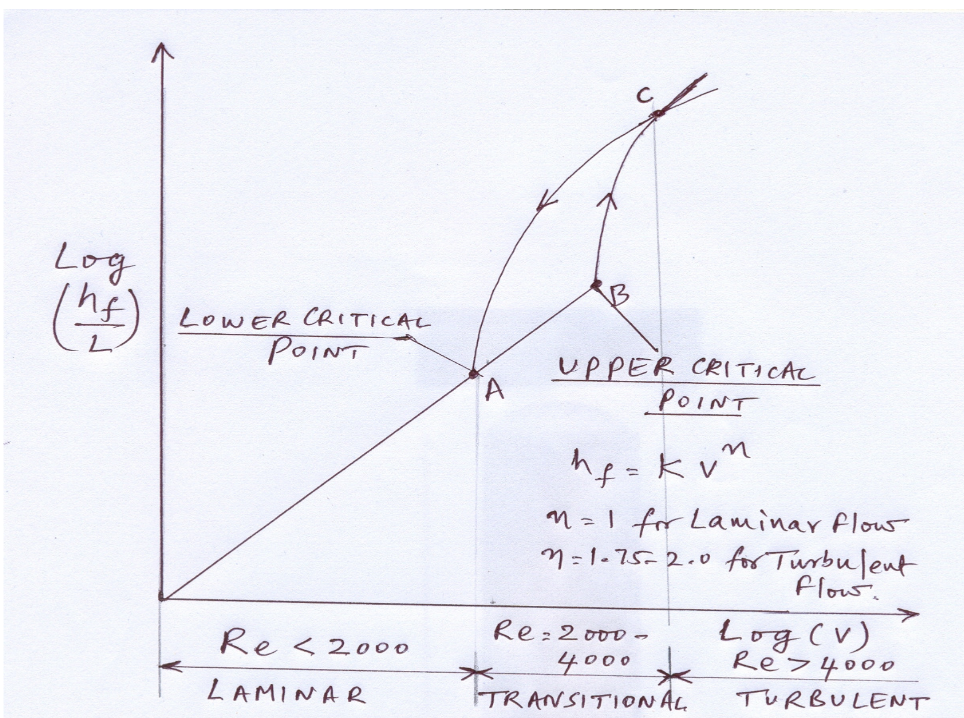


Fig. Types of Flow and Loss of Head

Loss of head (h_f) is measured in a pipe of length (L) for various values of flow velocity (v) in the pipe and (h_f/L) Vs (v) are plotted on a log – log scale. At low velocities, up to point (B), the curve is a straight line. (h_f) is proportional to (v) up to point (B). We see transition up to point (C). After (C) again, the curve obtained has a slope varying from 1.75 to 2.

Up to (B), it is one type of flow called the laminar flow in which (h_f) is proportional to (v). Beyond (C), it is another type of flow in which (h_f) is proportional to (V^n) where n=1.75 to 2. This is called turbulent flow. However, if the velocity is reduced from a high value, line BC is not retraced. Instead, the points lie along line CA.

Point (B) is called as higher (or upper) critical point and the corresponding velocity is called as upper critical velocity. Point (A) is called as lower critical point and the corresponding velocity is called as lower critical velocity. Reynolds Number, which is the ratio of inertia force to viscous force is the criterion which decides whether the flow is laminar or turbulent.

Re= ($\rho VL/\mu$)= (VL/ν). For pipes, L=d, the diameter of the pipe which is a characteristic dimension.

The Upper Critical Reynolds Number corresponding to point (B) is not definite. Its value depends upon how carefully the initial disturbance affecting the flow is prevented. Normally, Upper Critical Reynolds Number for pipe flow is about 4000. (Note: with proper precaution, values as high as 50,000 can be achieved.

The Lower Critical Reynolds Number corresponding to point (A) is definite. For a straight pipe, its value is about 2000. This Reynolds Number is the true Critical Reynolds Number, which is the dividing line between the laminar and turbulent flows. The Reynolds Number below which the flow is definitely laminar is called the Critical Reynolds Number (For pipe flow, Re (critical) around 2000.

LAMINAR FLOW

Definition: The flow in which the particles of fluid behave in orderly manner with out intermixing with each other and the flow takes place in number of sheets, layers or laminae, each sliding over the other is called as laminar flow.

Characteristics of Laminar Flow:

1. Particle of fluid behave in disciplined manner. No inter-mixing of particle.Flow takes place in layers which glide over one another.
2. Velocity of flow at a point is nearly constant in magnitude and direction.
3. Viscous force plays an important role in fluid flow (as compared to other forces).
4. Shear stress is obtained by the Newton's Law of Viscosity.
5. Any disturbance caused is quickly damped by viscous forces
6. Due to No-slip condition, velocity across the section is not uniform. Velocity gradient and hence, the shear stress gradient is established at right angles to the direction of flow.
7. Loss of head is proportional to the velocity of flow.
8. Velocity distribution is parabolic in nature (pipe flow)

Practical examples of laminar flow: Flow of oil in lubricating mechanisms, capillary tubes, blood flow in vein etc.,

SHEAR AND PRESSURE GRADIENT IN LAMINAR FLOW

Because of No-Slip Condition at the wall, different layers move over each other with different velocities in the flow near the wall. The relative motion between the layers gives rise to shear stress. Shear stress varies from layer to layer and it is maximum at the wall.

Shear stress (τ) = $\mu(du/dy)$

Shear stress gradient exists across the flow. Also, along the flow, pressure will vary to maintain the flow and pressure gradient exists along the flow.

RELATION BETWEEN SHEAR AND PRESSURE GRADIENTS IN LAMINAR FLOW

Consider the free body of the fluid element with sides (dx, dy, dz) as shown in the Fig. For ex, in the flow inside a pipe.

(τ) = Shear stress; p=pressure

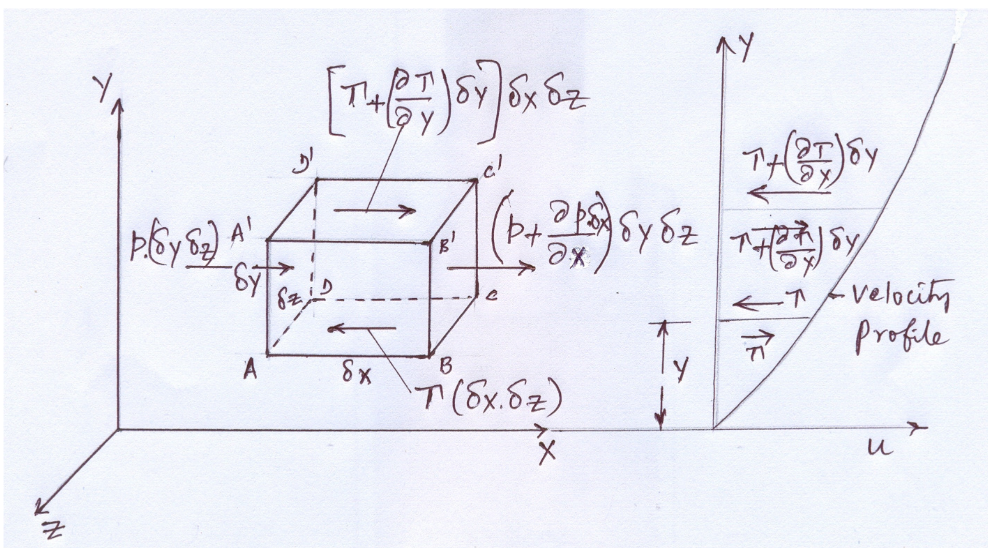


Fig. Shear and Pressure Gradients in Laminar Flow

Various forces acting on the element are shown in Fig. For steady uniform flow, there is no acceleration and the sum of forces acting in the direction of motion must be equal to zero. (Forces in +(x) direction are taken as positive)

Pressure forces + Shear forces =0

$[p.\,dydz - \{p + (\partial p/\partial x)dx\}dydz] + [\{\tau+(\partial\tau/\partial y)dy\}\,dxdz - \tau dxdz] = 0$

Simplifying,

$-(\partial p/\partial x)dxdydz + (\partial\tau/\partial y)dxdydz = 0$; Dividing by dxdydz, the volume of the parallelepiped,

$$(\partial p/\partial x) = (\partial\tau/\partial y)$$

For a two- dimensional steady uniform laminar flow, the pressure gradient in the direction of flow is equal to the shear stress gradient in the normal direction. Since p=p(x) and $\tau = \tau$ (y) only, and according to the Newton's law of viscosity, $(\tau) = \mu(du/dy)$; we get $(\partial p/\partial x) = \mu(\partial^2 u/\partial y^2)$

Problems on steady uniform laminar flows can be analyzed by integrating this equation.

LAMINAR FLOW THROUGH A CIRCULAR PIPE

Consider a steady laminar flow of fluid through a horizontal circular pipe as shown. Consider a concentric cylindrical element having radius (r) and length (dx)
Note: Shear stress resists motion.

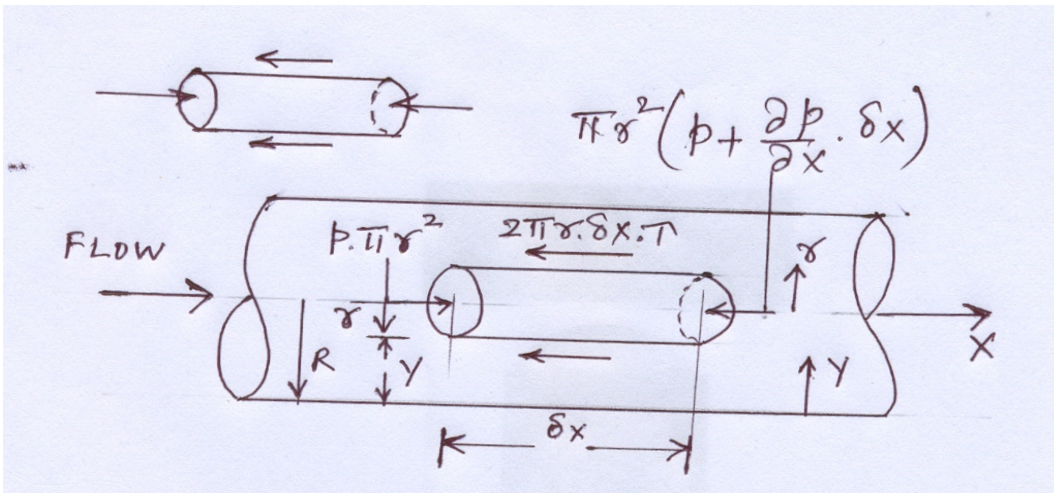


Fig. Laminar Flow through Circular Pipe

Since the flow is steady and uniform, there is no acceleration and the sum of all forces acting on the element in x-direction must be zero.

$p.\pi r^2 - (p+(\partial p/\partial x)dx)\,\pi r^2 - \tau 2\pi r dx = 0$

Or $\quad -(\partial p/\partial x)dx\,\pi r^2 - \tau 2\pi r dx = 0$

Or $\quad \tau = -(\partial p/\partial x)(r/2)$; Gives the variation of shear stress with respect to radius.

At the center, r=0, and shear stress is zero. At the pipe wall, r=R, the shear stress is maximum. $\tau_o = -(\partial p/\partial x)(R/2)$

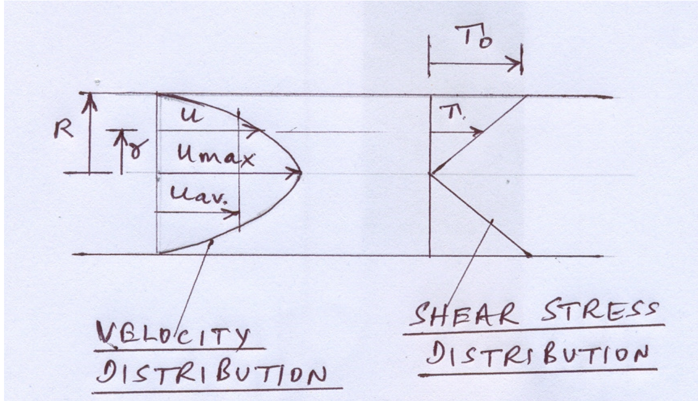


Fig. Velocity and Shear Stress Distributions

VELOCITY DISTRIBUTIONS

According to the Newton's Law of Viscosity,
$(\tau) = \mu(du/dy)$; But $y= (R- r)$ and $\partial y = -\partial r$
Therefore, $(\tau)= -\mu(\partial u/\partial r) = -(\partial p/\partial x)(r/2)$
Therefore, $(\partial u/\partial r) = (1/2\mu)(\partial p/\partial x)r$
Integrating w.r.t. (r) we get,
$u= (1/2\mu)(\partial p/\partial x)(r^2/2) +C$; where C=Constant
$(\partial p/\partial x)$ is independent of (r)
Now, let us find constant C
At r = R, that is at the wall, u=0.
$0=(1/4\mu)(\partial p/\partial x)R^2 +C$
Or $C= -(1/4\mu)(\partial p/\partial x) R^2$
$u=(1/4\mu)(\partial p/\partial x)(r^2-R^2)$ or
$u=(1/4\mu)(- \partial p/\partial x)(R^2-r^2)$

Since μ, $(\partial p/\partial x)$ and R are constant, u varies with square of r.
Thus, for steady laminar flow through a circular pipe, the velocity variation across the section is parabolic in nature.
At r=R, u=0
At r=0,i.e.,at center of the pipe, $u=u_{max}$
$u_{max} = (R^2/4\mu)(- \partial p/\partial x)$
$u=(1/4\mu)(- \partial p/\partial x)(R^2-r^2)$
$=(R^2/4\mu)(-\partial p/\partial x)[1-(r/R)^2]$
But $(R^2/4\mu)(- \partial p/\partial x)= u_{max}$ Therefore, $u= u_{max} [1-(r/R)^2]$
Gives the velocity distribution in the pipe, which is parabolic.

DISCHARGE AND AVERAGE VELOCITY

Discharge (q) across a section is found by integrating the discharge (dq) passing through an annular ring of width (dr) situated at a distance (r) from the center. Discharge through the annular ring=area of the ring X velocity at radius (r)

$dq=2\pi r\, dr\, [(1/4\mu)(-\partial p/\partial x)(R^2-r^2)]$

Total discharge,

$q = {}_0\int^R (R^2-r^2)\, r\, dr\, [(1/4\mu)(-\partial p/\partial x)\, 2\pi]$

$=(\pi/2\mu)(-\partial p/\partial x)\, [(R^2 r^2/2)-(r^4/4)]_0^R$

$=(\pi/2\mu)(-\partial p/\partial x)\, [(R^4/2)-(R^4/4)]$

$q= (\pi R^4/8\mu)(-\partial p/\partial x)$

Average velocity= u_{av}

$=(q/\text{area of pipe})=(q/\pi R^2)$

$u_{av}= (\pi R^4/8\mu)(-\partial p/\partial x)(1/\pi R^2)$

$= (R^2/8\mu)(-\partial p/\partial x)$; But $(R^2/4\mu)(-\partial p/\partial x)= u_{max}$. Therefore, $u_{av} =[u_{max}/2]$

Thus, in case of steady laminar flow through a circular pipe, average velocity is half of max. Velocity.

The radius at which the local velocity is equal to the average velocity is given by:

$u= u_{max}[1-(r/R)^2]= u_{av}= (u_{max}/2)$

$[r^2/R^2]= 0.5$ or $r=0.707R$

Thus the average velocity occurs at a radial distance of 0.707R from the center of the pipe.

Pressure drop over a given pipe length: We know that,

$u_{av} = (R^2/8\mu)(-\partial p/\partial x)$

$(-\partial p/\partial x)= [(8\mu\, u_{av})/R^2]$

$-dp=[(8\mu\, u_{av})/R^2]\, dx$ Integrating from 1 to 2

$\int -dp = \int [(8\mu\, u_{av})/R^2]\, dx$ At 1, $p=p_1$, $x=x_1$; At 2, $p=p_2$, $x=x_2$

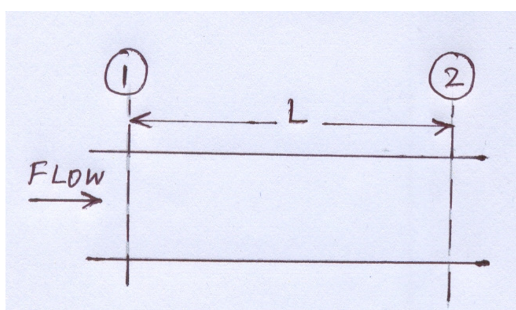


Fig. Pressure Drop over a Length of Pipe.

$(p_1 - p_2) = [(8\mu\, u_{av})/R^2](x_2 - x_1)$

$L= (x_2 - x_1)$

$(p_1 - p_2) = [(8\mu\, u_{av})/R^2](L)$ --------Eq.(1)

Since D=2R, $(p_1 - p_2) = (32\mu\, u_{av} L/D^2)$

Further, $u_{av} = [4q/\pi D^2]$; Substituting for u_{av} , We get

$(p_1 - p_2) = [128\mu qL/(\pi D^4)]$ -----Eq.(2)

Equations.(1) or (2) are called Hagen-Poiseuille's equation for steady uniform laminar flow through a circular pipe.

To obtain H-P equation, we can also use

$q = (\pi R^4 /8\mu)(-\partial p/\partial x)$

$-dp = (8\mu q/\pi R^4)dx = (128\mu q/\pi D^4)dx$

Integrating between (1) and (2)

$(p_1 - p_2) = [128\mu qL/\pi D^4]$

But $(4q/\pi D^2) = u_{av}$

Therefore,

$(p_1 - p_2) = [32\mu u_{av}L/D^2]$ - -----Eq.(3)

Another version of Hagen-Poiseuille's Equation

LAMINAR FLOW THROUGH A CIRCULAR PIPE

For a two- dimensional steady uniform laminar flow, the pressure gradient in the direction of flow is equal to the shear

LOSS OF HEAD

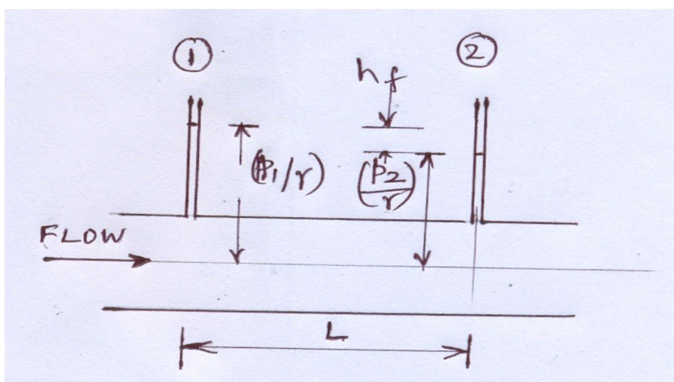


Fig. Loss of Head

Specific weight $(\gamma) = \rho g$

$(p_1 - p_2) = [32\mu u_{av}L/D^2]$

Loss of Head, $h_f = [(32\mu u_{av}L)/(\gamma D^2)]$

$= [(128\mu qL)/(\gamma\pi D^4)]$

FRICTION FACTOR (f)

The loss of head due to friction in pipe is given by Darcy- Weisbach equation
$h_f = [(4fLu_{av}^2)/(2gD)] = [(32\mu u_{av}L)/(\rho gD^2)]$
Simplifying, we get
$f = 16(\mu/rDu_{av}) = [16 / R_e]$; Therefore, the value of the friction factor for a steady laminar flow through a circular pipe, $f = [16 / R_e]$

POWER REQUIRED TO MAINTAIN THE FLOW

Power = Rate of doing work = (Force × Distance) /Time = Force × Velocity
Force = $(-\partial p/\partial x)AL$ where A= Area of Pipe and
L = Length of pipe.
Power (P)= $(-\partial p/\partial x)AUL$; where AU = q, the discharge, U= Average Flow Velocity.
$(-\partial p/\partial x) = (p_1 - p_2)/L$; Therefore,
Power (P) = $(p_1 - p_2)q$; But $(p_1 - p_2) = \rho gh_f$
Therefore, Power (P) = ρgqh_f

For laminar flow through an inclined pipe,

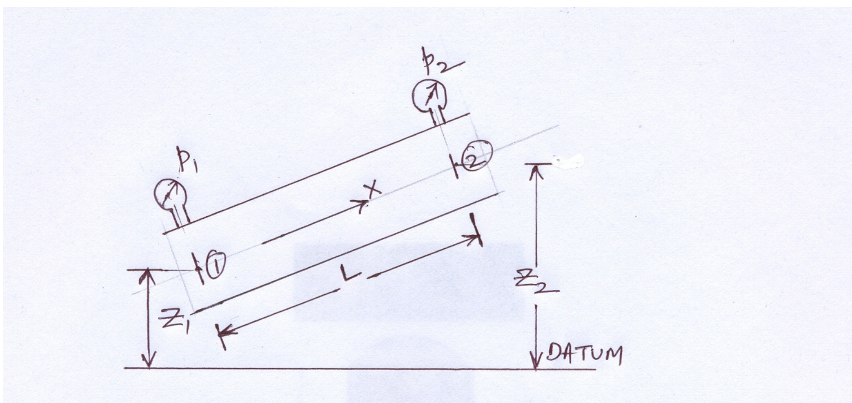


Fig. Laminar Flow through an Inclined Pipe,

We have $u=(1/4\mu)(-\partial p/\partial x)(R^2-r^2)$
Or $u=(1/4\mu)\rho g(-\partial h/\partial x)(R^2-r^2)$
Power (P) = $\rho gq(h_1 - h_2)$ where h_1 and h_2 are peizo-metric heads = $[(p/\rho g) + z]$

Problem-1

Calculate the loss of head in a pipe having a diameter of 15cm and a length of 2km. It carries oil of specific gravity 0.85 and viscosity of 6 Stokes at the rate of 30.48 lps (Assume laminar flow).

$(p_1 - p_2) = [128\mu qL/\pi D^4]$; $(p_1 - p_2) = \rho gqh_f$

$h_f = (p_1 - p_2)/\rho g = [128\mu qL/\rho g\pi D^4]$

$(\mu/\rho) = \nu = 6$ Stokes $= 6\times10^{-4}$ m^2/s;   (Stoke = 1cm^2/sec.)

Substituting,

$h_f = (128\times6\times10^{-4}\times0.03048\times2000)/(\pi\times9.81\times0.15^4)$

Loss of head (h_f) = 300 m

Problem - 2

Calculate the power required to maintain a laminar flow of an oil of viscosity 10P through a pipe of 100mm diameter at the rate of 10 lps if the length of the pipe is 1 km.   (assume laminar flow) (1 Ns/m^2 = 10 Poise)

$\Delta P = [128\mu qL/\pi D^4]$

$= (128\times1\times10\times10^{-3}\times10^3)/(\pi\times 0.1^4)$

$= 4.075\times10^6$ N/m^2

Power $= \Delta P\times q = 4.075\times10^6\times10\times10^{-3} = 40.75$kW

Problem-3.

Oil of viscosity 8P and specific gravity 1.2 flows through a horizontal pipe 80mm in diameter. If the pressure drop in 100m length of the pipe is 1500 kN/m^2, determine,

1. Rate of flow of oil in lpm.
2. The maximum velocity
3. The total frictional drag over 100m length of pipe
4. The power required to maintain flow.
5. The velocity gradient at the pipe wall.
6. The velocity and shear stress at 10mm from the wall.

We have $(-\partial p/\partial x)$

$= 1500\times1000/100 = 15{,}000$ N/m^2/m

Average velocity $= u_{av} = (R^2/8\mu)(-\partial p/\partial x) = (0.04^2/8\times0.8)(15{,}000) = 3.75$m/s.

Discharge (q) $= (\pi D^2/4)u_{av} = 0.01885$ m^3/s

= 18.85 lpm = 1131 lpm.   (D=0.08m)

Max. Velocity (u_{max}) $= 2\ u_{av} = 7.0$ m/s (at the center line)

Wall shear stress $(\tau_o) = -(\partial p/\partial x)(R/2) = 300$N/m^2

Total frictional drag for 100m-pipe length (F_D)

$= \tau_o\ \pi DL = 7540$N = 7.54kN   (L=100m)

Power required to maintain flow (P) $= F_D\times u_{av} = 28.275$ kW ;

Also, P= $q\Delta P = 0.01885\times1500 = 28.275$ kW.

Velocity gradient at pipe wall

$\tau_o = \mu(\partial u/\partial y)_{y=0}$

$(\partial u/\partial y)_{y=0} = (\tau_o/\mu) = 300/0.8 = 3.75$/s

(10P = 1 N-m/s^2)

Velocity and shear stress at 10mm from the wall (y=10mm, r=30mm)
At r=30mm, shear stress (τ) = τ_0 (30/40) = 225N/m^2
OR (τ) = ($-\partial p/\partial x$)(r/2) = 225N/m^2
Local velocity (at r=30mm)
$u = u_{max} [1-(r/R)^2]$ = 3.28m/s.

Problem-4

Oil is transported from a tanker to the shore at the rate of 0.6m^3/s using a pipe of 32cm diameter for a distance of 20km. If the oil has the viscosity of 0.1Nm/s^2 and density of 900 kg/m^3, calculate the power necessary to maintain flow.
$(p_1 - p_2) = [128\mu qL/\pi D^4]$
Power (P) = $(p_1 - p_2)$ q

LAMINAR FLOW BETWEEN PARALLEL PLATES – BOTH PLATES FIXED

Plates are at distance (B) apart. Consider a fluid element as shown with sides (dx, dy, dz). The flow is a steady and uniform. There is no acceleration.

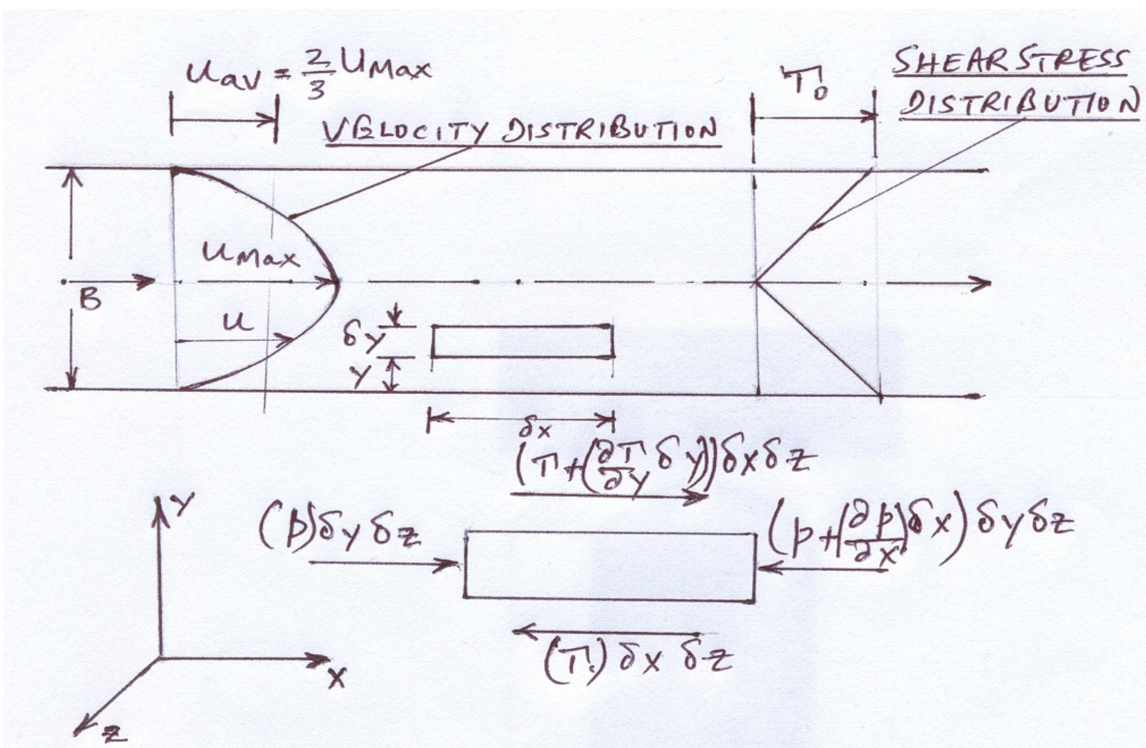


Fig. Laminar Flow through Parallel Plates

Sum of all forces in the direction of motion is zero.
Pressure forces + Shear forces =0
$[p.\, dydz - \{p + (\partial p/\partial x)dx\}dydz] + [\{\tau+(\partial\tau/\partial y)dy\}\, dxdz - \tau\, dxdz] = 0;$
Simplifying,
$-(\partial p/\partial x)dxdydz + (\partial\tau/\partial y)dxdydz = 0$; Dividing by $dxdydz$, The volume of the parallelepiped,

$$(\partial p/\partial x) = (\partial\tau/\partial y)$$

According to the Newton's law of viscosity,
$(\tau) = \mu(du/dy)$; Therefore, $(\partial p/\partial x) = \mu(\partial^2 u/\partial y^2)$
$(\partial^2 u/\partial y^2) = 1/\mu\ (\partial p/\partial x)$; Since $(\partial p/\partial x)$ is independent of (y), integrating the above equation, we get
$(\partial u/\partial y) = 1/\mu\ (\partial p/\partial x)y + C_1$; Integrating again,

$$u = 1/2\mu\ (\partial p/\partial x)y^2 + C_1 y + C_2$$

C_1 and C_2 are constants of integration.
At y=0, u=0; Therefore, $C_2 = 0$; At y=B (at the upper plate); u=0.
$0 = 1/2\mu\ (\partial p/\partial x)B^2 + C_1 B$ OR $C_1 = 1/2\mu\ (-\partial p/\partial x)B$; Substituting,
$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$

This equation shows that the velocity distribution for steady laminar flow between fixed parallel plates is parabolic.
$(\partial p/\partial x)$ – Pressure decreases in the direction of flow and $[-(\partial p/\partial x)]$ is a positive quantity.
Max. Velocity occurs mid-way between the plates and can be obtained using y=(B/2).
$U_{max} = B^2/8\mu\ (-\partial p/\partial x)$

DISCHARGE AND AVERAGE VELOCITY

Consider an elemental strip of height (dy) situated at a distance (y) from the bottom plate as shown. Consider unit width normal to the plane of paper.

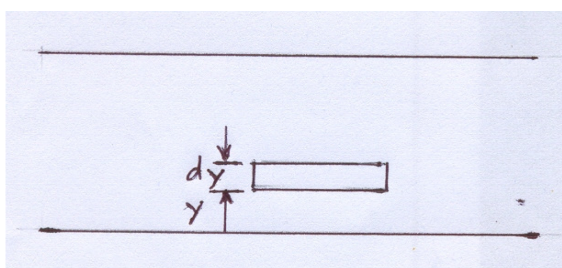


Fig. Discharge and Average Velocity

Velocity of fluid passing through the strip,
$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$
Discharge through the strip per unit width =
dq = Area ´ Velocity
$= dy \times 1 \times 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$
Discharge (q) per unit width of plate

$q = {}_0\int^B [1/2\mu\ (-\partial p/\partial x)\ [By - y^2]dy$

Integrating, $q = 1/2\mu\ (-\partial p/\partial x)[B^3/2 - B^3/3]$

OR $q = (B^3/12\mu)\ (-\partial p/\partial x)$

Average velocity = u_{av} = (q/Area)

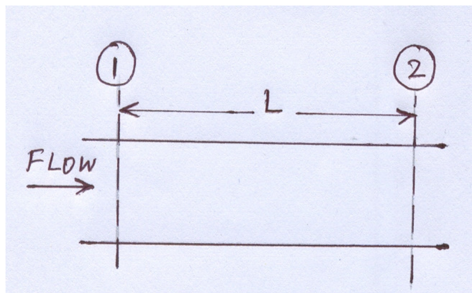
Area = $B \times 1$; $u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

Since $U_{max} = B^2/8\mu\ (-\partial p/\partial x)$; $u_{av} = (2/3)\ U_{max}$

In the case of steady laminar flow between two fixed parallel plates, the average velocity is equal to (2/3) maximum velocity

PRESSURE DROP OVER A GIVEN LENGTH OF PLATES

In the case of steady laminar flow between two fixed parallel plates, the average velocity is
equal to



$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

$(-\partial p) = [12\mu\ u_{av}/B^2]\ \partial x$ OR

Integrating between sections (1) and (2)

${}_1\int^2(-\partial p) = {}_1\int^2 [12\mu\ u_{av}/B^2]\ \partial x$

At (1), $x=x_1$, $p=p_1$; At(2), $x=x_2$, $p=p_2$

$(p_1 - p_2) = [12\mu\ u_{av}/B^2]\ (x_2 - x_1)$ OR

$(p_1 - p_2) = [12\mu\ u_{av}/B^2]\ L$ where $L = (x_2 - x_1)$

LOSS OF HEAD AND SHEAR STRESS

Loss of Head: $h_f = [(p_1 - p_2)/\rho\ g] = [12\mu\ u_{av}/\rho g B^2]\ L$

Shear stress: According to the Newton's law, $(\tau) = \mu(du/dy)$

$(\tau) = \mu(d/dy)[1/2\mu\ (-\partial p/\partial x)\ \{By - y^2\}]$

OR $(\tau) = \mu[1/2\mu\ (-\partial p/\partial x)\ \{B - 2y\}]$

OR $(\tau) = (-\partial p/\partial x)\ (B/2 - y)$

Shows the variation of shear stress with distance (y) - $(\tau) = 0$ at $y = B/2$, mid-way between the plates.

Shear stress is maximum at the plates –

$(\tau) = (\tau_0)$ at $y = 0$ or B.

$(\tau_0) = (-\partial p/\partial x)\ (B/2)$

Laminar Flow through Inclined Plates:

Replace $(\partial p/\partial x)$ by $\rho g(\partial h/\partial x)$ where

$h = z + (p/\rho g)$

$u = 1/2\mu\ [-\partial\ (p + \rho g z)/\partial x)\ \{By - y^2\}]$

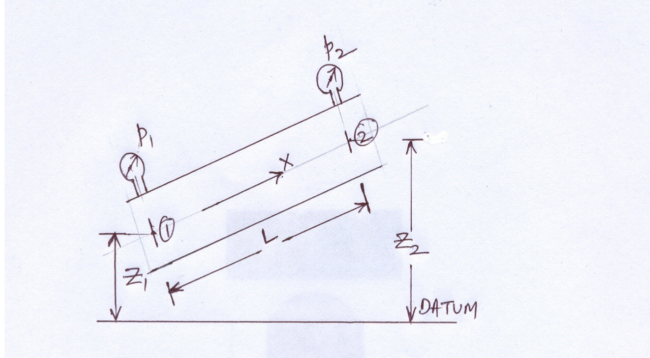


Fig. Laminar Flow through Inclined Plates

Problem-1
Oil of specific gravity 0.92 and dynamic viscosity 1.05 poise flows between two fixed parallel plates 12mm apart. If the mean velocity is 1.4m/s, calculate (a) maximum velocity (b) velocity and shear stress at a distance of 2mm from one of the plates and (c) loss of head over a distance of 25m.
For laminar flow between parallel plates, we have u_{av} = 1.4 m/s.

(a) $u_{max} = (3/2)\ u_{av} = 2.1$ m/s

(b) Velocity at 2mm from one of the plates:

$u = 1/2\mu\ (-\partial p/\partial x)\ [By - y^2]$; To get $(\partial p/\partial x)$

$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$; $(-\partial p/\partial x) = (12\mu u_{av}/B^2)$

$= (12\times1.05\times10^{-1}\times1.4)/(12\times10^{-3})^2$

= 12,250 $N/m^2/m$; Substituting we get

u [at y=2mm] = 1.167m/s

(c) Loss of head in 25m length:

$h_f = [12\mu\ u_{av}/\rho g B^2]\ L =$

$[(12\times0.105\times1.4)/(0.92\times1000\times9.81)(1.2\times10^{-2})^2]\ 25$

= 33.933m

Problem 2
Two parallel plates kept at 100mm apart have laminar flow of oil between them. The maximum velocity of flow is 1.5m/s. Calculate (a) Discharge per meter width (b) Shear stress at the plates (c) Pressure difference between two points 20m apart (d) Velocity gradient at the plates (e) Velocity at 20mm from the plate. Take viscosity of oil as 2.45 pa-s.

(a) Given u_{max} = 1.5m/s.; $u_{av} = 2/3\ u_{max}$ = 1m/s;

Discharge per unit width (q) = $(B\times1\times u_{av})$ = 0.1m/s/m width

(b) Shear stress at the plate:

$(\tau_0) = (-\partial p/\partial x)\ (B/2)$; Pa-s = N-s/m^2

$u_{av} = (B^2/12\mu)\ (-\partial p/\partial x)$

$(-\partial p/\partial x) = (12\mu u_{av})/B^2$ = 2490 $N/m^2/m$

(τ_0) = 147 N/m^2

(c) Pressure difference between two points 20m apart:

$\Delta p = [12\mu\, u_{av}/B^2]\, L = 58{,}800\ N/m^2$

(d) Velocity gradient at the plates: $(\tau_0) = \mu(du/dy)_{y=0}$

$(du/dy)_{y=0} = (\tau_0)/\mu = 147/2.45 = 60/s$

(e) Velocity at 20mm from plates: u(at y= 20mm)

$= 1/2\mu\, (-\partial p/\partial x)\, [By - y^2] = 0.96\ m/s$